Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Monday 12 November 2018 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

12 pages

8818 – 7201

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Consider two events, A and B, such that $\mathrm{P}(A) = \mathrm{P}(A' \cap B) = 0.4$ and $\mathrm{P}(A \cap B) = 0.1$.

(a) By drawing a Venn diagram, or otherwise, find $\mathrm{P}(A \cup B)$. [3]

(b) Show that the events A and B are not independent. [3]

2. [Maximum mark: 5]

A team of four is to be chosen from a group of four boys and four girls.

(a) Find the number of different possible teams that could be chosen. [3]

(b) Find the number of different possible teams that could be chosen, given that the team must include at least one girl and at least one boy. [2]

3. [Maximum mark: 7]

Consider the function $g(x) = 4\cos x + 1$, $a \le x \le \frac{\pi}{2}$ where $a < \frac{\pi}{2}$.

(a) For $a = -\frac{\pi}{2}$, sketch the graph of $y = g(x)$. Indicate clearly the maximum and minimum values of the function. [3]

(b) Write down the least value of a such that g has an inverse. [1]

(c) For the value of a found in part (b),

(i) write down the domain of g^{-1};

(ii) find an expression for $g^{-1}(x)$. [3]

4. [Maximum mark: 7]

Consider the following system of equations where $a \in \mathbb{R}$.

$$\begin{aligned} 2x + 4y - z &= 10 \\ x + 2y + az &= 5 \\ 5x + 12y &= 2a. \end{aligned}$$

(a) Find the value of a for which the system of equations does not have a unique solution. [2]

(b) Find the solution of the system of equations when $a = 2$. [5]

5. [Maximum mark: 6]

The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are defined by $\boldsymbol{a} = \begin{pmatrix} 1 \\ 1 \\ t \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} 0 \\ -t \\ 4t \end{pmatrix}$, where $t \in \mathbb{R}$.

(a) Find and simplify an expression for $\boldsymbol{a} \bullet \boldsymbol{b}$ in terms of t. [2]

(b) Hence or otherwise, find the values of t for which the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$ is obtuse. [4]

6. [Maximum mark: 6]

Use mathematical induction to prove that $\sum_{r=1}^{n} r(r!) = (n+1)! - 1$, for $n \in \mathbb{Z}^{+}$.

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7. [Maximum mark: 6]

Consider the curves C_1 and C_2 defined as follows

$$C_1 : xy = 4\,,\ x > 0$$
$$C_2 : y^2 - x^2 = 2\,,\ x > 0$$

(a) Using implicit differentiation, or otherwise, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ for each curve in terms of x and y. [4]

Let $\mathrm{P}(a\,,\,b)$ be the unique point where the curves C_1 and C_2 intersect.

(b) Show that the tangent to C_1 at P is perpendicular to the tangent to C_2 at P. [2]

8. [Maximum mark: 7]

Consider the equation $z^4 + az^3 + bz^2 + cz + d = 0$, where $a , b , c , d \in \mathbb{R}$ and $z \in \mathbb{C}$.
Two of the roots of the equation are $\log_2 6$ and $\mathrm{i}\sqrt{3}$ and the sum of all the roots is $3 + \log_2 3$.
Show that $6a + d + 12 = 0$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

Consider a triangle OAB such that O has coordinates $(0, 0, 0)$, A has coordinates $(0, 1, 2)$ and B has coordinates $(2b, 0, b-1)$ where $b < 0$.

(a) Find, in terms of b, a Cartesian equation of the plane Π containing this triangle. [5]

Let M be the midpoint of the line segment [OB].

(b) Find, in terms of b, the equation of the line L which passes through M and is perpendicular to the plane Π. [3]

(c) Show that L does not intersect the y-axis for any negative value of b. [7]

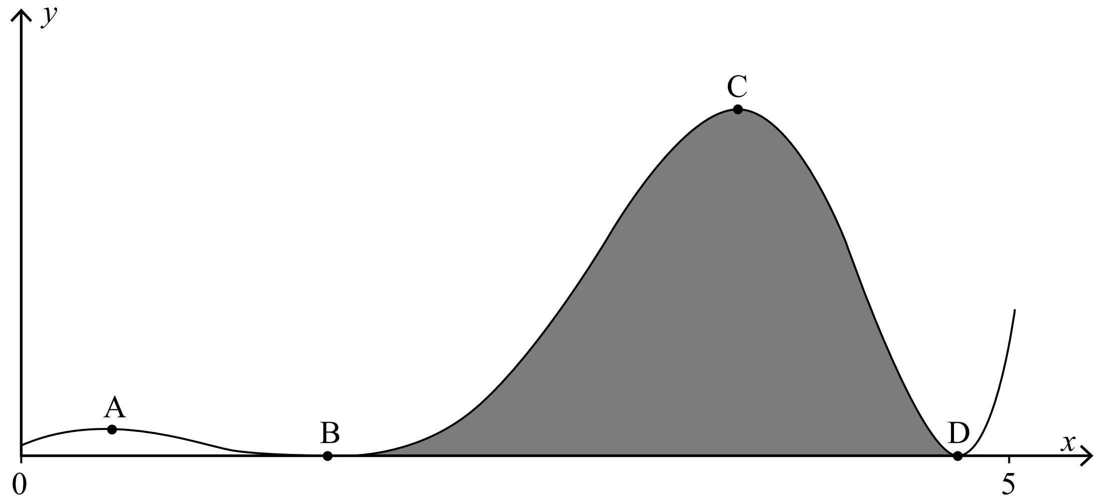
Do **not** write solutions on this page.

10. [Maximum mark: 19]

(a) Use integration by parts to show that $\int e^x \cos 2x dx = \frac{2e^x}{5}\sin 2x + \frac{e^x}{5}\cos 2x + c,\ c \in \mathbb{R}$. [5]

(b) Hence, show that $\int e^x \cos^2 x dx = \frac{e^x}{5}\sin 2x + \frac{e^x}{10}\cos 2x + \frac{e^x}{2} + c,\ c \in \mathbb{R}$. [3]

The function f is defined by $f(x) = e^x \cos^2 x$, where $0 \le x \le 5$. The curve $y = f(x)$ is shown on the following graph which has local maximum points at A and C and touches the x-axis at B and D.



(c) Find the x-coordinates of A and of C, giving your answers in the form $a + \arctan b$, where $a, b \in \mathbb{R}$. [6]

(d) Find the area enclosed by the curve and the x-axis between B and D, as shaded on the diagram. [5]

Do **not** write solutions on this page.

11. [Maximum mark: 16]

(a) Find the roots of $z^{24} = 1$ which satisfy the condition $0 < \arg(z) < \frac{\pi}{2}$, expressing your answers in the form $re^{i\theta}$, where $r, \theta \in \mathbb{R}^{+}$. [5]

(b) Let S be the sum of the roots found in part (a).

(i) Show that $\operatorname{Re} S = \operatorname{Im} S$.

(ii) By writing $\frac{\pi}{12}$ as $\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$, find the value of $\cos\frac{\pi}{12}$ in the form $\frac{\sqrt{a} + \sqrt{b}}{c}$, where a, b and c are integers to be determined.

(iii) Hence, or otherwise, show that $S = \frac{1}{2}\left(1 + \sqrt{2}\right)\left(1 + \sqrt{3}\right)(1 + i)$. [11]