

Intervals

Na prezentacji zostanie omówiony sposób zapisu przedziałów (**intervals**) i działania na nich. Przedziały są zbiorami, więc wszystkie działania na zbiorach możemy stosować do przedziałów

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We will use the following notation:

$\langle a, b \rangle$ denotes all real numbers x such that $a \leq x \leq b$

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(a, b) denotes all x such that $a < x < b$

So for example $(-1, 3] = \{x : x \in \mathbb{R} \wedge -1 < x \leq 3\}$.

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(a, b) denotes all x such that $a < x < b$

So for example $\langle -1, 3 \rangle = \{x : x \in \mathbb{R} \wedge -1 < x \leq 3\}$.

Note that because we will work in real numbers, I'll no longer write that x has to be a real number (in other words unless stated otherwise we assume that x is a real number).

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$\langle a, \infty \rangle$ denotes all x such that $a \leq x$

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Notation

$\langle a, \infty \rangle$ denotes all x such that $a \leq x$

(a, ∞) denotes all x such that $a < x$

$(-\infty, b]$ denotes all x such that $x \leq b$

$(-\infty, b)$ denotes all x such that $x < b$

Note that we never include ∞ (or $-\infty$) as it is not a number.

Remember that intervals are just sets of numbers (often infinite), so all the operations on sets can be used. We will practice those operations on the next slides.

Example 1

Let:

$$A = (1, 4) \quad B = (-\infty, 3)$$

Find $A \cup B$, $A \cap B$, $A - B$ oraz $B - A$.

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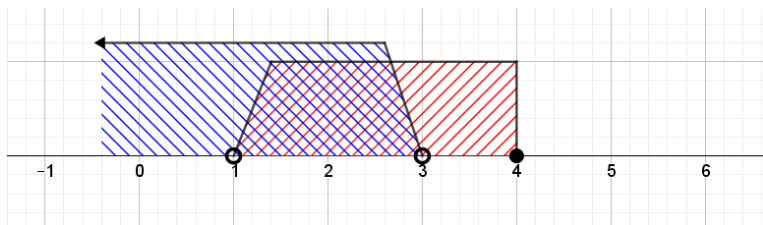
Find $A \cup B$, $A \cap B$, $A - B$ oraz $B - A$.

It is often helpful to mark both sets on a number line:

Example 1

$$A = (1, 4) \quad B = (-\infty, 3)$$

A is marked with red, B with blue.



Example 1

- $A \cup B$ is the union of the sets, so it is the part coloured by at least one of the colours.

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- $A - B$ is the difference between A and B , so it is the part coloured **only** in red.

Example 1

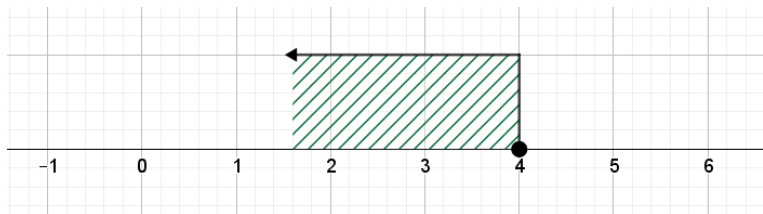
- $A \cup B$ is the union of the sets, so it is the part coloured by at least one of the colours.
- $A \cap B$ is the intersection, so it is the part coloured by both colours.
- $A - B$ is the difference between A and B , so it is the part coloured **only** in red.
- $B - A$ is the difference between B and A , so it is the part coloured **only** in blue.

Example 1

$$A \cup B = (-\infty, 4)$$

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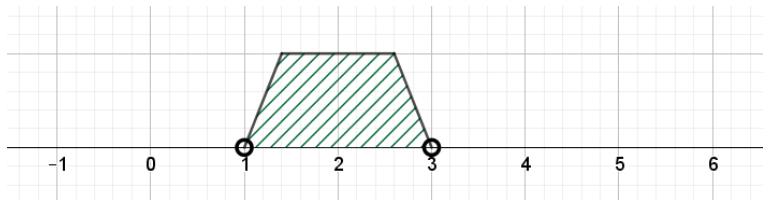


Example 1

$$A \cap B = (1, 3)$$

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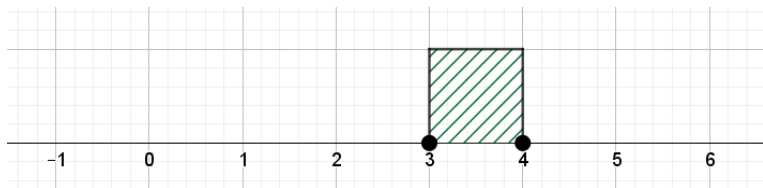


Example 1

$$A - B = \langle 3, 4 \rangle$$

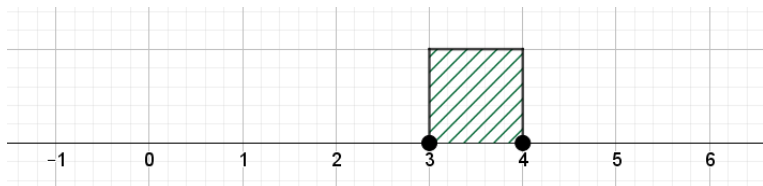
Example 1

$$A - B = \langle 3, 4 \rangle$$



Example 1

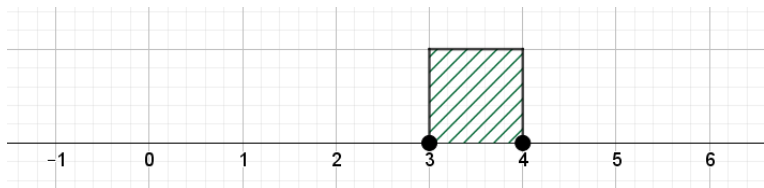
$$A - B = \langle 3, 4 \rangle$$



Why is 3 in this set?

Example 1

$$A - B = \langle 3, 4 \rangle$$



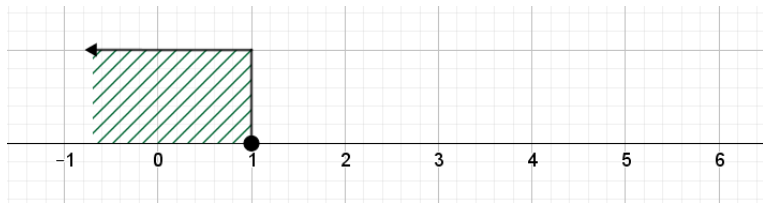
Why is 3 in this set? 3 belongs to $A - B$, since it belongs to A and doesn't belong to B . $B = (-\infty, 3)$, so 3 is outside of B .

Example 1

$$B - A = (-\infty, 1)$$

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Example 2

Let:

$$A = (0, 5) \quad B = \langle 1, 3 \rangle$$

Find $A \cup B$, $A \cap B$, $A - B$ oraz $B - A$.

Example 2

Let:

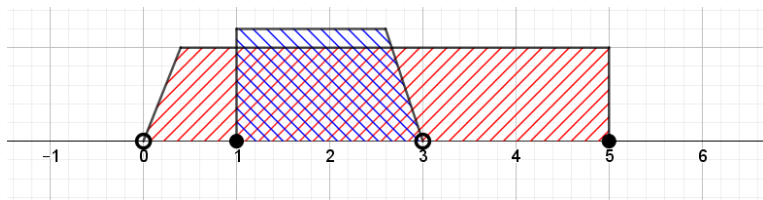
$$A = (0, 5) \quad B = \langle 1, 3 \rangle$$

Find $A \cup B$, $A \cap B$, $A - B$ oraz $B - A$.

Again it is useful to mark the sets on the number line.

Example 2

A is red, B is blue:

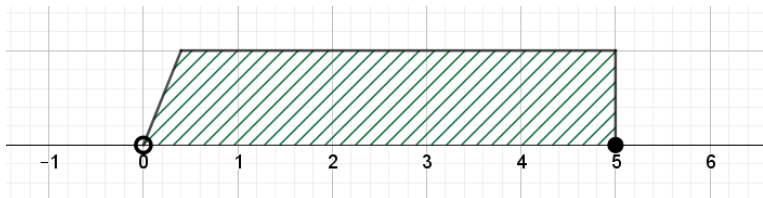


Example 2

$$A \cup B = (0, 5)$$

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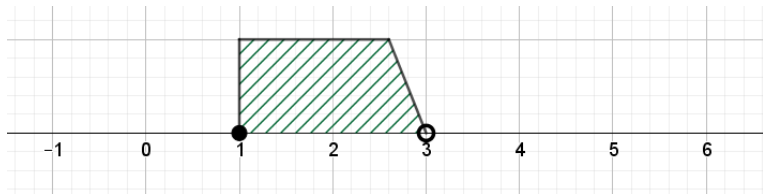


Example 2

$$A \cap B = \langle 1, 3 \rangle$$

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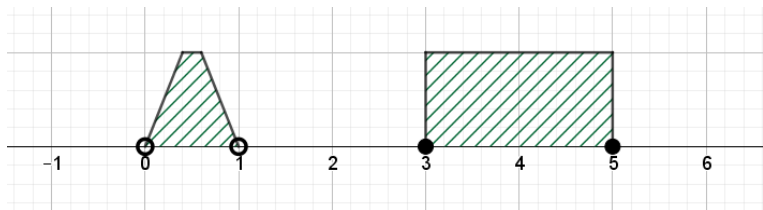


Example 2

$$A - B = (0, 1) \cup \langle 3, 5 \rangle$$

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Example 2

$$B - A = \emptyset$$

Example 3

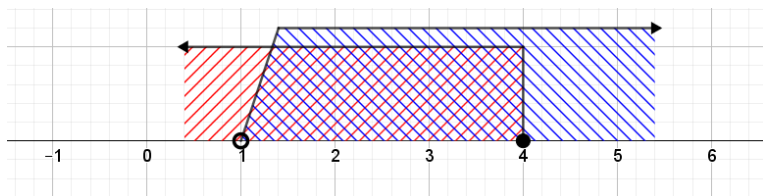
Let:

$$A = (-\infty, 4) \quad B = (1, \infty)$$

Find the sets A' , B' .

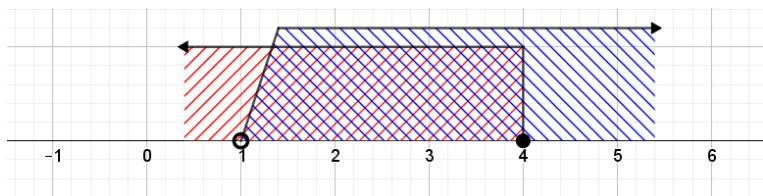
Example 3

We will use red for A and blue for B :



Example 3

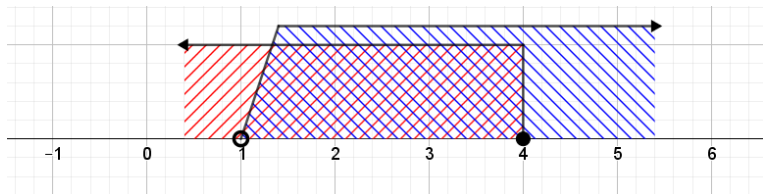
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- A' is the complement of A , so it is the part **not** coloured in red.

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- A' is the complement of A , so it is the part **not** coloured in red.
- B' is the complement of B , so it is the part **not** coloured in blue.

Example 3

$$A' = (4, \infty)$$

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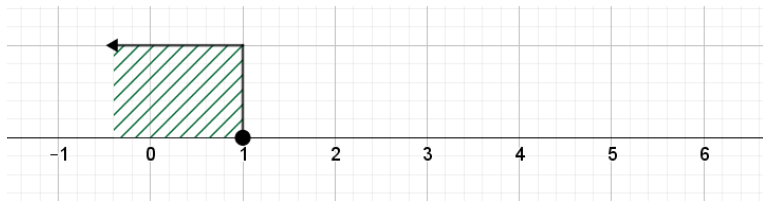


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The short test at the beginning of the class will be similar to the examples above.

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You may want to try questions 1.60, 1.61 and 1.62 from the exercise book.