Name:

1. (5 points)

   (a) Find the coordinates of the points of intersection of the line $2x - 3y + 4 = 0$ and the parabola $y^2 = 4x$.

   (b) Determine the value of c for which the line $2x - 3y + c = 0$ is tangent to the parabola. Find the coordinates of the point of tangency.

(c) Show that the equation of the normal at the point $P(p^2, 2p)$ (with $p \neq 0$) on the parabola is

$$y = -px + p^3 + 2p$$

(d) The normal cuts the parabola again at point Q. Find, in terms of p, the coordinates of point Q.

2. (8 points)

(a) Show that $x = 1$ is a solution to the equation:

$$8x^4 - 4x^3 - 8x^2 + 3x + 1 = 0$$

And hence factorize the polynomial $P(x) = 8x^4 - 4x^3 - 8x^2 + 3x + 1$ into a product of linear and cubic factor.

(b) Using the formula:

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

or otherwise, solve the equation:

$$\cos 4\theta = \cos 3\theta$$

for $0 \leqslant \theta \leqslant \pi$

(c) By applying double angle and compound angle formulas express both $\cos 4\theta$ and $\cos 3\theta$ in terms of $\cos\theta$ only.

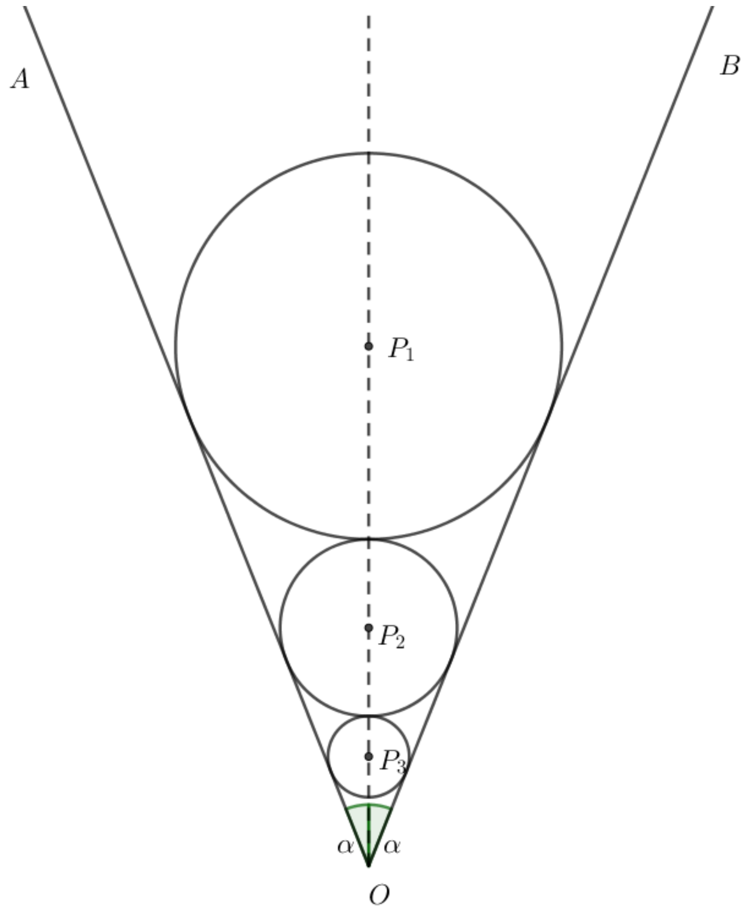
(d) Hence show that the solutions to the equation

$$8x^3 + 4x^2 - 4x - 1 = 0$$

are $\cos\frac{2\pi}{7}, \cos\frac{4\pi}{7}$ and $\cos\frac{6\pi}{7}$.

(e) Hence, or otherwise, find the value of $\sec\frac{2\pi}{7}\sec\frac{4\pi}{7}\sec\frac{6\pi}{7}$.

3. (7 points)



The diagram represents two straight lines OA and OB inclined at an angle 2α. The circle of centre P_1 has radius r and touches each of OA and OB. A sequence of circles is drawn, decreasing in radius, each touching OA, OB and its immediate predecessor.

(a) Prove that the radii of these circles are in geometric progression.

Let S_n denote the sum of the areas of the first n circles and S denote the sum to infinity of areas of all the circles.
(b) Prove that

$$S - S_n < \frac{1}{100} S$$

whenever

$$n > \frac{1}{\log_{10}\left(\frac{1+\sin\alpha}{1-\sin\alpha}\right)}$$

(c) Prove also that if the area of the first circle is equal to the sum of the areas of all the other circles then $\sin\alpha = 3 - 2\sqrt{2}$.