11 **a** ≈ 88 students
b $m \approx 24$
c

Time (t min)	*Frequency*
$5 \leqslant t < 10$	20
$10 \leqslant t < 15$	40
$15 \leqslant t < 20$	48
$20 \leqslant t < 25$	42
$25 \leqslant t < 30$	28
$30 \leqslant t < 35$	17
$35 \leqslant t < 40$	5

12 **a** $\sigma^2 \approx 63.0$, $\sigma \approx 7.94$ **b** $\sigma^2 \approx 0.969$, $\sigma \approx 0.984$

13 **a** $\overline{x} \approx 49.6$ matches, $\sigma \approx 1.60$ matches, $s \approx 1.60$ matches
b The claim is not justified, but a larger sample is needed.

14 **a** $\overline{x} \approx 33.6$ L **b** $\sigma \approx 7.63$ L, $s \approx 7.66$ L

15 **a** No, extreme values have less effect on the standard deviation of a larger population.
b **i** mean **ii** standard deviation
c A low standard deviation means that the weight of biscuits in each packet is, on average, close to 250 g.

REVIEW SET 13B

1 **a**

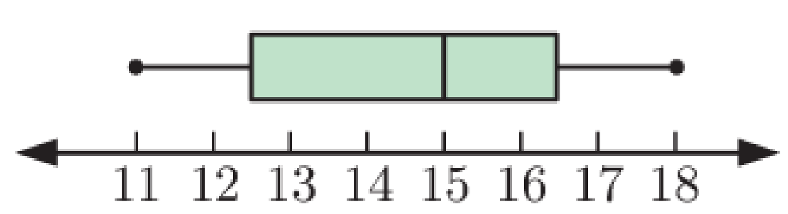
	mean (seconds)	median (seconds)
Week 1	≈ 16.0	16.3
Week 2	≈ 15.1	15.1
Week 3	≈ 14.4	14.3
Week 4	14.0	14.0

b Yes, Heike's mean and median times have gradually decreased each week which indicates that her speed has improved over the 4 week period.

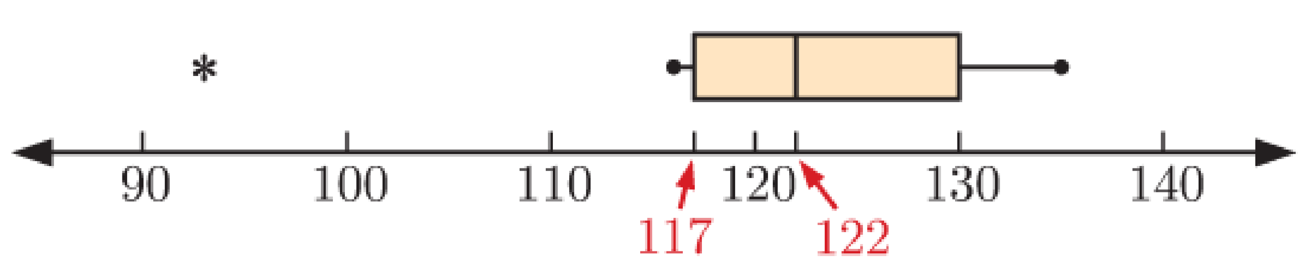
2 **a** 5 **b** 3.52 **c** 3.5 **3** **a** $x = 7$ **b** 6

4 $p = 7$, $q = 9$ (or $p = 9$, $q = 7$) **5** ≈ 414 patrons

6



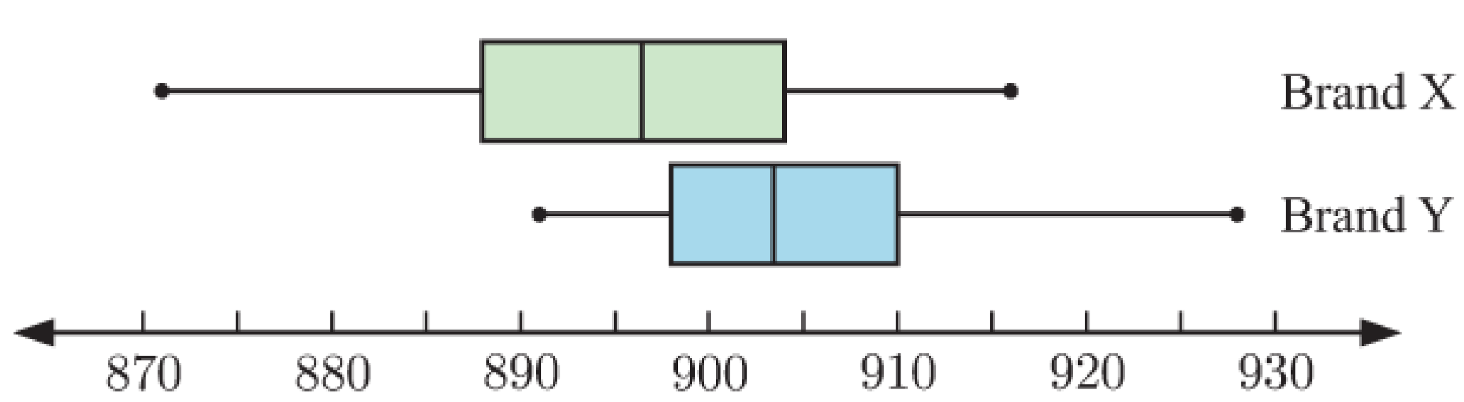
7 **a** $\sigma \approx 11.7$, $s \approx 12.4$ **b** $Q_1 = 117$, $Q_3 = 130$
c yes, 93
d



8 **a**

	Brand X	*Brand Y*
min	871	891
Q_1	888	898
median	896.5	903.5
Q_3	904	910
max	916	928
IQR	16	12

b



c **i** Brand Y, as the median is higher.
ii Brand Y, as the IQR is lower, so less variation.

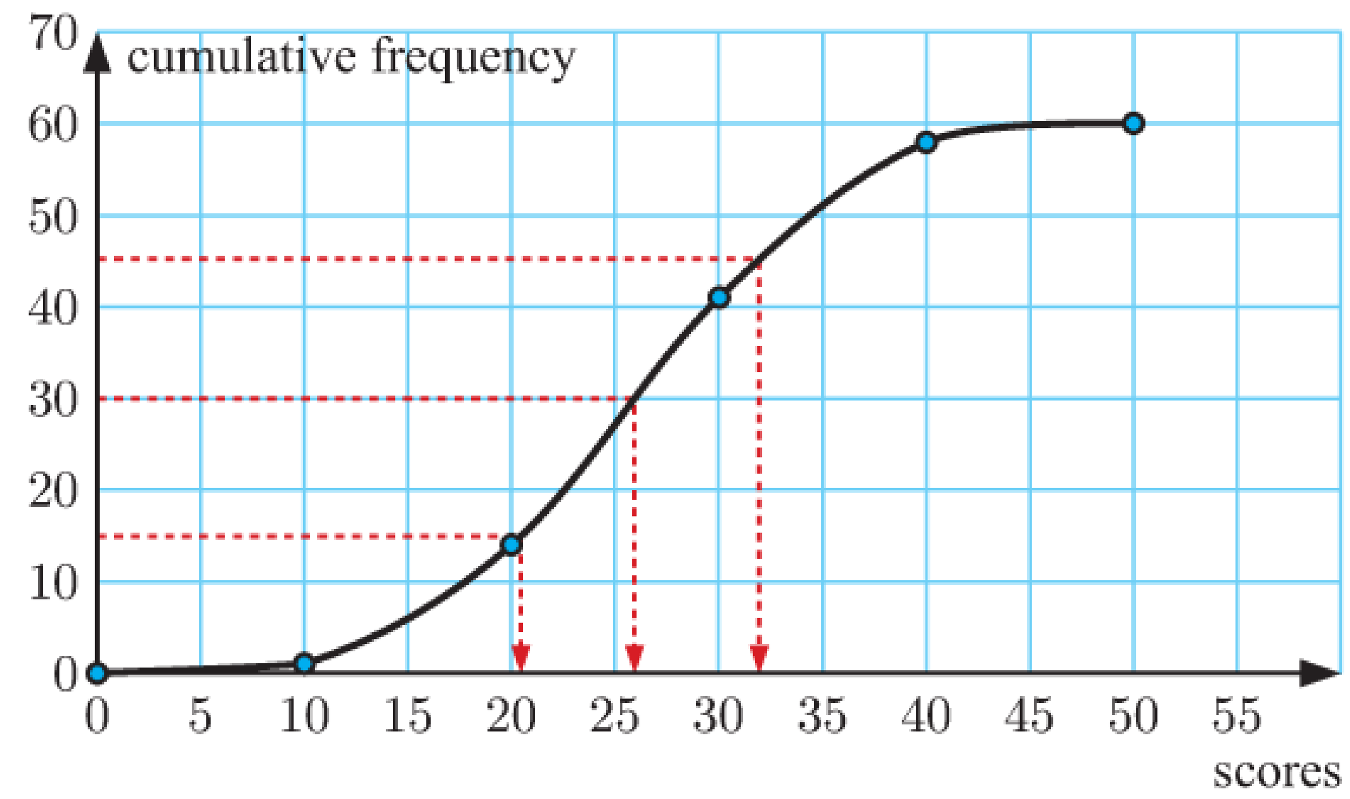
9 **a** $p = 12$, $m = 6$
b

Measure	*Value*
mode	9
median	9
range	4

c $\overline{x} = \frac{254}{30} = \frac{127}{15}$

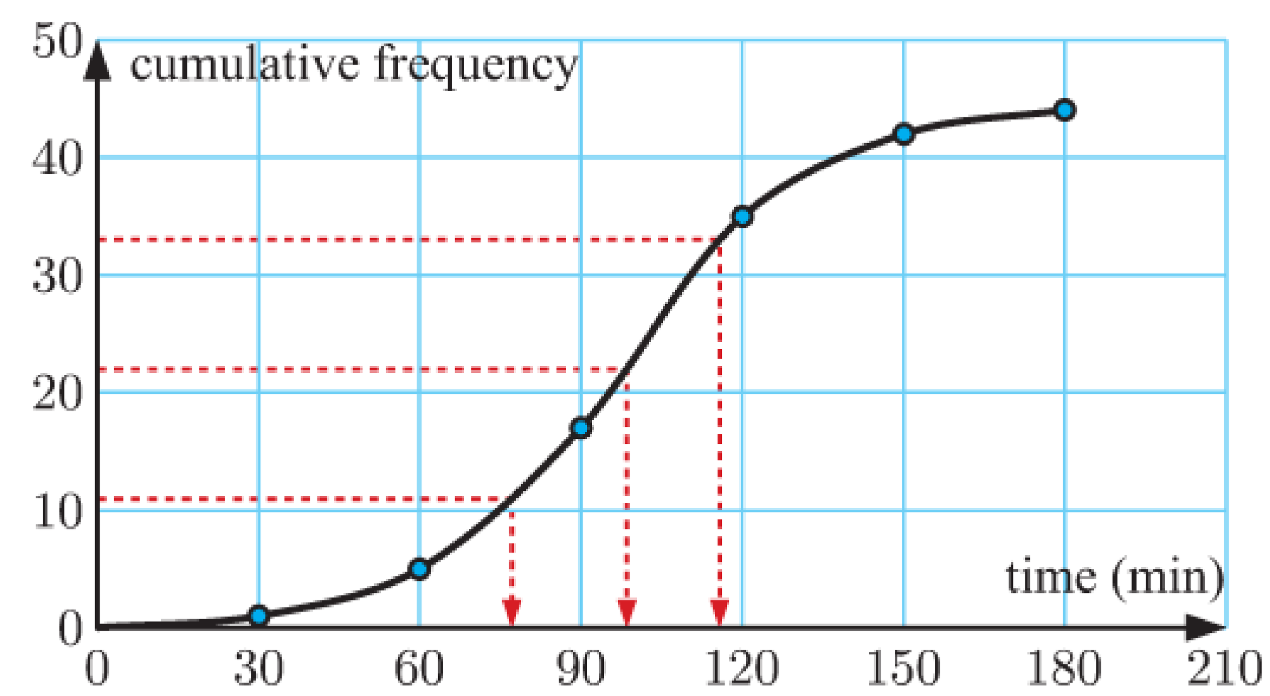
10 **a** ≈ 77 days **b** ≈ 12 days

11 **a**



b **i** median ≈ 26 **ii** IQR ≈ 11.5
iii $\overline{x} \approx 26.0$ **iv** $\sigma \approx 8.31$

12 **a** 44 players **b** $90 \leqslant t < 120$ min
c



d **i** ≈ 98.6 min **ii** ≈ 96.8 min **iii** no
e "…. between 77.2 and 115.7 minutes."

13 **a** $\overline{x} \approx$ €207.02 **b** $\sigma =$ €38.80, $s \approx$ €38.89

14 **a** Kevin: $\overline{x} = 41.2$ min; Felicity: $\overline{x} = 39.5$ min
b Kevin: $\sigma \approx 7.61$ min, $s \approx 7.81$ min;
Felicity: $\sigma \approx 9.22$ min, $s \approx 9.46$ min
c Felicity **d** Kevin

15 10 data values

EXERCISE 14A

1 **a** $y = x^2 - 3x + 1$

x	-2	-1	0	1	2
y	11	5	1	-1	-1

b $y = x^2 + 2x - 5$

x	-2	-1	0	1	2
y	-5	-6	-5	-2	3

c $y = 2x^2 - x + 3$

x	-4	-2	0	2	4
y	39	13	3	9	31

d $y = -3x^2 + 2x + 4$

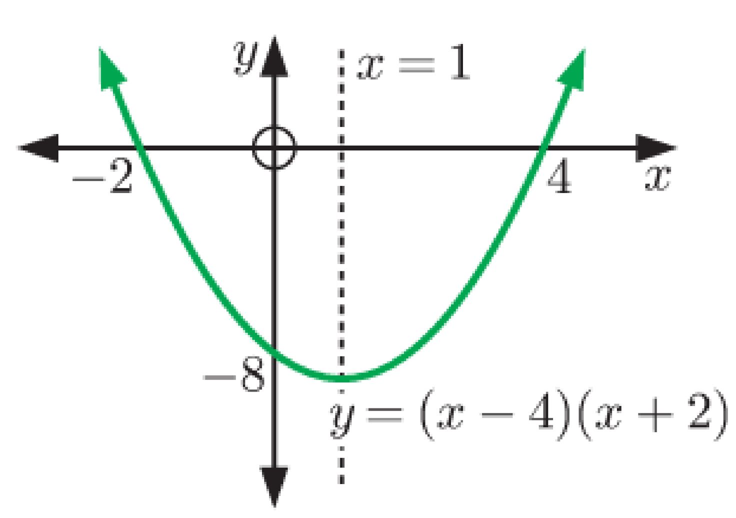
x	-4	-2	0	2	4
y	-52	-12	4	-4	-36

2 **a** no **b** yes **c** yes **d** yes **e** no **f** yes

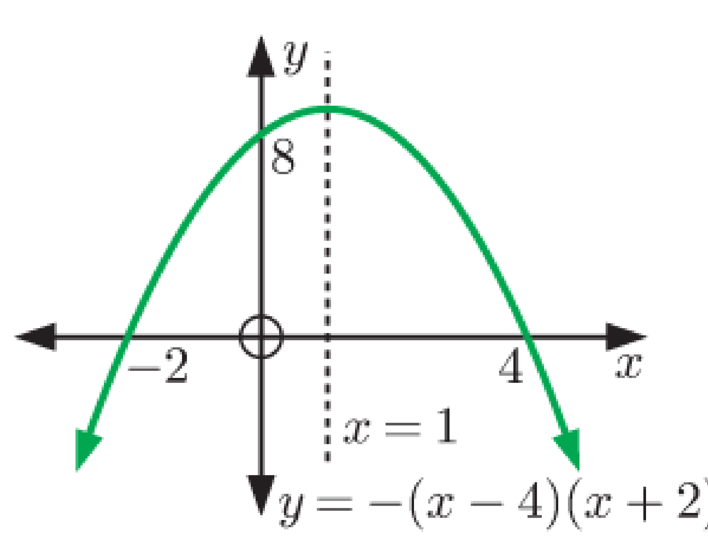
3 **a** $x = -1$ or -2 **b** $x = 2$ **c** $x = 1$ or 5
d $x = -3$ or $\frac{1}{2}$ **e** $x = -6$ or 1 **f** no real solutions

EXERCISE 14B.1

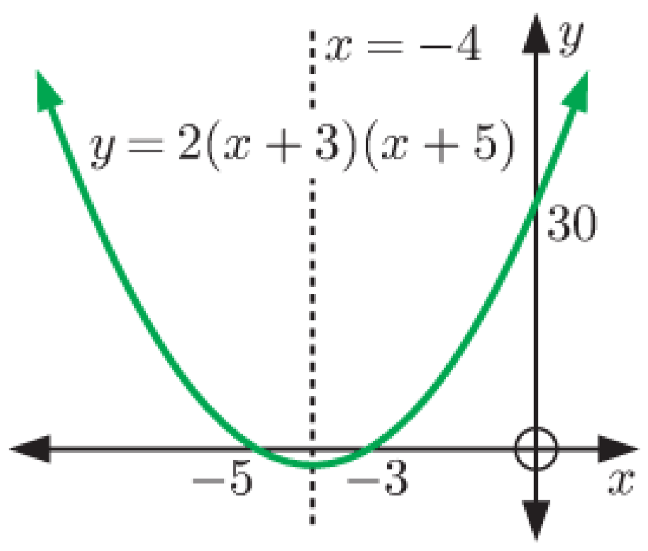
1 **a**



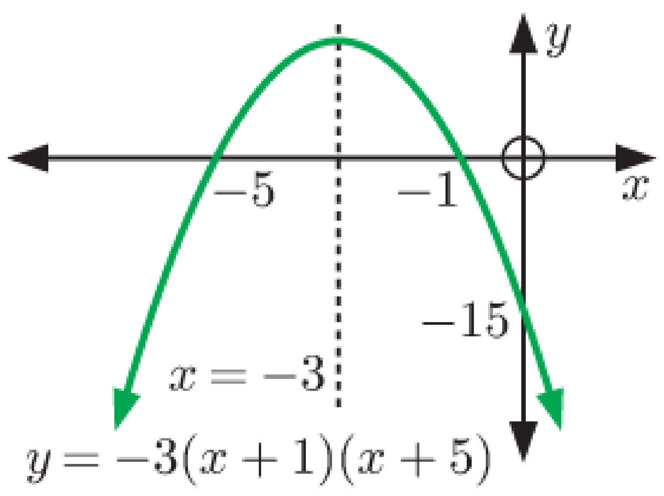
b



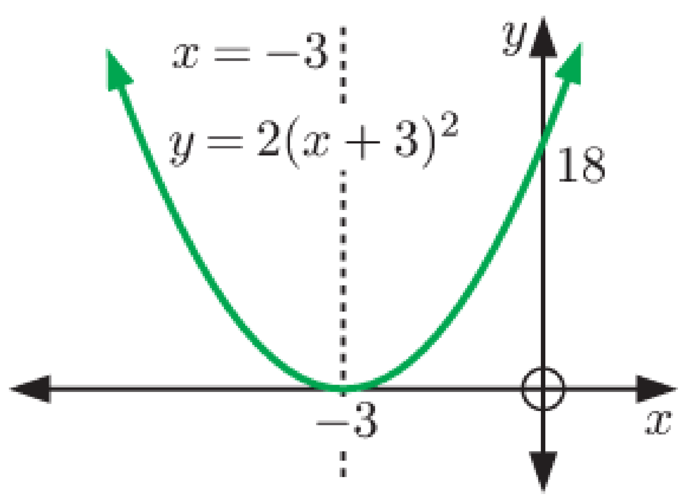
c



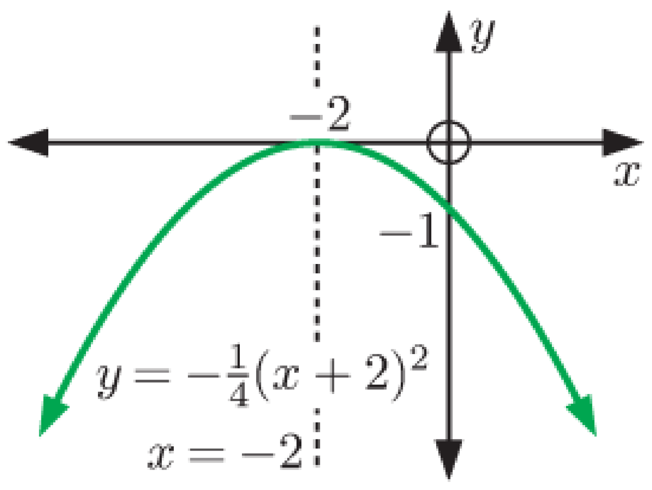
d



e

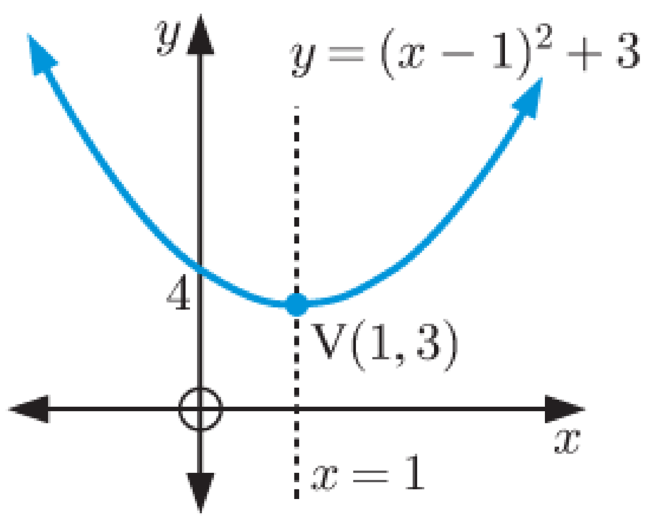


f

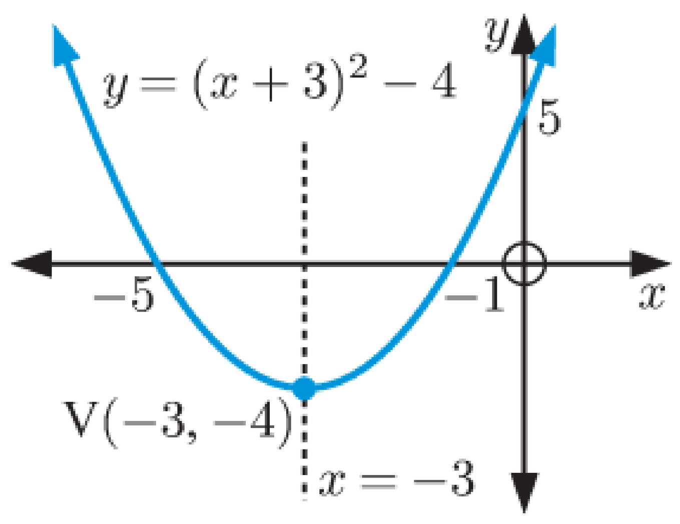


2 **a** C **b** E **c** B **d** F **e** G **f** H
g I **h** A **i** D

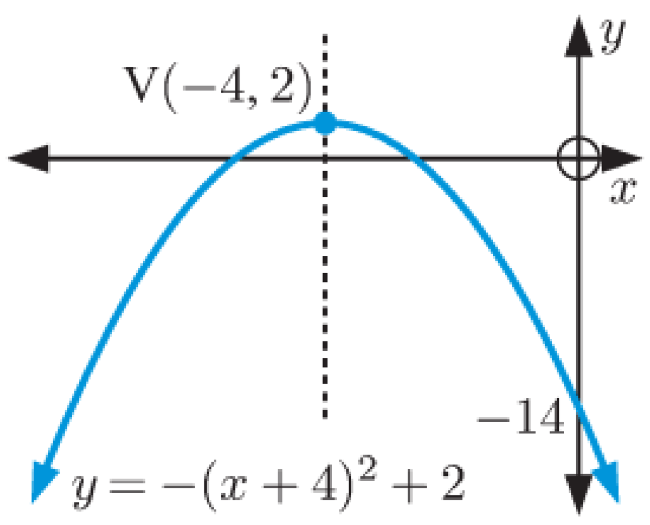
3 **a**



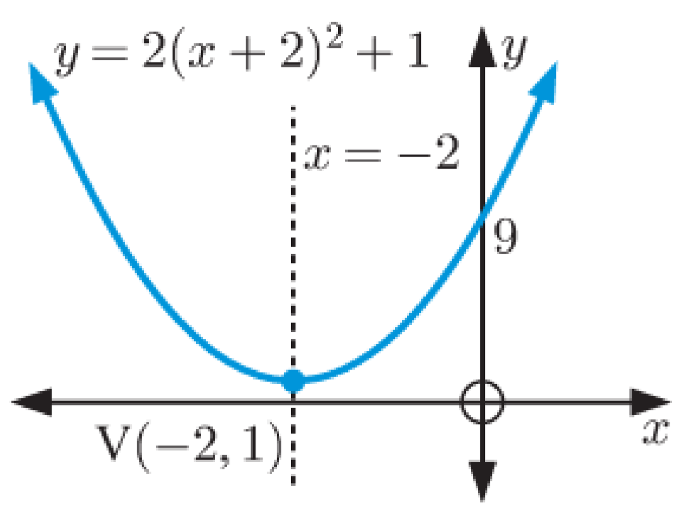
b



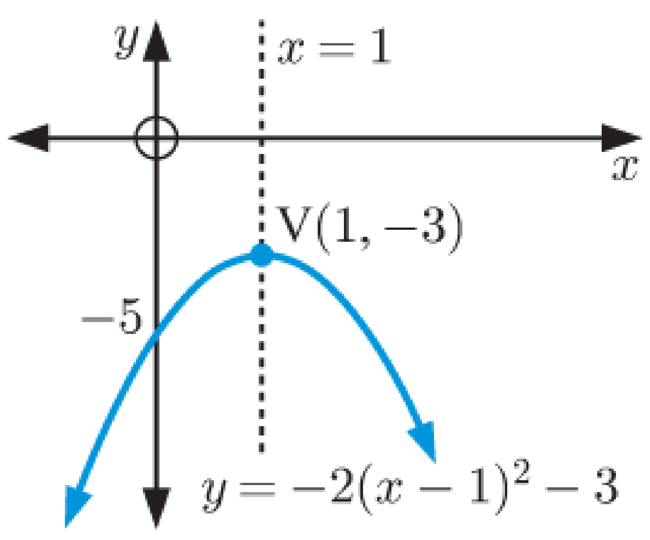
c



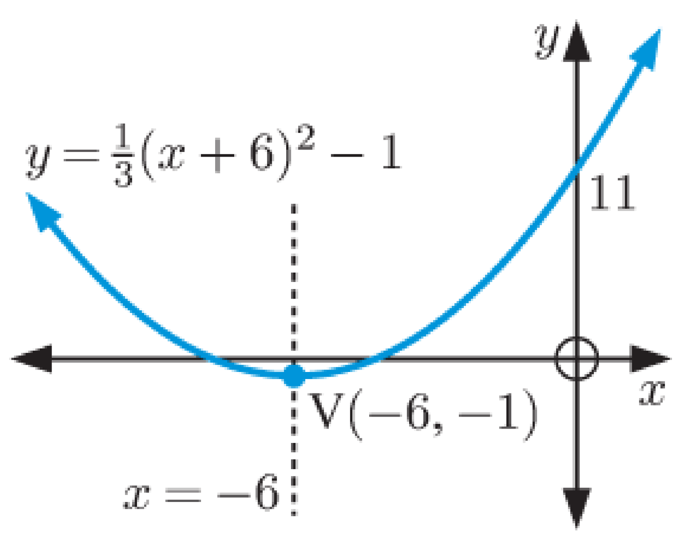
d



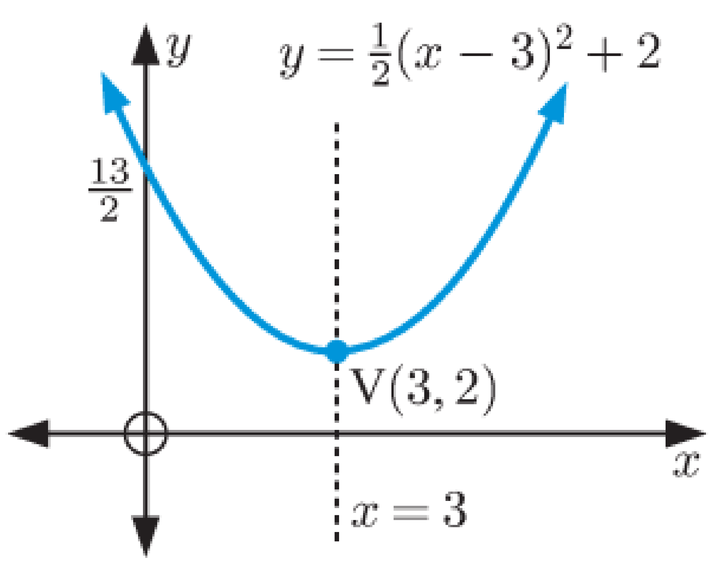
e



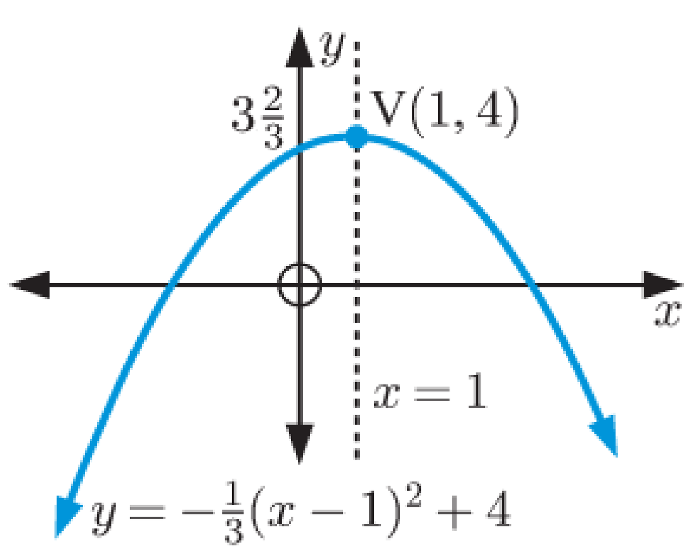
f



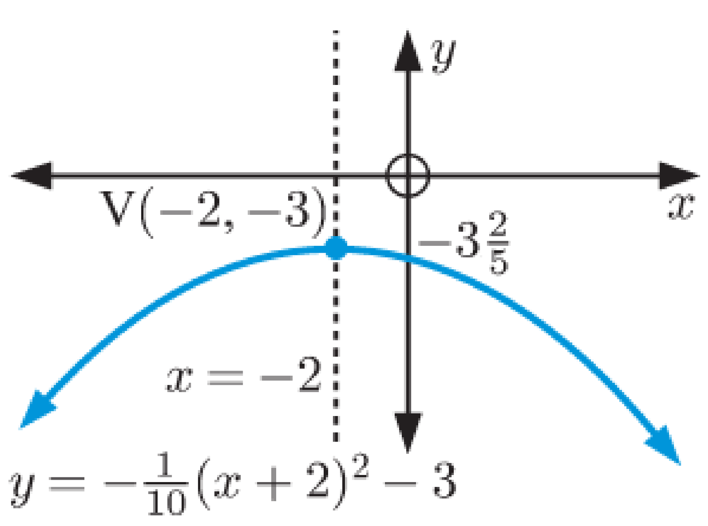
g



h

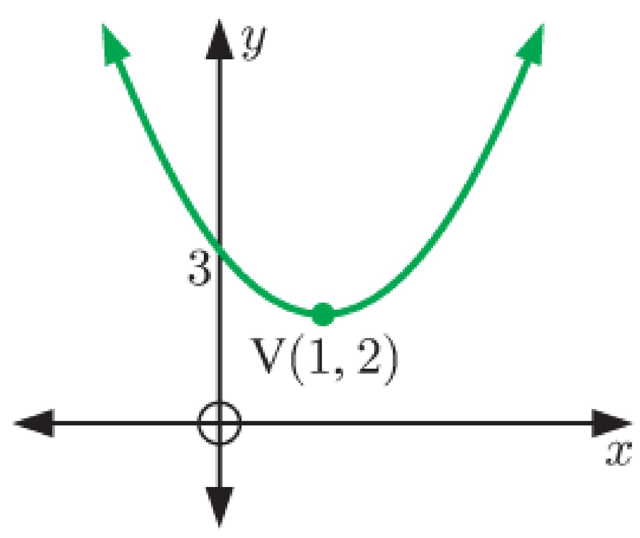


i

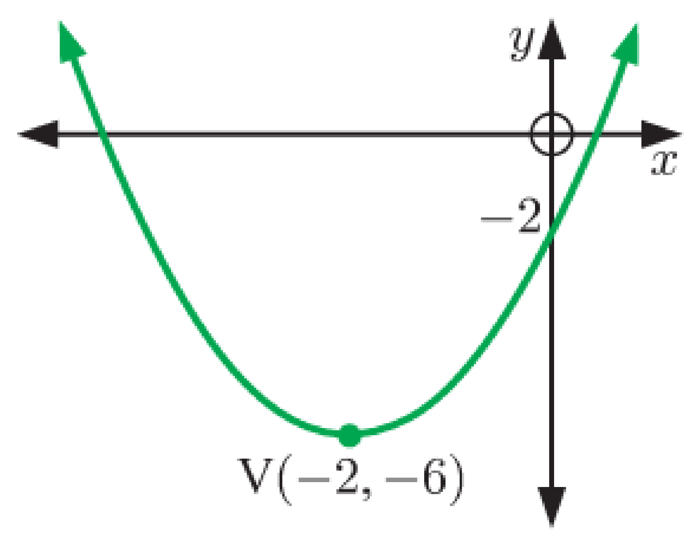


EXERCISE 14B.2

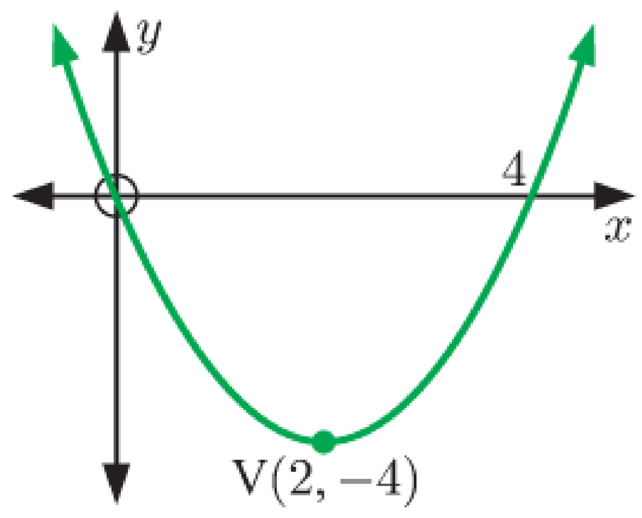
1 **a** $y = (x-1)^2 + 2$



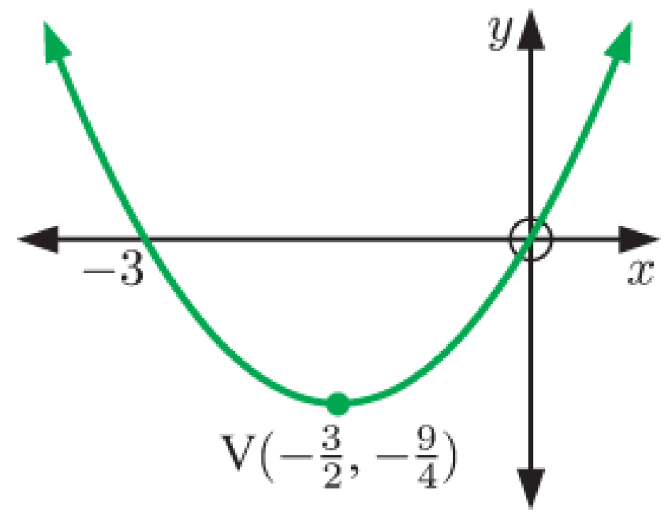
b $y = (x+2)^2 - 6$



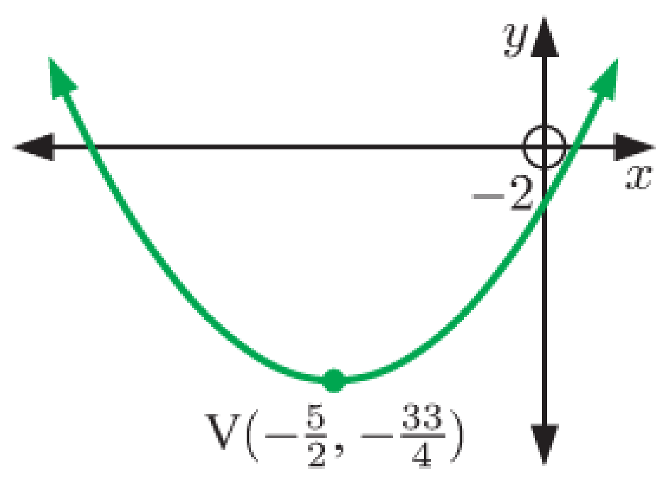
c $y = (x-2)^2 - 4$



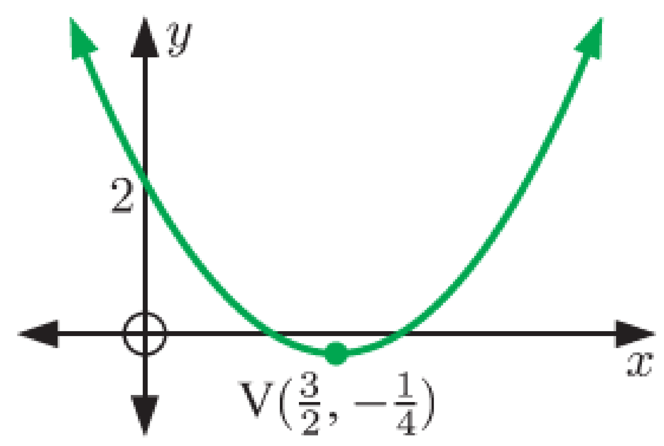
d $y = \left(x + \frac{3}{2}\right)^2 - \frac{9}{4}$



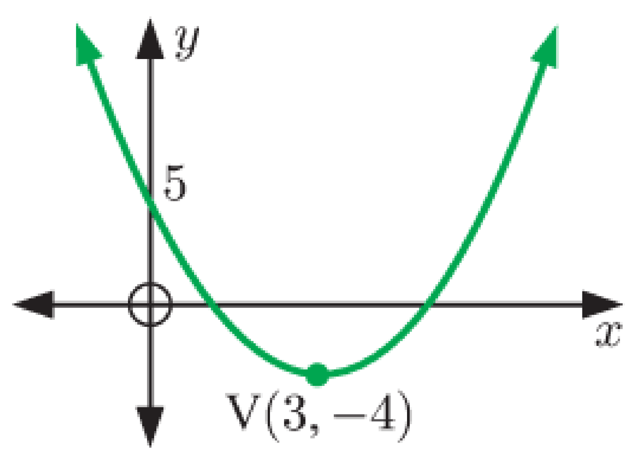
e $y = \left(x + \frac{5}{2}\right)^2 - \frac{33}{4}$



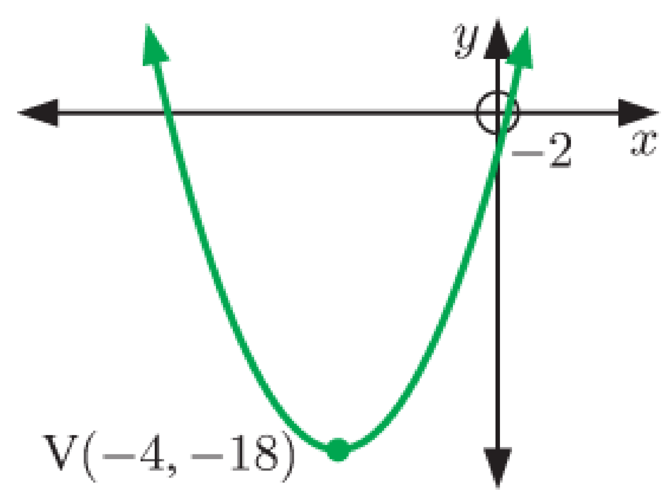
f $y = \left(x - \frac{3}{2}\right)^2 - \frac{1}{4}$



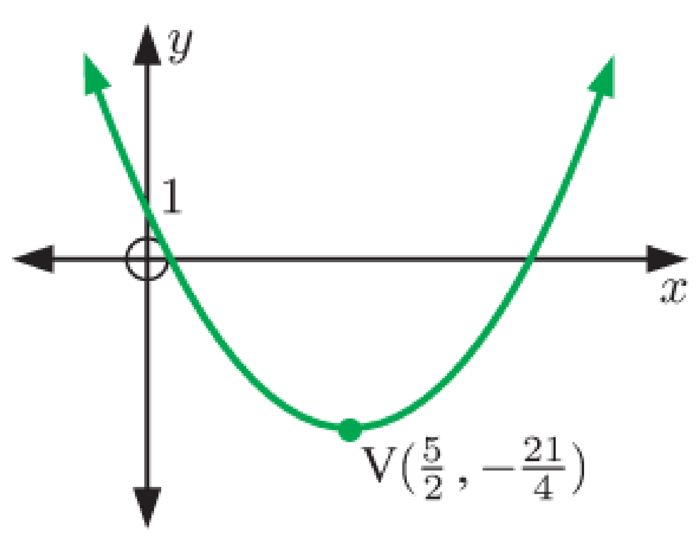
g $y = (x-3)^2 - 4$



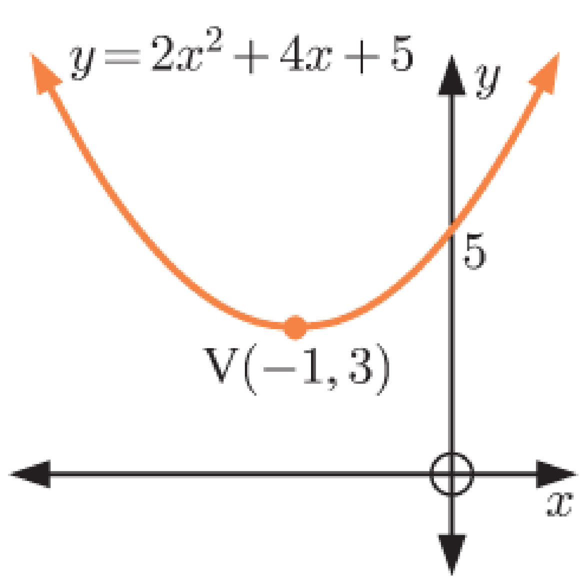
h $y = (x+4)^2 - 18$



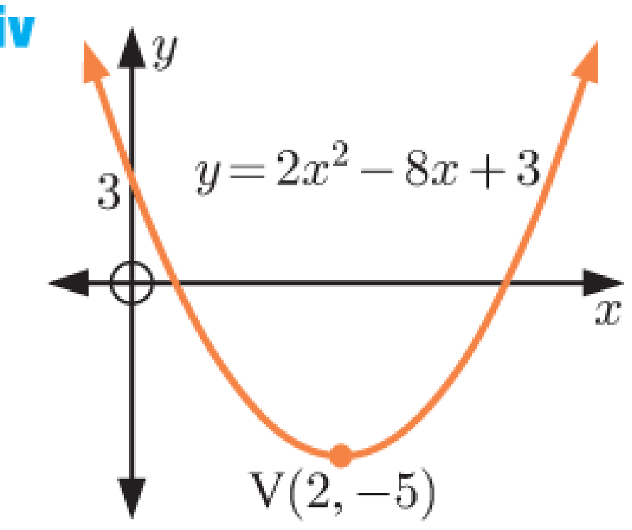
i $y = \left(x - \frac{5}{2}\right)^2 - \frac{21}{4}$



2 **a** **i** $y = 2(x+1)^2 + 3$
ii $(-1, 3)$ **iii** 5
iv

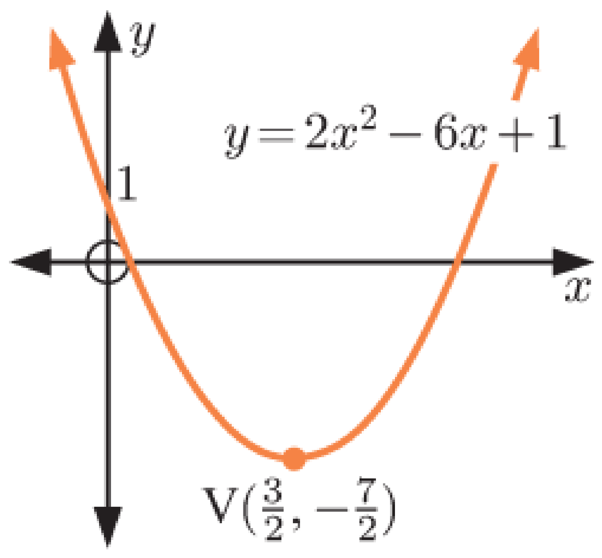


b **i** $y = 2(x-2)^2 - 5$
ii $(2, -5)$ **iii** 3
iv



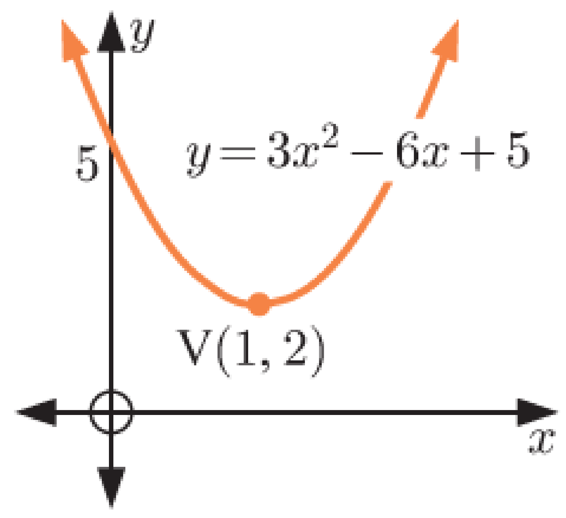
c **i** $y = 2(x - \frac{3}{2})^2 - \frac{7}{2}$ **ii** $(\frac{3}{2}, -\frac{7}{2})$ **iii** 1

iv



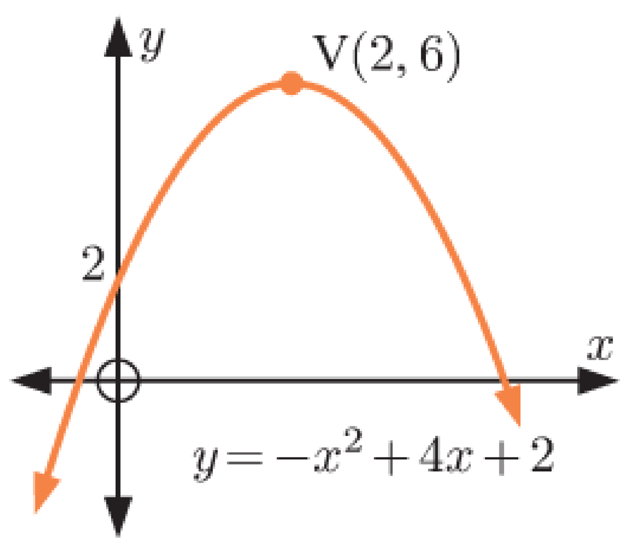
d **i** $y = 3(x - 1)^2 + 2$ **ii** $(1, 2)$ **iii** 5

iv



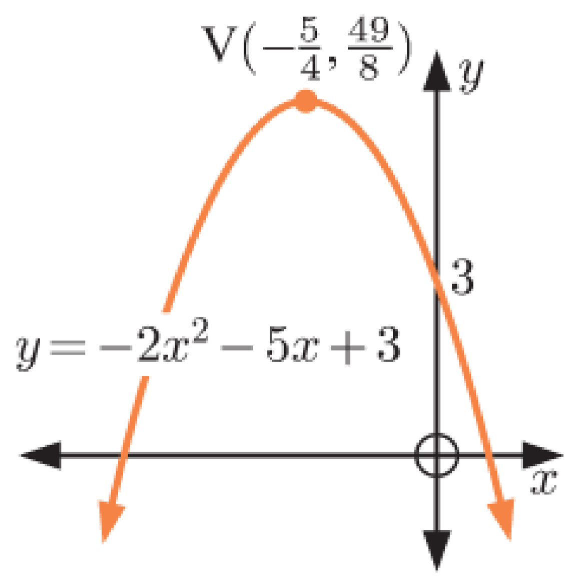
e **i** $y = -(x - 2)^2 + 6$ **ii** $(2, 6)$ **iii** 2

iv



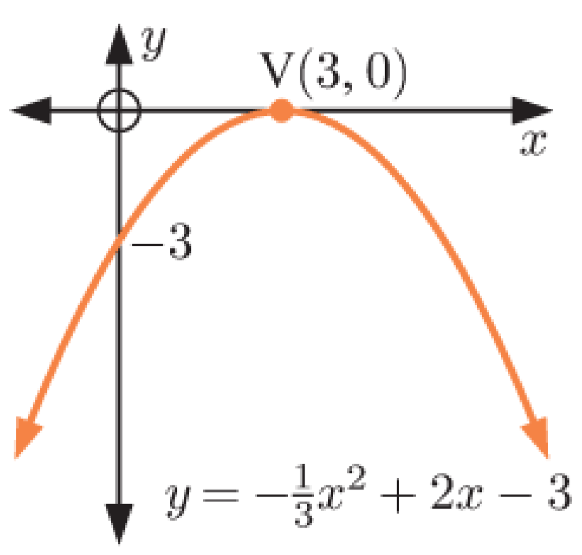
f **i** $y = -2(x + \frac{5}{4})^2 + \frac{49}{8}$ **ii** $(-\frac{5}{4}, \frac{49}{8})$ **iii** 3

iv



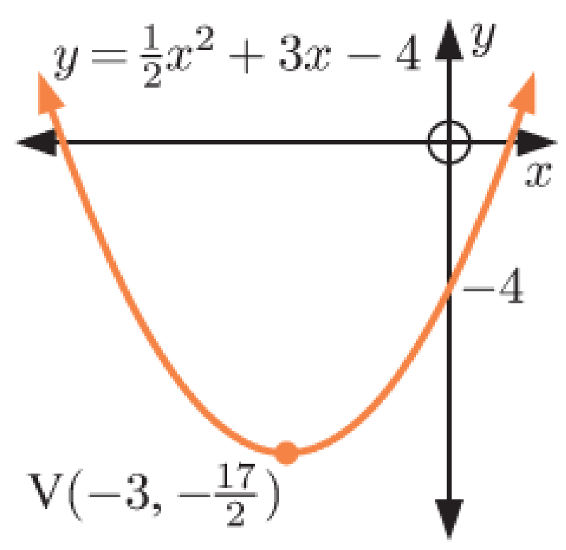
g **i** $y = -\frac{1}{3}(x - 3)^2$ **ii** $(3, 0)$ **iii** -3

iv



h **i** $y = \frac{1}{2}(x + 3)^2 - \frac{17}{2}$ **ii** $(-3, -\frac{17}{2})$ **iii** -4

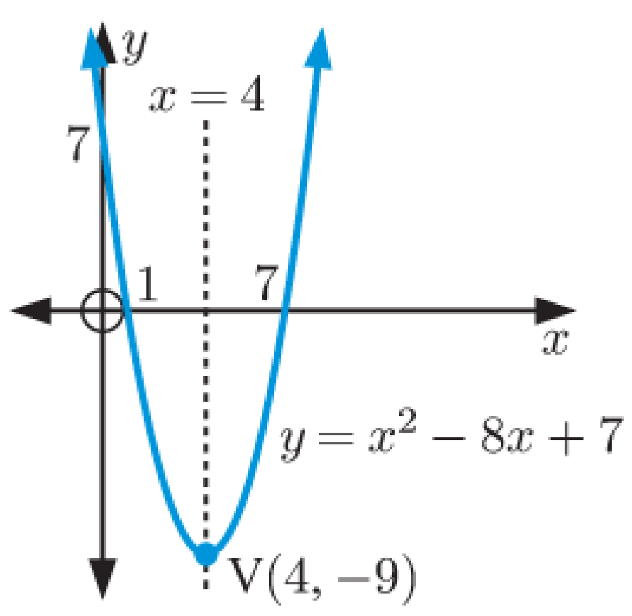
iv



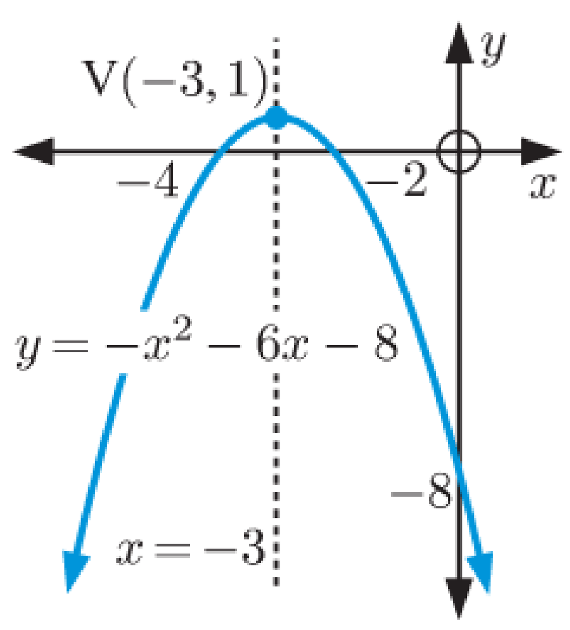
EXERCISE 14B.3

1 **a** **i** $(2, -2)$ **ii** minimum turning point
b **i** $(-1, -4)$ **ii** minimum turning point
c **i** $(0, 4)$ **ii** minimum turning point
d **i** $(0, 1)$ **ii** maximum turning point
e **i** $(-2, -15)$ **ii** minimum turning point
f **i** $(-2, -5)$ **ii** maximum turning point
g **i** $(-\frac{3}{2}, -\frac{11}{2})$ **ii** minimum turning point
h **i** $(\frac{5}{2}, -\frac{19}{2})$ **ii** minimum turning point
i **i** $(1, -\frac{9}{2})$ **ii** maximum turning point
j **i** $(14, -43)$ **ii** minimum turning point

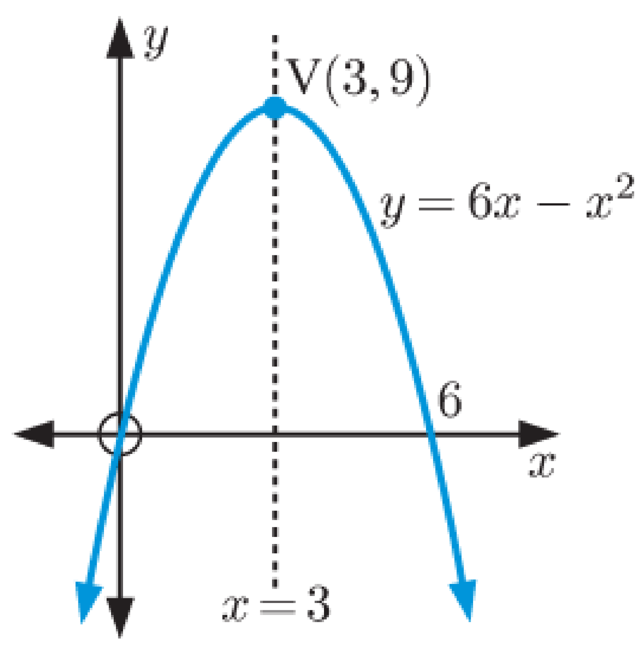
2 **a** **i** $x = 4$
ii $(4, -9)$
iii x-intercepts 1, 7, y-intercept 7
iv



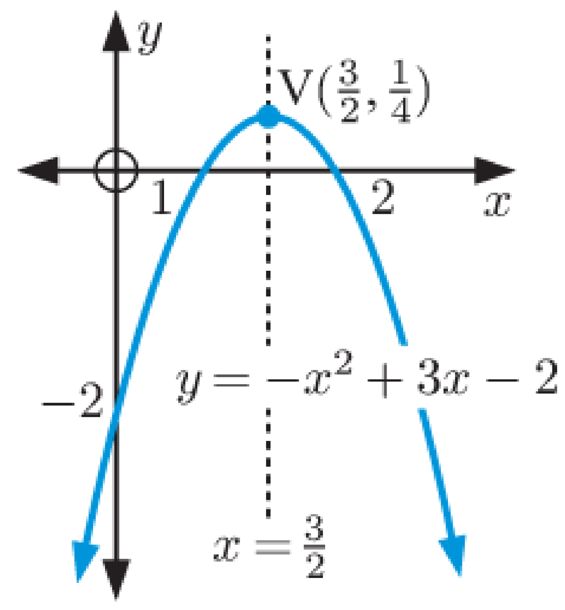
b **i** $x = -3$
ii $(-3, 1)$
iii x-int. -2, -4, y-intercept -8
iv



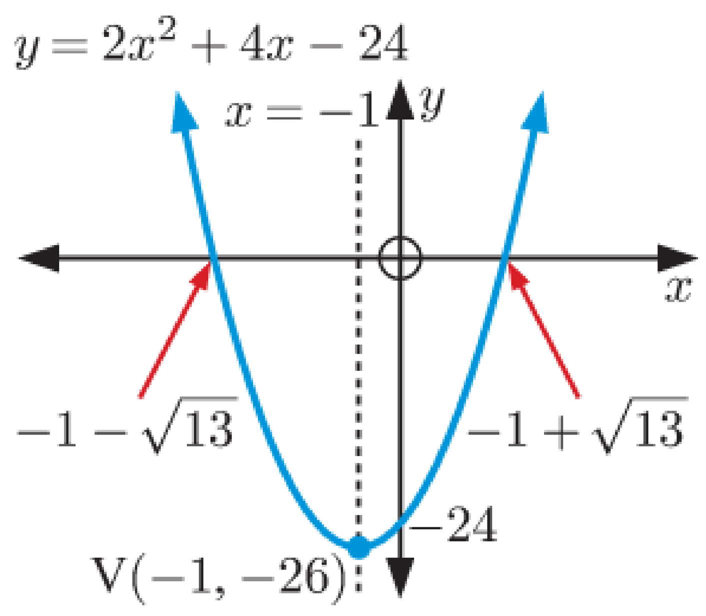
c **i** $x = 3$ **ii** $(3, 9)$
iii x-intercepts 0, 6, y-intercept 0
iv



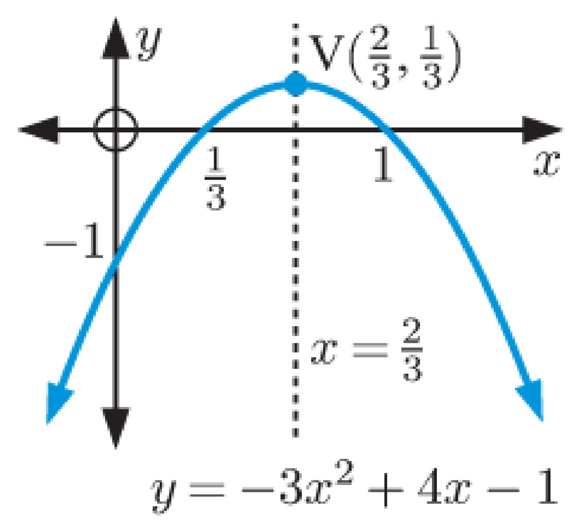
d **i** $x = \frac{3}{2}$ **ii** $(\frac{3}{2}, \frac{1}{4})$
iii x-intercepts 1, 2, y-intercept -2
iv



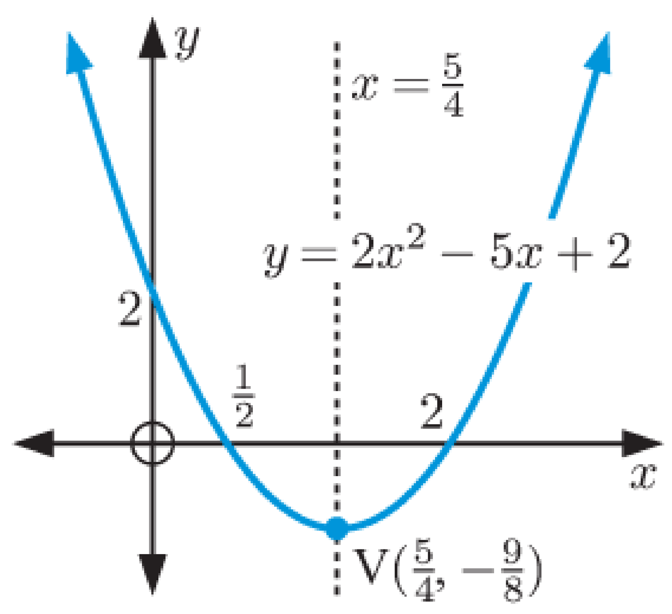
e **i** $x = -1$
ii $(-1, -26)$
iii x-int. $-1 \pm \sqrt{13}$, y-intercept -24
iv



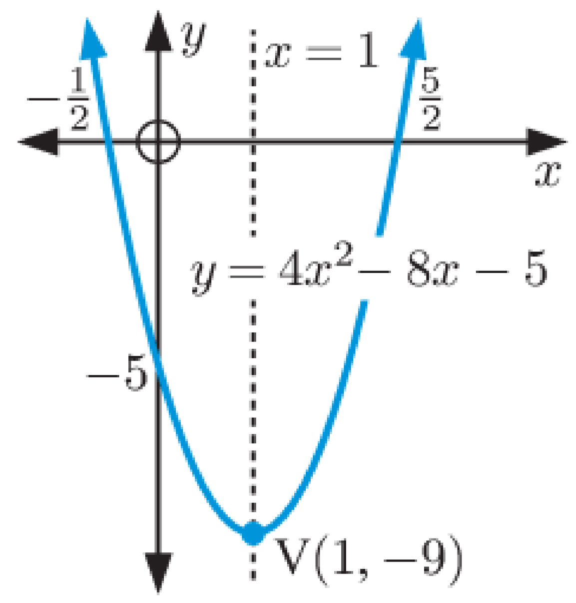
f **i** $x = \frac{2}{3}$
ii $(\frac{2}{3}, \frac{1}{3})$
iii x-intercepts $\frac{1}{3}$, 1, y-intercept -1
iv



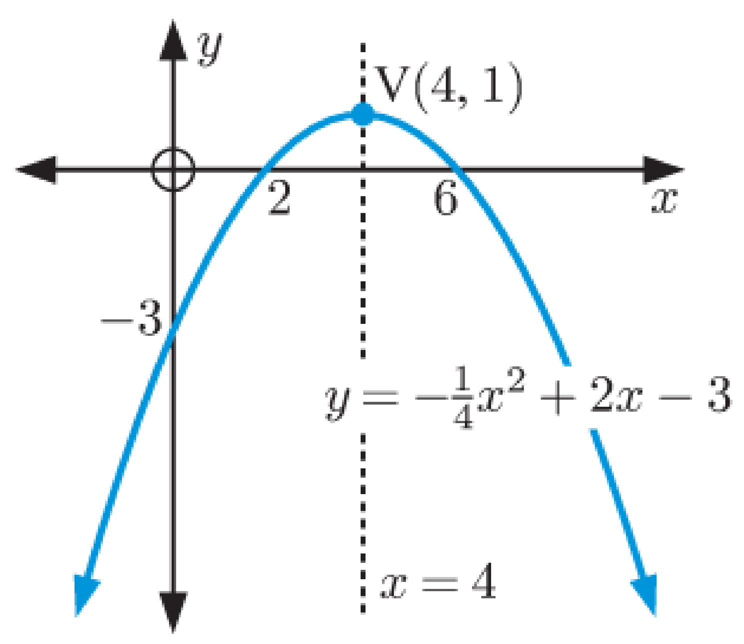
g **i** $x = \frac{5}{4}$
ii $(\frac{5}{4}, -\frac{9}{8})$
iii x-intercepts $\frac{1}{2}$, 2, y-intercept 2
iv



h **i** $x = 1$
ii $(1, -9)$
iii x-intercepts $-\frac{1}{2}$, $\frac{5}{2}$, y-intercept -5
iv



i **i** $x = 4$
ii $(4, 1)$
iii x-intercepts 2, 6, y-intercept -3
iv



3 **Hint:** $y = ax^2 + bx + c$ has vertex with x-coordinate $-\dfrac{b}{2a}$ and y-coordinate $a\left(-\dfrac{b}{2a}\right)^2 + b\left(-\dfrac{b}{2a}\right) + c$.

EXERCISE 14C

1 **a** $\Delta = 9$ which is > 0, graph cuts x-axis twice; is concave up.
b $\Delta = 12$ which is > 0, graph cuts x-axis twice; is concave up.
c $\Delta = -12$ which is < 0, graph lies entirely below the x-axis; is concave down, negative definite.
d $\Delta = 57$ which is > 0, graph cuts x-axis twice; is concave up.

e $\Delta = 0$, graph touches x-axis; is concave up.

f $\Delta = 17$ which is > 0, graph cuts x-axis twice; is concave down.

g $\Delta = 121$ which is > 0, graph cuts x-axis twice; is concave up.

h $\Delta = 25$ which is > 0, graph cuts x-axis twice; is concave down.

i $\Delta = 0$, graph touches x-axis; is concave up.

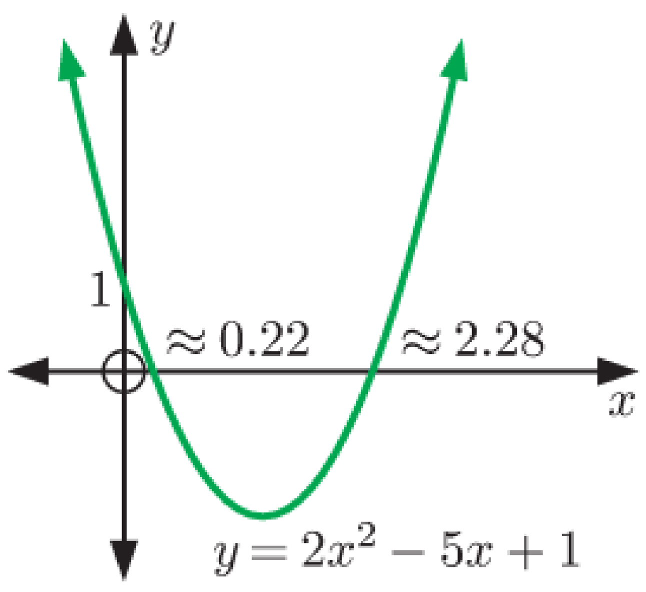
2 **a** concave up

b $\Delta = 17$ which is > 0
$\therefore$ cuts x-axis twice

c x-intercepts
≈ 0.22 and 2.28

d y-intercept is 1

e

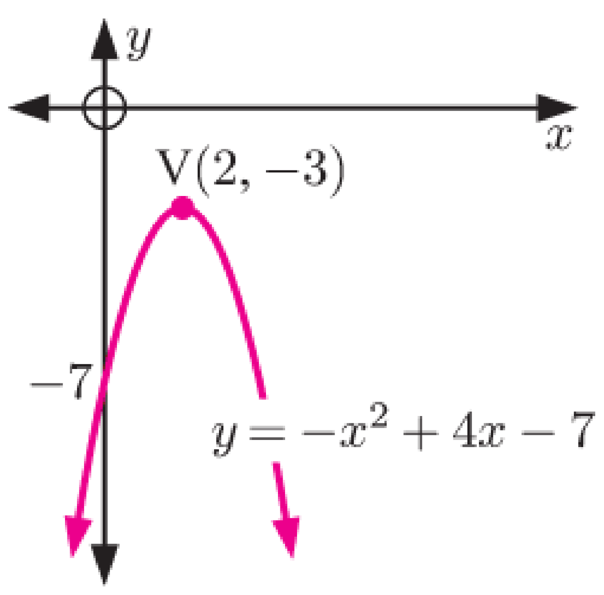


3 **a** $\Delta = -12$ which is < 0
$\therefore$ does not cut x-axis

b negative definite, since
$a < 0$ and $\Delta < 0$

c vertex is $(2, -3)$,
y-intercept is -7

d



4 **a** $a = 2$ which is > 0 and $\Delta = -40$ which is < 0
$\therefore$ positive definite.

b $a = -2$ which is < 0 and $\Delta = -23$ which is < 0
$\therefore$ negative definite.

c $a = 1$ which is > 0 and $\Delta = -15$ which is < 0
$\therefore$ positive definite so $x^2 - 3x + 6 > 0$ for all x.

d $a = -1$ which is < 0 and $\Delta = -8$ which is < 0
$\therefore$ negative definite so $4x - x^2 - 6 < 0$ for all x.

5

Constant	a	b	c	d	e	f	Δ_1	Δ_2
Sign	+	−	+	−	+	0	−	+

6 **a** **i** $k < \frac{9}{4}$ **ii** $k = \frac{9}{4}$ **iii** $k > \frac{9}{4}$

b **i** $k < 4$ **ii** $k = 4$ **iii** $k > 4$

c **i** $k > -\frac{4}{3}$ **ii** $k = -\frac{4}{3}$ **iii** $k < -\frac{4}{3}$

7 $a = 3$ which is > 0 and $\Delta = k^2 + 12$ which is always > 0
$\{$as $k^2 \geqslant 0$ for all $k\}$ $\therefore$ cannot be positive definite.

8 $k = -2$, the graph touches the x-axis in this case.

EXERCISE 14D

1 **a** $y = 2(x-1)(x-2)$ **b** $y = 3(x-2)^2$

c $y = (x-1)(x-3)$ **d** $y = -(x-3)(x+1)$

e $y = -3(x-1)^2$ **f** $y = -2(x+2)(x-3)$

2 **a** $y = \frac{3}{2}(x-2)(x-4)$ **b** $y = -\frac{1}{2}(x+4)(x-2)$

c $y = -\frac{4}{3}(x+3)^2$

3 **a** $y = 3x^2 - 18x + 15$ **b** $y = -4x^2 + 6x + 4$

c $y = -x^2 + 6x - 9$ **d** $y = 4x^2 + 16x + 16$

e $y = \frac{3}{2}x^2 - 6x + \frac{9}{2}$ **f** $y = -\frac{1}{3}x^2 + \frac{2}{3}x + 5$

4 **a** $y = -(x-2)^2 + 4$ **b** $y = 2(x-2)^2 - 1$

c $y = \frac{1}{3}(x+3)^2 - 4$ **d** $y = -2(x-3)^2 + 8$

e $y = \frac{2}{3}(x-4)^2 - 6$ **f** $y = -\frac{5}{9}(x+2)^2 + 5$

g $y = -2(x-2)^2 + 3$ **h** $y = \frac{3}{2}(x+4)^2 + 3$

i $y = 2(x - \frac{1}{2})^2 - \frac{3}{2}$

5 $y = 3$

EXERCISE 14E

1 **a** $(1, 7)$ and $(2, 8)$ **b** $(4, 5)$ and $(-3, -9)$

c $(3, 0)$ (touching) **d** graphs do not meet

2 **a** $(0.586, 5.59)$ and $(3.41, 8.41)$

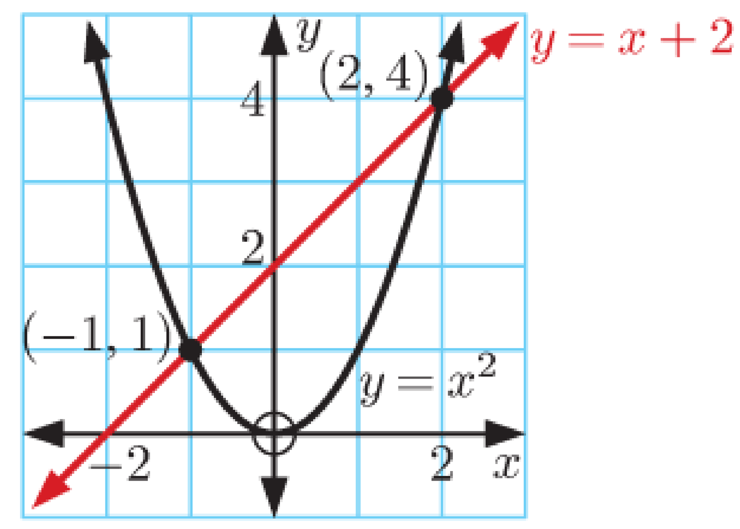
b $(3, -4)$ (touching) **c** graphs do not meet

d $(-2.56, -18.8)$ and $(1.56, 1.81)$

3 **a** $(-1, 1)$ and $(2, 4)$

c $x < -1$ or $x > 2$

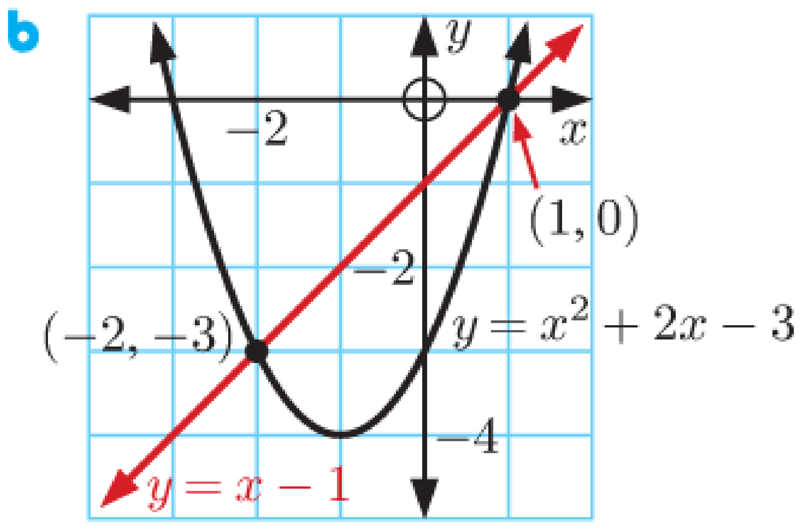
b



4 **a** $(-2, -3)$ and $(1, 0)$

c $x < -2$ or $x > 1$

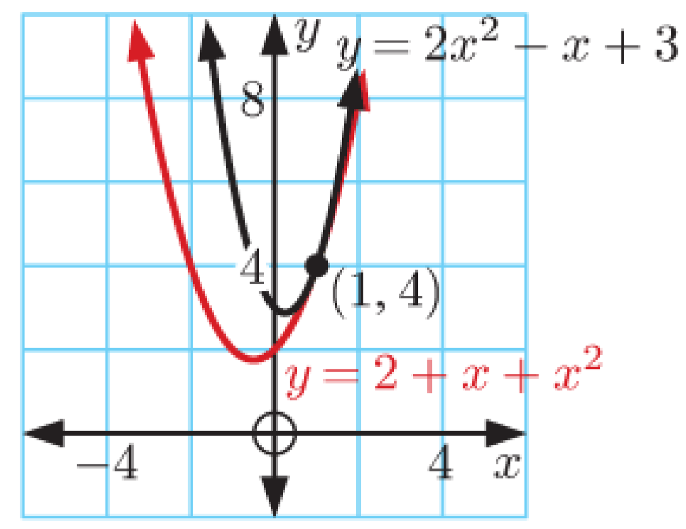
b



5 **a** $(1, 4)$

c $x \in \mathbb{R}$, $x \neq 1$

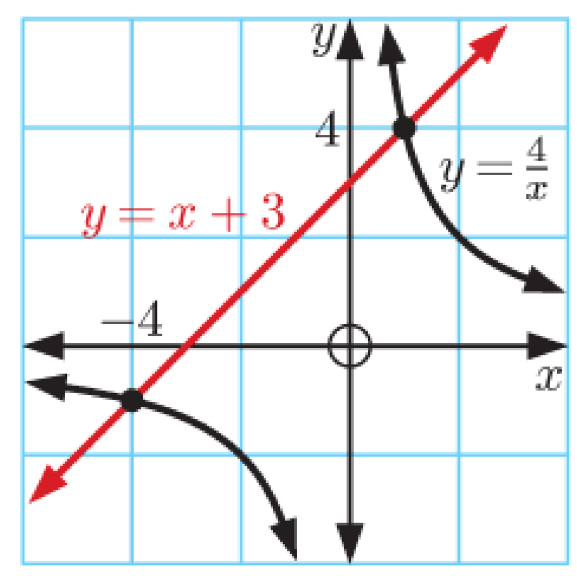
b



6 **a** $x = -4$ or 1

c $x < -4$ or
$0 < x < 1$

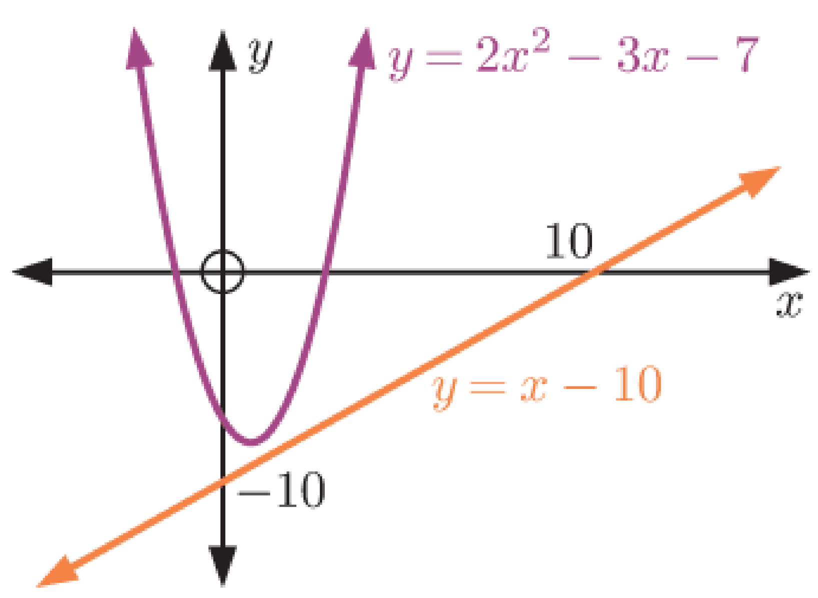
b



7 $c = -9$ **8** $m = 0$ or -8 **9** -1 or 11

10 **a** $c < -9$

b example: $c = -10$



12 **a** $c > -2$ **b** $c = -2$ **c** $c < -2$

13 **Hint:** A straight line through $(0, 3)$ will have an equation of the form $y = mx + 3$.

14 $b = 8$, $c = -14$ **15** **a** $c = a^2$, $m \in \mathbb{R}$ **b** $m = 2a$

EXERCISE 14F

1 7 and −5 or −7 and 5 **2** 5 or $\frac{1}{5}$ **3** 14

4 18 and 20 or −18 and −20

5 15 and 17 or −15 and −17 **6** 15 sides **7** ≈ 3.48 cm

8 **b** 6 cm by 6 cm by 7 cm **9** ≈ 11.2 cm square

10 no **12** ≈ 61.8 km h^{-1} **13** 32 elderly citizens

14 **a** $y = -\frac{8}{9}x^2 + 8$
b No, as the tunnel is only 4.44 m high when it is the same width as the truck.

15 **a** $h = -5(t-2)^2 + 80$ **b** 75 m **c** 6 seconds

EXERCISE 14G

1 **a** min. -1, when $x = 1$ **b** max. 8, when $x = -1$
c max. $8\frac{1}{3}$, when $x = \frac{1}{3}$ **d** min. $-1\frac{1}{8}$, when $x = -\frac{1}{4}$
e min. $4\frac{15}{16}$, when $x = \frac{1}{8}$ **f** max. $6\frac{1}{8}$, when $x = \frac{7}{4}$

2 **a** 40 refrigerators **b** €4000

4 500 m by 250 m

5 **a** $41\frac{2}{3}$ m by $41\frac{2}{3}$ m **b** 50 m by $31\frac{1}{4}$ m

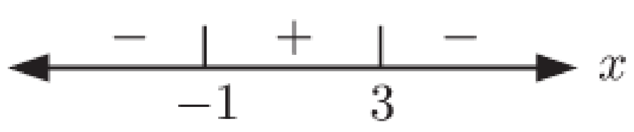
6 **b** $3\frac{1}{8}$ units

7 **a** $y = 6 - \frac{3}{4}x$ **b** 3 cm by 4 cm

8 $m = \dfrac{\sum_{i=1}^{n} a_i b_i}{\sum_{i=1}^{n} a_i^2}$

9 $y = x^4 - 2(a^2+b^2)x^2 + (a^2-b^2)^2$
least value $= -4a^2b^2$

EXERCISE 14H.1

1 **a**


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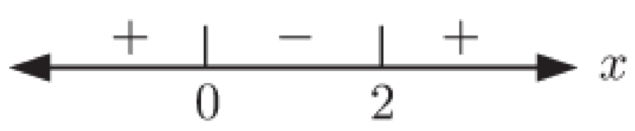
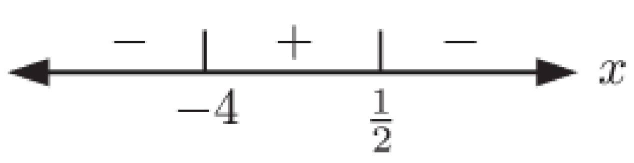
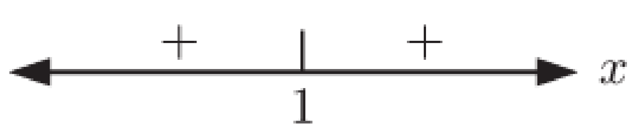


c


d


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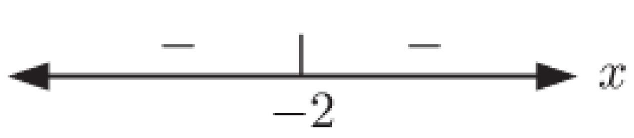
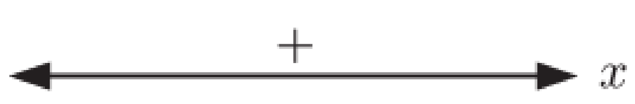
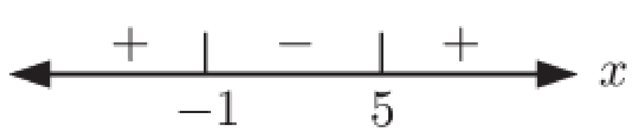


f


2 **a**


b


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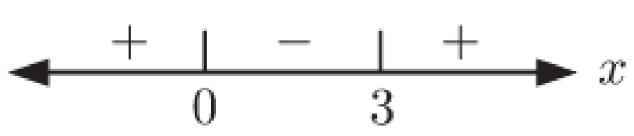

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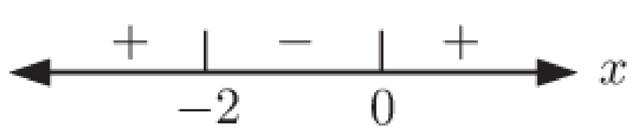

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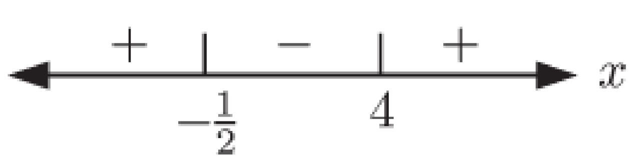

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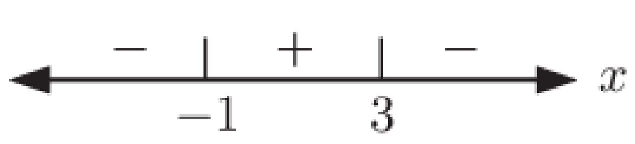

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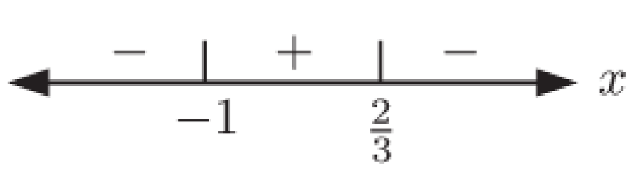
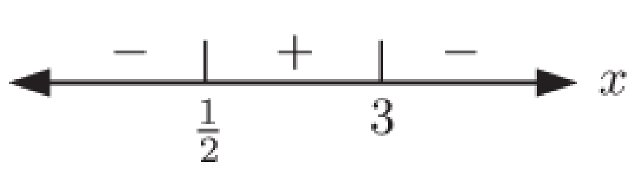


h


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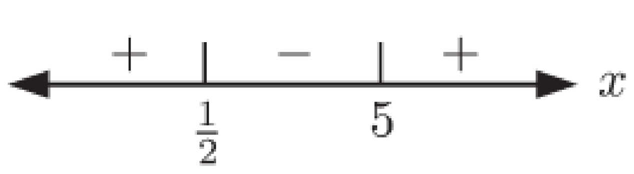
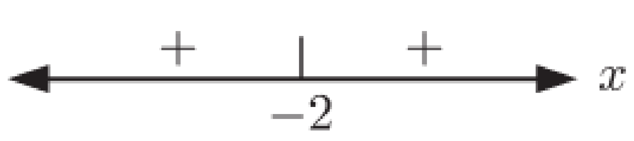


3 **a**


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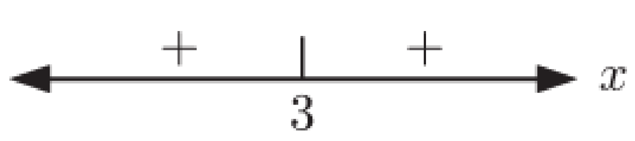

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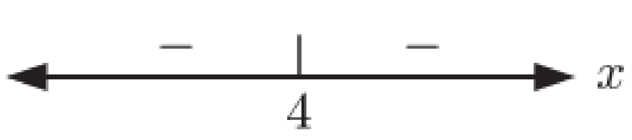

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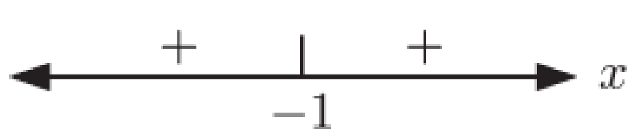

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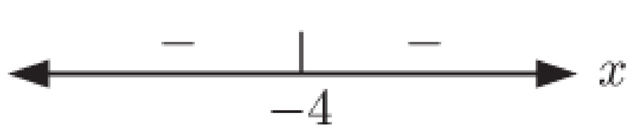

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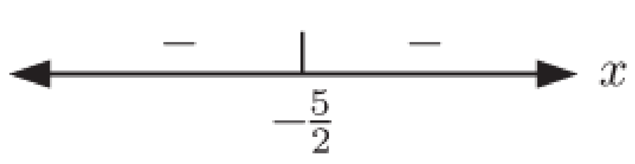
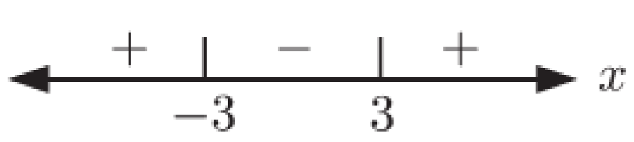


4 **a**


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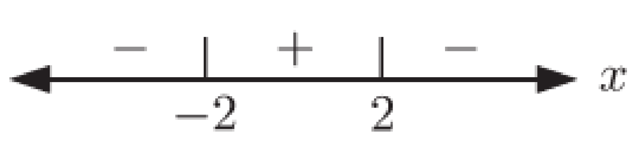

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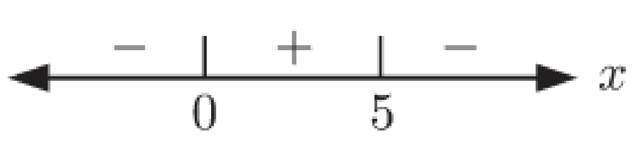

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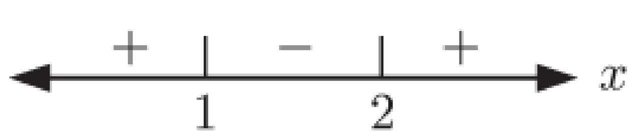

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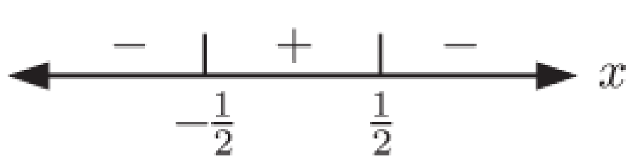

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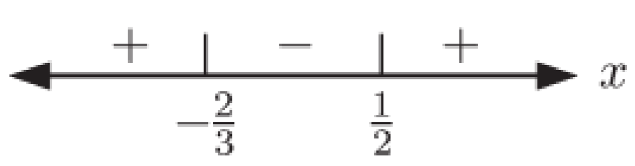

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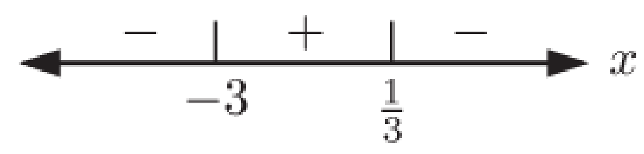

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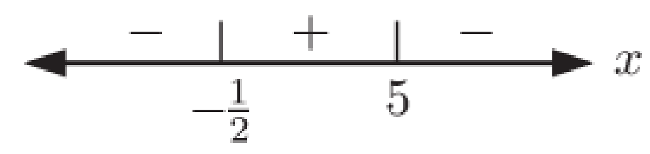

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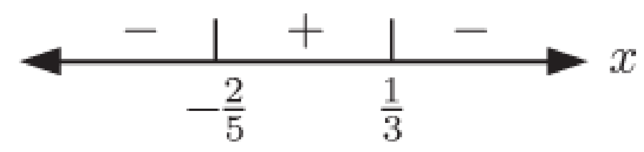
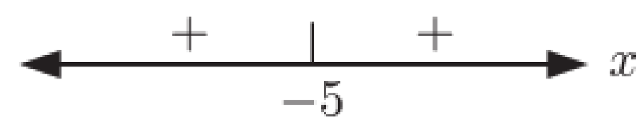


5 **a**


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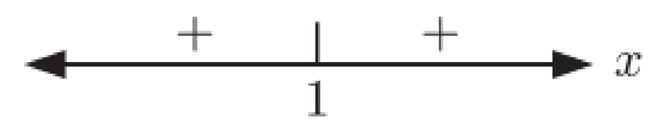

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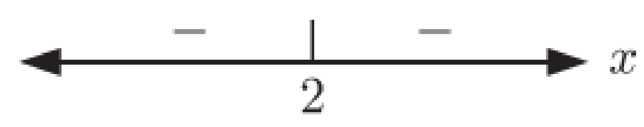

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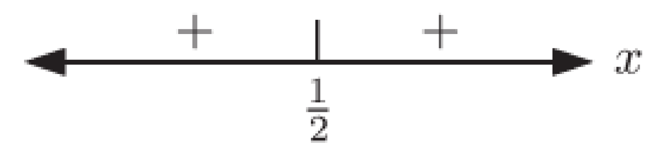

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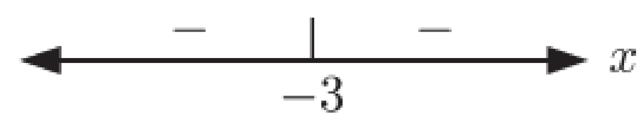

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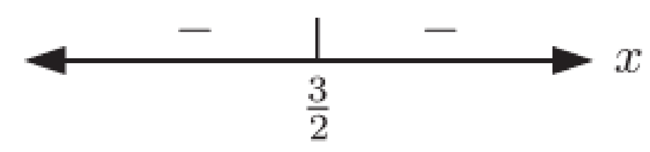


EXERCISE 14H.2

1 **a** $-5 \leqslant x \leqslant 2$ **b** $-3 \leqslant x \leqslant 2$ **c** no solutions
d all $x \in \mathbb{R}$ **e** $-\frac{1}{2} < x < 3$ **f** $-\frac{3}{2} < x < 4$

2 **a** $x \leqslant 0$ or $x \geqslant 1$ **b** $-\frac{2}{3} < x < 0$ **c** $x \neq -2$
d $-5 \leqslant x \leqslant 3$ **e** $x < -2$ or $x > 6$ **f** $-4 < x < 1$

3 **a** $x \leqslant 0$ or $x \geqslant 3$ **b** $-2 < x < 2$
c $x \leqslant -\sqrt{2}$ or $x \geqslant \sqrt{2}$ **d** $-3 \leqslant x \leqslant 7$
e $x < 5$ or $x > 6$ **f** $x < -6$ or $x > 7$
g $x \leqslant -1$ or $x \geqslant \frac{3}{2}$ **h** no solutions
i $-\frac{3}{2} < x < \frac{1}{3}$ **j** $x < -\frac{4}{3}$ or $x > 4$
k $x \neq 1$ **l** $\frac{1}{3} \leqslant x \leqslant \frac{1}{2}$ **m** $x < -\frac{1}{6}$ or $x > 1$
n $x \leqslant -\frac{1}{4}$ or $x \geqslant \frac{2}{3}$ **o** $x < \frac{3}{2}$ or $x > 3$

4 **a** **i** $k < -8$ or $k > 0$ **ii** $k = -8$ or 0
iii $-8 < k < 0$
b **i** $-1 < k < 1,\ k \neq 0$ **ii** $k = -1$ or 1
iii $k < -1$ or $k > 1$
c **i** $k < -6$ or $k > 2$ **ii** $k = -6$ or $k = 2$
iii $-6 < k < 2$

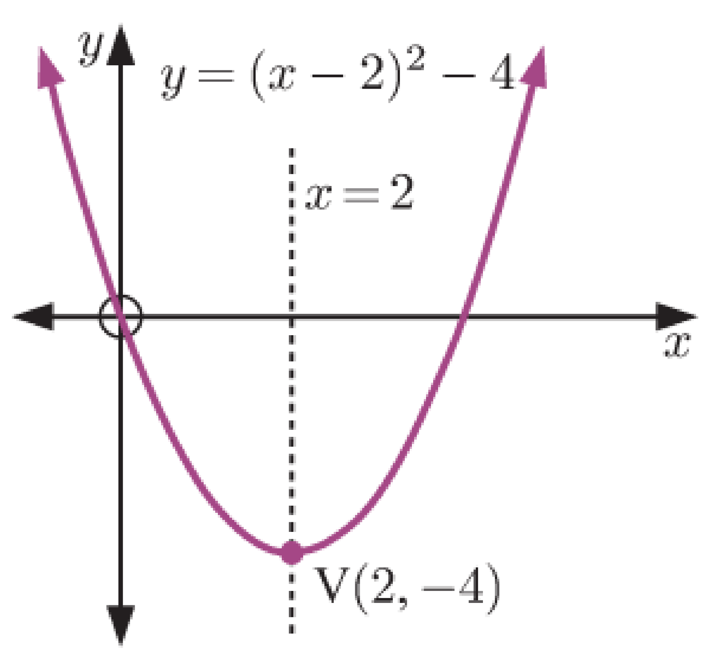
5 **a** **i** $k < -2$ or $k > 6$ **ii** $k = -2$ or $k = 6$
iii $-2 < k < 6$
b **i** $k < -\frac{13}{9}$ or $k > 3$ **ii** $k = -\frac{13}{9}$ or $k = 3$
iii $-\frac{13}{9} < k < 3$
c **i** $-\frac{4}{3} < k < 0,\ k \neq -1$ **ii** $k = -\frac{4}{3}$ or $k = 0$
iii $k < -\frac{4}{3}$ or $k > 0$

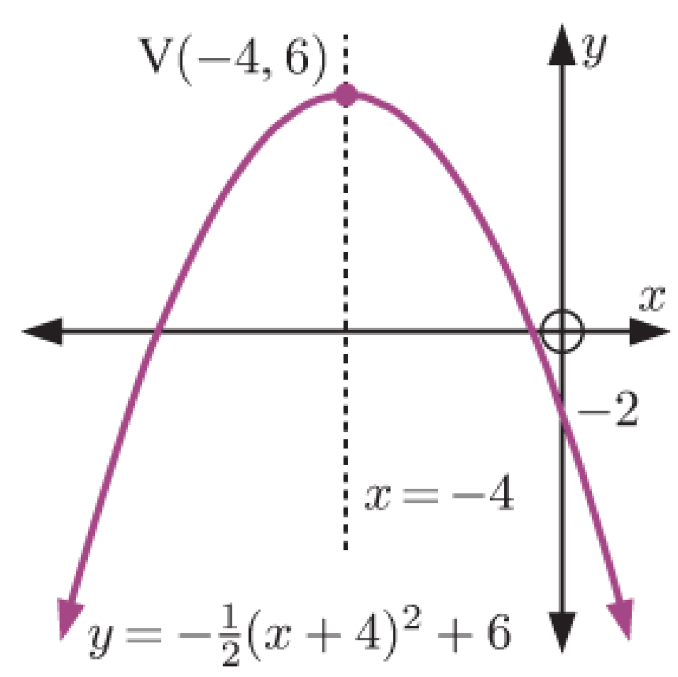
6 **a** $m > 3$ **b** $m < -1$

7 **a** $m < -1$ or $m > 7$ **b** $m = -1$ or $m = 7$
c $-1 < m < 7$

8 **a** $a < 6 - 2\sqrt{10}$ or $a > 6 + 2\sqrt{10}$ **b** $a = 6 \pm 2\sqrt{10}$
c $6 - 2\sqrt{10} < a < 6 + 2\sqrt{10}$

REVIEW SET 14A

1 **a**


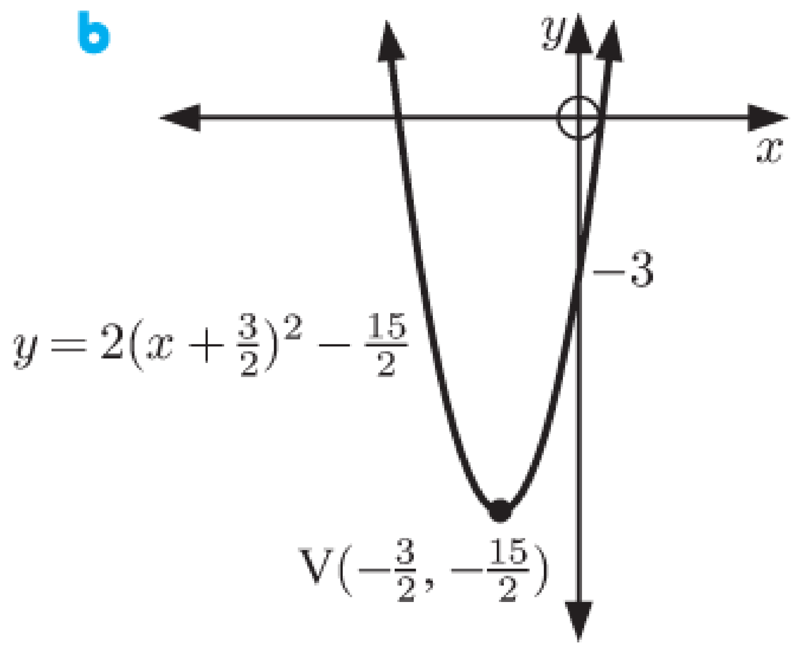
b


2 (4, 4) and (−3, 18) **3** $k < -3\frac{1}{8}$

4 **a** $m = \frac{9}{8}$ **b** $m < \frac{9}{8}$ **c** $m > \frac{9}{8}$ **5** $\frac{6}{5}$ or $\frac{5}{6}$

6 **Hint:** Let the line have equation $y = mx + 10$.

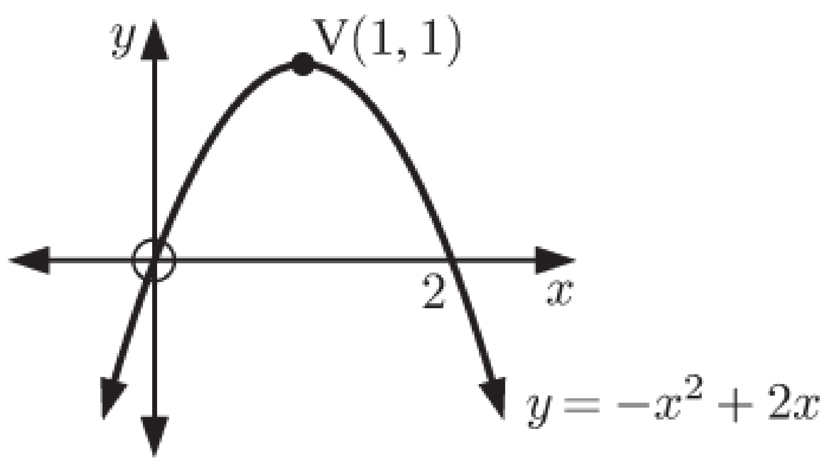
7 **a** $y = 2(x + \frac{3}{2})^2 - \frac{15}{2}$ **b**



8 **a** $y = \frac{20}{9}(x - 2)^2 - 20$ **b** $y = -\frac{2}{7}(x - 1)(x - 7)$

c $y = \frac{2}{9}(x + 3)^2$

9



10 $\frac{1}{2}$

11 **a** **i** $\Delta > 0$ **ii** $a < 0$

b **i** $A(-m, 0)$, $B(-n, 0)$ **ii** $x = \dfrac{-m-n}{2}$

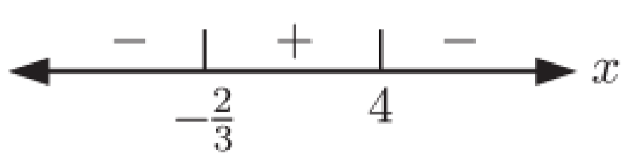
13 $y = -4x^2 + 4x + 24$ **14** $k = \frac{3}{2}$

15 **a** $c = 8$ **b** $3a + b = -3$, $a - b = -5$

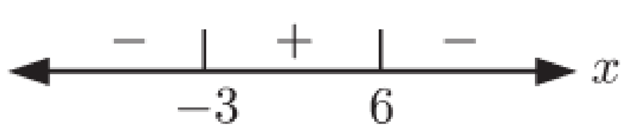
c $a = -2$, $b = 3$, $y = -2x^2 + 3x + 8$

16 $m = -5$ or 19 **17** 21 m

18 **a**



b



19 **a** $x < -2$ or $x > 3$ **b** $-1 \leqslant x \leqslant 5$

c $x < -\frac{5}{2}$ or $x > 2$

20 **a** $k < 6 - 2\sqrt{5}$ or $k > 6 + 2\sqrt{5}$ **b** $k = 6 \pm 2\sqrt{5}$

c $6 - 2\sqrt{5} < k < 6 + 2\sqrt{5}$

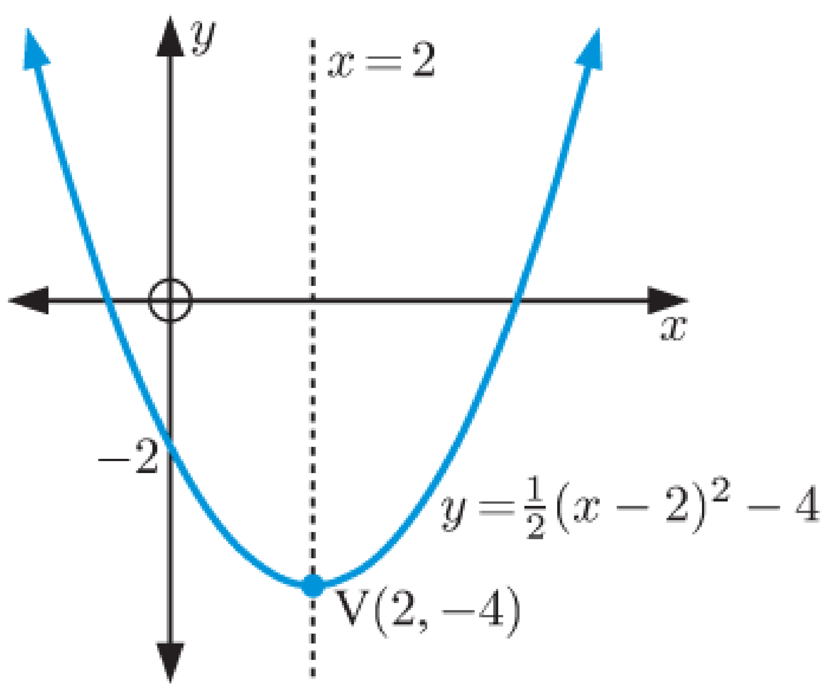
REVIEW SET 14B

1 **a** $x = 2$

b (2, −4)

c −2

d



2 $x = \frac{4}{3}$, $V(1\frac{1}{3}, 12\frac{1}{3})$

3 **a** $\Delta = 65$, the graph cuts the x-axis twice

b $\Delta = 97$, the graph cuts the x-axis twice

4 $y = -6(x - 2)^2 + 25$

5 **a** $y = -\frac{2}{5}(x + 5)(x - 1)$ **b** $(-2, 3\frac{3}{5})$, $x = -2$

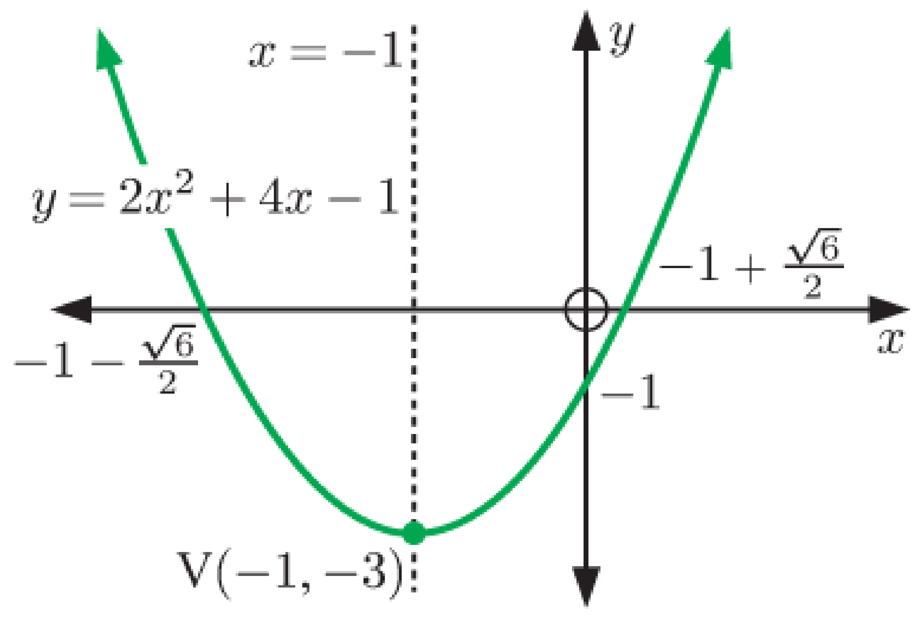
6 **a** $x = -1$

b (−1, −3)

c x-int. $-1 \pm \frac{\sqrt{6}}{2}$

y-intercept −1

d



7 **a** $y = 2x^2 - 12x + 18$ **b** $y = -\frac{1}{2}x^2 + \frac{1}{2}x + 3$

c $y = x^2 + 7x - 3$ **d** $y = -2x^2 + 12x - 3$

8 **a** $c > -6$

b For example, when $c = -2$, points of intersection are (−1, −5) and (3, 7).

9 **a** minimum is $5\frac{2}{3}$ when $x = -\frac{2}{3}$

b maximum is $5\frac{1}{8}$ when $x = -\frac{5}{4}$

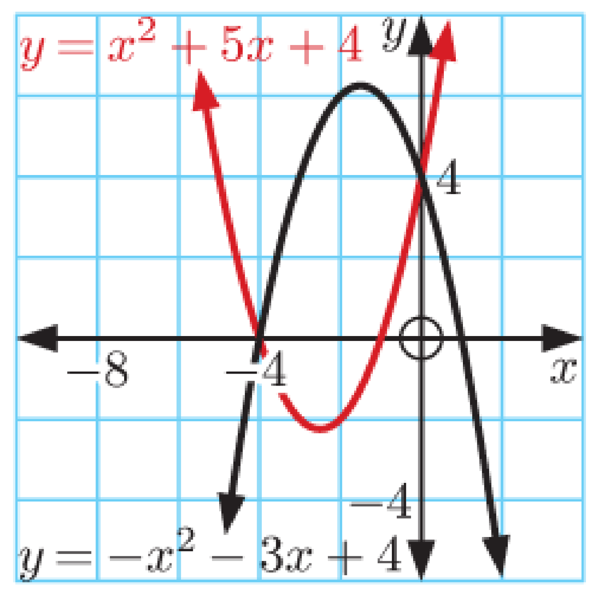
10 **a** $y = 3x^2 - 3x - 18$ **b** −18 **c** $(\frac{1}{2}, -18\frac{3}{4})$

11 **a** $m = -2$, $n = 4$ **b** $k = 7$ **12** ≈ 13.5 cm square

13 **a** $x = -4$ or 0

c $x < -4$ or $x > 0$

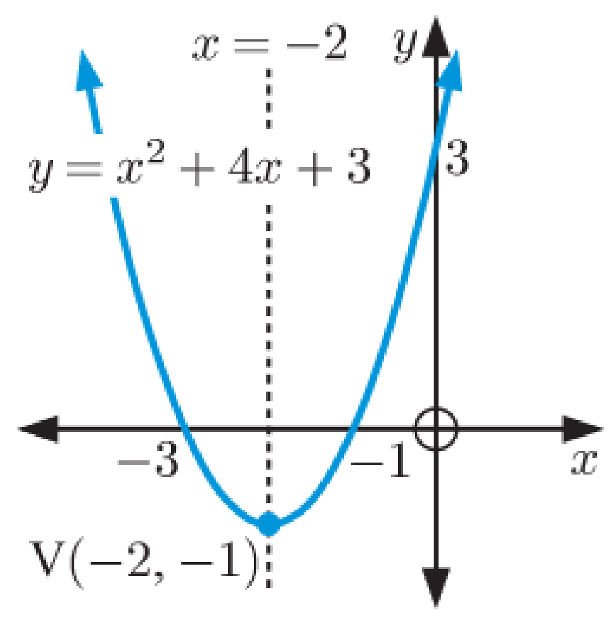
b



14 **a** **i** $y = (x + 2)^2 - 1$

ii $y = (x + 3)(x + 1)$

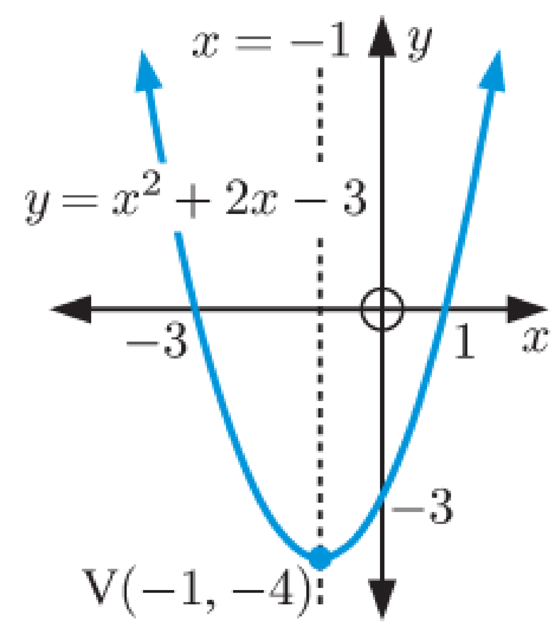
iii



b **i** $y = (x + 1)^2 - 4$

ii $y = (x + 3)(x - 1)$

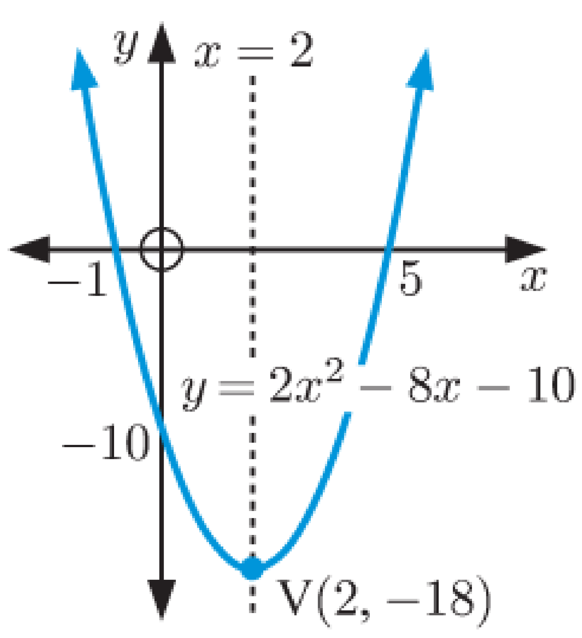
iii



c **i** $y = 2(x - 2)^2 - 18$

ii $y = 2(x - 5)(x + 1)$

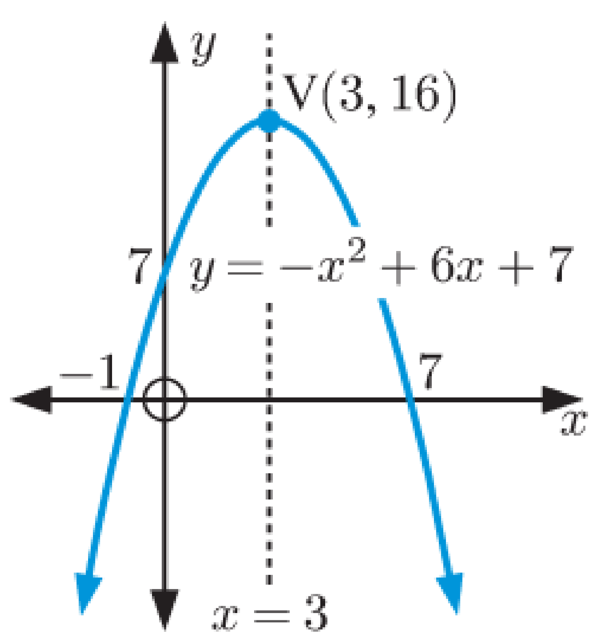
iii



d **i** $y = -(x - 3)^2 + 16$

ii $y = -(x - 7)(x + 1)$

iii

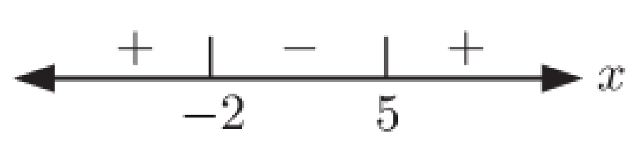


15 **a** $k = \pm 12$ **b** (0, 4)

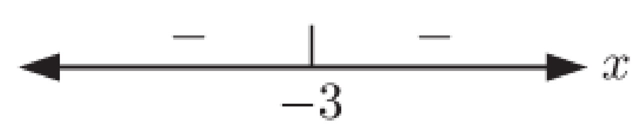
16 **b** $37\frac{1}{2}$ m by $33\frac{1}{3}$ m **c** 1250 m^2

17 **b** \$60, revenue is \$2400 per day

18 **a**



b



19 **a** $0 < x < \frac{3}{4}$ **b** $x \leqslant -1$ or $x \geqslant \frac{5}{2}$

c $x \leqslant \frac{1}{3}$ or $x \geqslant \frac{3}{2}$

20 **a** $-\frac{25}{2} < m < \frac{1}{2}$, $m \neq 0$ **b** $m = -\frac{25}{2}$ or $m = \frac{1}{2}$
c $m < -\frac{25}{2}$ or $m > \frac{1}{2}$

EXERCISE 15A

1 **a** Is a function, since for every value of x there is only one corresponding value of y.
b Is not a function. When $x = 2$, $y = 1$ or 0.

2 **a** Is a function, since for any value of x there is at most one value of y.
b Is a function, since for any value of x there is at most one value of y.
c Is not a function. If $x^2 + y^2 = 9$, then $y = \pm\sqrt{9 - x^2}$. So, for example, for $x = 2$, $y = \pm\sqrt{5}$.

3 **a** function **b** not a function **c** function
d not a function

4 Not a function as a 2 year old child could pay \$0 or \$20.

5 No, because a vertical line (the y-axis) would cut the relation more than once.

6 No. A vertical line is not a function. It will not pass the "vertical line" test.

7 **a** $y^2 = x$ is a relation but not a function.
$y = x^2$ is a function (and a relation).
$y^2 = x$ has a horizontal axis of symmetry (the x-axis).
$y = x^2$ has a vertical axis of symmetry (the y-axis).
Both $y^2 = x$ and $y = x^2$ have vertex $(0, 0)$.
$y^2 = x$ is a rotation of $y = x^2$ clockwise through 90° about the origin *or* $y^2 = x$ is a reflection of $y = x^2$ in the line $y = x$.
b **i** The part of $y^2 = x$ in the first quadrant.
ii $y = \sqrt{x}$ is a function as any vertical line cuts the graph at most once.

8 **a** Both curves are functions since any vertical line will cut each curve at most once.
b $y = \sqrt[3]{x}$

EXERCISE 15B

1 **a** 2 **b** 2 **c** -16 **d** -68 **e** $\frac{17}{4}$

2 **a** -3 **b** 3 **c** 3 **d** -3 **e** $\frac{15}{2}$

3 **a** **i** $-\frac{7}{2}$ **ii** $-\frac{3}{4}$ **iii** $-\frac{4}{9}$ **b** $x = 4$ **c** $x = \frac{9}{5}$

4 **a** $7 - 3a$ **b** $7 + 3a$ **c** $-3a - 2$ **d** $7 - 6a$
e $1 - 3x$ **f** $7 - 3x - 3h$

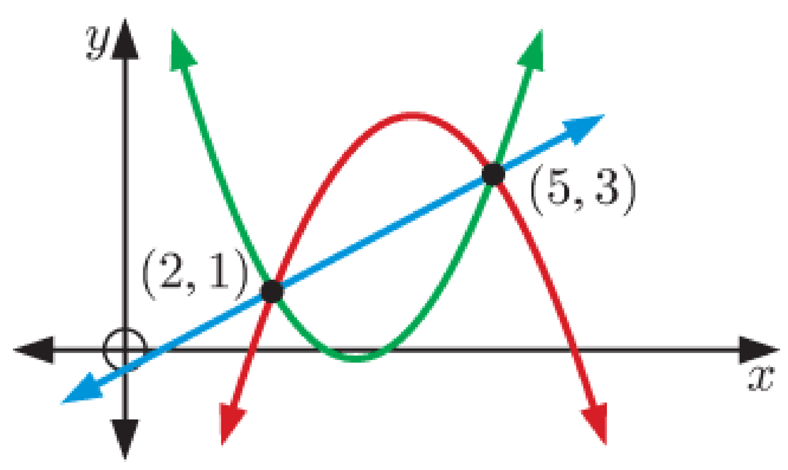
5 **a** $2x^2 + 19x + 43$ **b** $2x^2 - 11x + 13$
c $2x^2 - 3x - 1$ **d** $2x^4 + 3x^2 - 1$
e $18x^2 + 9x - 1$ **f** $2x^2 + (4h + 3)x + 2h^2 + 3h - 1$

6 **a** $9x^2$ **b** $\frac{x^2}{4}$ **c** $3x^2$ **d** $2x^2 - 4x + 7$

7 **a** $-\frac{1}{x}$ **b** $\frac{2}{x}$ **c** $\frac{2 + 3x}{x}$ **d** $\frac{2x + 1}{x - 1}$

8 f is the function which converts x into $f(x)$ whereas $f(x)$ is the value of the function at any value of x.

9 **Note:** Other answers are possible.



10 $f(x) = -2x + 5$

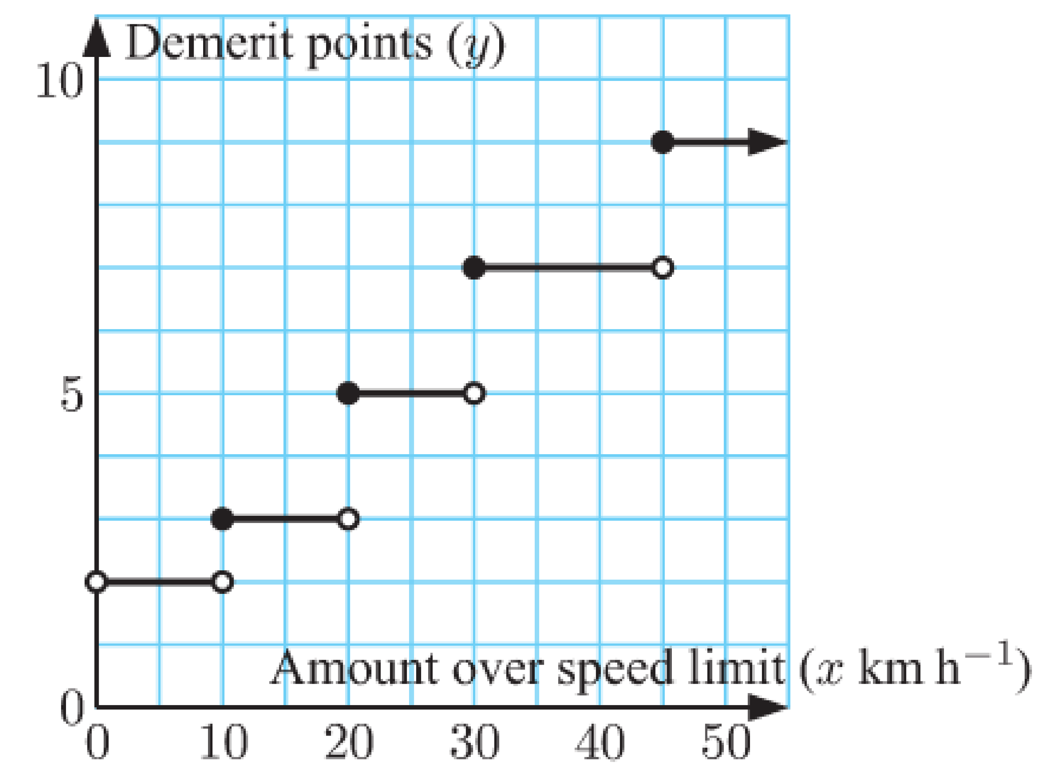
11 **a** $H(30) = 800$. After 30 minutes the balloon is 800 m high.
b $t = 20$ or 70. After 20 minutes and after 70 minutes the balloon is 600 m high.
c $0 \leqslant t \leqslant 80$ **d** 0 m to 900 m

12 $a = 3$, $b = -2$ **13** $a = 3$, $b = -1$, $c = -4$

14 **a** $V(4) = 5400$; $V(4)$ is the value of the photocopier in pounds after 4 years.
b $t = 6$. After 6 years the value of the photocopier is £3600.
c £9000 **d** $0 \leqslant t \leqslant 10$

EXERCISE 15C

1 **a**



b Yes, since for every value of x, there is at most one value of y.
c Domain is $\{x \mid x > 0\}$, Range is $\{2, 3, 5, 7, 9\}$

2 **a** At any moment in time there can be only one temperature, so the graph is a function.
b Domain is $\{t \mid 0 \leqslant t \leqslant 30\}$, Range is $\{T \mid 15 \leqslant T \leqslant 25\}$

3 **a** Domain is $\{x \mid -1 < x \leqslant 5\}$, Range is $\{y \mid 1 < y \leqslant 3\}$
b Domain is $\{x \mid x \neq 2\}$, Range is $\{y \mid y \neq -1\}$
c Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid 0 < y \leqslant 2\}$
d Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid y \leqslant \frac{25}{4}\}$
e Domain is $\{x \mid x \geqslant -4\}$, Range is $\{y \mid y \geqslant -3\}$
f Domain is $\{x \mid x \neq \pm 2\}$,
Range is $\{y \mid y \leqslant -1$ or $y > 0\}$

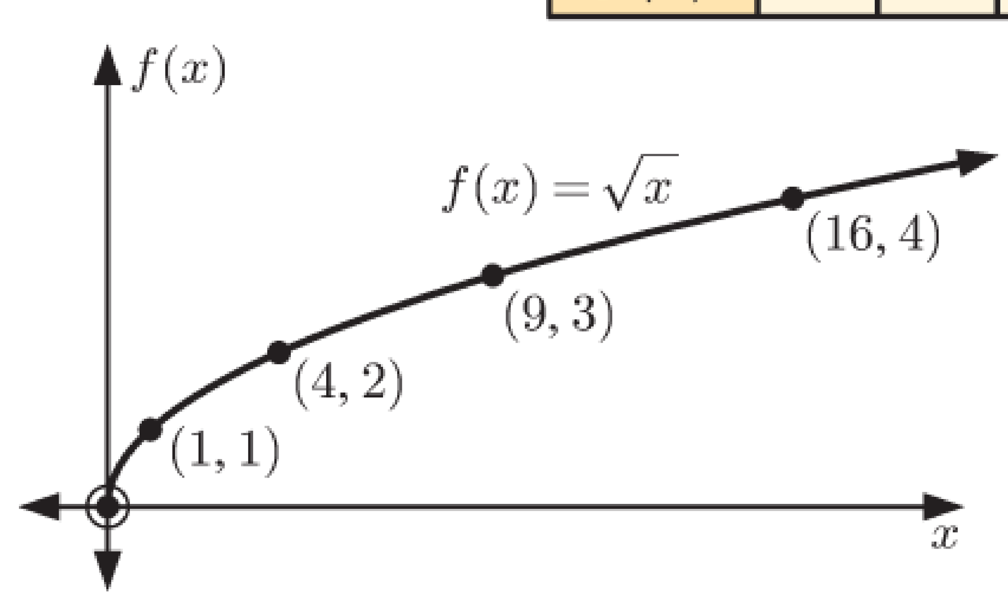
4 **a** true **b** false **c** true **d** true

5 **a** $\{y \mid y \geqslant 0\}$ **b** $\{y \mid y \leqslant 0\}$ **c** $\{y \mid y \geqslant 2\}$
d $\{y \mid y \leqslant 0\}$ **e** $\{y \mid y \leqslant 1\}$ **f** $\{y \mid y \geqslant 3\}$
g $\{y \mid y \geqslant -\frac{9}{4}\}$ **h** $\{y \mid y \leqslant 9\}$ **i** $\{y \mid y \leqslant \frac{25}{12}\}$

6 **a** $\{x \mid x \geqslant 0\}$ **b**

x	0	1	4	9	16
$f(x)$	0	1	2	3	4

c



d $\{y \mid y \geqslant 0\}$

7 **a** Domain is $\{x \mid x \geqslant -6\}$, Range is $\{y \mid y \geqslant 0\}$
b Domain is $\{x \mid x \neq 0\}$, Range is $\{y \mid y > 0\}$
c Domain is $\{x \mid x \neq -1\}$, Range is $\{y \mid y \neq 0\}$
d Domain is $\{x \mid x > 0\}$, Range is $\{y \mid y < 0\}$
e Domain is $\{x \mid x \neq 3\}$, Range is $\{y \mid y \neq 0\}$
f Domain is $\{x \mid x \leqslant 4\}$, Range is $\{y \mid y \geqslant 0\}$