# Introduction to sequences

You should think of a sequences simply as a list of numbers. For example:

$$1, 4, 7, 10, 13, \ldots$$

is an example of a sequence. The dots ... indicate that the pattern continues.

In the example above we will say that the first term of the sequence is 1, the second term is 4, the third term is 7 etc. We will use the following notation:

$$u_1 = 1, \qquad u_2 = 4, \qquad u_3 = 7$$

So for example $u_8$ denotes the 8th term of a given sequence and $u_{100}$ denotes the 100th term.

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# Example 1

Consider the sequence:

$$-4, 1, 6, 11, \ldots$$

Write down the value of the second, fourth and fifth term.

The first two are easy, we can just copy the values $u_2 = 1$ and $u_4 = 11$. Now we need to recognize the pattern, each time we add 5, so the next term will be 16, so $u_5 = 16$.

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# Recoginzing patterns

Consider the following sequences. In each case recognize the pattern and find the following term.

$$8, 12, 16, 20, \ldots$$

We add 4 each time, so the next term will be 24.

$$3, 6, 12, 24, \ldots$$

We multiply by 2 each time, so the next term will be 48.

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# Recoginzing patterns

$$1, 4, 9, 16, \dots$$

We take consecutive square numbers, so the next term will be 25.

$$2, 3, 5, 7, \dots$$

We take consecutive prime numbers, so the next term will be 11.

$$10, 7, 4, 1, \dots$$

We subtract 3 each time, so the next term will be $-2$.

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$$2, -4, 8, -16, \ldots$$

We multiply by -2 each time, so the next term will be 32.

$$\frac{1}{1}, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \ldots$$

The numerator increases by 1 and the denominator increases by 2, so the next term will be $\frac{5}{9}$.

$$1, 1, 2, 3, 5, 8, \ldots$$

This is the famous Fibonacci sequence, each term is the sum of the two preceding ones, so the next term will be 13.

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We will focus our studies on two types of sequences: arithmetic and geometric.

### Arithmetic sequence

A sequence is arithmetic if the difference of the consecutive terms is constant, that is:

$$u_{n+1} - u_n = const.$$

This definition is in fact very easy, it simple means that each time we add the same amount.

This common difference between consecutive terms is usually denoted with the letter $d$.

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Consider the sequence:

$$8, 15, 22, 29, \dots$$

This is an arithmetic sequence because the difference between consecutive terms is constant:

$$15 - 8 = 7$$
$$22 - 15 = 7$$
$$29 - 22 = 7$$

In this example we have the first term $u_1 = 8$ and the common difference $d = 7$. In other words, to get to the next term we need to add 7 to the preceding one. We can easily calculate the $u_5 = 29 + 7 = 36$ and $u_6 = 36 + 7 = 43$ etc.

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# Arithmetic sequences

Examples:

$$5, 7, 9, 11, \ldots$$

is an arithmetic sequence with $u_1 = 5$ and common difference $d = 2$.

$$9, 20, 31, 42, \ldots$$

is an arithmetic sequence with the first term $u_1 = 9$ and common difference $d = 11$.

$$-5, -1, 3, 7, \ldots$$

is an arithmetic sequence with the first term $u_1 = -5$ and common difference $d = 4$.

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# Arithmetic sequences

$$12, 7, 2, -3, \dots$$

is an arithmetic sequence with $u_1 = 12$ and common difference $d = -5$.

$$4, -8, -20, -32, \dots$$

is an arithmetic sequence with the first term $u_1 = 4$ and common difference $d = -12$.

$$-1, -2, -3, -4, \dots$$

is an arithmetic sequence with the first term $u_1 = -1$ and common difference $d = -1$.

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is an arithmetic sequence with the first term $u_1 = -1$ and common difference $d = -1$.

$$2, 4, 8, 16, \dots$$

is **not** an arithmetic sequence, because the difference is not constant
$4 - 2 \neq 8 - 4$

$$1, 4, 9, 16, \dots$$

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$4 - 1 \neq 9 - 4$

$$\frac{1}{1}, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \dots$$

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# Test

There might be a short test at the beginning of the next class. You will be given a few sequences and you will need to decide if they are arithmetic or not, and if they are write down the first term and the common difference.