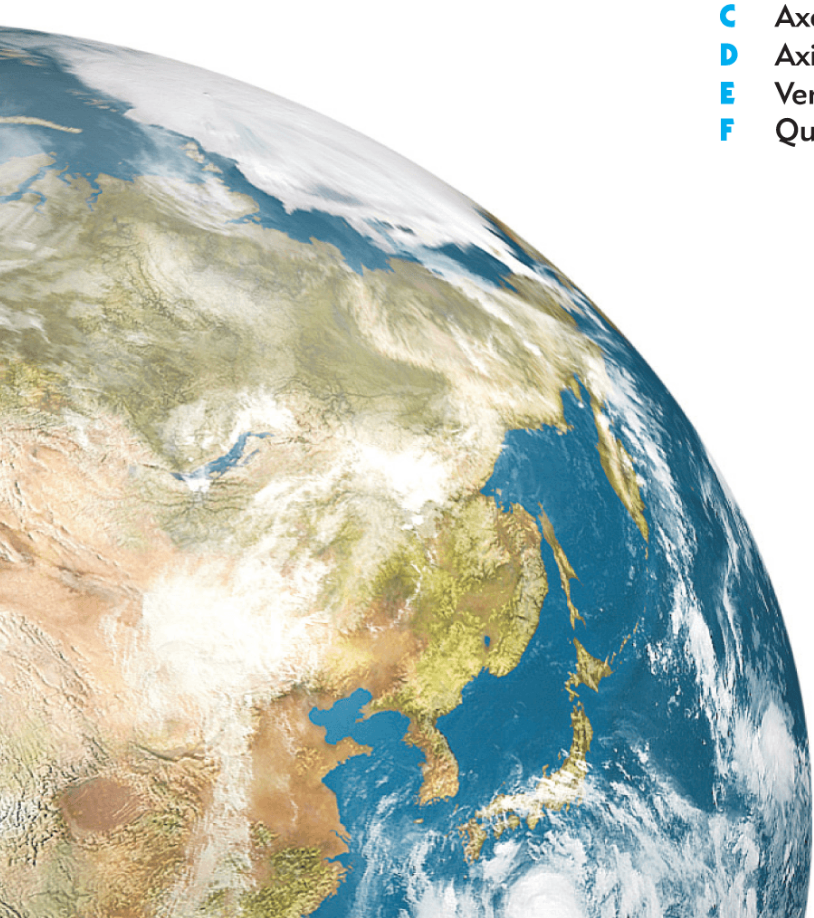
Chapter 20

# Quadratic functions

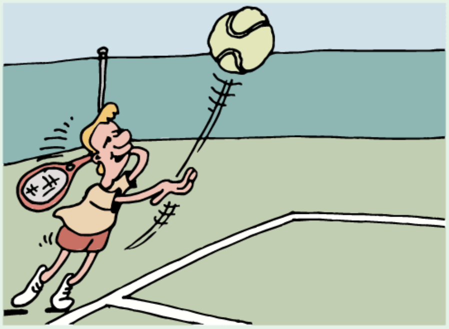
Contents:





## OPENING PROBLEM

Tennis player Bradley tosses the ball in the air before he serves it. The ball's height above the ground $t$ seconds after it is tossed is given by the function $H(t) = -5t^2 + 6t + 2$ metres.



**Things to think about:**

**a** How high was the ball when it was released?

**b** What was the maximum height reached by the tennis ball?

**c** Bradley hits the ball when it is 3 metres above the ground, and on its way down. How long after Bradley releases the ball does he hit it?

In this chapter we consider relationships between variables which are **quadratic** in nature. These relationships can be described algebraically using **quadratic functions**.

# A QUADRATIC FUNCTIONS

A **quadratic function** is a relationship between two variables which can be written in the form $y = ax^2 + bx + c$ where $x$ and $y$ are the variables, and $a$, $b$, and $c$ are constants, $a \neq 0$.

Using function notation, $y = ax^2 + bx + c$ can be written as $f(x) = ax^2 + bx + c$.

### FINDING $y$ GIVEN $x$

For any value of $x$, the corresponding value of $y$ can be found by substitution into the function equation.

**Example 1** Self Tutor

Suppose $y = 2x^2 + 4x - 5$. Find the value of $y$ when:

**a** $x = 0$ **b** $x = 3$

**a** When $x = 0$,
$$y = 2(0)^2 + 4(0) - 5 = 0 + 0 - 5 = -5$$

**b** When $x = 3$,
$$y = 2(3)^2 + 4(3) - 5 = 18 + 12 - 5 = 25$$

### FINDING $x$ GIVEN $y$

When we substitute a value for $y$, we are left with a quadratic equation which we need to solve for $x$. Since the equation is quadratic, there may be 0, 1, or 2 possible values for $x$.

**Example 2** Self Tutor

Suppose $y = x^2 - 6x + 8$. Find the value(s) of $x$ for which:

**a** $y = 15$ **b** $y = -1$

**a** When $y = 15$,
$$x^2 - 6x + 8 = 15$$
$$\therefore\ x^2 - 6x - 7 = 0$$
$$\therefore\ (x+1)(x-7) = 0$$
$$\therefore\ x = -1 \text{ or } x = 7$$

**b** When $y = -1$,
$$x^2 - 6x + 8 = -1$$
$$\therefore\ x^2 - 6x + 9 = 0$$
$$\therefore\ (x-3)^2 = 0$$
$$\therefore\ x = 3$$

**Example 3** Self Tutor

A stone is thrown into the air. Its height above the ground is given by the function $h(t) = -5t^2 + 30t + 2$ metres, where $t$ is the time in seconds from when the stone is thrown.

**a** How high is the stone above the ground after 3 seconds?

**b** From what height above the ground was the stone released?

**c** At what times is the stone 27 m above the ground?

**a**
$$h(3) = -5(3)^2 + 30(3) + 2$$
$$= -45 + 90 + 2$$
$$= 47$$
$\therefore$ the stone is 47 m above the ground.

**b** The stone was released when $t = 0$ s.
Now $h(0) = -5(0)^2 + 30(0) + 2 = 2$
$\therefore$ the stone was released from 2 m above ground level.

**c** When $h(t) = 27$,
$$-5t^2 + 30t + 2 = 27$$
$$\therefore\ -5t^2 + 30t - 25 = 0$$
$$\therefore\ t^2 - 6t + 5 = 0$$
$$\therefore\ (t-1)(t-5) = 0$$
$$\therefore\ t = 1 \text{ or } 5$$
$\therefore$ the stone is 27 m above the ground after 1 second and after 5 seconds.

## EXERCISE 20A

**1** Which of the following are quadratic functions?

**a** $y = 15x - 8$ **b** $y = \frac{1}{3}x^2 + 6$ **c** $3y + 2x^2 - 7 = 0$ **d** $y = 15x^3 + 2x - 16$

**2** For each of the following functions, find the value of $y$ for the given value of $x$:

**a** $y = x^2 + 5x - 14$ when $x = 2$ **b** $y = 2x^2 + 9x$ when $x = -5$

**c** $y = -2x^2 + 3x - 6$ when $x = 3$ **d** $y = 4x^2 + 7x + 10$ when $x = -2$

**3** State whether the following quadratic functions are satisfied by the given ordered pairs:

**a** $y = 6x^2 - 10$ $(0, 4)$ **b** $y = 2x^2 - 5x - 3$ $(4, 9)$

**c** $y = -4x^2 + 6x$ $(-\frac{1}{2}, -4)$ **d** $y = -7x^2 + 9x + 11$ $(-1, -6)$

**e** $y = 3x^2 - 11x + 20$ $(2, -10)$ **f** $y = -3x^2 + x + 6$ $(\frac{1}{3}, 4)$

**4** For each of the following quadratic functions, find the value(s) of $x$ for the given value of $y$:

**a** $y = x^2 + 6x + 10$ when $y = 1$
**b** $y = x^2 + 5x + 8$ when $y = 2$
**c** $y = x^2 - 5x + 1$ when $y = -3$
**d** $y = 3x^2$ when $y = -3$

**5** Find the value(s) of $x$ for which:

**a** $f(x) = 3x^2 - 3x + 6$ takes the value $6$
**b** $f(x) = x^2 - 2x - 7$ takes the value $-4$
**c** $f(x) = -2x^2 - 13x + 3$ takes the value $-4$
**d** $f(x) = 2x^2 - 10x + 1$ takes the value $-11$

**6** An object is projected into the air with a velocity of 80 $\text{m s}^{-1}$. Its height after $t$ seconds is given by the function $h(t) = 80t - 5t^2$ metres.

**a** Calculate the height of the object after:
**i** 1 second **ii** 3 seconds **iii** 5 seconds.

**b** Calculate the time(s) at which the height of the object is:
**i** 140 m **ii** 0 m.

**c** Explain your answers in part **b**.

**7** A cake manufacturer finds that the profit from making $x$ cakes per day is given by the function $P(x) = -\frac{1}{2}x^2 + 36x - 40$ dollars.

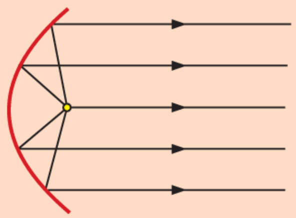
**a** Calculate the profit if **i** 0 cakes **ii** 20 cakes are made per day.
**b** How many cakes need to be made per day to achieve a profit of $270?

## B GRAPHS OF QUADRATIC FUNCTIONS

The graphs of all quadratic functions are **parabolas**. The parabola is one of the **conic sections**.

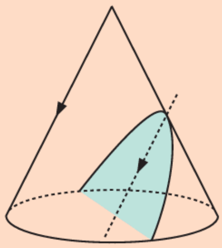
### HISTORICAL NOTE — CONIC SECTIONS

**Conic sections** are curves which can be obtained by cutting a cone with a plane. The Ancient Greek mathematicians were fascinated by conic sections.



The name parabola comes from the Greek word for **thrown** because when an object is thrown, its path makes a parabolic arc.

There are many other examples of parabolas in everyday life. For example, parabolic mirrors are used in car headlights, heaters, satellite dishes, and radio telescopes, because of their special geometric properties.

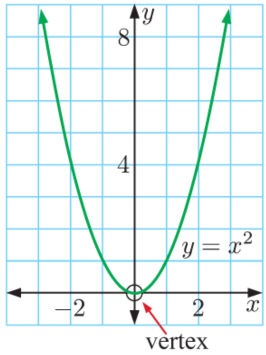


You may like to explore the conic sections for yourself by cutting an icecream cone. Cutting parallel to the side produces a parabola, as shown in the diagram.

The simplest quadratic function is $y = x^2$. Its graph can be drawn from a table of values.

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $y$ | $9$ | $4$ | $1$ | $0$ | $1$ | $4$ | $9$ |

We can see that the graph has a minimum turning point at $(0, 0)$. We call this the **vertex** of the parabola.




**Example 4** — Self Tutor

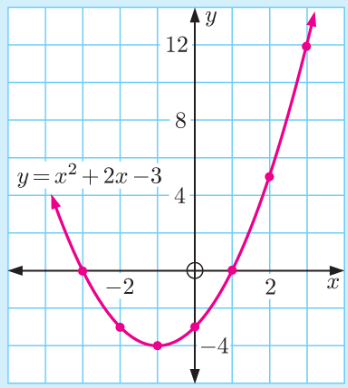
Draw the graph of $y = x^2 + 2x - 3$ using a table of values from $x = -3$ to $x = 3$.

Consider $f(x) = x^2 + 2x - 3$

Now, $f(-3) = (-3)^2 + 2(-3) - 3$
$= 9 - 6 - 3$
$= 0$

We can do the same for the other values of $x$:

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $y$ | $0$ | $-3$ | $-4$ | $-3$ | $0$ | $5$ | $12$ |




## EXERCISE 20B.1

**1** Using a table of values from $x = -3$ to $x = 3$, draw the graph of:

- **a** $y = x^2 - 2x + 8$
- **b** $f(x) = -x^2 + 2x + 1$
- **c** $y = 2x^2 + 3x$
- **d** $y = -2x^2 + 4$
- **e** $y = x^2 + x + 4$
- **f** $f(x) = -x^2 + 4x - 9$

Use the **graphing package** or a **graphics calculator** to check your graphs.

GRAPHING PACKAGE

GRAPHICS CALCULATOR INSTRUCTIONS

**2** **a** Use tables of values to graph $y = 2x^2 - x - 3$ and $y = -x^2 + 2x + 3$ on the same set of axes. *Hence* find the values of $x$ for which $2x^2 - x - 3 = -x^2 + 2x + 3$.

**b** Solve algebraically: $2x^2 - x - 3 = -x^2 + 2x + 3$.

## USING TRANSFORMATIONS TO GRAPH QUADRATICS

By observing how a quadratic function is related to $f(x) = x^2$, we can transform the graph of $y = x^2$ to produce the graph of the function.

## INVESTIGATION 1 GRAPHS OF QUADRATIC FUNCTIONS

In this Investigation we consider different forms of quadratic functions, and how the form of the quadratic affects its graph. You can use either the graphing package or your graphics calculator to draw the graphs.

GRAPHING PACKAGE

GRAPHICS CALCULATOR INSTRUCTIONS

### Part 1: Graphs of the form $y = x^2 + k$

**What to do:**

**1** Graph the two functions on the same set of axes, and observe the coordinates of the vertex of each function.

**a** $y = x^2$ and $y = x^2 + 2$  **b** $y = x^2$ and $y = x^2 - 2$

**c** $y = x^2$ and $y = x^2 + 4$  **d** $y = x^2$ and $y = x^2 - 4$

**2** What effect does the value of $k$ have on:

**a** the position of the graph  **b** the shape of the graph?

**3** What transformation is needed to graph $y = x^2 + k$ from $y = x^2$?

### Part 2: Graphs of the form $y = (x - h)^2$

**What to do:**

**1** Graph the two functions on the same set of axes, and observe the coordinates of the vertex of each function.

**a** $y = x^2$ and $y = (x - 2)^2$  **b** $y = x^2$ and $y = (x + 2)^2$

**c** $y = x^2$ and $y = (x - 4)^2$  **d** $y = x^2$ and $y = (x + 4)^2$

**2** What effect does the value of $h$ have on:

**a** the position of the graph  **b** the shape of the graph?

**3** What transformation is needed to graph $y = (x - h)^2$ from $y = x^2$?

### Part 3: Graphs of the form $y = (x - h)^2 + k$

**What to do:**

**1** Graph the two functions on the same set of axes, and observe the coordinates of the vertex of each function.

**a** $y = x^2$ and $y = (x - 2)^2 + 3$  **b** $y = x^2$ and $y = (x + 4)^2 - 1$

**c** $y = x^2$ and $y = (x - 5)^2 - 2$  **d** $y = x^2$ and $y = (x + 1)^2 + 5$

**2** Copy and complete:

- The graph of $y = (x - h)^2 + k$ is the same shape as the graph of ......
- The graph of $y = (x - h)^2 + k$ is found by translating $y = x^2$ ...... units horizontally and ...... units vertically. This is a translation through $\begin{pmatrix} \ldots\ldots \\ \ldots\ldots \end{pmatrix}$.
- The vertex of the graph of $y = (x - h)^2 + k$ is at (......, ......).

**Part 4: Graphs of the form** $y = ax^2$, $a \neq 0$

**What to do:**

**1** Graph the two functions on the same set of axes, and observe the coordinates of the vertex of each function.

**a** $y = x^2$ and $y = 2x^2$ **b** $y = x^2$ and $y = 4x^2$ **c** $y = x^2$ and $y = \frac{1}{2}x^2$

**d** $y = x^2$ and $y = -x^2$ **e** $y = x^2$ and $y = -2x^2$ **f** $y = x^2$ and $y = -\frac{1}{2}x^2$

**2** These functions are all members of the family $y = ax^2$. What effect does $a$ have on:

**a** the position of the graph **b** the shape of the graph

**c** the direction in which the graph opens?

**Part 5: Graphs of the form** $y = a(x-h)^2 + k$, $a \neq 0$

**What to do:**

**1** Graph the two functions on the same set of axes, and observe the coordinates of the vertex of each function.

**a** $y = 2x^2$ and $y = 2(x-1)^2 + 3$ **b** $y = -x^2$ and $y = -(x+2)^2 - 1$

**c** $y = \frac{1}{2}x^2$ and $y = \frac{1}{2}(x-3)^2 - 2$ **d** $y = -3x^2$ and $y = -3(x+1)^2 + 4$

**2** Copy and complete:

- The graph of $y = a(x-h)^2 + k$ has the same shape and opens in the same direction as the graph of ......
- The graph of $y = a(x-h)^2 + k$ is found by translating $y = ax^2$ ...... units horizontally and ...... units vertically. This is a translation through $\begin{pmatrix} \ldots\ldots \\ \ldots\ldots \end{pmatrix}$.

From the **Investigation** you should have discovered the following important facts:

- Graphs of the form $y = x^2 + k$ have the same shape as the graph of $y = x^2$.

  The graph of $y = x^2$ is translated through $\begin{pmatrix} 0 \\ k \end{pmatrix}$ to give the graph of $y = x^2 + k$.
- Graphs of the form $y = (x-h)^2$ have the same shape as the graph of $y = x^2$.

  The graph of $y = x^2$ is translated through $\begin{pmatrix} h \\ 0 \end{pmatrix}$ to give the graph of $y = (x-h)^2$.
- Graphs of the form $y = (x-h)^2 + k$ have the same shape as the graph of $y = x^2$.

  The graph of $y = x^2$ is translated through $\begin{pmatrix} h \\ k \end{pmatrix}$ to give the graph of $y = (x-h)^2 + k$.

  The vertex is shifted to $(h, k)$.
- If $a > 0$, $y = ax^2$ opens upwards. 

  If $a < 0$, $y = ax^2$ opens downwards. 

  If $a < -1$ or $a > 1$, then $y = ax^2$ is 'thinner' than $y = x^2$.
  If $-1 < a < 1$, $a \neq 0$, then $y = ax^2$ is 'wider' than $y = x^2$.

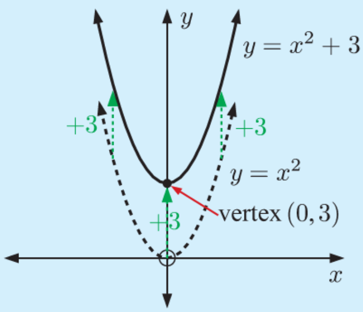
- $y = a(x-h)^2 + k$
  - $a > 0$ (opens upwards), $a < 0$ (opens downwards)
  - $a < -1$ or $a > 1$, thinner than $y = x^2$
  - $-1 < a < 1$, $a \neq 0$, wider than $y = x^2$
  - vertical translation of $k$ units:
    if $k > 0$ it goes up
    if $k < 0$ it goes down
  - horizontal translation of $h$ units:
    if $h > 0$ it goes right
    if $h < 0$ it goes left

**Example 5** — Self Tutor

Sketch each of the following functions on the same set of axes as $y = x^2$. In each case state the coordinates of the vertex.

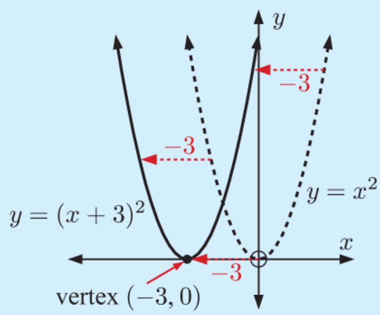
**a** $y = x^2 + 3$ **b** $y = (x+3)^2$

**a** We draw $y = x^2$, then translate it 3 units upwards.




The vertex is at (0, 3).

**b** We draw $y = x^2$, then translate it 3 units to the left.




The vertex is at (−3, 0).

## EXERCISE 20B.2

**1** Sketch each of the following functions on the same set of axes as $y = x^2$. Use a separate set of axes for each part, and in each case state the coordinates of the vertex.

GRAPHING PACKAGE

**a** $y = x^2 - 3$ **b** $y = x^2 - 1$ **c** $y = x^2 + 2$

**d** $y = x^2 - 5$ **e** $y = x^2 + 5$ **f** $y = x^2 - \frac{1}{2}$

Use a **graphics calculator** or **graphing package** to check your answers.

**2** Sketch each of the following functions on the same set of axes as $y = x^2$. Use a separate set of axes for each part, and in each case state the coordinates of the vertex.

**a** $y = (x-3)^2$ **b** $y = (x+1)^2$ **c** $y = (x-2)^2$

**d** $y = (x-5)^2$ **e** $y = (x+5)^2$ **f** $y = (x-\frac{3}{2})^2$

Use a **graphics calculator** or **graphing package** to check your answers.

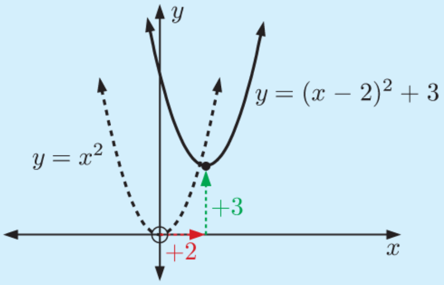
**Example 6** **Self Tutor**

Sketch each of the following functions on the same set of axes as $y = x^2$. In each case state the coordinates of the vertex.

**a** $y = (x-2)^2 + 3$

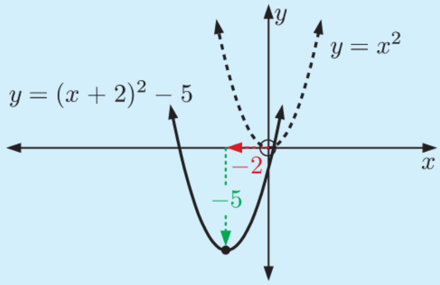
**b** $y = (x+2)^2 - 5$

**a** We draw $y = x^2$, then translate it 2 units to the right and 3 units upwards.




The vertex is at $(2, 3)$.

**b** We draw $y = x^2$, then translate it 2 units to the left and 5 units downwards.




The vertex is at $(-2, -5)$.

**3** Sketch each of the following functions on the same set of axes as $y = x^2$. Use a separate set of axes for each part, and in each case state the coordinates of the vertex.

**a** $y = (x-1)^2 + 3$ **b** $y = (x-2)^2 - 1$ **c** $y = (x+1)^2 + 4$

**d** $y = (x+2)^2 - 3$ **e** $y = (x+3)^2 - 2$ **f** $y = (x-3)^2 + 3$

Use a **graphics calculator** or **graphing package** to check your answers.

**Example 7** **Self Tutor**

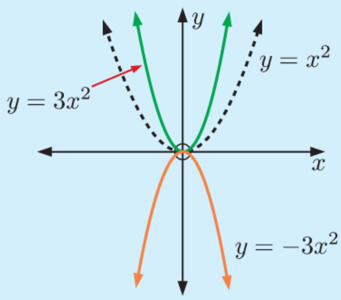
Sketch $y = x^2$ on a set of axes and hence sketch:

**a** $y = 3x^2$

**b** $y = -3x^2$

**a** $y = 3x^2$ is 'thinner' than $y = x^2$.

**b** $y = -3x^2$ has the same shape as $y = 3x^2$, but opens downwards.




**4** Sketch each of the following functions on the same set of axes as $y = x^2$. Comment on the shape of the graph, and the direction in which the graph opens.

**GRAPHING PACKAGE**

**a** $y = 5x^2$ **b** $y = -5x^2$ **c** $y = \frac{1}{3}x^2$

**d** $y = -\frac{1}{3}x^2$ **e** $y = -4x^2$ **f** $y = \frac{1}{4}x^2$

Use a **graphics calculator** or **graphing package** to check your answers.

**Example 8** **Self Tutor**

Sketch the graph of $y = -(x-2)^2 - 3$ from the graph of $y = x^2$, and hence state the coordinates of its vertex.

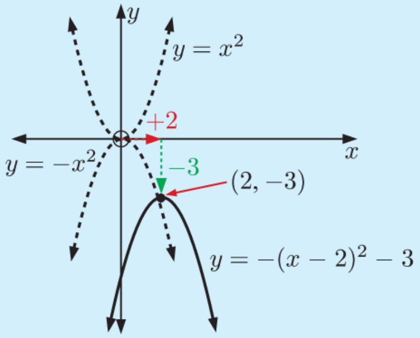
$y = -(x-2)^2 - 3$

reflect in $x$-axis; horizontal translation 2 units right; vertical translation 3 units down

We start with $y = x^2$, then reflect it in the $x$-axis to give $y = -x^2$.

We then translate $y = -x^2$ 2 units to the right and 3 units down.

The vertex of $y = -(x-2)^2 - 3$ is $(2, -3)$.




**5** Sketch each of the following functions on the same set of axes as $y = x^2$. In each case, state the coordinates of the vertex.

**a** $y = -(x-1)^2 + 3$ **b** $y = 2x^2 + 4$ **c** $y = -(x-2)^2 + 4$

**d** $y = 3(x+1)^2 - 4$ **e** $y = \frac{1}{2}(x+3)^2$ **f** $y = -\frac{1}{2}(x+3)^2 + 1$

**g** $y = -2(x+4)^2 + 3$ **h** $y = 2(x-3)^2 + 5$ **i** $y = \frac{1}{2}(x-2)^2 - 1$

Use a **graphics calculator** or **graphing package** to check your answers.

**6** Match the following quadratic functions with their graphs:

**a** $y = -(x+2)^2 - 3$

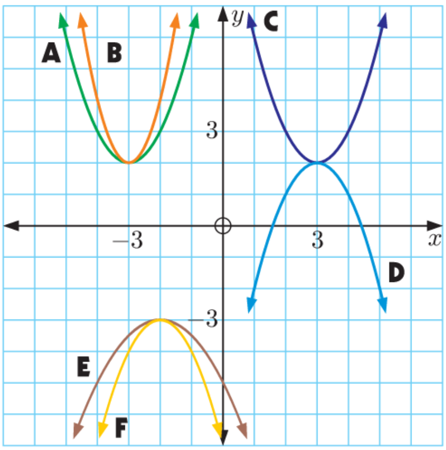
**b** $y = (x-3)^2 + 2$

**c** $y = 2(x+3)^2 + 2$

**d** $y = -(x-3)^2 + 2$

**e** $y = -\frac{1}{2}(x+2)^2 - 3$

**f** $y = (x+3)^2 + 2$

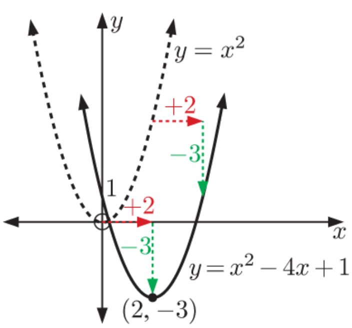



## COMPLETING THE SQUARE

Suppose we want to graph the quadratic function $y = x^2 - 4x + 1$. This function is not written in the form $y = (x-h)^2 + k$, but we can convert it into this form by **completing the square**.

Consider
$$y = x^2 - 4x + 1$$
$$\therefore\ y = x^2 - 4x + 2^2 + 1 - 2^2$$
$$\therefore\ y = (x-2)^2 - 3$$

So, the graph of $y = x^2 - 4x + 1$ can be found by translating $y = x^2$ 2 units to the right and 3 units downwards.



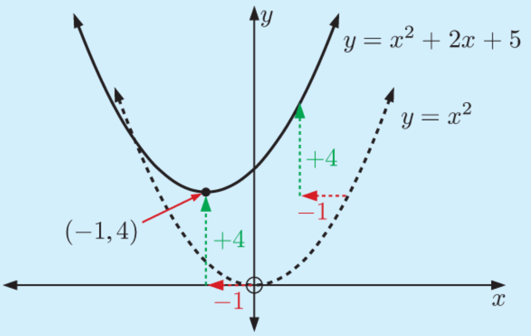
**Example 9** — Self Tutor

Write $y = x^2 + 2x + 5$ in the form $y = (x-h)^2 + k$.
Hence sketch $y = x^2 + 2x + 5$, stating the coordinates of the vertex.

$y = x^2 + 2x + 5$
$\therefore\ y = x^2 + 2x + 1^2 + 5 - 1^2$
$\therefore\ y = (x+1)^2 + 4$

translate 1 unit left; translate 4 units up

The vertex is at $(-1, 4)$.




## EXERCISE 20B.3

**1** Write the following quadratics in the form $y = (x-h)^2 + k$.
Hence sketch each function, stating the coordinates of the vertex.

**a** $y = x^2 + 2x + 4$ **b** $y = x^2 - 6x + 3$
**c** $y = x^2 + 4x - 1$ **d** $y = x^2 - 2x + 5$
**e** $y = x^2 - 2x$ **f** $y = x^2 + 5x$
**g** $y = x^2 + 5x - 3$ **h** $y = x^2 - 3x + 3$ **i** $y = x^2 - 5x + 2$

Use a **graphics calculator** or **graphing package** to check your answers.

GRAPHING PACKAGE

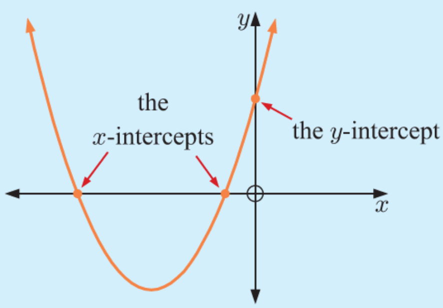
**2** Write the following quadratics in the form $y = a(x-h)^2 + k$.
Hence sketch each function, stating the coordinates of the vertex.

**a** $y = 2x^2 + 10x + 8$ **b** $y = -x^2 + x + 6$ **c** $y = 3x^2 - 6x - 24$
**d** $y = -2x^2 + 6x + 8$ **e** $y = 2x^2 - 8x - 3$ **f** $y = -3x^2 - 6x + 2$

Use a **graphics calculator** or **graphing package** to check your answers.

# C AXES INTERCEPTS

- An ***x*-intercept** of a function is a value of $x$ where its graph meets the $x$-axis.
  $x$-intercepts are found by letting $y$ be 0 in the equation of the function.
- A ***y*-intercept** of a function is a value of $y$ where its graph meets the $y$-axis.
  $y$-intercepts are found by letting $x$ be 0 in the equation of the function.



## INVESTIGATION 2 — AXES INTERCEPTS

**What to do:**

GRAPHING PACKAGE

GRAPHICS CALCULATOR INSTRUCTIONS

**1** For each of the following quadratic functions, use a **graphing package** or **graphics calculator** to:

- **i** draw the graph
- **ii** find the $y$-intercept
- **iii** find any $x$-intercepts.

**a** $y = x^2 - 3x - 4$ **b** $y = -x^2 + 2x + 8$ **c** $y = 2x^2 - 3x$

**d** $y = -2x^2 + 2x - 3$ **e** $y = (x-1)(x-3)$ **f** $y = -(x+2)(x-3)$

**g** $y = 3(x+1)(x+4)$ **h** $y = 2(x-2)^2$ **i** $y = -3(x+1)^2$

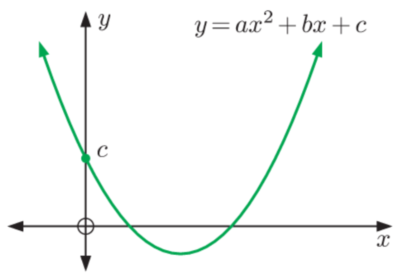
**2** **a** State the $y$-intercept of a quadratic function in the form $y = ax^2 + bx + c$.

**b** State the $x$-intercepts of a quadratic function in the form:

- **i** $y = a(x-\alpha)(x-\beta)$
- **ii** $y = a(x-\alpha)^2$

## THE $y$-INTERCEPT

For a quadratic function in the form $y = ax^2 + bx + c$, the $y$-intercept is the constant term $c$.



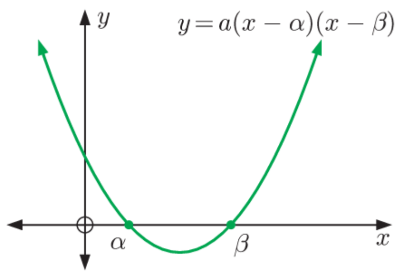

**Proof:**

If $x = 0$ then $y = a(0)^2 + b(0) + c$

$\therefore\ y = c$

## THE $x$-INTERCEPTS

For a quadratic function in the form $y = a(x-\alpha)(x-\beta)$, the $x$-intercepts are $\alpha$ and $\beta$.




**Proof:**

If $y = 0$ then $a(x-\alpha)(x-\beta) = 0$

$\therefore\ x = \alpha$ or $\beta$ {since $a \neq 0$}

$x$-intercepts are therefore easy to find when the quadratic is in **factorised** form.

**Example 10** — Self Tutor

Find the $x$-intercepts of:

**a** $y = 2(x-3)(x+2)$ **b** $y = -(x-4)^2$

**a** When $y = 0$,

$2(x-3)(x+2) = 0$

$\therefore\ x = 3$ or $x = -2$

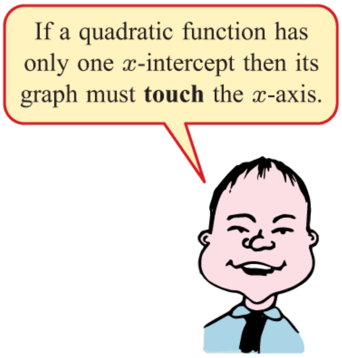
$\therefore$ the $x$-intercepts are 3 and $-2$.

**b** When $y = 0$,

$-(x-4)^2 = 0$

$\therefore\ x = 4$

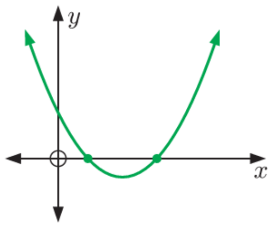
$\therefore$ the $x$-intercept is 4.



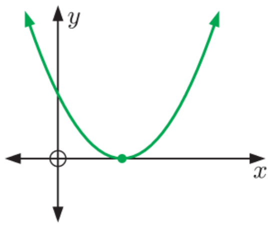
## FACTORISING TO FIND $x$-INTERCEPTS

> For any quadratic function of the form $y = ax^2 + bx + c$, the $x$-intercepts can be found by solving the equation $ax^2 + bx + c = 0$.

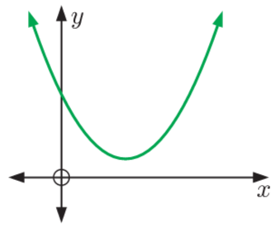
You will recall from **Chapter 11** that quadratic equations may have *two solutions*, *one solution*, or *no solutions*. These solutions correspond to the *two $x$-intercepts*, *one $x$-intercept*, or *no $x$-intercepts* found when the graph of the corresponding quadratic function is drawn.




**two $x$-intercepts**




**one $x$-intercept**




**no $x$-intercepts**

**Example 11** Self Tutor

Find the $x$-intercept(s) of the quadratic function:

**a** $y = x^2 - 6x + 9$ **b** $y = -x^2 - x + 6$

**a** When $y = 0$,

$x^2 - 6x + 9 = 0$

$\therefore (x - 3)^2 = 0$

$\therefore x = 3$

$\therefore$ the $x$-intercept is 3.

**b** When $y = 0$,

$-x^2 - x + 6 = 0$

$\therefore x^2 + x - 6 = 0$

$\therefore (x + 3)(x - 2) = 0$

$\therefore x = -3$ or $2$

$\therefore$ the $x$-intercepts are $-3$ and $2$.

## EXERCISE 20C.1

**1** For the following functions, state the $y$-intercept:

**a** $y = x^2 + 3x + 3$ **b** $y = x^2 - 5x + 2$ **c** $f(x) = 2x^2 + 7x - 8$

**d** $y = 3x^2 - x + 1$ **e** $f(x) = -x^2 + 3x + 6$ **f** $y = -2x^2 + 5 - x$

**g** $y = 6 - x - x^2$ **h** $f(x) = 8 + 2x - 3x^2$ **i** $y = 5x - x^2 - 2$

**2** For the following functions, find the $x$-intercepts:

**a** $y = (x - 3)(x + 1)$ **b** $f(x) = -(x - 2)(x - 4)$ **c** $y = 2(x + 3)(x + 2)$

**d** $y = -3(x - 4)(x - 5)$ **e** $y = 2(x + 3)^2$ **f** $f(x) = -5(x - 1)^2$

**3** For the following functions, find the $x$-intercepts:

**a** $y = x^2 - 9$ **b** $y = 25 - x^2$ **c** $y = x^2 - 6x$

**d** $f(x) = x^2 + 7x + 10$ **e** $y = x^2 + x - 12$ **f** $y = 4x - x^2$

**g** $y = -x^2 - 6x - 8$ **h** $f(x) = -2x^2 - 4x - 2$ **i** $y = 4x^2 - 24x + 36$

**Example 12** — Self Tutor

Use the quadratic formula to find the $x$-intercepts of the quadratic function $y = x^2 - 2x - 5$.

When $y = 0$,

$x^2 - 2x - 5 = 0$

$$\therefore \quad x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$$

$$\therefore \quad x = \frac{2 \pm \sqrt{4 + 20}}{2}$$

$$\therefore \quad x = \frac{2 \pm \sqrt{24}}{2}$$

$$\therefore \quad x = \frac{2 \pm 2\sqrt{6}}{2}$$

$$\therefore \quad x = 1 \pm \sqrt{6}$$

$\therefore$ the $x$-intercepts are $1 + \sqrt{6}$ and $1 - \sqrt{6}$.

**4** Use the quadratic formula to find the $x$-intercepts of the following functions:

**a** $y = x^2 - 4x + 1$ **b** $y = x^2 + 4x - 3$ **c** $y = -x^2 + 6x - 4$

**d** $f(x) = 3x^2 - 7x - 2$ **e** $f(x) = 2x^2 - x - 5$ **f** $f(x) = -4x^2 + 9x - 3$

**Example 13** — Self Tutor

Sketch the graphs of the following functions by considering:

**i** the value of $a$ **ii** the $y$-intercept **iii** the $x$-intercepts.

**a** $y = x^2 - 2x - 3$ **b** $y = -2(x+1)(x-2)$ **c** $y = 2(x-3)^2$

**a** $y = x^2 - 2x - 3$

**i** $a = 1$ which is $> 0$, so the parabola opens upwards.

**ii** When $x = 0$, $y = -3$

$\therefore$ the $y$-intercept is $-3$.

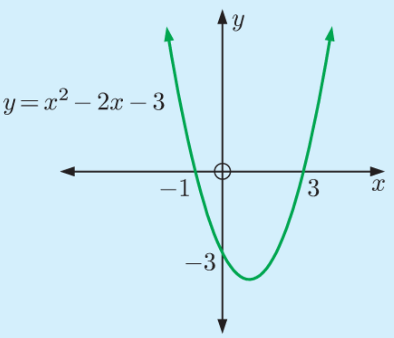
**iii** When $y = 0$,

$x^2 - 2x - 3 = 0$

$\therefore \quad (x-3)(x+1) = 0$

$\therefore \quad x = 3$ or $x = -1$

$\therefore$ the $x$-intercepts are 3 and $-1$.




**b** $y = -2(x+1)(x-2)$

**i** $a = -2$ which is $< 0$, so the parabola opens downwards.

**ii** When $x = 0$,

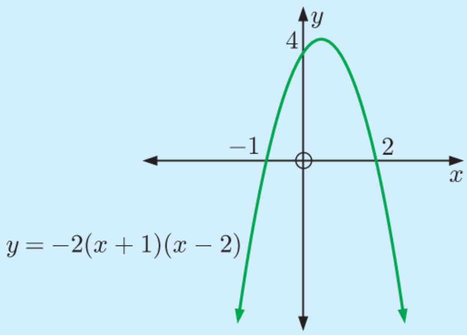
$y = -2(0+1)(0-2)$

$= -2 \times 1 \times -2$

$= 4$

$\therefore$ the $y$-intercept is 4.

**iii** When $y = 0$,
$-2(x+1)(x-2) = 0$
$\therefore \quad x = -1 \quad \text{or} \quad x = 2$
$\therefore$ the $x$-intercepts are $-1$ and $2$.

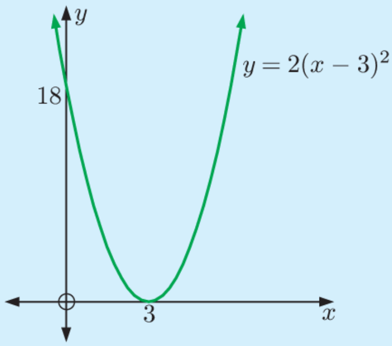



**c** $y = 2(x-3)^2$

**i** $a = 2$ which is $> 0$, so the parabola opens upwards.

**ii** When $x = 0$, $y = 2(0-3)^2 = 18$
$\therefore$ the $y$-intercept is 18.

**iii** When $y = 0$, $2(x-3)^2 = 0$
$\therefore \quad x = 3$
$\therefore$ the $x$-intercept is 3.
There is only one $x$-intercept, which means the graph *touches* the $x$-axis.




**5** Sketch the graph of the quadratic function which has:

**a** $x$-intercepts $-1$ and $1$, and $y$-intercept $-1$
**b** $x$-intercepts $-3$ and $1$, and $y$-intercept $2$
**c** $x$-intercepts $2$ and $5$, and $y$-intercept $-4$
**d** $x$-intercept $2$ and $y$-intercept $4$.

**6** Sketch the graphs of the following quadratic functions by considering:

**i** the value of $a$ **ii** the $y$-intercept **iii** the $x$-intercepts.

**a** $y = x^2 - 4x + 4$
**b** $f(x) = (x-1)(x+3)$
**c** $y = 2(x+2)^2$
**d** $f(x) = -(x-2)(x+1)$
**e** $y = -3(x+1)^2$
**f** $y = -3(x-4)(x-1)$
**g** $y = 2(x+3)(x+1)$
**h** $y = -2x^2 - 3x + 5$
**i** $f(x) = -x^2 + 8x - 10$

## ACTIVITY

Click on the icon to practise matching a quadratic function with its graph.

QUADRATIC FUNCTIONS

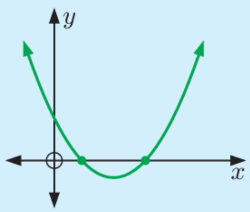
## THE DISCRIMINANT AND THE QUADRATIC GRAPH (EXTENSION)

In **Chapter 11**, we saw that the discriminant of the quadratic equation $ax^2 + bx + c = 0$ is $\Delta = b^2 - 4ac$. We used $\Delta$ to determine whether the equation has real solutions.

We can therefore use $\Delta$ to determine how many $x$-intercepts a quadratic function has.

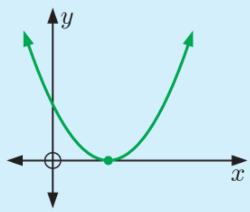
For a quadratic function $f(x) = ax^2 + bx + c$, we consider the discriminant $\Delta = b^2 - 4ac$.

- $\Delta > 0$



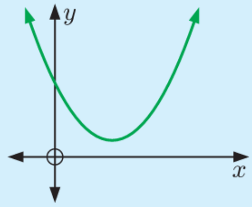

**two $x$-intercepts**

- $\Delta = 0$




**one $x$-intercept**

- $\Delta < 0$




**no $x$-intercepts**

**Example 14** — Self Tutor

Use the discriminant to determine the relationship between the graph and the $x$-axis:

**a** $y = x^2 - 5x + 3$ **b** $y = -3x^2 + x - 2$

**a** $a = 1, \quad b = -5, \quad c = 3$

$\therefore \quad \Delta = b^2 - 4ac$
$= (-5)^2 - 4(1)(3)$
$= 13$

Since $\Delta > 0$, the graph cuts the $x$-axis twice.

**b** $a = -3, \quad b = 1, \quad c = -2$

$\therefore \quad \Delta = b^2 - 4ac$
$= 1^2 - 4(-3)(-2)$
$= -23$

Since $\Delta < 0$, the graph does not cut the $x$-axis.

## EXERCISE 20C.2

**1** Use the discriminant to determine the relationship between the graph and the $x$-axis:

**a** $y = x^2 - 2x - 7$ **b** $y = 2x^2 - 3x + 6$ **c** $y = -x^2 + 4x - 2$

**d** $y = x^2 - 10x + 25$ **e** $y = 3x^2 + 2$ **f** $y = -2x^2 + 12x - 18$

**2** Consider the quadratic function $y = x^2 - 8x + 14$.

**a** Write the function in the form $y = (x - h)^2 + k$.

**b** Hence, sketch the function.

**c** How many $x$-intercepts does the function appear to have?

**d** Use the discriminant to check your answer to **c**.

**3** Consider the quadratic function $y = x^2 + 2x + 5$.

**a** Write the function in the form $y = (x - h)^2 + k$.

**b** Hence, sketch the function.

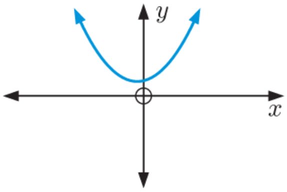
**c** How many $x$-intercepts does the function appear to have?

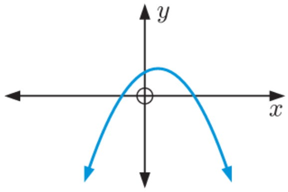
**d** Use the discriminant to check your answer to **c**.

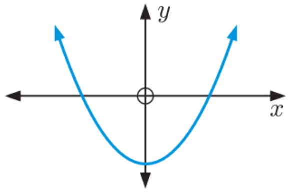
**4** Match each description of a quadratic function $f(x) = ax^2 + bx + c$ with its graph:

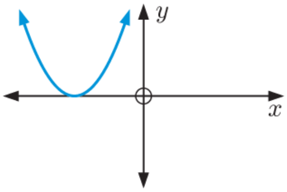
**a** $a = 1, \ \Delta = 7$ **b** $a = -2, \ \Delta = 0$ **c** $a = \frac{1}{2}, \ \Delta = -5$

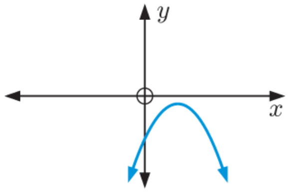
**d** $a = -1, \ \Delta = 11$ **e** $a = -\frac{2}{3}, \ \Delta = -6$ **f** $a = 2, \ \Delta = 0$

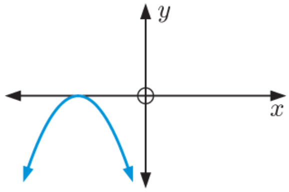
**A**



**B**



**C**



**D**



**E**



**F**



**5** **a** Determine the relationship between $y = x^2 - 4x + c$ and the $x$-axis for the case where:

**i** $c = 3$ **ii** $c = 4$ **iii** $c = 5$.

**b** Using technology, sketch each of the graphs in **a** on the same set of axes.

**6** Consider a quadratic function $y = ax^2 + bx + c$ where $a > 0$ and $c < 0$. Explain why the graph must cut the $x$-axis twice:

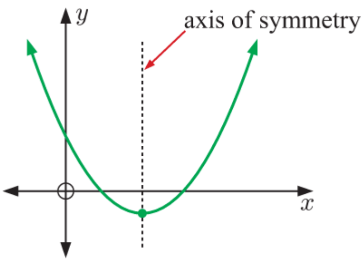
**a** by considering the graphical significance of $a > 0$ and $c < 0$

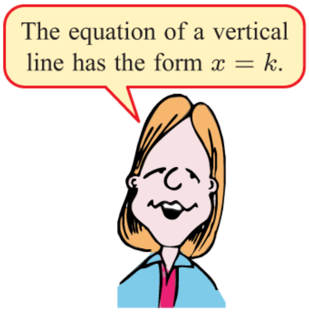
**b** using the discriminant.

## D AXIS OF SYMMETRY

The graphs of all quadratic functions are symmetrical about a vertical line passing through the vertex. This line is called the **axis of symmetry**.

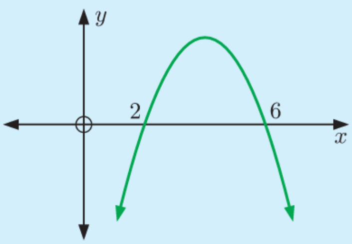
If the graph has two $x$-intercepts, then the axis of symmetry is midway between them.



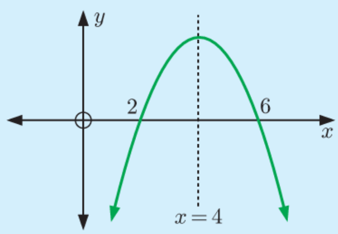




**Example 15** **Self Tutor**

Find the equation of the axis of symmetry for the quadratic graph below.




The $x$-intercepts are 2 and 6, and 4 is midway between 2 and 6.




The axis of symmetry is $x = 4$.

If the quadratic does not have any $x$-intercepts, or if we do not know the $x$-intercepts, we can use the rule below to find the axis of symmetry:

The equation of the axis of symmetry of $y = ax^2 + bx + c$ is $x = \frac{-b}{2a}$.

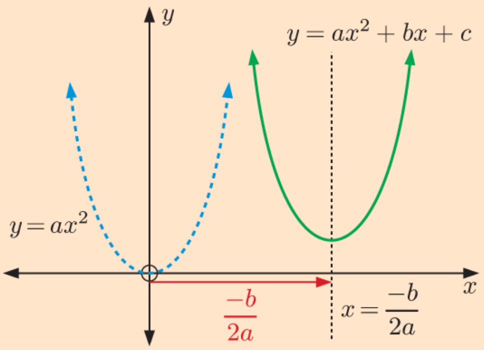
**Proof:**

$y = ax^2 + bx + c$

$\therefore\ y = a\left(x^2 + \frac{b}{a}x\right) + c$

$\therefore\ y = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) + c - a\left(\frac{b}{2a}\right)^2$

$\therefore\ y = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$




The horizontal shift from $y = ax^2$ to $y = ax^2 + bx + c$ is $\frac{-b}{2a}$ units.

$\therefore$ the axis of symmetry of $y = ax^2 + bx + c$ has equation $x = \frac{-b}{2a}$.

**Example 16** — Self Tutor

Find the equation of the axis of symmetry of $y = 2x^2 + 3x + 1$.

$y = 2x^2 + 3x + 1$ has $a = 2$, $b = 3$, $c = 1$.

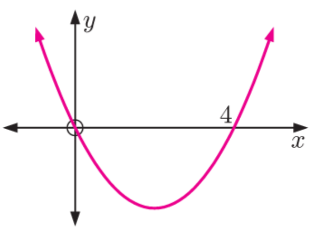
Now $\frac{-b}{2a} = \frac{-3}{2 \times 2} = -\frac{3}{4}$

$\therefore$ the axis of symmetry has equation $x = -\frac{3}{4}$.

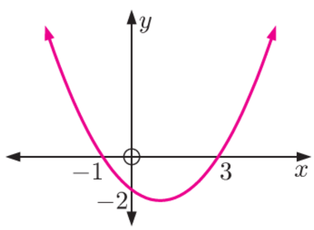
## EXERCISE 20D

**1** For each of the following graphs, find the equation of the axis of symmetry:

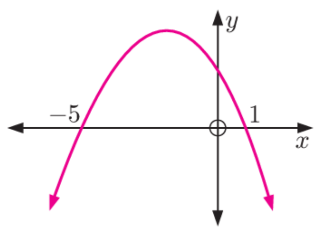
**a**



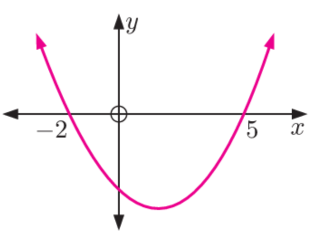

**b**



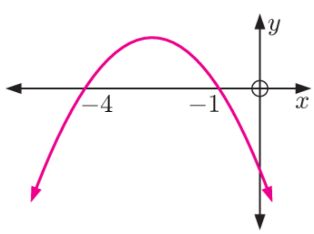

**c**



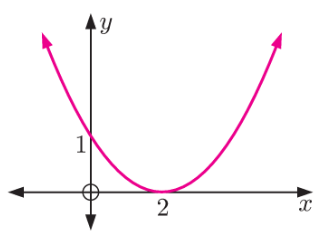

**d**




**e**




**f**



**2** For each of the following quadratic functions, find the equation of the axis of symmetry:

**a** $y = (x-2)(x-4)$ **b** $y = -(x+1)(x-5)$ **c** $y = 2(x+3)(x-3)$

**d** $y = x(x+5)$ **e** $y = -3(x+4)^2$ **f** $y = 4(x+6)(x-9)$

**3** Determine the equation of the axis of symmetry for the following quadratic functions:

**a** $y = x^2 + 4x + 1$ **b** $y = 2x^2 - 6x + 3$ **c** $f(x) = 3x^2 + 4x - 1$

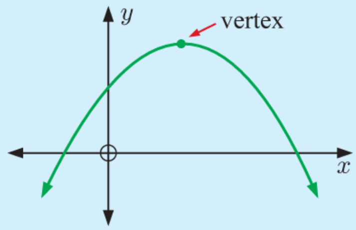
**d** $y = -x^2 - 4x + 5$ **e** $y = -2x^2 + 5x + 1$ **f** $f(x) = \frac{1}{2}x^2 - 10x + 2$

**g** $y = \frac{1}{3}x^2 + 4x$ **h** $f(x) = 100x - 4x^2$ **i** $y = -\frac{1}{10}x^2 + 30x$

## E VERTEX

The **vertex** or **turning point** of the quadratic function $y = ax^2 + bx + c$ is the point at which the function has:

- a **maximum value** for $a < 0$  , *or*
- a **minimum value** for $a > 0$  .




Since the vertex lies on the axis of symmetry, its $x$-coordinate will be $\dfrac{-b}{2a}$.

The $y$-coordinate can be found by substituting this value for $x$ into the function.

**Example 17** **Self Tutor**

Consider the quadratic function $y = -x^2 + 2x + 3$.

**a** Find the axes intercepts. **b** Find the equation of the axis of symmetry.

**c** Find the coordinates of the vertex.

**d** Sketch the function, showing all important features.

**a** When $x = 0$, $y = 3$

$\therefore$ the $y$-intercept is 3.

When $y = 0$, $-x^2 + 2x + 3 = 0$

$\therefore x^2 - 2x - 3 = 0$

$\therefore (x-3)(x+1) = 0$

$\therefore x = 3$ or $-1$

$\therefore$ the $x$-intercepts are 3 and $-1$.

**b** $a = -1$, $b = 2$, $c = 3$

$\therefore \dfrac{-b}{2a} = \dfrac{-2}{-2} = 1$

$\therefore$ the axis of symmetry is $x = 1$.

**c** When $x = 1$,

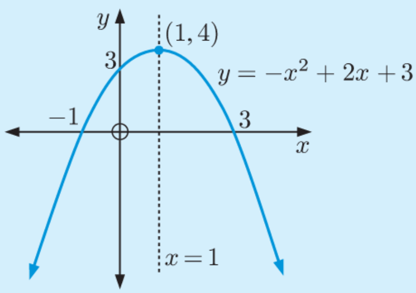
$y = -(1)^2 + 2(1) + 3$

$= -1 + 2 + 3$

$= 4$

$\therefore$ the vertex is $(1, 4)$.

**d**



## EXERCISE 20E

**1** For each of the following quadratic functions:

- **i** Find the coordinates of the vertex.
- **ii** Determine whether the vertex is a maximum or minimum turning point.
- **iii** Find the range of the function.

**a** $y = x^2 - 4x + 2$ **b** $y = x^2 + 2x - 3$
**c** $f(x) = 2x^2 + 4$ **d** $y = -3x^2 + 1$
**e** $y = -x^2 - 4x - 4$ **f** $y = 2x^2 - 10x + 3$

The vertex is called the **maximum turning point** or the **minimum turning point**, depending on whether the graph opens downwards or upwards.

**2** For each of the following quadratic functions:

- **i** Find the axes intercepts.
- **ii** Find the equation of the axis of symmetry.
- **iii** Find the coordinates of the vertex.
- **iv** Hence, sketch the graph of the function.

**a** $y = x^2 - 2x - 8$ **b** $y = 4x - x^2$ **c** $y = x^2 + 3x$
**d** $f(x) = x^2 + 4x + 4$ **e** $y = x^2 + 3x - 4$ **f** $y = -x^2 + 2x - 1$
**g** $y = 2x^2 + 5x - 3$ **h** $f(x) = -3x^2 - 4x + 4$ **i** $y = x^2 - 6x + 3$

### Example 18 — Self Tutor

Consider the quadratic function $y = 2(x-2)(x+4)$.

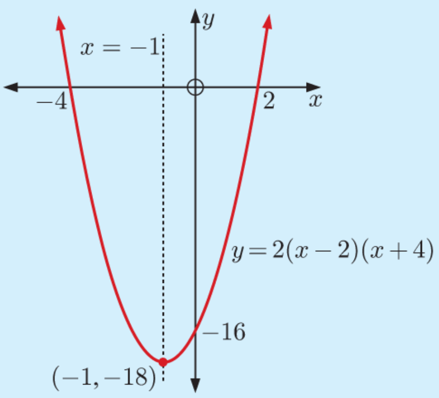
**a** Find the axes intercepts.
**b** Find the equation of the axis of symmetry.
**c** Find the coordinates of the vertex.
**d** Sketch the function, showing all important features.

**a** When $x = 0$, $y = 2 \times -2 \times 4 = -16$
$\therefore$ the $y$-intercept is $-16$.
When $y = 0$, $2(x-2)(x+4) = 0$
$\therefore x = 2$ or $x = -4$
$\therefore$ the $x$-intercepts are 2 and $-4$.

**b** The axis of symmetry is halfway between the $x$-intercepts, and $-1$ is halfway between 2 and $-4$.
$\therefore$ the axis of symmetry is $x = -1$.

**c** When $x = -1$,
$y = 2(-1-2)(-1+4)$
$= 2 \times -3 \times 3$
$= -18$
$\therefore$ the vertex is $(-1, -18)$.

**d**



**3** For each of the following quadratic functions:

- **i** Find the axes intercepts.
- **ii** Find the equation of the axis of symmetry.
- **iii** Find the coordinates of the vertex.
- **iv** Hence, sketch the graph of the function.

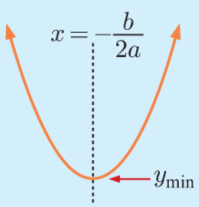
**a** $f(x) = x(x-2)$
**b** $y = 2(x-3)^2$
**c** $y = -(x-1)(x+3)$
**d** $y = -2(x-1)^2$
**e** $f(x) = -5(x+2)(x-2)$
**f** $y = 2(x+1)(x+4)$

## F QUADRATIC OPTIMISATION

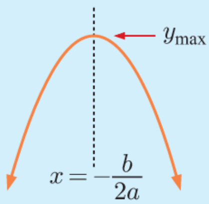
The process of finding the **maximum** or **minimum** value of a function is called **optimisation**.

For the quadratic function $y = ax^2 + bx + c$:

- If $a > 0$, the **minimum** value of $y$ occurs when $x = -\frac{b}{2a}$.




- If $a < 0$, the **maximum** value of $y$ occurs when $x = -\frac{b}{2a}$.




Optimisation is a very useful tool when looking at such issues as:

- maximising profits
- minimising costs
- maximising heights reached.

**Example 19** **Self Tutor**

The height of a rocket $t$ seconds after it is fired upwards is given by $H(t) = 100t - 5t^2$ metres, $t \geqslant 0$.

**a** How long does the rocket take to reach its maximum height?

**b** Find the maximum height reached by the rocket.

**c** How long does it take for the rocket to fall back to earth?

**a** $H(t) = 100t - 5t^2$

$\therefore$ $H(t) = -5t^2 + 100t$

Now $a = -5$ which is $< 0$, so the shape of the graph is  .

The maximum height is reached when $t = \frac{-b}{2a} = \frac{-100}{2(-5)} = 10$

$\therefore$ the maximum height is reached after 10 seconds.

**b** $H(10) = 100(10) - 5(10)^2$
$= 500$

$\therefore$ the maximum height reached is 500 m.

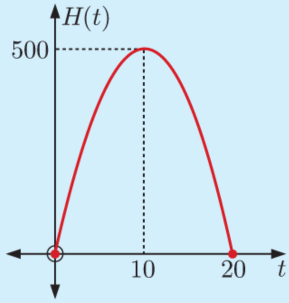
**c** The rocket falls back to earth when $H(t) = 0$

$\therefore\ -5t^2 + 100t = 0$

$\therefore\ -5t(t - 20) = 0$

$\therefore\ t = 0$ or $20$

$\therefore$ the rocket falls back to earth after 20 seconds.




## EXERCISE 20F

**1** The height of a ball $t$ seconds after it is kicked upwards is given by $H(t) = 20t - 5t^2$ metres.

**a** How long does the ball take to reach its maximum height?

**b** Find the maximum height reached by the ball.

**c** How long does it take for the ball to hit the ground?

**2** A manufacturer finds that the profit €$P$ from assembling $x$ bicycles per day is given by $P(x) = -x^2 + 50x - 200$.

**a** How many bicycles should be assembled per day to maximise the profit?

**b** Find the maximum profit.

**c** What is the loss made if no bicycles are assembled in a day? Suggest why this loss would be made.

**3** The driver of a car travelling downhill applied the brakes. The speed of the car, $t$ seconds after the brakes were applied, is given by $s(t) = -6t^2 + 12t + 60$ km h$^{-1}$.

**a** How fast was the car travelling when the driver applied the brakes?

**b** After how many seconds did the car reach its maximum speed?

**c** Find the maximum speed reached.

**4** The hourly profit obtained from operating a fleet of $n$ taxis is given by $P(n) = 120n - 200 - 2n^2$ dollars.


**a** What number of taxis gives the maximum hourly profit?

**b** Find the maximum hourly profit.

**c** How much money is lost per hour if no taxis are on the road?

**5** The temperature $T$°C in a greenhouse $t$ hours after 7:00 pm is given by $T(t) = \frac{1}{4}t^2 - 6t + 25$ for $t \leqslant 20$.

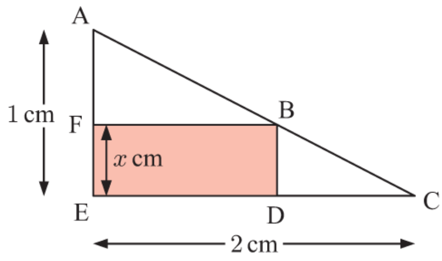
**a** Find the temperature in the greenhouse at 7:00 pm.

**b** At what time is the temperature at a minimum?

**c** Find the minimum temperature in the greenhouse for $0 \leqslant t \leqslant 20$.

**6** Answer the questions posed in the **Opening Problem** on page **432**.

**7** Infinitely many rectangles may be inscribed within the triangle ACE shown. One of them is illustrated.
Suppose EF $= x$ cm.




**a** Show that triangles ABF and ACE are similar.
**b** Show that BF $= 2(1 - x)$ cm.
**c** Show that the area of rectangle BDEF is given by $A = -2x^2 + 2x$ cm$^2$.
**d** **i** Find $x$ such that the area of the rectangle is maximised.
**ii** What is the maximum area?

## REVIEW SET 20A

**1** For the quadratic function $y = x^2 - 3x - 15$, find:

**a** the value of $y$ when $x = 4$
**b** the values of $x$ when $y = 3$.

**2** Sketch each of the following functions on the same set of axes as $y = x^2$:

**a** $y = 3x^2$
**b** $y = (x - 2)^2 + 1$
**c** $y = -(x + 3)^2 - 2$

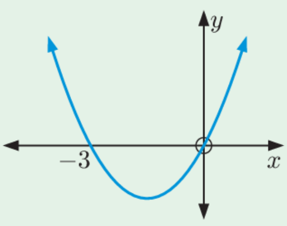
**3** **a** Write the quadratic $y = x^2 - 4x + 10$ in the form $y = (x - h)^2 + k$.
**b** Hence sketch the function, stating the coordinates of the vertex.

**4** Find the $x$-intercepts of:

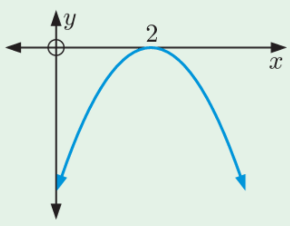
**a** $y = 5x(x + 4)$
**b** $y = 2x^2 + 6x - 56$

**5** Find the equation of the axis of symmetry for:

**a**




**b**




**6** Consider the quadratic function $y = -2(x - 1)(x + 3)$.

**a** Find the:
**i** direction the parabola opens
**ii** $y$-intercept
**iii** $x$-intercepts
**iv** equation of the axis of symmetry.

**b** Sketch the function, showing all of the above features.

**7** Find the vertex of each of the following quadratic functions:

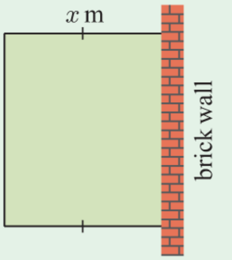
**a** $y = x^2 - 8x - 3$
**b** $y = -4x^2 + 4x - 3$

**8** Consider the function $y = x^2 - 2x - 15$.

**a** Find the:
**i** $y$-intercept
**ii** $x$-intercepts
**iii** equation of the axis of symmetry
**iv** coordinates of the vertex.

**b** Sketch the function, showing all of the above features.

**9** A vegetable gardener has 40 m of fencing to enclose a rectangular garden plot where one side is an existing brick wall. Suppose the plot is $x$ m wide as shown.




**a** Show that the area enclosed is given by $A = -2x^2 + 40x$ $\text{m}^2$.

**b** Find $x$ such that the vegetable garden has the maximum possible area.

**c** Find the maximum possible area.

**10** Determine the relationship between the graph and the $x$-axis for:

**a** $y = x^2 - 2x + 3$ **b** $y = -2x^2 + 5x - 3$ **c** $y = 5x^2 - 10x + 5$

## REVIEW SET 20B

**1** Find the values of $x$ for which $f(x) = x^2 + x - 12$ takes the value 30.

**2** Determine whether the ordered pair $(2, 5)$ satisfies the quadratic function $f(x) = x^2 - 3x + 8$.

**3** Draw the graphs of $y = x^2$ and $y = (x+2)^2 + 5$ on the same set of axes.

**4** Use the quadratic formula to find the $x$-intercepts of:

**a** $y = 3x^2 - x - 5$ **b** $y = -x^2 + 2x + 6$

**5** Draw the graph of $y = 3(x-2)^2$, showing the axes intercepts and the coordinates of the vertex.

**6** Determine the equation of the axis of symmetry for the following quadratic functions:

**a** $f(x) = (x+3)(x-5)$ **b** $f(x) = 3x^2 - 5x + 2$

**7** **a** Find the vertex of the quadratic function $f(x) = -x^2 + 4x - 7$.

**b** Hence, find the range of the function.

**8** Consider the quadratic function $f(x) = (x-1)(x-4)$.

**a** Find $f(-1)$. **b** Find the axes intercepts.

**c** Find the equation of the axis of symmetry.

**d** Find the coordinates of the vertex.

**e** Sketch the graph of $y = f(x)$, showing all of the above features.

**9** Suppose $f(x) = (x+2)(x+6)$ and $g(x) = -x^2 - 8x - 20$.

**a** Find the axes intercepts of each function.

**b** Show that the two functions have the same vertex.

**c** State the range of each function.

**d** Sketch the functions on the same set of axes.

**10** Suppose a quadratic function $y = ax^2 + bx + c$ has $c = 0$. Explain why the graph must cut the $x$-axis at least once:

**a** by considering the graphical significance of $c = 0$

**b** using the discriminant.