Name:

1. (5 points) Consider the curve given by the equation:

$$\sqrt{x} + \sqrt{y} = 4$$

Find the area of the triangle enclosed by the tangent to the curve at $x = 1$ and the axes.

2. (5 points) Consider the function

$$f(x) = 2^{-x}$$

Let P be a point on the graph of f and A the point on the x-axis with same x-coordinate as P. Let B be the point where the tangent to the graph of f at P intersects the x-axis. Show that the length AB is independent of the choice of P.

3. (5 points) Suppose that α is an acute angle such that $\tan\alpha = \dfrac{1}{5}$.

(a) Find the value of:

(i) $\tan 2\alpha$

(ii) $\tan 4\alpha$

(iii) $\tan(4\alpha - \frac{\pi}{4})$

(b) Hence, or otherwise, show that:

$$\frac{\pi}{4} = 4\arctan\left(\frac{1}{5}\right) - \arctan\left(\frac{1}{239}\right)$$

4. (5 points)

(a) Find the stationary point on the graph of $y = \ln x - x$ and deduce that

$$\ln x \leqslant x - 1$$

for $x > 0$ with equality only when $x = 1$.

(b) Find the stationary point on the graph of $y = \ln x + \frac{1}{x}$ and deduce that

$$\ln x \geqslant \frac{x-1}{x}$$

for $x > 0$ with equality only when $x = 1$.

(c) By putting $x = \frac{z}{y}$ where $0 < y < z$, deduce Napier's inequality:

$$\frac{1}{z} < \frac{\ln z - \ln y}{z - y} < \frac{1}{y}$$