1. A rocket is rising vertically at a speed of 300 m s^{-1} when it is 800 m directly above the launch site. Calculate the rate of change of the distance between the rocket and an observer, who is 600 m from the launch site and on the same horizontal level as the launch site.
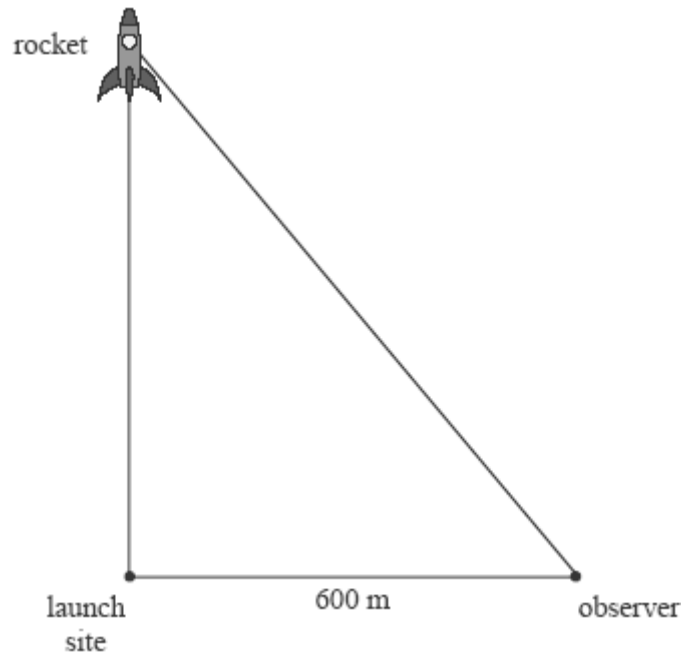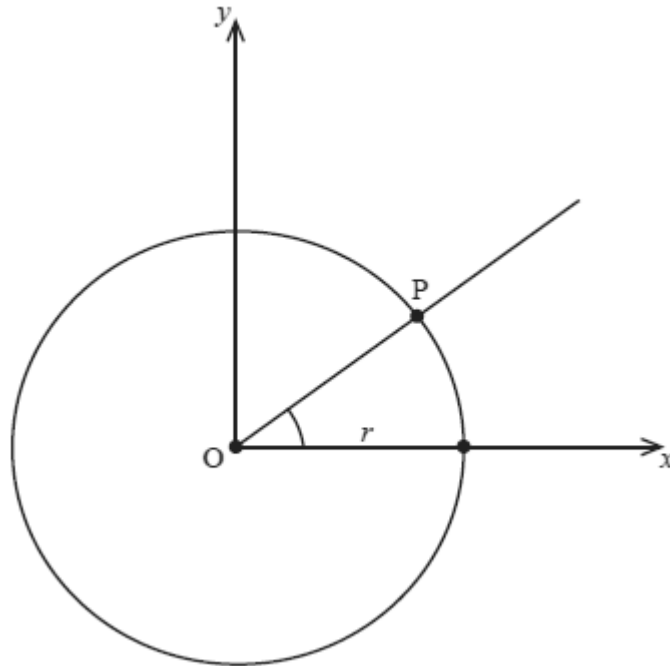

rocket

launch
site

600 m

observer

diagram not to scale

(Total 6 marks)

2. The diagram below shows a circle with centre at the origin O and radius $r > 0$.

A point P(x, y), $(x > 0, y > 0)$ is moving round the circumference of the circle.

Let $m = \tan\left(\arcsin\dfrac{y}{r}\right)$.

(a) Given that $\dfrac{dy}{dt} = 0.001r$, show that $\dfrac{dm}{dt} = \left(\dfrac{r}{10\sqrt{r^2 - y^2}}\right)^3$.

(6)

(b) State the geometrical meaning of $\dfrac{dm}{dt}$.

(1)
(Total 7 marks)

3. Consider the function $f(x) = \dfrac{\ln x}{x}$, $0 < x < e^2$.

(a) (i) Solve the equation $f'(x) = 0$.

(ii) Hence show the graph of f has a local maximum.

(iii) Write down the range of the function f.

(5)

(b) Show that there is a point of inflexion on the graph and determine its coordinates.

(5)

(c) Sketch the graph of $y = f(x)$, indicating clearly the asymptote, x-intercept and the local maximum.

(3)

(d) Now consider the functions $g(x) = \dfrac{\ln|x|}{x}$ and $h(x) = \dfrac{\ln|x|}{|x|}$, where $0 < |x| < e^2$.

(i) Sketch the graph of $y = g(x)$.

(ii) Write down the range of g.

(iii) Find the values of x such that $h(x) > g(x)$.

(6)
(Total 19 marks)

4. Consider $f(x) = \dfrac{x^2 - 5x + 4}{x^2 + 5x + 4}$.

(a) Find the equations of all asymptotes of the graph of f.

(4)

(b) Find the coordinates of the points where the graph of f meets the x and y axes.

(2)

(c) Find the coordinates of

    (i) the maximum point and justify your answer;

    (ii) the minimum point and justify your answer.

(10)

(d) Sketch the graph of f, clearly showing all the features found above.

(3)

(e) **Hence**, write down the number of points of inflexion of the graph of f.

(1)
(Total 20 marks)

5. A family of cubic functions is defined as $f_k(x) = k^2x^3 - kx^2 + x,\ k \in \mathbb{Z}^+$.

(a) Express in terms of k

    (i) $f'_k(x)$ and $f''_k(x)$;

    (ii) the coordinates of the points of inflexion P_k on the graphs of f_k.

(6)

(b) Show that all P_k lie on a straight line and state its equation.

(2)

(c) Show that for all values of k, the tangents to the graphs of f_k at P_k are parallel, and find the equation of the tangent lines.

(5)
(Total 13 marks)