Name:

SECTION A

1. (4 points) 12 and $-\frac{4}{9}$ are the second and fifth term of a geometric sequence. Find the sum to infinity of this sequence.

2. (4 points) Triangle ABC has $\angle ACB = 42^\circ$, $BC = 1.74\ cm$ and and area of $1.19\ cm^2$.

   (a) Find AC.

   (b) Find BC

3. (4 points) The following frequency distribution of marks has mean 4.5

Mark	1	2	3	4	5	6	7
Frequency	2	4	6	9	x	9	4

(a) Find the value of x.

(b) Write down the standard deviation of the marks.

(c) Find the interquartile range of the marks.

4. (5 points) Given that $\log_2 x = p$ and $\log_5 x = q$, express in terms of p and q:

(a) $\log_x 10$

(b) $\log x$

5. (5 points) Consider the function $f(x) = \dfrac{|x-1|+3}{|x+1|-2}$.

(a) Sketch the graph of f, clearly indicating any asymptotes and axes intercepts.

(b) Solve the inequality $f(x) > 2$.

6. (7 points) Consider the function $f(x) = \dfrac{2e^x}{1+3e^x}$.

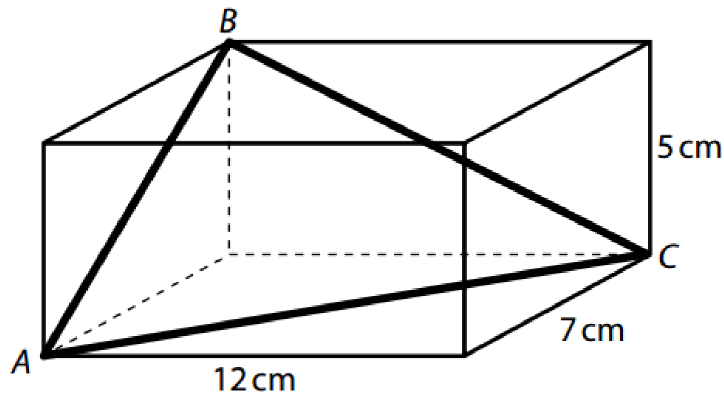
(a) Find $f^{-1}(x)$.

(b) Find the domain and range of $f^{-1}(x)$.

7. (5 points) For the rectangular box shown, find:

(a) the measure of $\angle ABC$,

(b) the area of triangle ABC.



8. (4 points)

(a) Solve the equation

$$a^x = e^{2x+1}$$

where $a > 0$, giving your for x in terms of a.

(b) For what value of a does the above equation have no solution?

9. (6 points) Given the following system of equations:

$$\begin{cases} \sin x + \cos y = 1 \\ x = e^{\frac{y}{2}} + 1 \end{cases}$$

(a) Express y in terms of x in both equations.

(b) Hence solve the system of $0 < x < \pi$ and $0 < y < \pi$.

10. (6 points) The radius and height of a cylinder are both equal to x cm. The curved surface area of the cylinder is increasing at a constant rate of 10 cm^2/s. When $x = 2$, find the rate of change of:

    (a) the radius of the cylinder,

    (b) the volume of the cylinder.

11. (8 points) Consider the function $f(x) = \ln x \sin x$.

(a) State the largest possible domain of f.

(b) Find $f'(x)$.

(b) By writing $f(x)$ as $\dfrac{\ln x}{\csc x}$, find $\lim\limits_{x \to 0^+} f(x)$.

SECTION B

12. (14 points) Two boats A and B start moving from the same point P. Boat A moves in a straight line at 20 km/h and boat B moves in a straight line at 32 km/h. The angle between their paths is $70°$.

(a) Find the distance between the two boats when t, the time after they started moving, is 2.5 $hours$.

(b) Find the rate of change of the distance between the boats when $t = 2.5$ $hours$.

(c) When $t = 2.5\ hours$, boat A continues along the same path and with the same speed, boat B changes its path (its speed is unchanged) so that it will meet boat A at some point X. Calculate the time t at which both boats will meet and the angle by which the boat B needs to turn in order to meet boat A.

13. (14 points) Consider the function $f(x) = \dfrac{\log x}{x}$.

(a) State the largest possible domain of the function.

(b) Calculate the coordinates of the stationary point of f and show that this point is a maximum.

(c) Find the range of f on the domain found in (a). Justify your answer.

(d) Hence (without calculating any numerical values) prove that:

$$e^{\pi} > \pi^{e}$$

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14. (24 points) Consider the function

$$y(t) = e^{-\lambda t}\sin(pt + \alpha)$$

where λ, p and α are constants.

(a) Show that

$$\frac{d^2y}{dt^2} + 2\lambda\frac{dy}{dt} + (\lambda^2 + p^2)y = 0$$

(b) Given that $\frac{dy}{dt} = -2p$ and $\frac{d^2y}{dt^2} = -3p$, when $\alpha = 0$ and $t = \frac{\pi}{p}$, calculate the value of λ and show that

$$p = \frac{3\pi}{4\ln 2}$$

(c) Now consider a different case when $\frac{dy}{dt} = 0$ at $t = 0$ and λ is not specified. Show that

(i) $\lambda = p \cot \alpha$;

(ii) the values of t at which $\frac{dy}{dt} = 0$ are in an arithmetic progression, and find the common difference;

(iii) the values of y at which $\dfrac{dy}{dt} = 0$ are in a geometric progression, and find the common ratio in terms of α.