Divisibility

Na tej prezentacji omówione zostanie dzielenie w zbiorze liczb naturalnych.

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Rational numbers $\mathbb{Q} = \{\frac{p}{q} \mid p, q \in \mathbb{Z} \text{ and } q \neq 0\}$. In other words rational numbers are numbers that **can be** written as a fraction where both the numerator and the denominator are integers and the denominator is not 0.

Divisibility (Podzielność)

We say that an integer n is divisible (podzielny) by an integer m, if there exists an integer k, such that $n = k \cdot m$.

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We will write $m \not| n$ to indicate that n is not divisible by m. So for instance we have $8 \not| 42$, because there is no integer k such that $42 = k \cdot 8$.

Definitions

Prime numbers

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A prime number is a natural number, which has exactly two positive divisors (1 and itself).

Try and prove (or at least convince yourself) that these two definitions are indeed equivalent.

Prime numbers

There are 25 prime numbers less than 100:

$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,$$
$$43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97$$

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Theorem

There are infinitely many prime numbers.

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Theorem

There are infinitely many prime numbers.

We've proved the above theorem in class - it's very important, make sure that you understands the proof.

Prime numbers - bonus material

Some of you may find it interesting to think of the distribution of prime numbers.

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The question is how large can the gap between two consecutive prime numbers be?

Composite numbers

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Note that 1 is neither prime nor composite.

Fundamental Theorem of Arithmetic

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Every natural number greater than 1 is either prime or can be uniquely written as a product of prime numbers.

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Example: $20 = 2 \times 2 \times 5$.

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For example we have $20 = 4 \times 5$, but also $20 = 2 \times 10$, so 20 can be written as a product of two natural numbers in more than one way.

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For example we have $20 = 4 \times 5$, but also $20 = 2 \times 10$, so 20 can be written as a product of two natural numbers in more than one way.

However if we wanted to express 20 as a product of prime numbers then it can be done and it can be done in only one way: $20 = 2 \times 2 \times 5$

Prime decomposition

In order to decompose a number into its prime factors we do the following. We divide the given number by the least possible prime number that divides it. We then repeat this for the quotient (result of the division) until our quotient is 1.

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Lets decompose 20 into prime. The least prime number that divides 20 is **2**. $20 \div 2 = 10$. Now we work with 10. The least prime number that divides 10 is again **2**. $10 \div 2 = 5$. Now 5, the least prime number that divides 5 is **5** itself. $5 \div 5 = 1$. We got to 1, we're done.

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Finally we have: $20 = 2 \times 2 \times 5$.

Prime decomposition

The whole thing can be written as:

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So (for the fourth time): $20 = 2 \times 2 \times 5$.

Prime decomposition - examples

Write 378 as a product of prime numbers.

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$$\begin{array}{r|l} 378 & 2 \\ 189 & 3 \\ 63 & 3 \\ 21 & 3 \\ 7 & 7 \\ 1 & \end{array}$$

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$$\begin{array}{c|c} 378 & 2 \\ 189 & 3 \\ 63 & 3 \\ 21 & 3 \\ 7 & 7 \\ 1 & \end{array}$$

So: $378 = 2 \times 3 \times 3 \times 3 \times 7$.

Prime decomposition - examples

Write 14300 as a product of prime numbers.

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14300	2
7150	2
3575	5
715	5
143	11
13	13
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So: $14300 = 2 \times 2 \times 5 \times 5 \times 11 \times 13$.

Now we move on to divisibility rules. In other words - how do we know that some number is divisible by, say, 3?

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- $2|n$ iff the units digit (cyfra jendości) of n is 0,2,4,6 or 8.

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- $9|n$ iff the sum of the digits of n is divisible by 9.
- $11|n$ iff the difference between the sums of every second digit and the sum of the remaining digits is divisible by 11.

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Note that iff stands for *if and only if*.

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- not divisible by 4, the last two digits make up a number 21, which is not divisible by 4.

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- not divisible by 5, last digit not 0 or 5;
- not divisible by 6, because it's not divisible by 2 (and 3);

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- not divisible by 5, last digit not 0 or 5;
- not divisible by 6, because it's not divisible by 2 (and 3);
- not divisible by 8, 321 is not divisible by 8 and also the number is not divisible by 2 (and 4);

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- not divisible by 5, last digit not 0 or 5;
- not divisible by 6, because it's not divisible by 2 (and 3);
- not divisible by 8, 321 is not divisible by 8 and also the number is not divisible by 2 (and 4);
- not divisible by 9, the sum of its digits is 16, which is not divisible by 9. And also the number is not divisible by 3;

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- not divisible by 5, last digit not 0 or 5;
- not divisible by 6, because it's not divisible by 2 (and 3);
- not divisible by 8, 321 is not divisible by 8 and also the number is not divisible by 2 (and 4);
- not divisible by 9, the sum of its digits is 16, which is not divisible by 9. And also the number is not divisible by 3;
- divisible by 11, we have $(1+3+3+1)-(2+4+2)=0$, which is divisible by 11.

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The number 659442 is:

- divisible by 2, the last digits 2;
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Divisibility rules - example

The number 659442 is:

- divisible by 2, the last digits 2;
- divisible by 3, the sum of all its digits is $6 + 5 + 9 + 4 + 4 + 2 = 30$, which is divisible by 3;
- not divisible by 4, the last two digits make up a number 42, which is not divisible by 4.

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- not divisible by 4, the last two digits make up a number 42, which is not divisible by 4.
- not divisible by 5, last digit not 0 or 5;

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- not divisible by 5, last digit not 0 or 5;
- divisible by 6, because it's divisible by both 2 and 3;

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- not divisible by 5, last digit not 0 or 5;
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- not divisible by 5, last digit not 0 or 5;
- divisible by 6, because it's divisible by both 2 and 3;
- not divisible by 8, 442 is not divisible by 8 and also the number is not divisible by 4;
- not divisible by 9, the sum of its digits is 30, which is not divisible by 9.
- not divisible by 11, we have $(6 + 9 + 4) - (5 + 4 + 2) = 8$, which is not divisible by 11.

Now we want to practice writing numbers given the information about their divisibility.

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Recall that an even number (liczba parzysta) is an integer divisible by 2 and an odd number (liczba nieparzysta) is an integer which is not divisible by 2.

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Write down an integer x if

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$x = 2k \qquad k \in \mathbb{Z}$

b) x is an odd number.
$x = 2k + 1 \qquad k \in \mathbb{Z}$

c) x is a product of three consecutive even numbers.
$x = (2k - 2) \times 2k \times (2k + 2) \qquad k \in \mathbb{Z}$

d) x is a product of three consecutive odd numbers.
$x = (2k - 1) \times (2k + 1) \times (2k + 3) \qquad k \in \mathbb{Z}$

Example 2 - divisibility

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d) x is divisible by 2 and 6
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e) x is divisible by 4 and 6
$x = 12k \qquad k \in \mathbb{Z}$

Important reminder

Remember, if a number is divisible by m and n, then it is divisible by $lcm(m, n)$, but not necessarily by mn.

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Remember, if a number is divisible by m and n, then it is divisible by $lcm(m, n)$, but not necessarily by mn.

So for instance if a number is divisible by 4 and 6, then it doesn't mean that it is divisible by 24. In fact 12 is both divisible by 4 and 6, but of course it's not divisible by 24.

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Example 4 - remainders

Write down three consecutive integers such that

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Note: you could have also written for example:
$6k-5, 6k+1, 6k+7 \qquad k \in \mathbb{Z}$

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Note: you could have also written for example:
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b) the remainder when they are divided by 13 is 5 .
$13k + 5, 13k + 18, 13k + 31 \qquad k \in \mathbb{Z}$

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b) the remainder when they are divided by 13 is 5 .
$13k+5, 13k+18, 13k+31 \qquad k \in \mathbb{Z}$

Note: again, another possible way would be:
$13k-8, 13k+5, 13k+18 \qquad k \in \mathbb{Z}$

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Find three consecutive odd numbers whose sum is 159

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$$(2k-1)+(2k+1)+(2k+3)=159$$

$$6k=156$$

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$$2k-1=2\times 26-1=51$$

The numbers are 51, 53 and 55.

Example 6

Find four consecutive numbers, which when divide by 4 give remainder equal to 3, and their sum is 116.

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$$(4k-1)+(4k+3)+(4k+7)+(4k+11)=116$$

$$16k=96$$

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$$(4k-1)+(4k+3)+(4k+7)+(4k+11)=116$$

$$16k=96$$

$$k=6$$

$$4k-1=4\times 6-1=23$$

The numbers are 23, 27, 31 and 35.

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$$(2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1 = 2m + 1$$

where $m = 2k^2 + 2k$, and so m is a integer.

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where $m = 2k^2 + 2k$, and so m is a integer. So $(2k+1)^2$ is in the form $2m+1$, with m integer, so it is odd.

□

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$$n^2 = (3k + 2)^2 = 9k^2 + 12k + 4 = 3(3k^2 + 4k + 1) + 1 = 3m + 1$$

where $m = 3k^2 + 4k + 1$, so it's an integer.

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where $m = 3k^2 + 4k + 1$, so it's an integer. So n^2 is of the form $3m + 1$, with m integer, so it has a remainder of 1, when divided by 3.

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where $n = 2km + k + m$, so it's an integer.

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where $n = 2km + k + m$, so it's an integer. So we got the form $2n + 1$, with n being an integer, which means that we got an odd number.

□

Example 9

Show that a square number is divisible by 4 or has remainder 1 when divided by 4.

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$$(4k)^2 = 16k^2 = 4 \times 4k^2 = 4m$$
$$(4k+1)^2 = 16k^2 + 8k + 1 = 4(4k^2 + 2k) + 1 = 4n + 1$$
$$(4k+2)^2 = 16k^2 + 16k + 4 = 4(4k^2 + 4k + 1) = 4s$$
$$(4k+3)^2 = 16k^2 + 24k + 9 = 4(4k^2 + 6k + 2) + 1 = 4t + 1$$

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$$(4k+3)^2 = 16k^2 + 24k + 9 = 4(4k^2 + 6k + 2) + 1 = 4t + 1$$

in each case we have a number of the form $4m$ or $4m+1$, so we either have a number divisible by 4 or that gives remainder 1 when divided by 1.

Short test

The class will begin with a short test. You may be asked to write a number as a product of primes or check whether a given number is divisible by 2,3,4 and so on.

If there are any questions or doubts, you can email me at T.J.Lechowski@gmail.com