20 **a** $-\frac{25}{2} < m < \frac{1}{2}$, $m \neq 0$ **b** $m = -\frac{25}{2}$ or $m = \frac{1}{2}$
c $m < -\frac{25}{2}$ or $m > \frac{1}{2}$

EXERCISE 15A

1 **a** Is a function, since for every value of x there is only one corresponding value of y.
b Is not a function. When $x = 2$, $y = 1$ or 0.

2 **a** Is a function, since for any value of x there is at most one value of y.
b Is a function, since for any value of x there is at most one value of y.
c Is not a function. If $x^2 + y^2 = 9$, then $y = \pm\sqrt{9 - x^2}$. So, for example, for $x = 2$, $y = \pm\sqrt{5}$.

3 **a** function **b** not a function **c** function
d not a function

4 Not a function as a 2 year old child could pay \$0 or \$20.

5 No, because a vertical line (the y-axis) would cut the relation more than once.

6 No. A vertical line is not a function. It will not pass the "vertical line" test.

7 **a** $y^2 = x$ is a relation but not a function.
$y = x^2$ is a function (and a relation).
$y^2 = x$ has a horizontal axis of symmetry (the x-axis).
$y = x^2$ has a vertical axis of symmetry (the y-axis).
Both $y^2 = x$ and $y = x^2$ have vertex $(0, 0)$.
$y^2 = x$ is a rotation of $y = x^2$ clockwise through 90° about the origin *or* $y^2 = x$ is a reflection of $y = x^2$ in the line $y = x$.
b **i** The part of $y^2 = x$ in the first quadrant.
ii $y = \sqrt{x}$ is a function as any vertical line cuts the graph at most once.

8 **a** Both curves are functions since any vertical line will cut each curve at most once.
b $y = \sqrt[3]{x}$

EXERCISE 15B

1 **a** 2 **b** 2 **c** -16 **d** -68 **e** $\frac{17}{4}$

2 **a** -3 **b** 3 **c** 3 **d** -3 **e** $\frac{15}{2}$

3 **a** **i** $-\frac{7}{2}$ **ii** $-\frac{3}{4}$ **iii** $-\frac{4}{9}$ **b** $x = 4$ **c** $x = \frac{9}{5}$

4 **a** $7 - 3a$ **b** $7 + 3a$ **c** $-3a - 2$ **d** $7 - 6a$
e $1 - 3x$ **f** $7 - 3x - 3h$

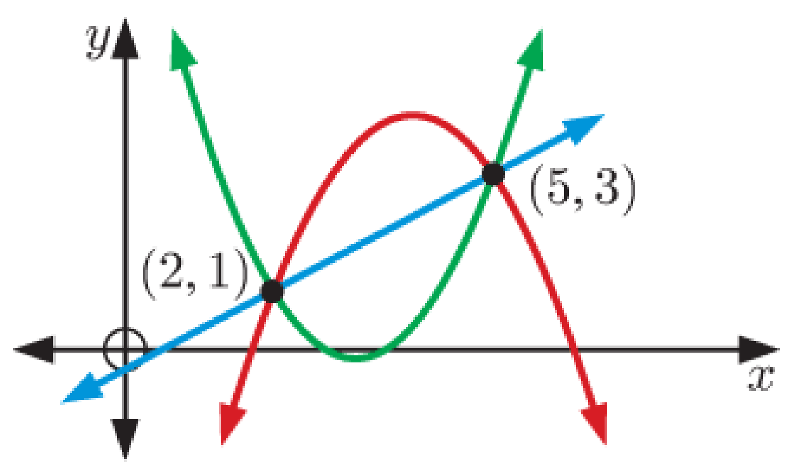
5 **a** $2x^2 + 19x + 43$ **b** $2x^2 - 11x + 13$
c $2x^2 - 3x - 1$ **d** $2x^4 + 3x^2 - 1$
e $18x^2 + 9x - 1$ **f** $2x^2 + (4h + 3)x + 2h^2 + 3h - 1$

6 **a** $9x^2$ **b** $\frac{x^2}{4}$ **c** $3x^2$ **d** $2x^2 - 4x + 7$

7 **a** $-\frac{1}{x}$ **b** $\frac{2}{x}$ **c** $\frac{2 + 3x}{x}$ **d** $\frac{2x + 1}{x - 1}$

8 f is the function which converts x into $f(x)$ whereas $f(x)$ is the value of the function at any value of x.

9 **Note:** Other answers are possible.



10 $f(x) = -2x + 5$

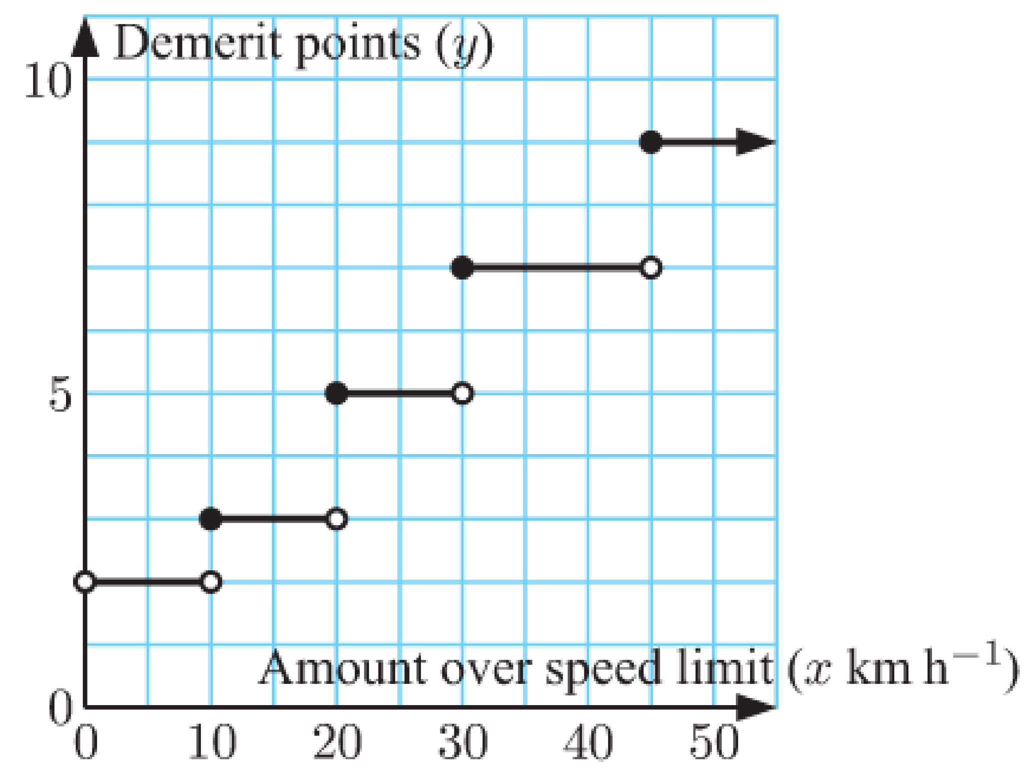
11 **a** $H(30) = 800$. After 30 minutes the balloon is 800 m high.
b $t = 20$ or 70. After 20 minutes and after 70 minutes the balloon is 600 m high.
c $0 \leqslant t \leqslant 80$ **d** 0 m to 900 m

12 $a = 3$, $b = -2$ **13** $a = 3$, $b = -1$, $c = -4$

14 **a** $V(4) = 5400$; $V(4)$ is the value of the photocopier in pounds after 4 years.
b $t = 6$. After 6 years the value of the photocopier is £3600.
c £9000 **d** $0 \leqslant t \leqslant 10$

EXERCISE 15C

1 **a**



b Yes, since for every value of x, there is at most one value of y.
c Domain is $\{x \mid x > 0\}$, Range is $\{2, 3, 5, 7, 9\}$

2 **a** At any moment in time there can be only one temperature, so the graph is a function.
b Domain is $\{t \mid 0 \leqslant t \leqslant 30\}$, Range is $\{T \mid 15 \leqslant T \leqslant 25\}$

3 **a** Domain is $\{x \mid -1 < x \leqslant 5\}$, Range is $\{y \mid 1 < y \leqslant 3\}$
b Domain is $\{x \mid x \neq 2\}$, Range is $\{y \mid y \neq -1\}$
c Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid 0 < y \leqslant 2\}$
d Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid y \leqslant \frac{25}{4}\}$
e Domain is $\{x \mid x \geqslant -4\}$, Range is $\{y \mid y \geqslant -3\}$
f Domain is $\{x \mid x \neq \pm 2\}$,
Range is $\{y \mid y \leqslant -1$ or $y > 0\}$

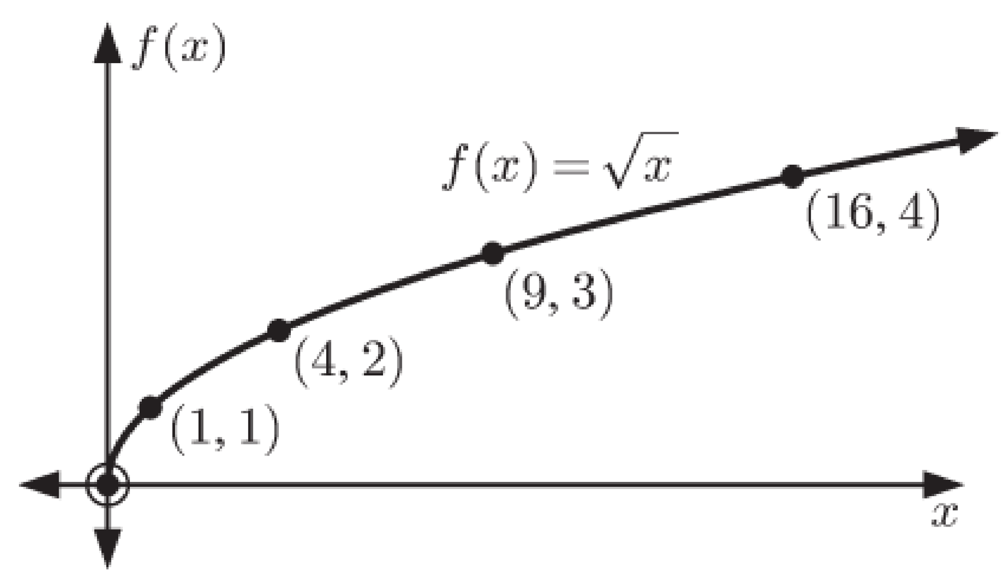
4 **a** true **b** false **c** true **d** true

5 **a** $\{y \mid y \geqslant 0\}$ **b** $\{y \mid y \leqslant 0\}$ **c** $\{y \mid y \geqslant 2\}$
d $\{y \mid y \leqslant 0\}$ **e** $\{y \mid y \leqslant 1\}$ **f** $\{y \mid y \geqslant 3\}$
g $\{y \mid y \geqslant -\frac{9}{4}\}$ **h** $\{y \mid y \leqslant 9\}$ **i** $\{y \mid y \leqslant \frac{25}{12}\}$

6 **a** $\{x \mid x \geqslant 0\}$ **b**

x	0	1	4	9	16
$f(x)$	0	1	2	3	4

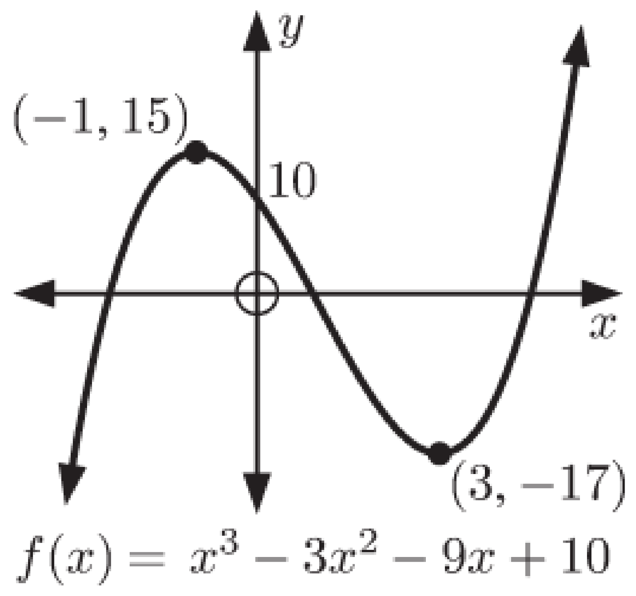
c



d $\{y \mid y \geqslant 0\}$

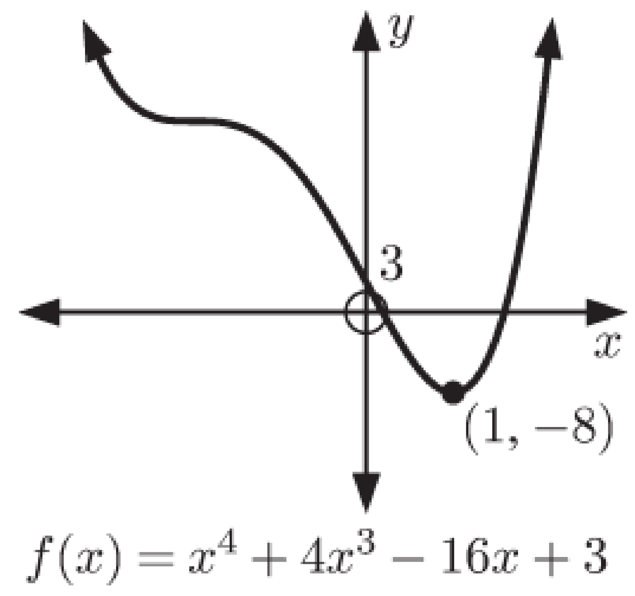
7 **a** Domain is $\{x \mid x \geqslant -6\}$, Range is $\{y \mid y \geqslant 0\}$
b Domain is $\{x \mid x \neq 0\}$, Range is $\{y \mid y > 0\}$
c Domain is $\{x \mid x \neq -1\}$, Range is $\{y \mid y \neq 0\}$
d Domain is $\{x \mid x > 0\}$, Range is $\{y \mid y < 0\}$
e Domain is $\{x \mid x \neq 3\}$, Range is $\{y \mid y \neq 0\}$
f Domain is $\{x \mid x \leqslant 4\}$, Range is $\{y \mid y \geqslant 0\}$

8 **a**



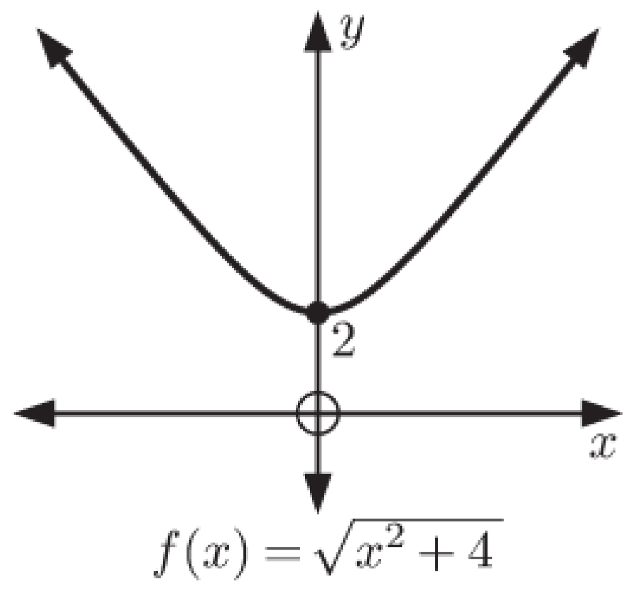
Domain is $\{x \mid x \in \mathbb{R}\}$,
Range is $\{y \mid y \in \mathbb{R}\}$

b



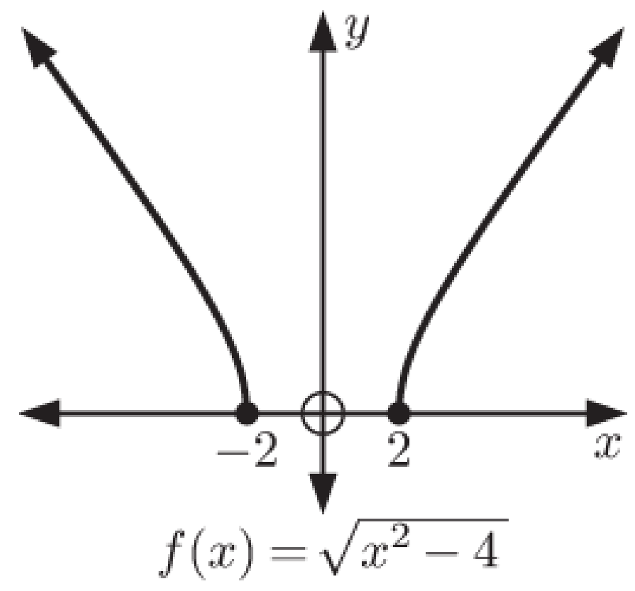
Domain is $\{x \mid x \in \mathbb{R}\}$,
Range is $\{y \mid y \geqslant -8\}$

c



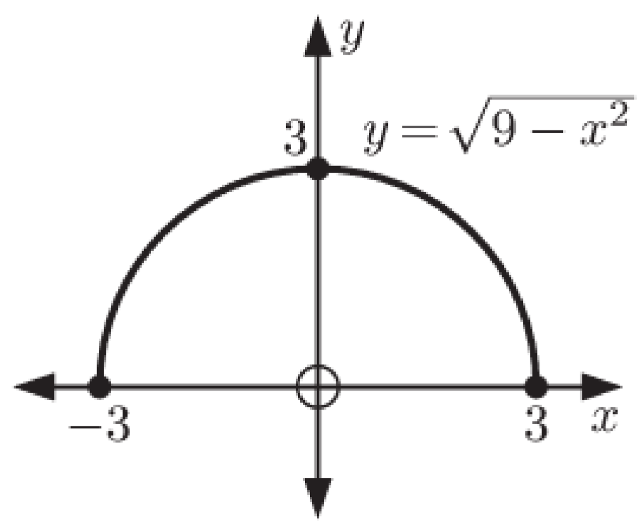
Domain is $\{x \mid x \in \mathbb{R}\}$,
Range is $\{y \mid y \geqslant 2\}$

d



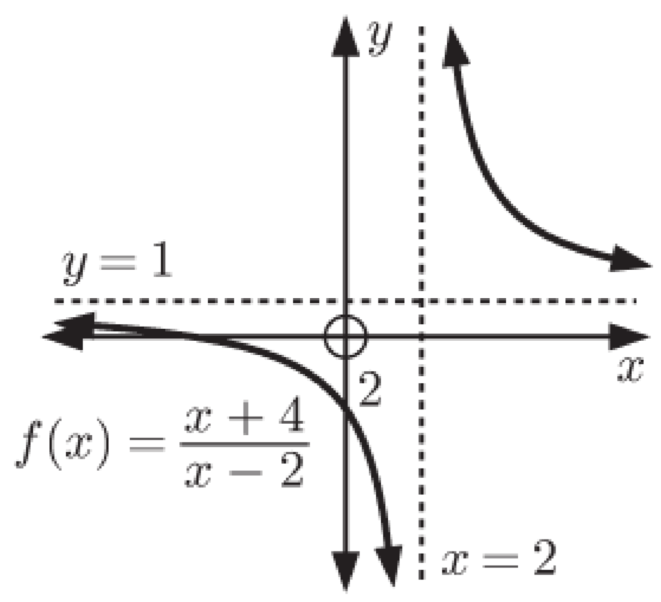
Domain is $\{x \mid x \leqslant -2$ or $x \geqslant 2\}$,
Range is $\{y \mid y \geqslant 0\}$

e



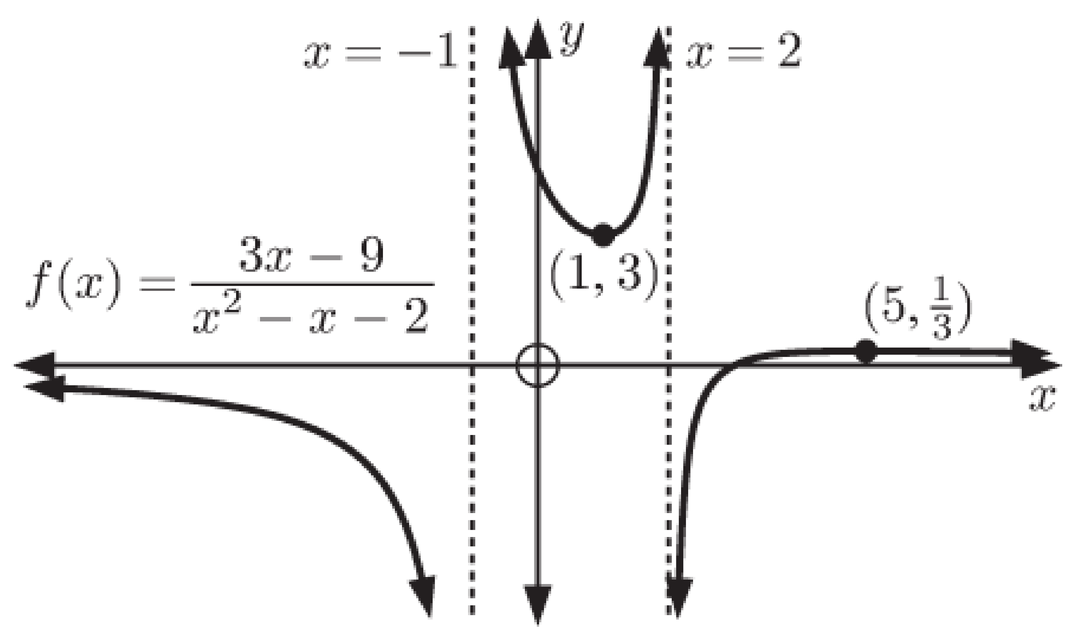
Domain is
$\{x \mid -3 \leqslant x \leqslant 3\}$,
Range is $\{y \mid 0 \leqslant y \leqslant 3\}$

f



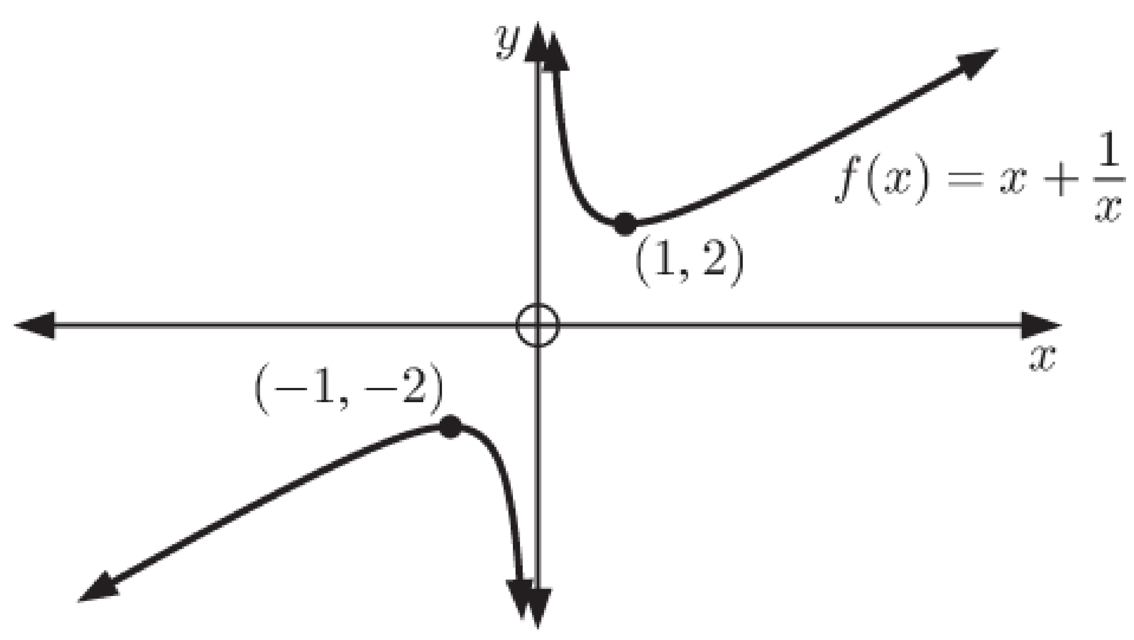
Domain is $\{x \mid x \neq 2\}$,
Range is $\{y \mid y \neq 1\}$

g



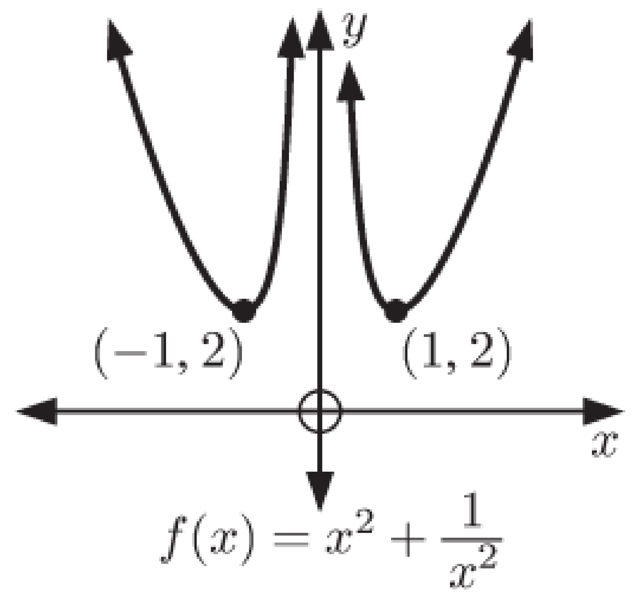
Domain is $\{x \mid x \neq -1$ or $2\}$,
Range is $\{y \mid y \leqslant \frac{1}{3}$ or $y \geqslant 3\}$

h



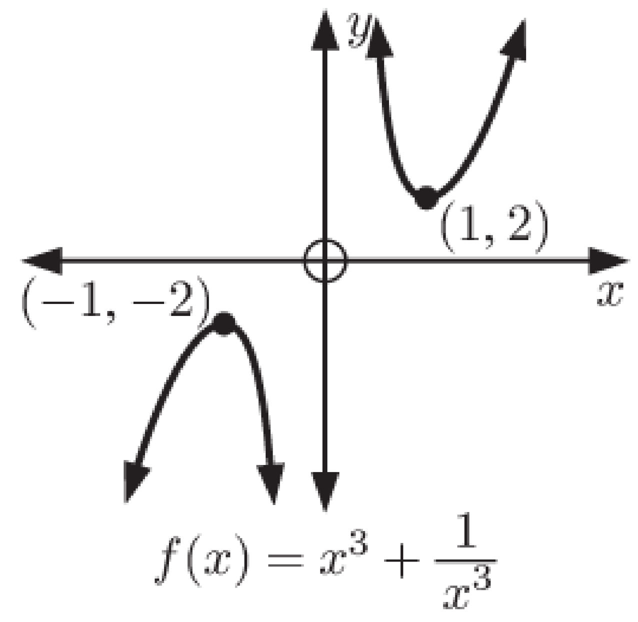
Domain is $\{x \mid x \neq 0\}$,
Range is $\{y \mid y \leqslant -2$ or $y \geqslant 2\}$

i



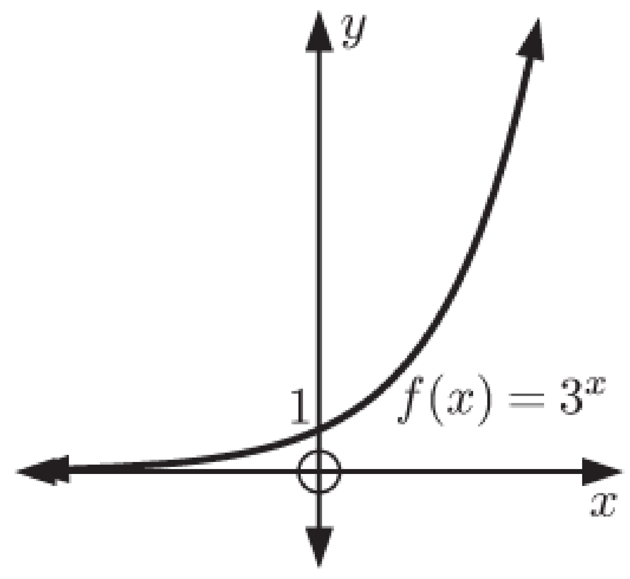
Domain is $\{x \mid x \neq 0\}$,
Range is $\{y \mid y \geqslant 2\}$

j



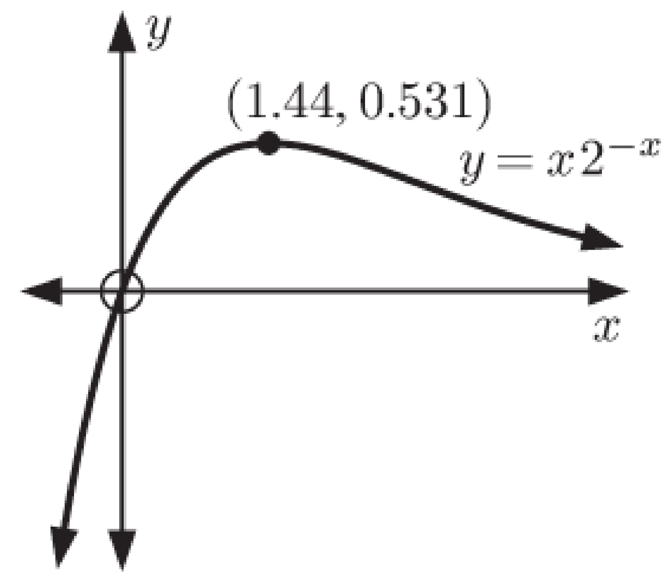
Domain is $\{x \mid x \neq 0\}$,
Range is $\{y \mid y \leqslant -2$ or $y \geqslant 2\}$

k



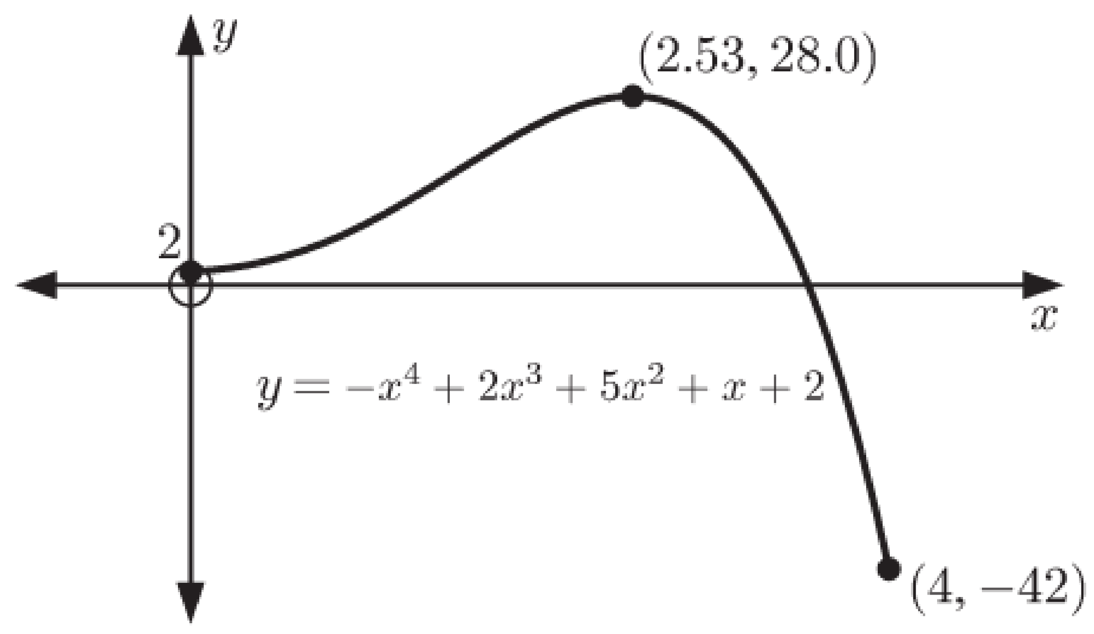
Domain is $\{x \mid x \in \mathbb{R}\}$,
Range is $\{y \mid y > 0\}$

l



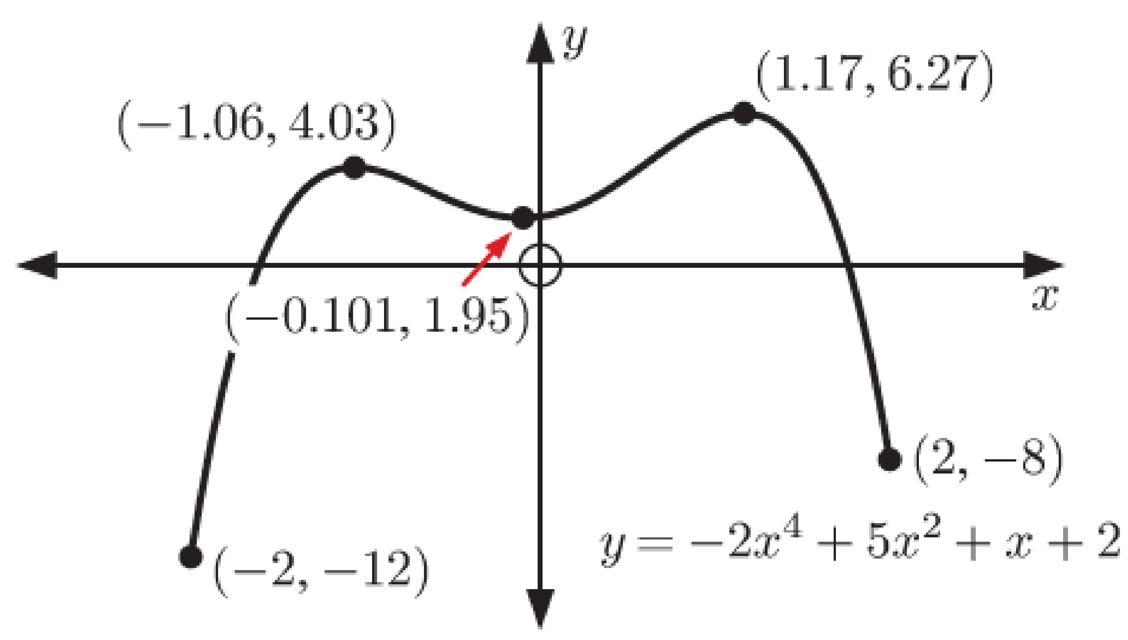
Domain is $\{x \mid x \in \mathbb{R}\}$,
Range is $\{y \mid y \leqslant 0.531\}$

9 **a**



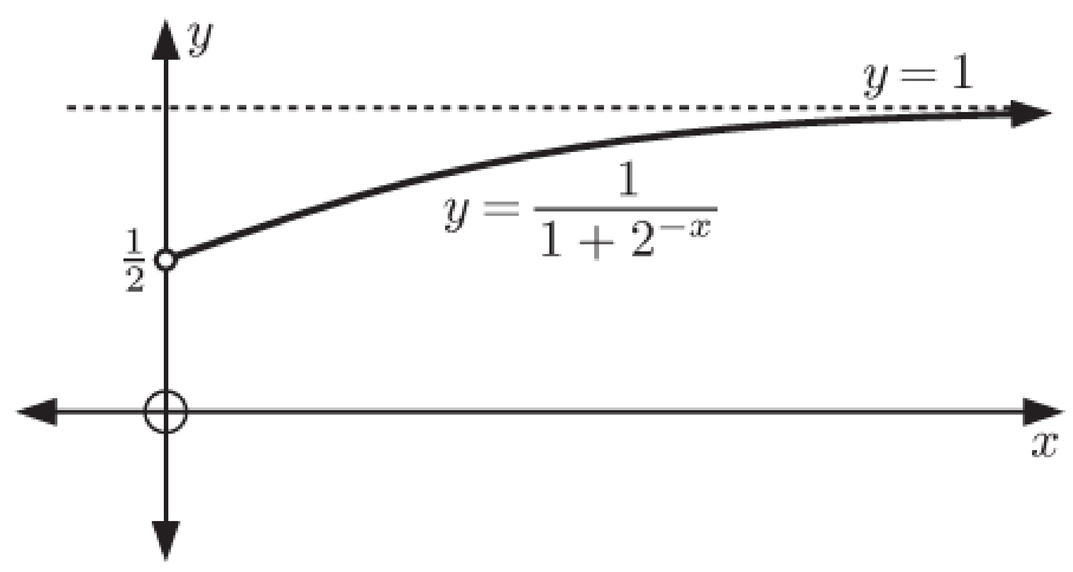
Range is $\{y \mid -42 \leqslant y \leqslant 28.0\}$

b



Range is $\{y \mid -12 \leqslant y \leqslant 6.27\}$

c



Range is $\{y \mid \frac{1}{2} < y < 1\}$

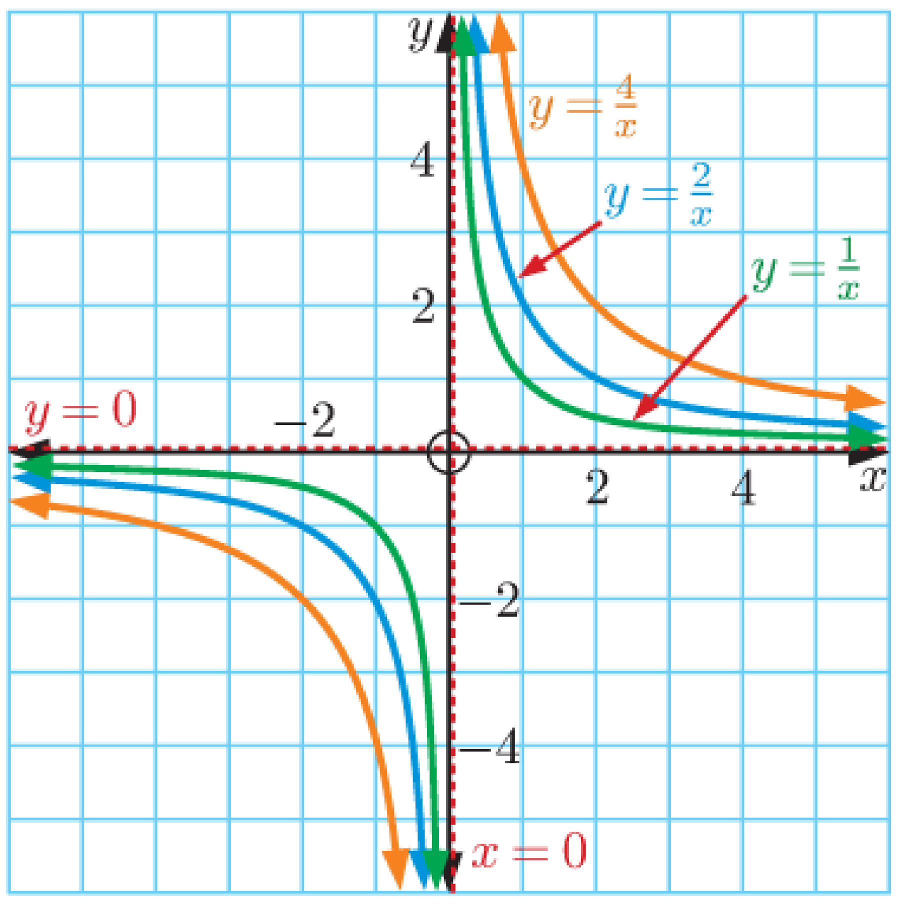
10 **a** $k \geqslant \frac{25}{4}$ **b** Range is $\{y \mid y \geqslant \sqrt{k - \frac{25}{4}}\}$

11 **a** Domain is $\{x \mid -2 \leqslant x \leqslant 2\}$
Range is $\{y \mid -2 \leqslant y \leqslant 2\}$

b Domain is $\{-2, -1, 0, 1, 2\}$
Range is $\{-2, -\sqrt{3}, 0, \sqrt{3}, 2\}$

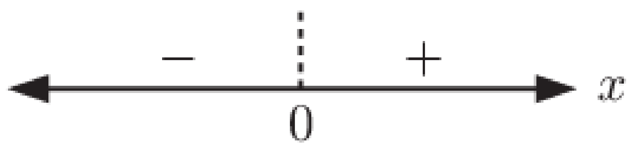
EXERCISE 15D.1

1 **a**

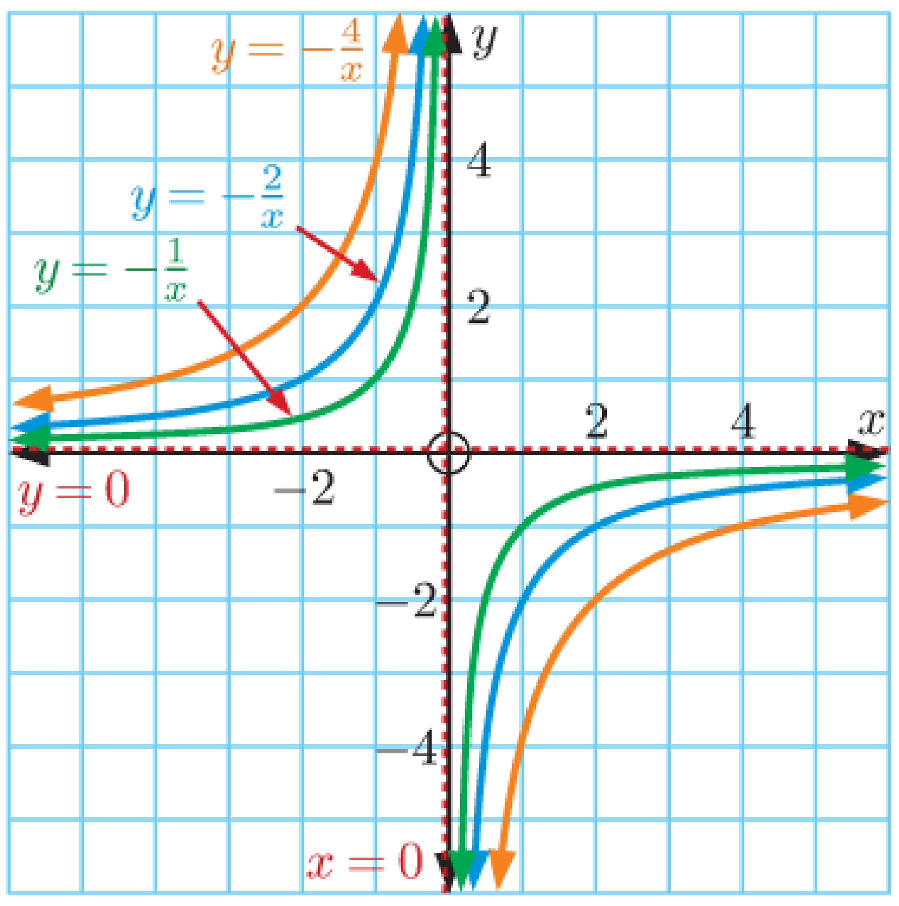


b **i** As k becomes larger the graphs move further from the origin.

ii quadrants 1 and 3 **iii**

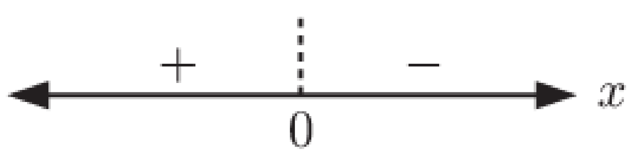


2 **a**



b **i** As $|k|$ becomes larger, the graphs move further from the origin.

ii quadrants 2 and 4 **iii**



3 **a** $\{x \mid x \neq 0\}$ **b** $\{y \mid y \neq 0\}$ **c** $x = 0$ **d** $y = 0$

4 **a** $y = \dfrac{6}{x}$ **b** $y = \dfrac{15}{x}$ **c** $y = -\dfrac{36}{x}$

EXERCISE 15D.2

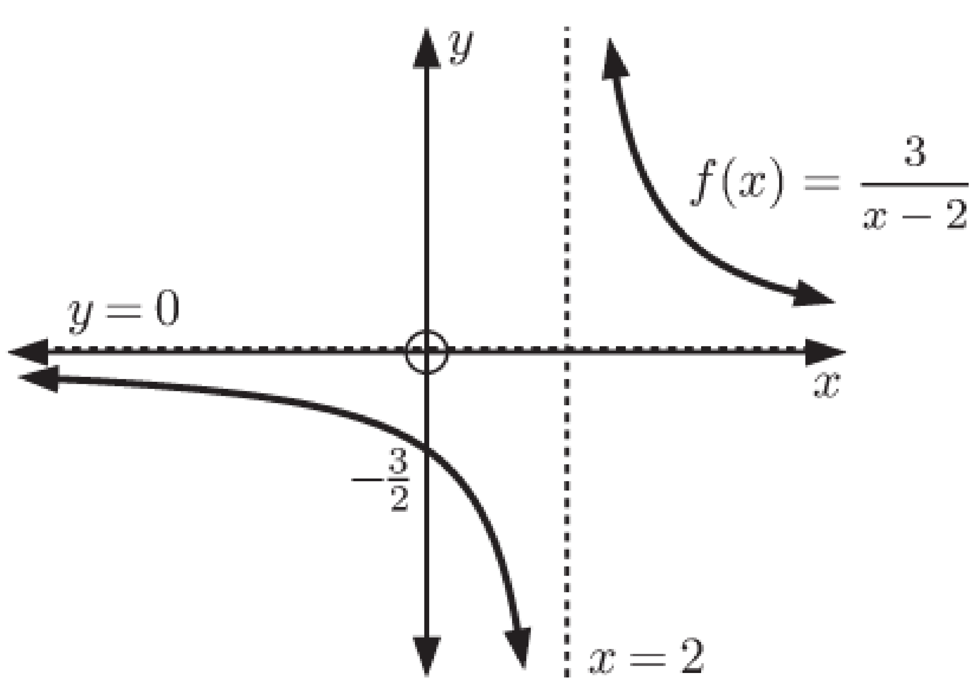
1 **a** **i** vertical asymptote $x = 2$, horizontal asymptote $y = 0$

ii Domain is $\{x \mid x \neq 2\}$, Range is $\{y \mid y \neq 0\}$

iii no x-intercept, y-intercept $-\frac{3}{2}$

iv as $x \to 2^-$, $f(x) \to -\infty$
as $x \to 2^+$, $f(x) \to \infty$
as $x \to -\infty$, $f(x) \to 0^-$
as $x \to \infty$, $f(x) \to 0^+$

v



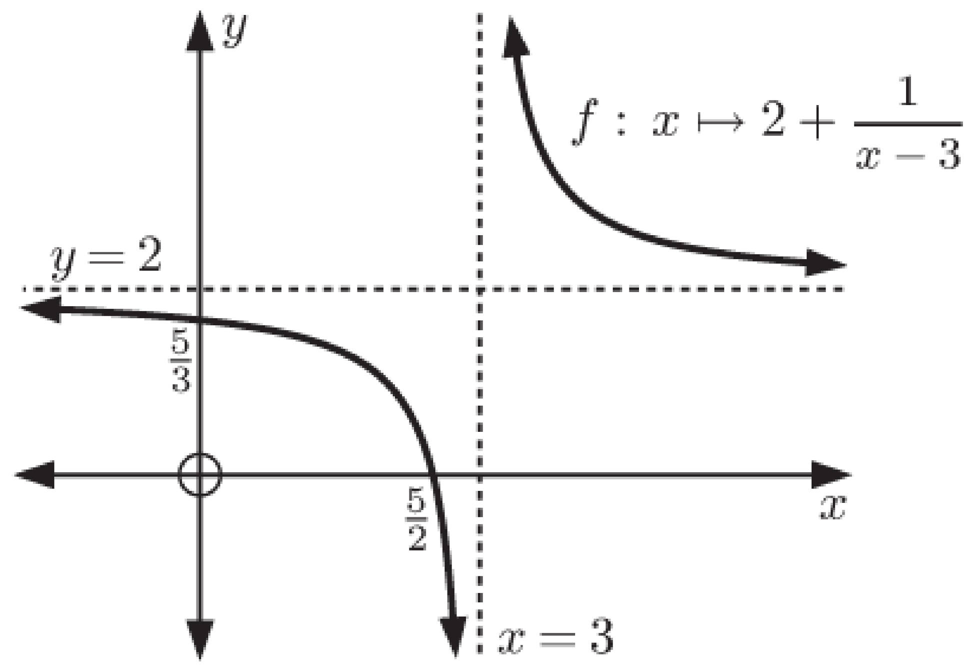
b **i** vertical asymptote $x = 3$, horizontal asymptote $y = 2$

ii Domain is $\{x \mid x \neq 3\}$, Range is $\{y \mid y \neq 2\}$

iii x-intercept $\frac{5}{2}$, y-intercept $\frac{5}{3}$

iv as $x \to 3^-$, $f(x) \to -\infty$
as $x \to 3^+$, $f(x) \to \infty$
as $x \to -\infty$, $f(x) \to 2^-$
as $x \to \infty$, $f(x) \to 2^+$

v



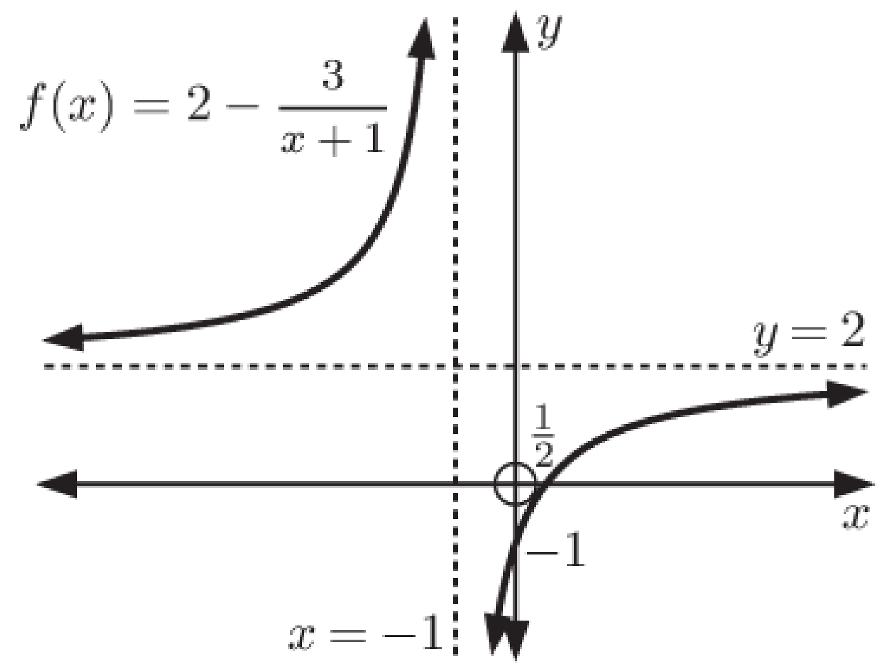
c **i** vertical asymptote $x = -1$, horizontal asymptote $y = 2$

ii Domain is $\{x \mid x \neq -1\}$, Range is $\{y \mid y \neq 2\}$

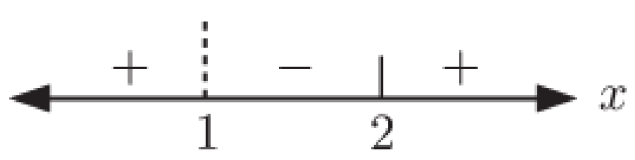
iii x-intercept $\frac{1}{2}$, y-intercept -1

iv as $x \to -1^-$, $f(x) \to \infty$
as $x \to -1^+$, $f(x) \to -\infty$
as $x \to -\infty$, $f(x) \to 2^+$
as $x \to \infty$, $f(x) \to 2^-$

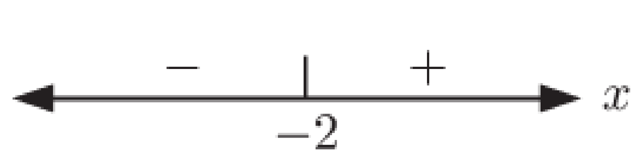
v



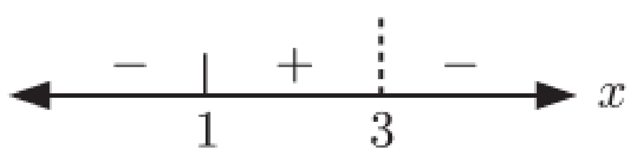
2 **a**



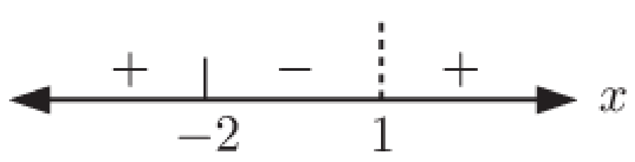
b



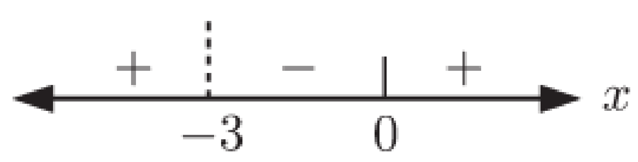
c



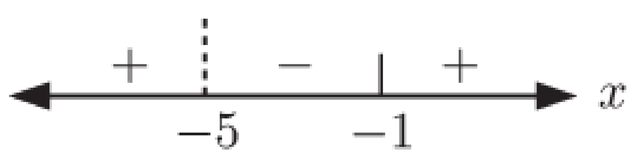
3 **a**



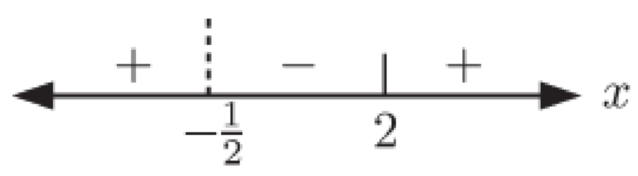
b



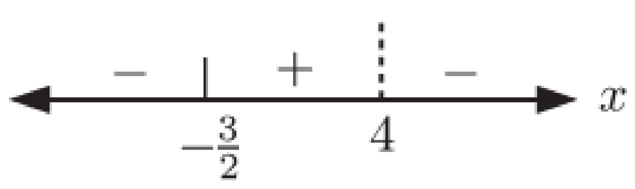
c



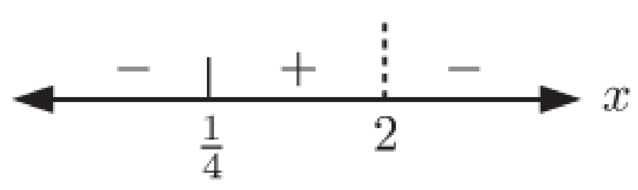
d



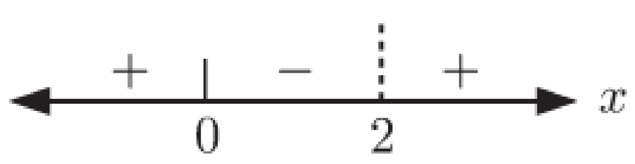
e



f



g



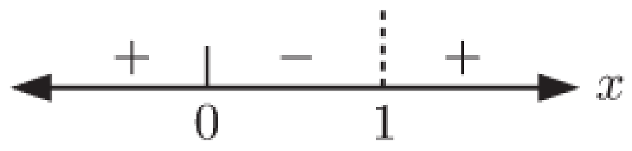
h

4 **a** **i** vertical asymptote is $x = 1$

ii x-intercept 0, y-intercept 0

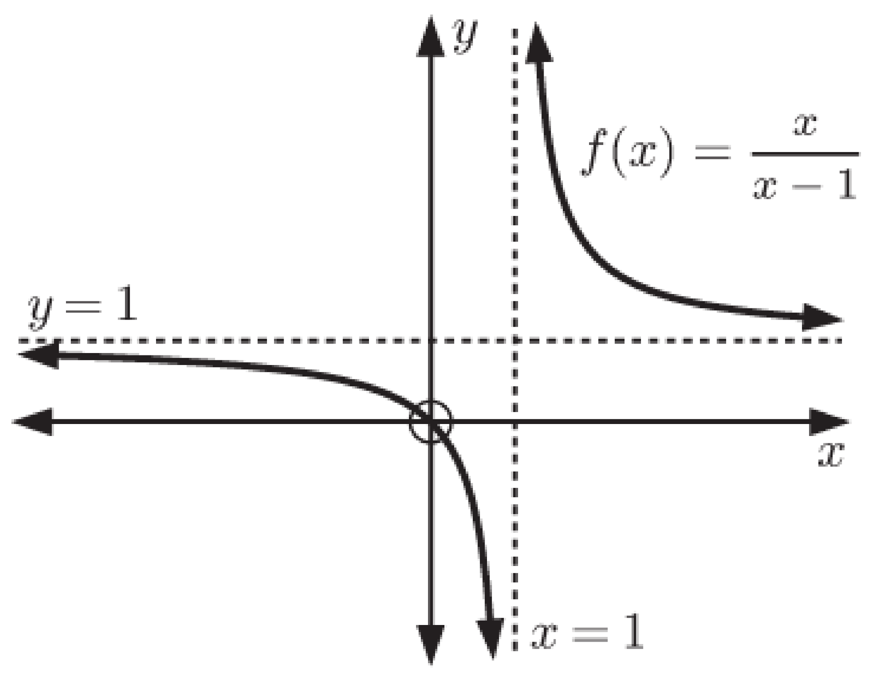
iii $f(x) = 1 + \dfrac{1}{x-1}$, horizontal asymptote is $y = 1$

iv



v as $x \to 1^-$, $f(x) \to -\infty$
as $x \to 1^+$, $f(x) \to \infty$
as $x \to -\infty$, $f(x) \to 1^-$
as $x \to \infty$, $f(x) \to 1^+$

vi

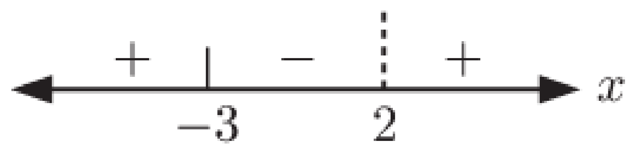


b **i** vertical asymptote is $x = 2$

ii x-intercept -3, y-intercept $-\frac{3}{2}$

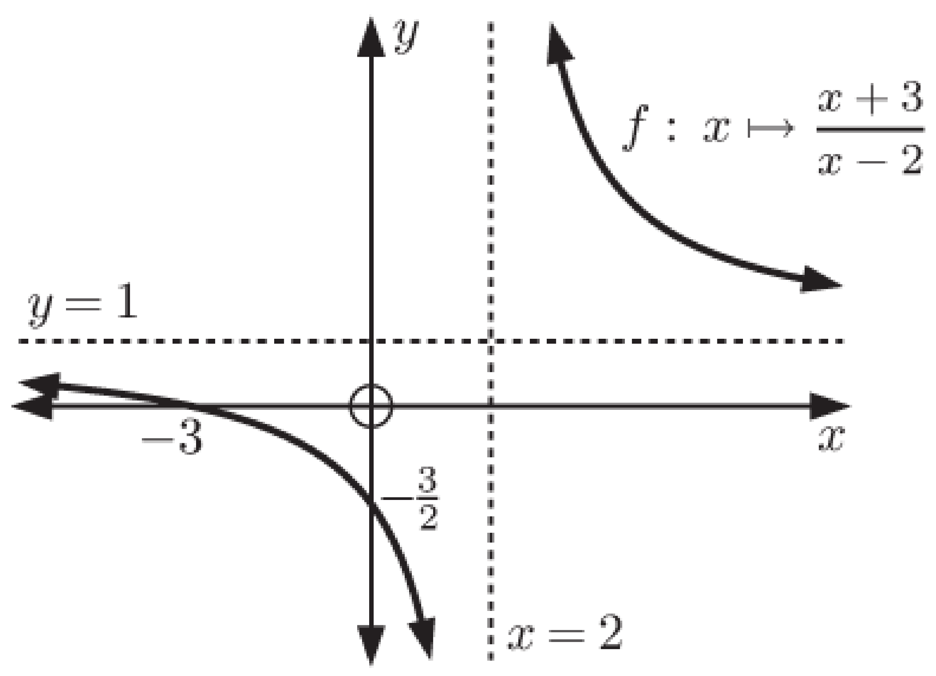
iii $f(x) = 1 + \dfrac{5}{x-2}$, horizontal asymptote is $y = 1$

iv



v as $x \to 2^-$, $f(x) \to -\infty$
as $x \to 2^+$, $f(x) \to \infty$
as $x \to -\infty$, $f(x) \to 1^-$
as $x \to \infty$, $f(x) \to 1^+$

vi

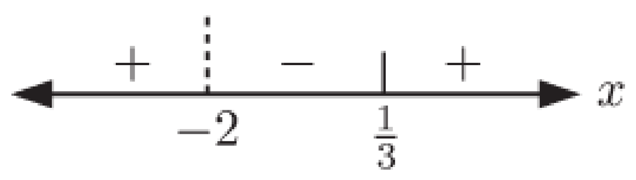


c **i** vertical asymptote is $x = -2$

ii x-intercept $\frac{1}{3}$, y-intercept $-\frac{1}{2}$

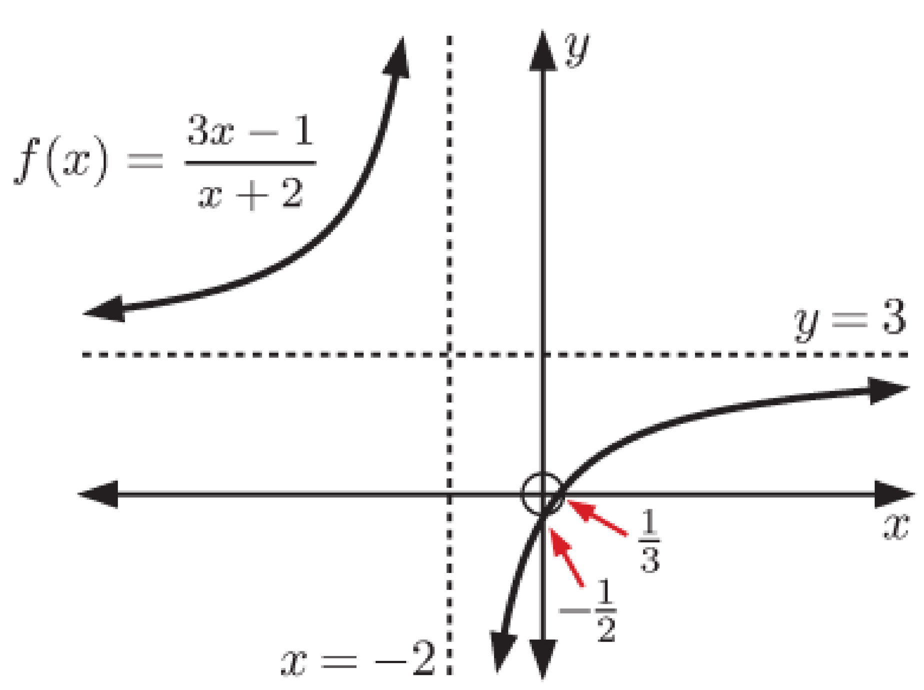
iii $f(x) = 3 - \dfrac{7}{x+2}$, horizontal asymptote is $y = 3$

iv



v as $x \to -2^-$, $f(x) \to \infty$
as $x \to -2^+$, $f(x) \to -\infty$
as $x \to -\infty$, $f(x) \to 3^+$
as $x \to \infty$, $f(x) \to 3^-$

vi

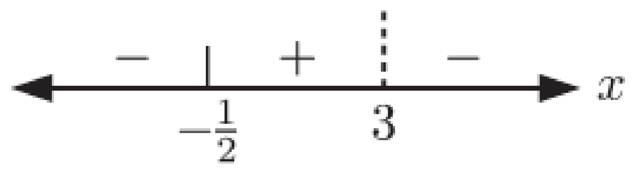


d **i** vertical asymptote is $x = 3$

ii x-intercept $-\frac{1}{2}$, y-intercept $\frac{1}{3}$

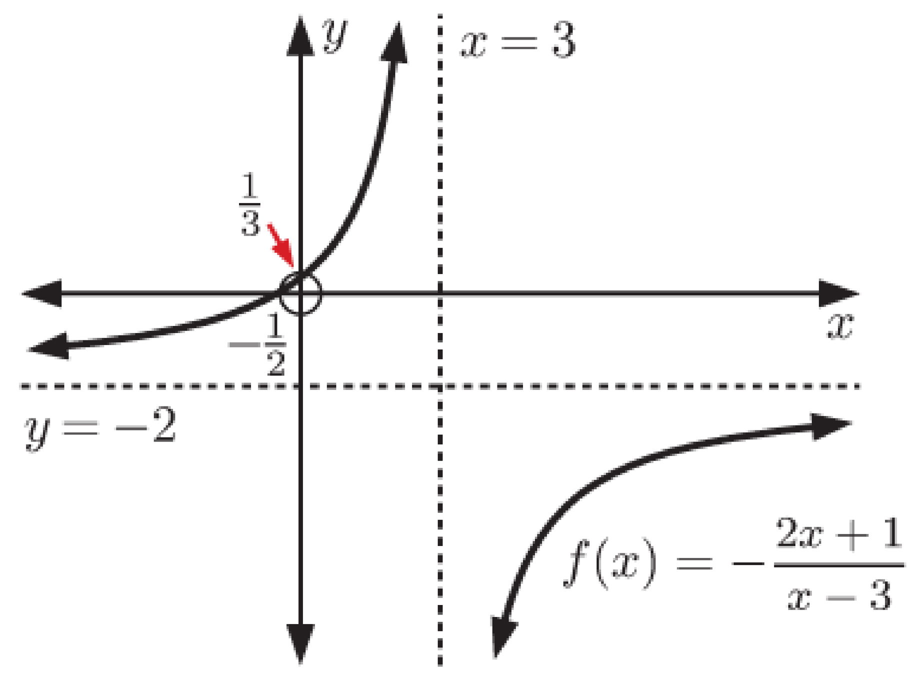
iii $f(x) = -2 - \dfrac{7}{x-3}$, horizontal asymptote is $y = -2$

iv



v as $x \to 3^-$, $f(x) \to \infty$
as $x \to 3^+$, $f(x) \to -\infty$
as $x \to -\infty$, $f(x) \to -2^+$
as $x \to \infty$, $f(x) \to -2^-$

vi

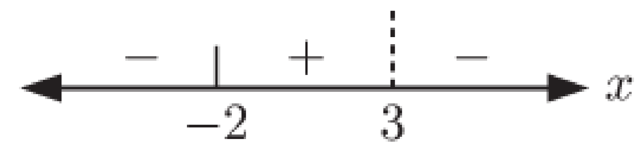


e **i** vertical asymptote is $x = 3$

ii x-intercept -2, y-intercept $\frac{4}{3}$

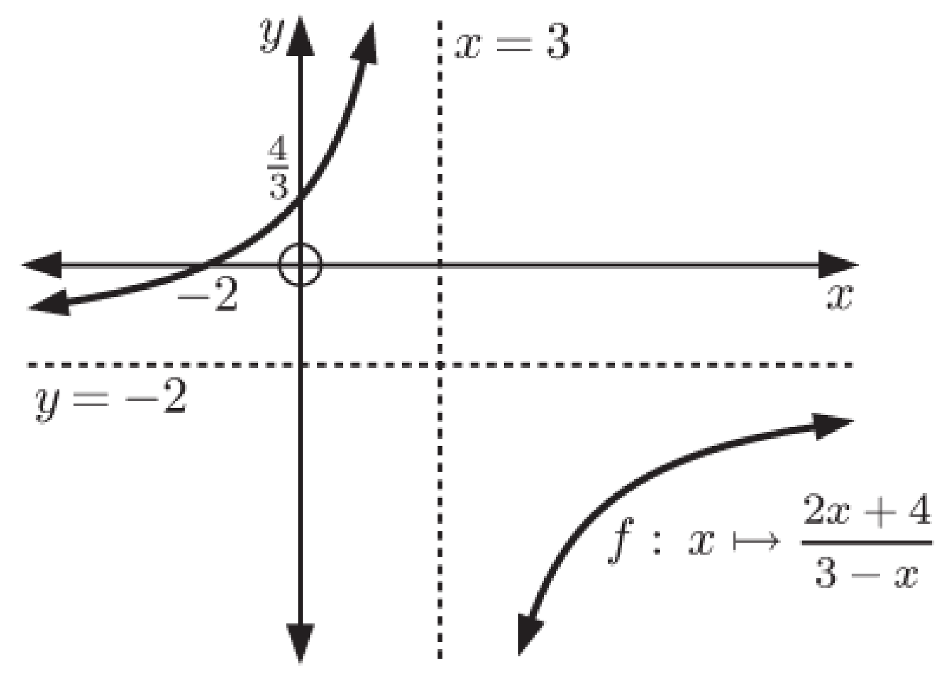
iii $f(x) = -2 + \dfrac{10}{3-x}$, horizontal asymptote is $y = -2$

iv



v as $x \to 3^-$, $f(x) \to \infty$
as $x \to 3^+$, $f(x) \to -\infty$
as $x \to -\infty$, $f(x) \to -2^+$
as $x \to \infty$, $f(x) \to -2^-$

vi

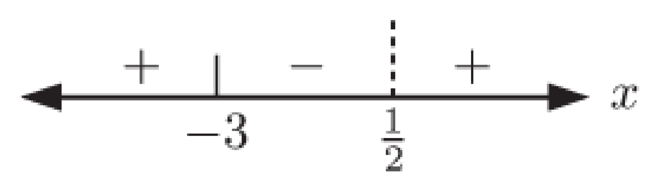


f **i** vertical asymptote is $x = \frac{1}{2}$

ii x-intercept -3, y-intercept -3

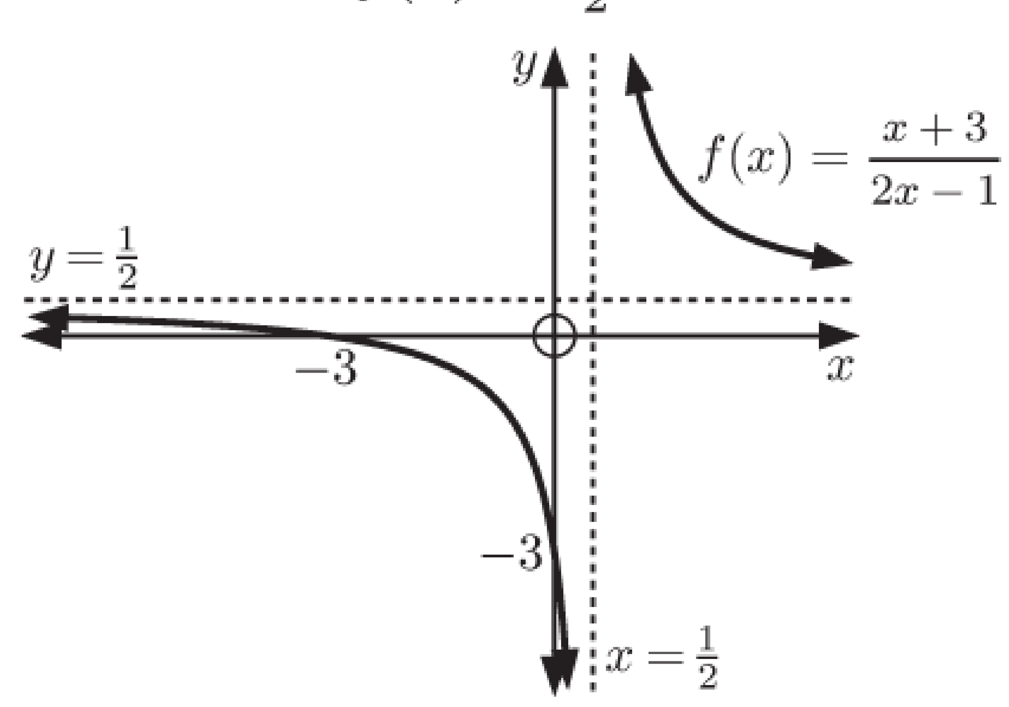
iii $f(x) = \frac{1}{2} + \dfrac{7}{4x-2}$, horizontal asymptote is $y = \frac{1}{2}$

iv

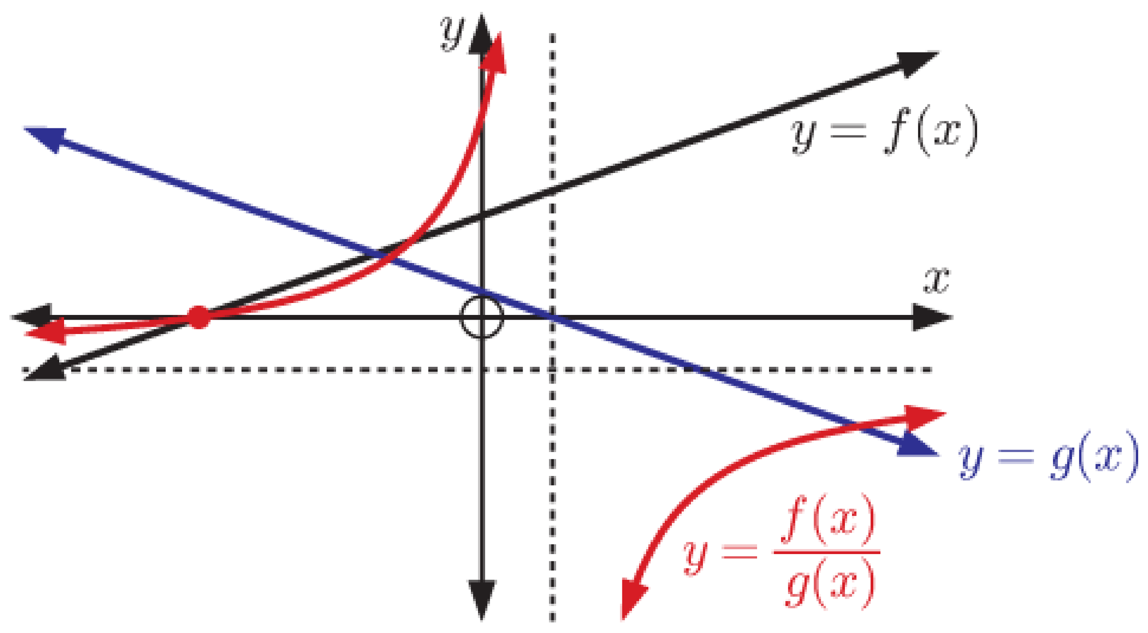


v as $x \to \frac{1}{2}^-$, $f(x) \to -\infty$
as $x \to \frac{1}{2}^+$, $f(x) \to \infty$
as $x \to -\infty$, $f(x) \to \frac{1}{2}^-$
as $x \to \infty$, $f(x) \to \frac{1}{2}^+$

vi



5



6 **a** Domain is $\{x \mid x \neq -\frac{d}{c}\}$

b vertical asymptote is $x = -\frac{d}{c}$

c x-intercept is $-\frac{b}{a}$, y-intercept is $\frac{b}{d}$

d $\dfrac{ax+b}{cx+d} = \dfrac{\frac{a}{c}(cx+d) - \frac{ad}{c} + b}{cx+d}$ and so on

As $x \to \infty$, $\dfrac{b - \frac{ad}{c}}{cx+d} \to 0$.

$\therefore$ the horizontal asymptote is $y = \frac{a}{c}$.

EXERCISE 15E

1 **a** $-2 - 2x^2$ **b** $1 + 4x^2$ **c** -10 **d** -4

2 **a** $-4x^2 - 16x - 13$ **b** $10 - 2x^2$ **c** 14 **d** $-\frac{73}{16}$

3 **a** $25x - 42$ **b** $\sqrt{8}$ **c** -7 **d** 2

4 **a** **i** $x^2 - 6x + 10$ **ii** $2 - x^2$ **b** $x = \pm\frac{1}{\sqrt{2}}$

5 **a** $(f \circ g)(x) = 9 - \sqrt{x^2 + 4}$

Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid y \leqslant 7\}$

b 53

c $(f \circ f)(x) = 9 - \sqrt{9 - \sqrt{x}}$

Domain is $\{x \mid 0 \leqslant x \leqslant 81\}$, Range is $\{y \mid 6 \leqslant y \leqslant 9\}$

6 **a** $-6x - 9$ **b** $x = -1$

7 **a** **i** $1 - 9x^2$ **ii** $1 + 6x - 3x^2$ **b** $x = -\frac{1}{9}$

8 **a** $(f \circ g)(x) = \dfrac{1}{x-3}$

Domain is $\{x \mid x \neq 3\}$, Range is $\{y \mid y \neq 0\}$

b $(f \circ g)(x) = -\dfrac{1}{x^2 + 3x + 2}$

Domain is $\{x \mid x \neq -1,\ x \neq -2\}$

Range is $\{y \mid y \geqslant 4,\ y < 0\}$

9 **a** $f \circ g = \{(2, 7), (5, 2), (7, 5), (9, 9)\}$

b $g \circ f = \{(0, 2), (1, 0), (2, 1), (3, 3)\}$

10 **a** $(f \circ g)(x) = \dfrac{4x - 2}{3x - 1}$, Domain is $\{x \mid x \neq \frac{1}{3}$ or $1\}$

b $(g \circ f)(x) = 2x + 5$, Domain is $\{x \mid x \neq -2\}$

c $(g \circ g)(x) = x$, Domain is $\{x \mid x \neq 1\}$

11 **a** Let $x = 0$, $\therefore$ $b = d$ and so

$ax + b = cx + b$

$\therefore$ $ax = cx$ for all x

Let $x = 1$, $\therefore$ $a = c$

b $(f \circ g)(x) = [2a]x + [2b + 3] = 1x + 0$ for all x

$\therefore$ $2a = 1$ and $2b + 3 = 0$

c Yes, $\{(g \circ f)(x) = [2a]x + [3a + b]\}$

12 **a** $(f \circ g)(x) = \sqrt{1 - x^2}$

b Domain is $\{x \mid -1 \leqslant x \leqslant 1\}$, Range is $\{y \mid 0 \leqslant y \leqslant 1\}$

c $(g \circ f)(x) = 1 - x$

d Domain is $\{x \mid x \leqslant 1\}$, Range is $\{y \mid y \geqslant 0\}$

13 **a** $R_g \cap D_f \neq \varnothing$

b Domain is $\{x \mid x \in D_g,\ g(x) \in D_f\}$

14 **a** $V \circ D = 6800 - 400t$

This is the value of Mila's car t years after purchase.

b 4400; the value of Mila's car 6 years after purchase is \$4400.

15 **a** **i** $T \circ S$ **ii** $S \circ T$ **b** €715

16 **a** $V = 2000 - 20t$

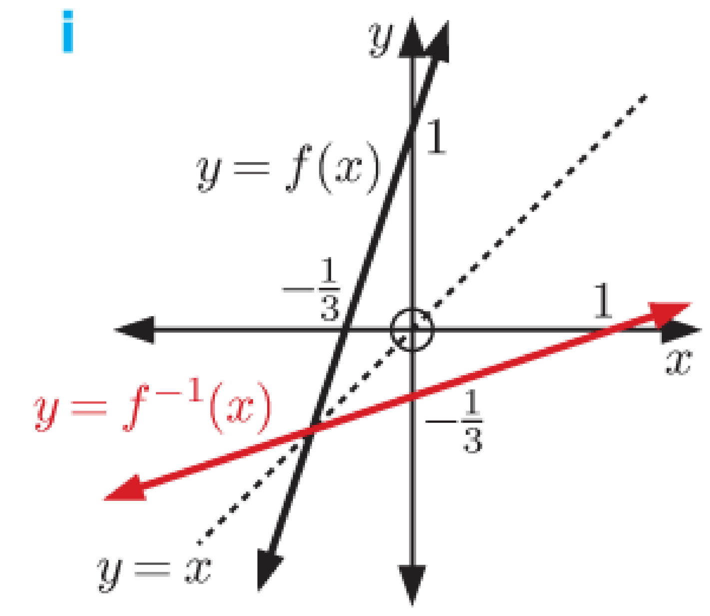
c $H \circ V = \sqrt[3]{\dfrac{24\,000 - 240t}{\pi}}$

This is the height of the solution after t minutes.

d $(H \circ V)(30) \approx 17.5$; the height of the solution after 30 minutes is about 17.5 cm.

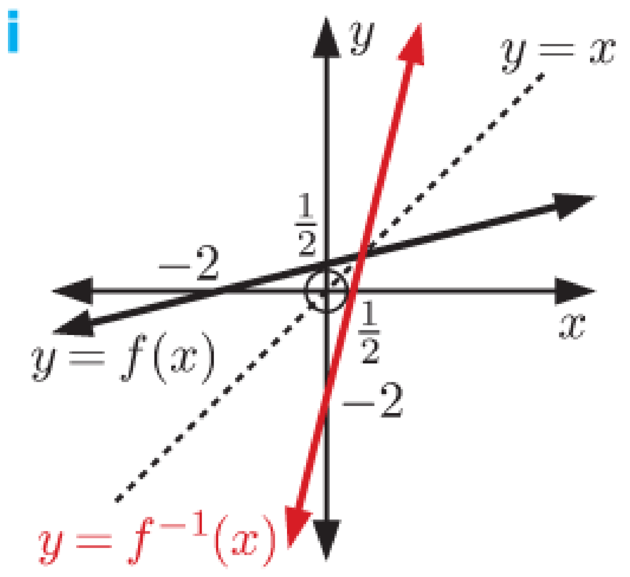
EXERCISE 15F

1 **a** **i**



ii, iii $f^{-1}(x) = \dfrac{x-1}{3}$

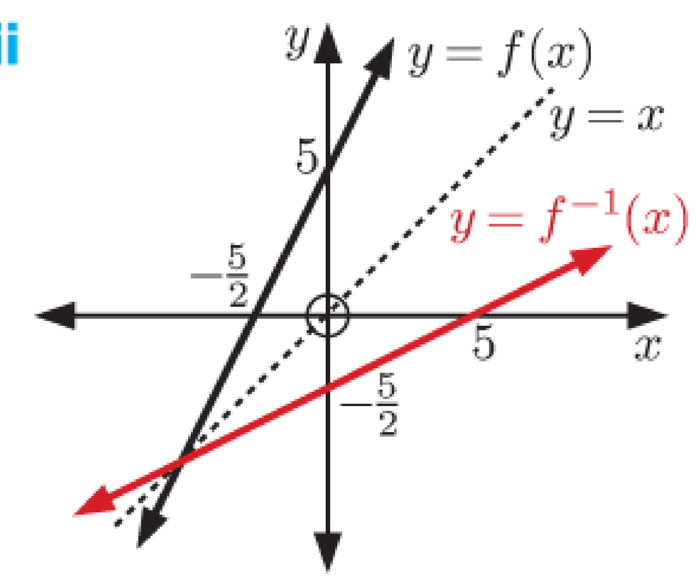
b **i**



ii, iii $f^{-1}(x) = 4x - 2$

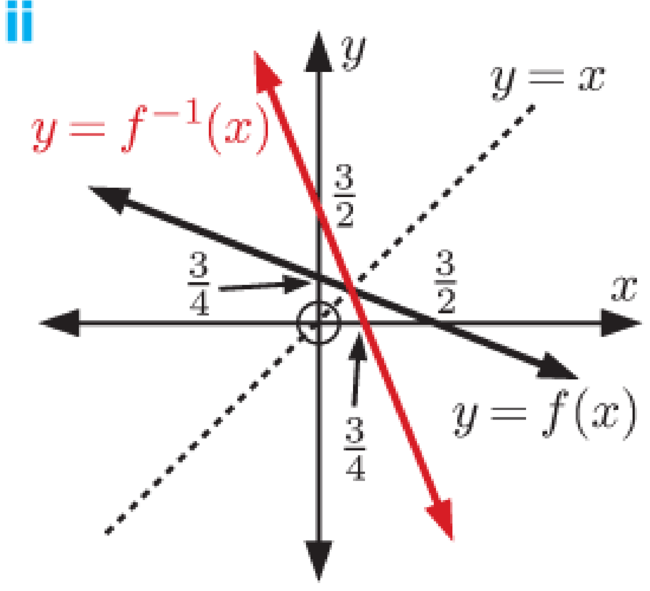
2 **a** **i** $f^{-1}(x) = \dfrac{x-5}{2}$

ii

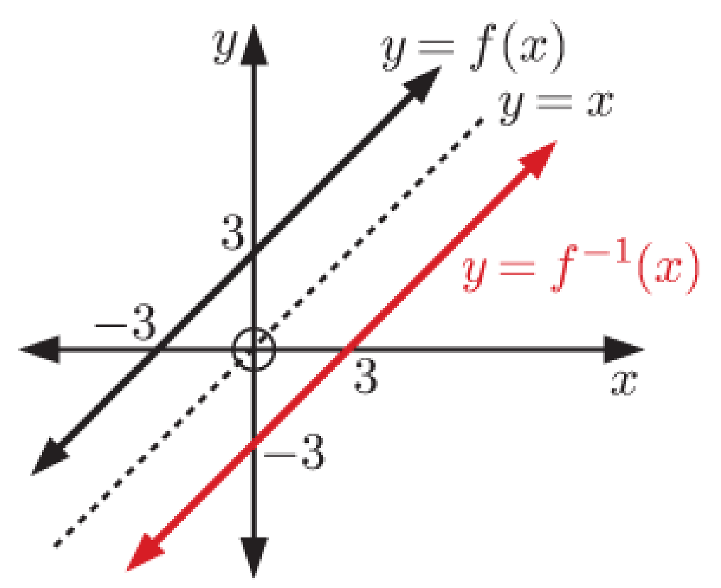


b **i** $f^{-1}(x) = -2x + \frac{3}{2}$

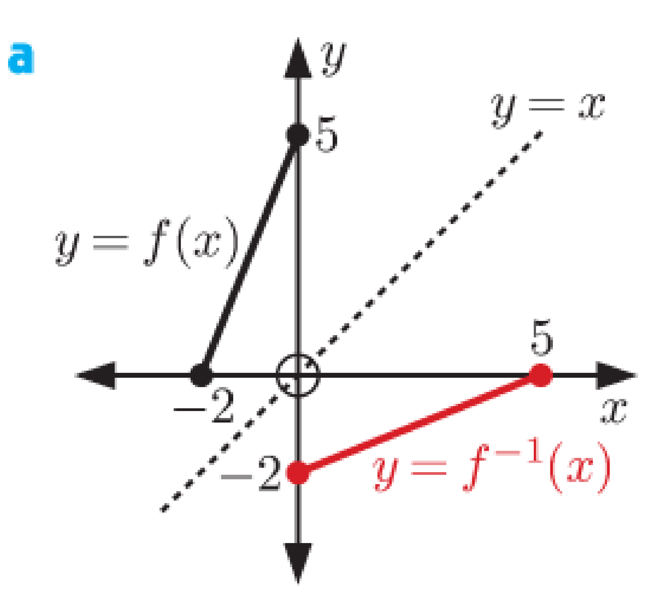
ii



c **i** $f^{-1}(x) = x - 3$ **ii**



3 **a**



f:

Domain is $\{x \mid -2 \leqslant x \leqslant 0\}$

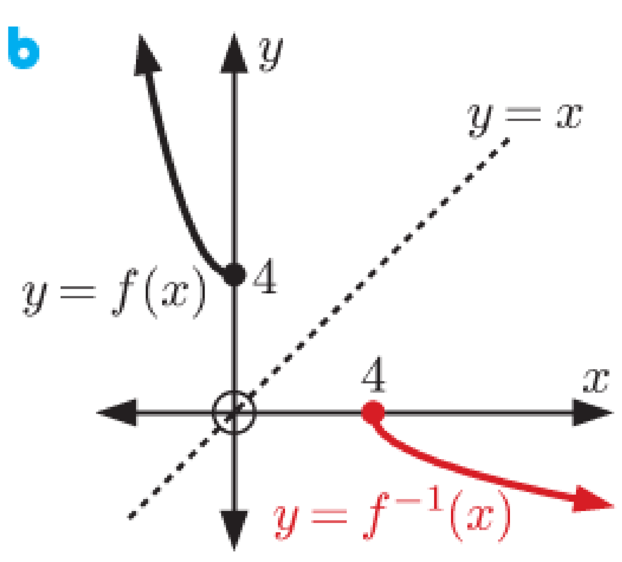
Range is $\{y \mid 0 \leqslant y \leqslant 5\}$

f^{-1}:

Domain is $\{x \mid 0 \leqslant x \leqslant 5\}$

Range is $\{y \mid -2 \leqslant y \leqslant 0\}$

b



f:

Domain is $\{x \mid x \leqslant 0\}$

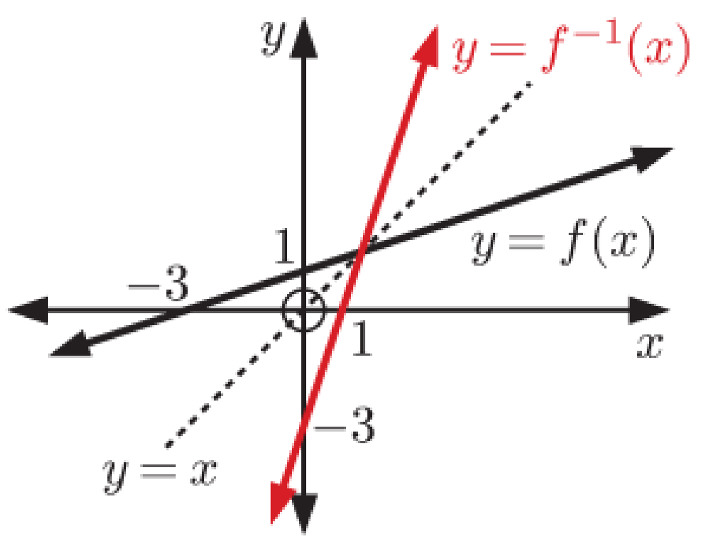
Range is $\{y \mid y \geqslant 4\}$

f^{-1}:

Domain is $\{x \mid x \geqslant 4\}$

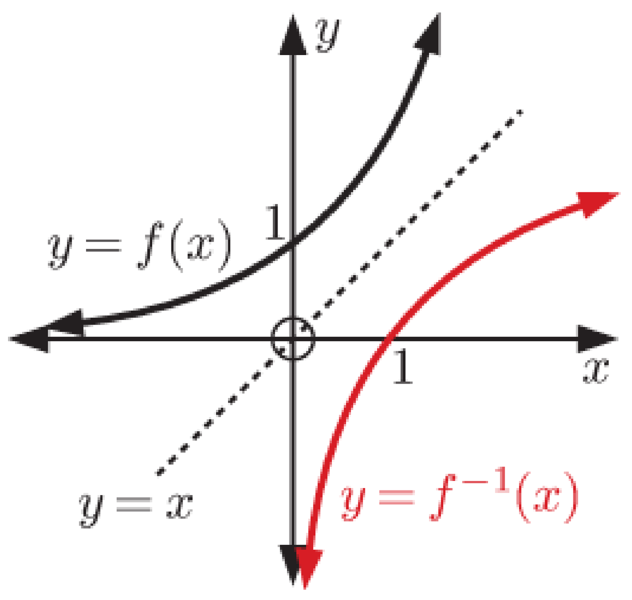
Range is $\{y \mid y \leqslant 0\}$

c



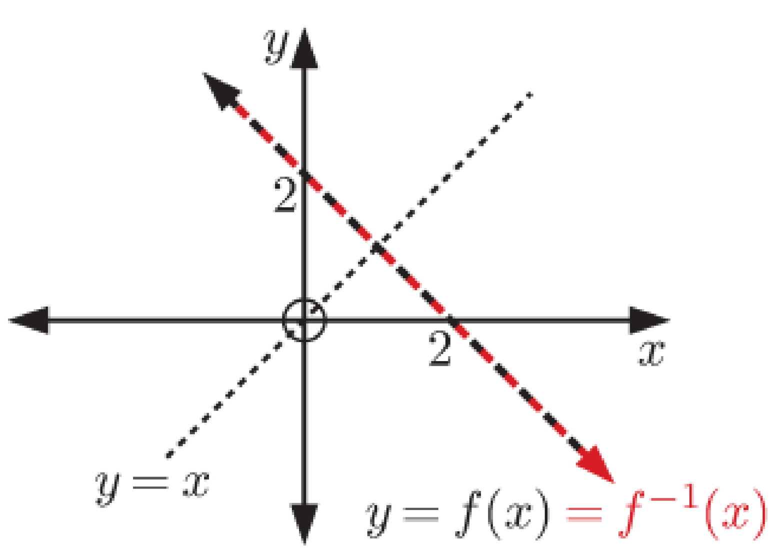
f:
Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y \in \mathbb{R}\}$
f^{-1}:
Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y \in \mathbb{R}\}$

d



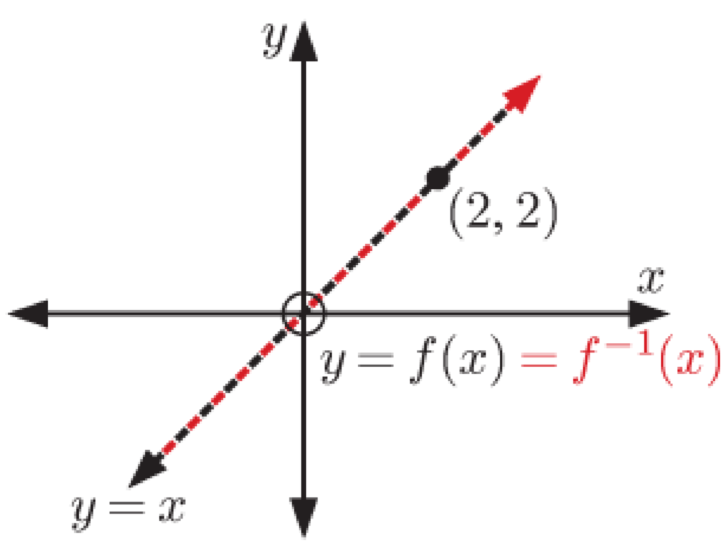
f:
Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y > 0\}$
f^{-1}:
Domain is $\{x \mid x > 0\}$
Range is $\{y \mid y \in \mathbb{R}\}$

e



f:
Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y \in \mathbb{R}\}$
f^{-1}:
Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y \in \mathbb{R}\}$

f



f:
Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y \in \mathbb{R}\}$
f^{-1}:
Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y \in \mathbb{R}\}$

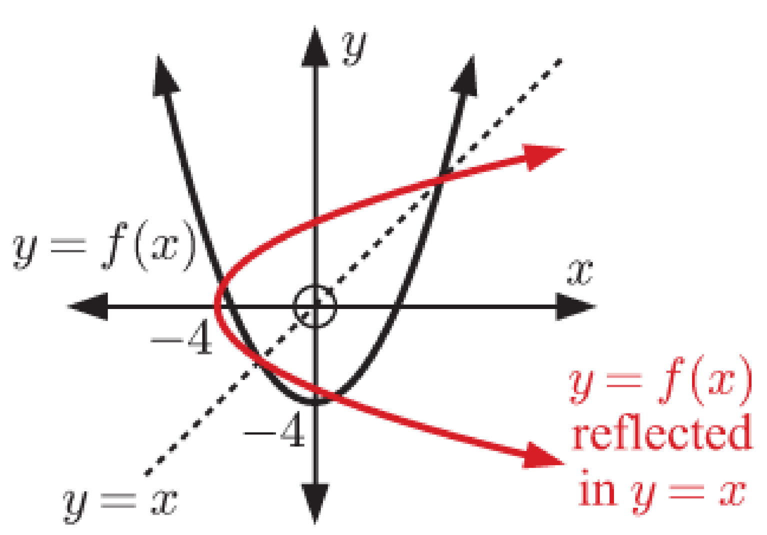
4 $(f^{-1})^{-1}(x) = 2x - 5 = f(x)$

5 **a** $\{(2, 1), (4, 2), (5, 3)\}$ **b** inverse does not exist
c $\{(1, 2), (0, -1), (2, 0), (3, 1)\}$
d $\{(-1, -1), (0, 0), (1, 1)\}$

6 $f(x) = x$ and $f(x) = -x + c,\ c \in \mathbb{R}$

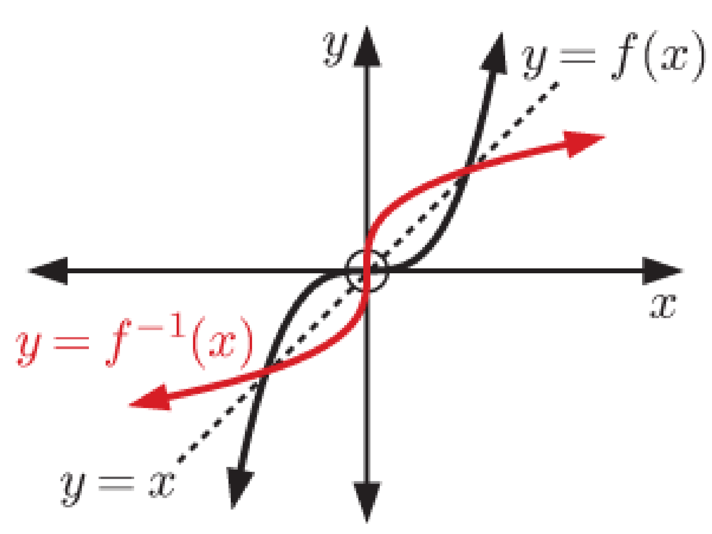
7 Range is $\{y \mid -2 \leqslant y < 3\}$

8 **a**



b no **c** yes, it is $f^{-1} : x \mapsto \sqrt{x + 4}$

9



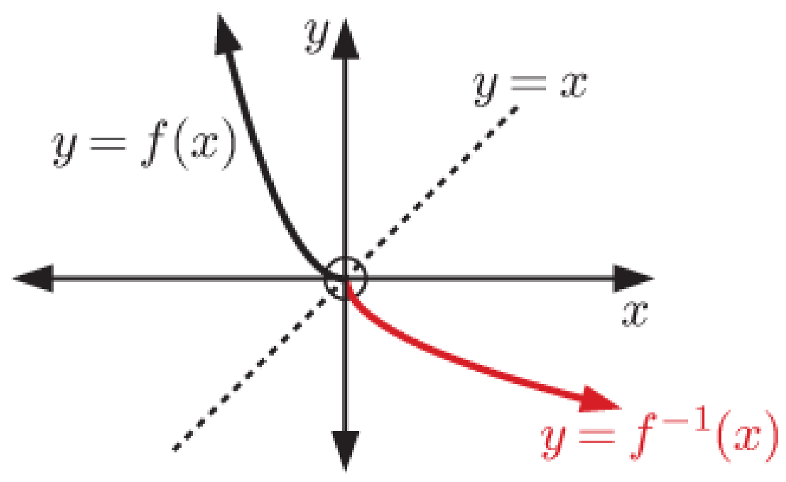
10 f is $y = \dfrac{1}{x},\ x \neq 0$ $\therefore$ f^{-1} is $x = \dfrac{1}{y}$

$\therefore$ $y = \dfrac{1}{x}$

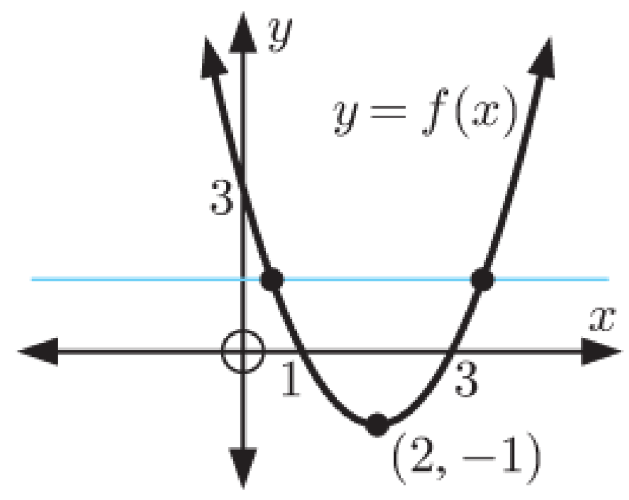
$\therefore$ $f = f^{-1}$
$\therefore$ f is self-inverse.

11 **a** The inverse function must also be a function and must therefore satisfy the vertical line test, which it can only do if the original function satisfies the horizontal line test.
b **i** is the only one.
c **i** Domain is $\{x \mid x \geqslant 1\}$ or $\{x \mid x \leqslant 1\}$
ii Domain is $\{x \mid x \geqslant 1\}$ or $\{x \mid x \leqslant -2\}$

12 **a** $f^{-1}(x) = -\sqrt{x}$ **b**



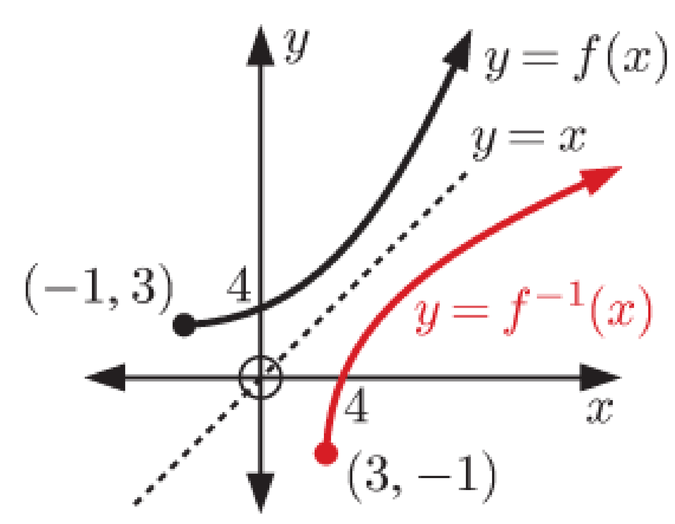
13 **a**



Every vertical line cuts the graph once. So, it is a function.
A horizontal line above the vertex cuts the graph **twice**. So, it does not have an inverse.

b For $x \geqslant 2$, all horizontal lines cut the graph at most once.
$\therefore$ $g(x)$ has an inverse.
Hint: Inverse is $x = y^2 - 4y + 3$ for $y \geqslant 2$
c g: Domain is $\{x \mid x \geqslant 2\}$, Range is $\{y \mid y \geqslant -1\}$
g^{-1}: Domain is $\{x \mid x \geqslant -1\}$, Range is $\{y \mid y \geqslant 2\}$
d **Hint:** Find $gg^{-1}(x)$ and $g^{-1}g(x)$ and show that they both equal x.

14 **a** $f^{-1}(x) = \sqrt{x - 3} - 1,\ x \geqslant 3$
b



c f:
Domain is $\{x \mid x \geqslant -1\}$
Range is $\{y \mid y \geqslant 3\}$
f^{-1}:
Domain is $\{x \mid x \geqslant 3\}$
Range is $\{y \mid y \geqslant -1\}$

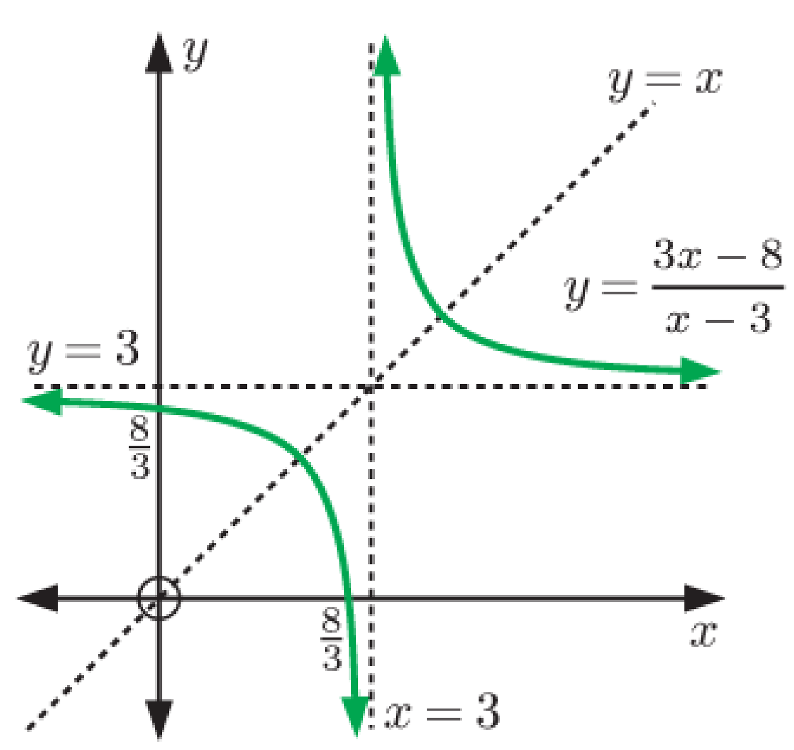
15 **a** $f^{-1}(x) = 3 - \sqrt{13 - x}$
b f: Domain is $\{x \mid x \leqslant 3\}$, Range is $\{y \mid y \leqslant 13\}$
f^{-1}: Domain is $\{x \mid x \leqslant 13\}$, Range is $\{y \mid y \leqslant 3\}$

16 **a** $k = \frac{5}{2}$
b **i** $f^{-1}(x) = \dfrac{5 - \sqrt{2x + 13}}{2}$
ii Domain is $\{x \mid x \geqslant -\frac{13}{2}\}$, Range is $\{y \mid y \leqslant \frac{5}{2}\}$

17 **a** $g^{-1}(x) = 8 - 2x$ **b** $x = 10$
c $f^{-1}(-3) - g^{-1}(6) = -4 - (-4) = 0$ **d** $x = 3$

18 **a** $8x - 6$ **b** $k = 10$
c $(f^{-1} \circ g^{-1})(x) = \dfrac{x + 3}{8}$ and $(g \circ f)^{-1}(x) = \dfrac{x + 3}{8}$

19 **a**



$y = \dfrac{3x - 8}{x - 3}$ is symmetrical about $y = x$
$\therefore$ f is a self-inverse function.

b $f^{-1}(x) = \dfrac{3x-8}{x-3}$ and $f(x) = \dfrac{3x-8}{x-3}$

$\therefore\ f = f^{-1}\quad \therefore\ f$ is a self-inverse function

20 $d = -a$ **21** **a** $\mathrm{B}(f(x), x)$

22 **a** Domain is $\{x \mid x \geqslant 0\}$

b No, as $f(x)$ does not pass the horizontal line test.

c **i** $g^{-1}(x) = (3 - \sqrt{x+8})^2$

ii g: Domain is $\{x \mid 0 \leqslant x \leqslant 9\}$
Range is $\{y \mid -8 \leqslant y \leqslant 1\}$

g^{-1}: Domain is $\{x \mid -8 \leqslant x \leqslant 1\}$
Range is $\{y \mid 0 \leqslant y \leqslant 9\}$

d **i** $h^{-1}(x) = (3 + \sqrt{x+8})^2$

ii h: Domain is $\{x \mid x \geqslant 9\}$
Range is $\{y \mid y \geqslant -8\}$

h^{-1}: Domain is $\{x \mid x \geqslant -8\}$
Range is $\{y \mid y \geqslant 9\}$

iii $x = -8$

REVIEW SET 15A

1 **a** **i** Domain is $\{x \mid x \in \mathbb{R}\}$ **ii** Range is $\{y \mid y > -4\}$
iii yes, it is a function

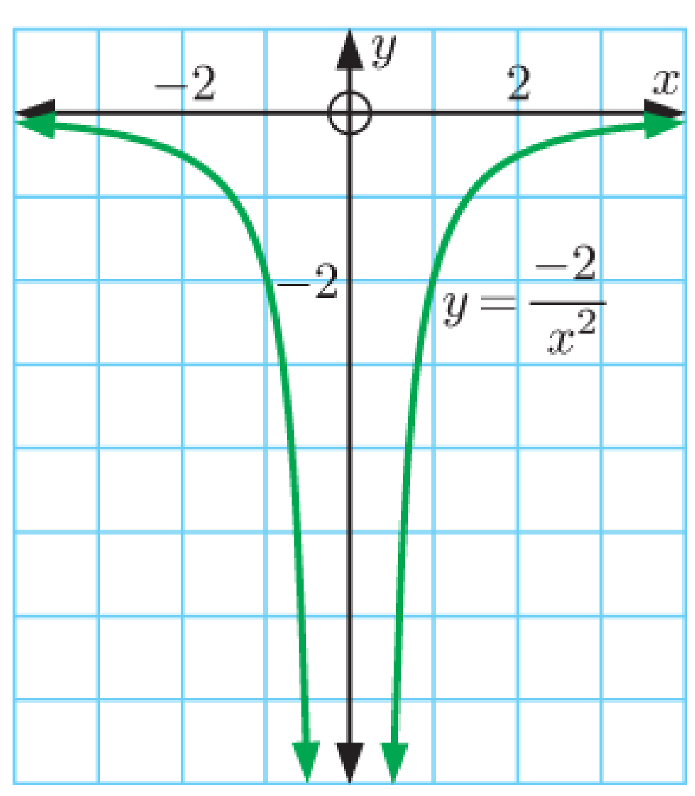
b **i** Domain is $\{x \mid x \in \mathbb{R}\}$
ii Range is $\{y \mid y \leqslant -1 \text{ or } y \geqslant 1\}$
iii no, not a function

c **i** Domain is $\{x \mid x \in \mathbb{R}\}$
ii Range is $\{y \mid -5 \leqslant y \leqslant 5\}$ **iii** yes, it is a function

2 **a** 0 **b** -15 **c** $-\frac{5}{4}$ **3** $a = -6,\ b = 13$

4 **a** $x = 0$

b



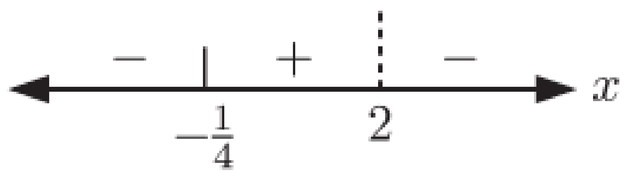
c Domain is $\{x \mid x \neq 0\}$
Range is $\{y \mid y < 0\}$

5 **a** $f(-3) = (-3)^2 = 9,\quad g(-\frac{4}{3}) = 1 - 6(-\frac{4}{3}) = 9$
b $x = -4$

6 **a** Domain is $\{x \mid x \geqslant -4\}$, Range is $\{y \mid y \geqslant 0\}$
b Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid y \leqslant 1\}$
c Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid y \geqslant -\frac{1}{8}\}$

7 **a** $y = -\dfrac{20}{x}$ **b** $y = \dfrac{60}{x}$

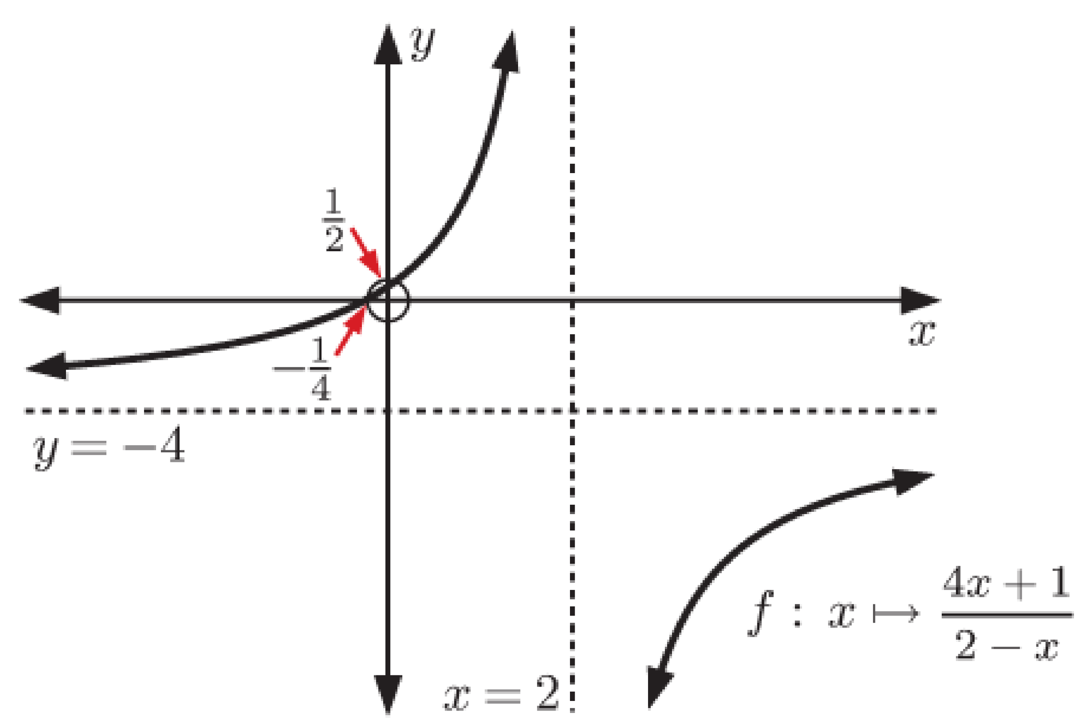
8 **a** vertical asymptote $x = 2$, horizontal asymptote $y = -4$
b Domain is $\{x \mid x \neq 2\}$, Range is $\{y \mid y \neq -4\}$
c



as $x \to 2^-$, $f(x) \to \infty$ as $x \to -\infty$, $f(x) \to -4^+$
as $x \to 2^+$, $f(x) \to -\infty$ as $x \to \infty$, $f(x) \to -4^-$

d x-intercept $-\frac{1}{4}$, y-intercept $\frac{1}{2}$

e



9 **a** $6x - 3$ **b** $x = 1$

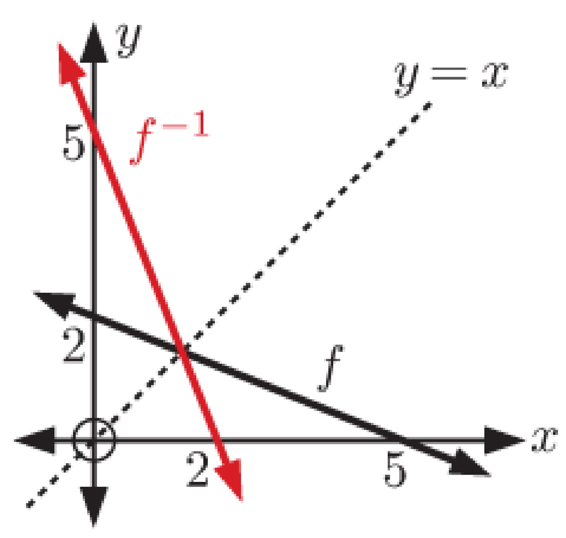
10 **a** $1 - 2\sqrt{x}$ **b** $\sqrt{1-2x}$ **c** 3

11 **a** $(f \circ g)(x) = \sqrt{x^2 - 1}$
Domain is $\{x \mid x \leqslant -1 \text{ or } x \geqslant 1\}$
Range is $\{y \mid y \geqslant 0\}$

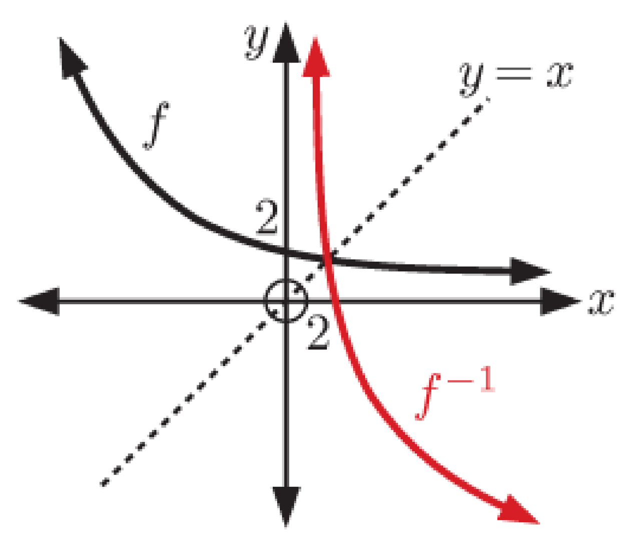
b $(g \circ f)(x) = x - 1$
Domain is $\{x \mid x \geqslant -2\}$, Range is $\{y \mid y \geqslant -3\}$

12 $a = 1,\ b = -1$

13 **a**

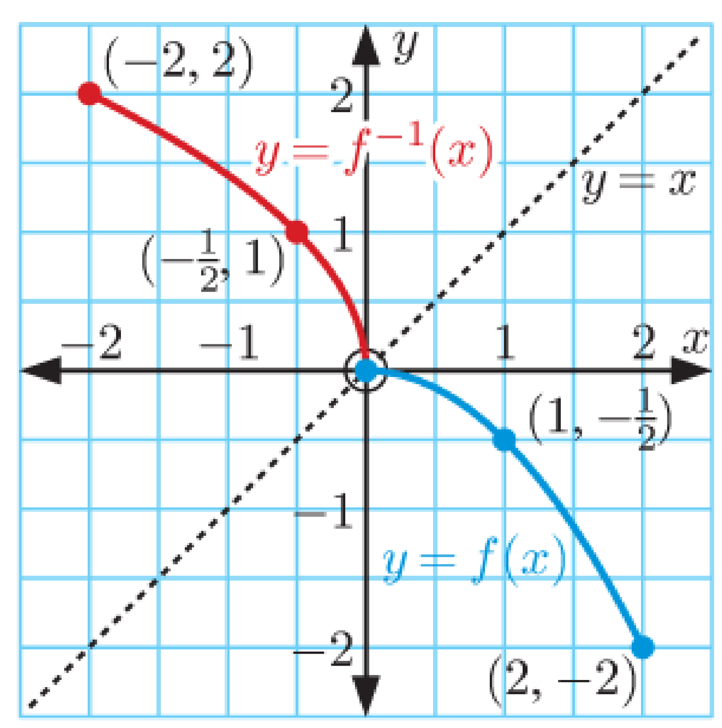


b



14 **a** $f^{-1}(x) = \dfrac{x-2}{4}$ **b** $f^{-1}(x) = \dfrac{3-4x}{5}$

15 **a**

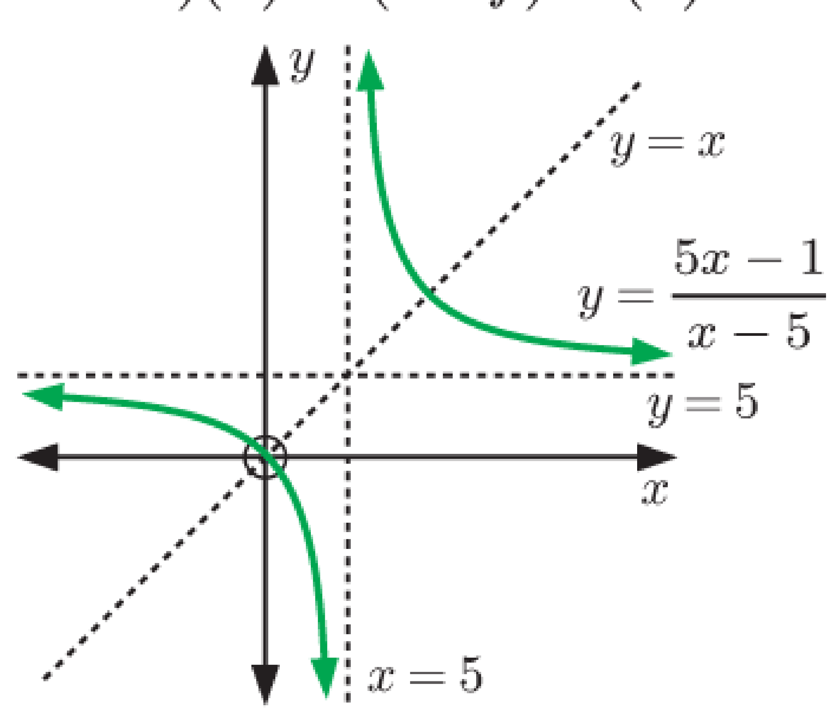


b Range is $\{y \mid 0 \leqslant y \leqslant 2\}$

c **i** $x = \sqrt{3}$ **ii** $x = -\frac{1}{2}$

16 $(f^{-1} \circ h^{-1})(x) = (h \circ f)^{-1}(x) = x - 2$

17 **a**



$y = \dfrac{5x-1}{x-5}$ is symmetrical about $y = x$

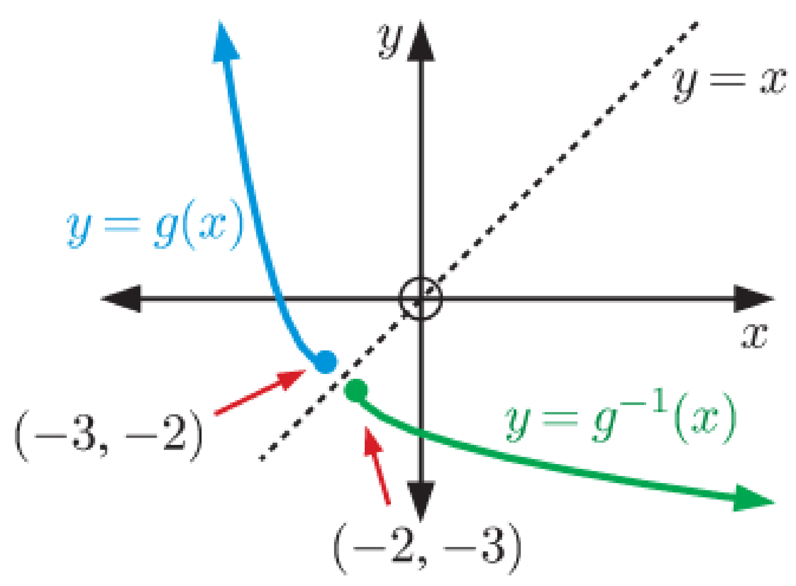
$\therefore\ f$ is a self-inverse function.

b $f^{-1}(x) = \dfrac{5x-1}{x-5}$ and $f(x) = \dfrac{5x-1}{x-5}$

$\therefore\ f = f^{-1}\quad \therefore\ f$ is a self-inverse function.

18 **a** -4 **b** 1

19 **a, d**



b Any horizontal line cuts the graph at most once.
c $g^{-1}(x) = -3 - \sqrt{x+2},\ x \geqslant -2$
e Range of g is $\{y \mid y \geqslant -2\}$
f Domain of g^{-1} is $\{x \mid x \geqslant -2\}$,
Range of g^{-1} is $\{y \mid y \leqslant -3\}$

REVIEW SET 15B

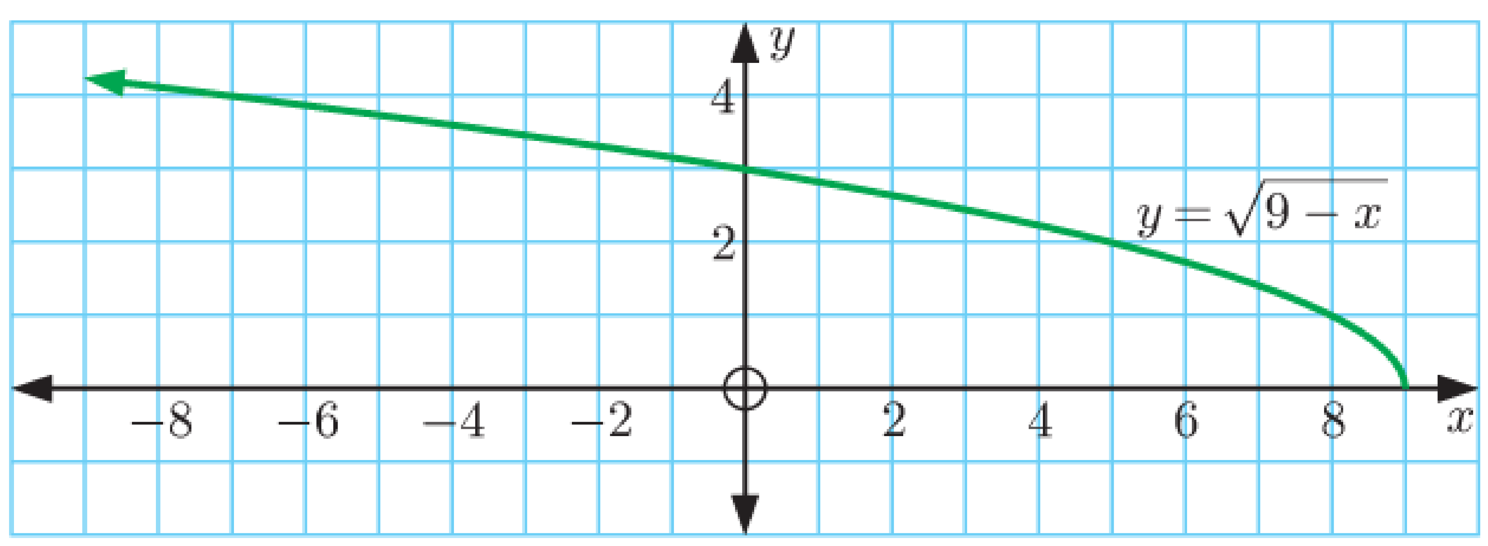
1 **a** Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid y \geqslant -4\}$
b Domain is $\{x \mid x \geqslant -2\}$, Range is $\{y \mid 1 \leqslant y < 3\}$
c Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid y = -1,\ 1, \text{ or } 2\}$

2 **a** $x^2 - x - 2$ **b** $16x^2 - 12x$

3 **a** is a function **b** is not a function

4 **a** Domain is $\{x \mid x \neq \frac{1}{2}\}$, Range is $\{y \mid y \neq 10\}$
b Domain is $\{x \mid x \geqslant -7\}$, Range is $\{y \mid y \geqslant 0\}$

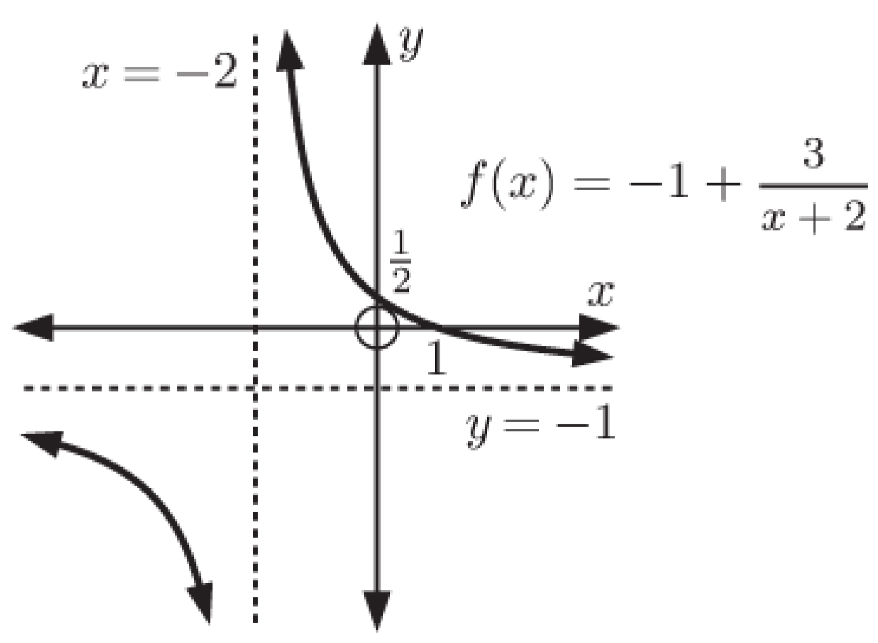
5 **a**



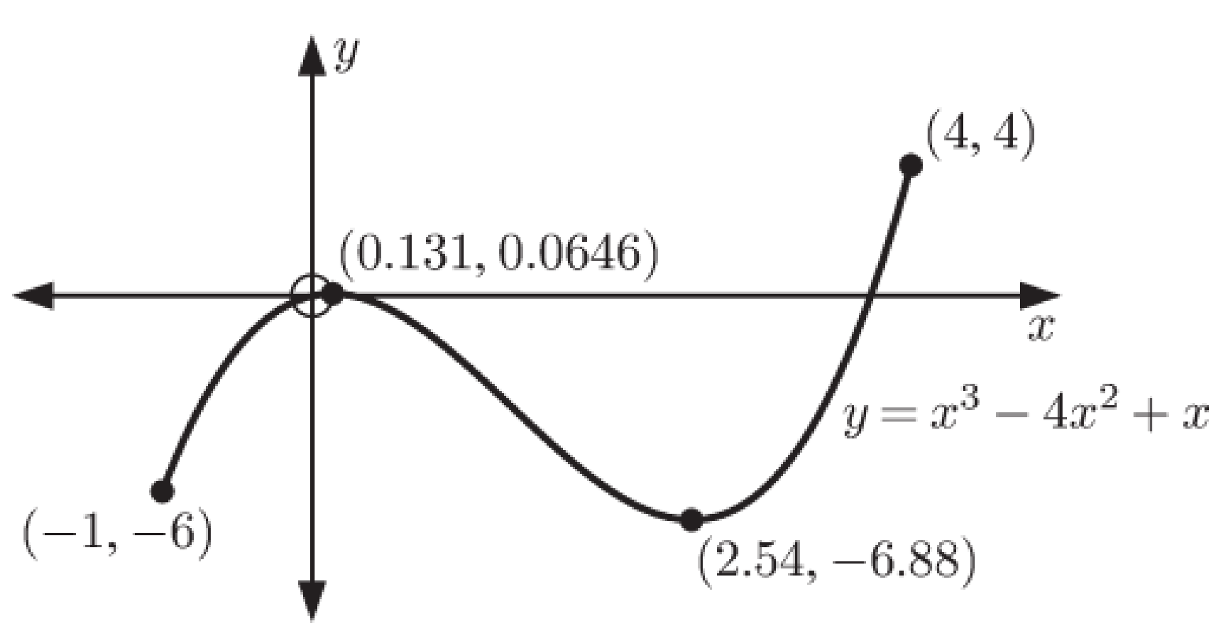
b It is a function.
c Domain is $\{x \mid x \leqslant 9\}$, Range is $\{y \mid y \geqslant 0\}$

6 $a = -2$ **7** $a = 1,\ b = -6,\ c = 5$

8 **a** vertical asymptote is $x = -2$,
horizontal asymptote is $y = -1$
b Domain is $\{x \mid x \neq -2\}$, Range is $\{y \mid y \neq -1\}$
c x-intercept is 1, y-intercept is $\frac{1}{2}$
d as $x \to -2^-$, $f(x) \to -\infty$ as $x \to -\infty$, $f(x) \to -1^-$
as $x \to -2^+$, $f(x) \to \infty$ as $x \to \infty$, $f(x) \to -1^+$
e



9



Range is $\{y \mid -6.88 \leqslant y \leqslant 4\}$

10 **a** $-4x^2 + 4x + 2$ **b** $5 - 2x^2$ **c** 2

11 $(f \circ g)(x) = \dfrac{1}{(x^2 - 4x + 3)^2}$

Domain is $\{x \mid x \neq 3,\ x \neq 1\}$, Range is $\{y \mid y > 0\}$

12 **a** **i** $6x^2 - 3x + 5$ **ii** $18x^2 + 57x + 45$
b $x = -\frac{5}{11}$

13 **a** $D \circ S = 4.9t^2$
This is the distance travelled by the object after t seconds.
b $(D \circ S)(5) = 122.5$ m
The object has travelled 122.5 m after 5 seconds.

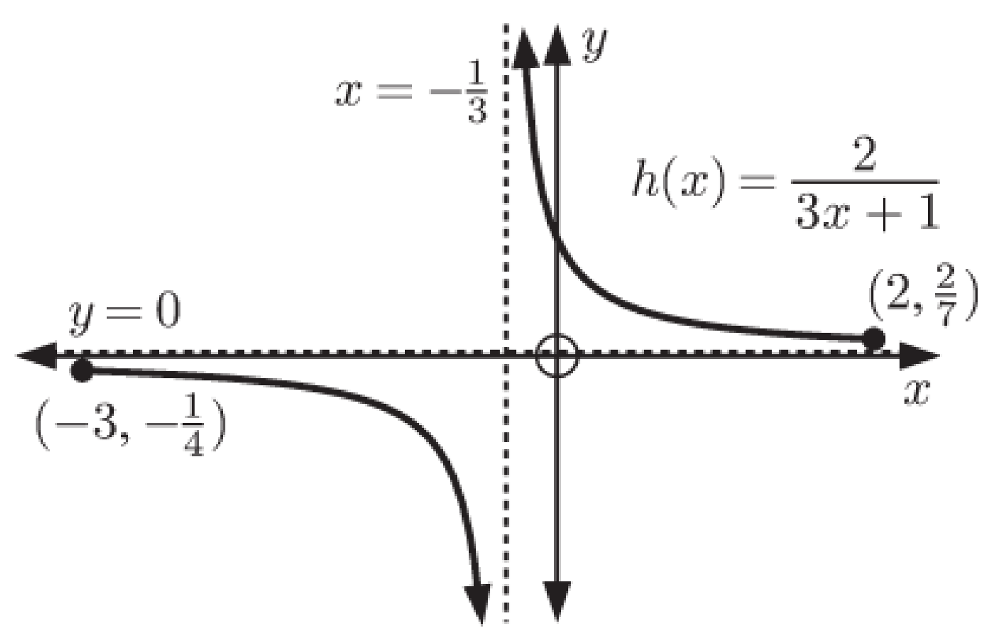
14 $f^{-1}(x) = \dfrac{2x + 29}{5}$

15 $(f^{-1} \circ h^{-1})(x) = (h \circ f)^{-1}(x) = \dfrac{4x + 6}{15}$

16 **a** $(g \circ f)(x) = \dfrac{2}{3x + 1}$ **b** $x = -\frac{1}{2}$

c **i** vertical asymptote $x = -\frac{1}{3}$,
horizontal asymptote $y = 0$

ii



iii Range is $\{y \mid y \leqslant -\frac{1}{4} \text{ or } y \geqslant \frac{2}{7}\}$

17 16

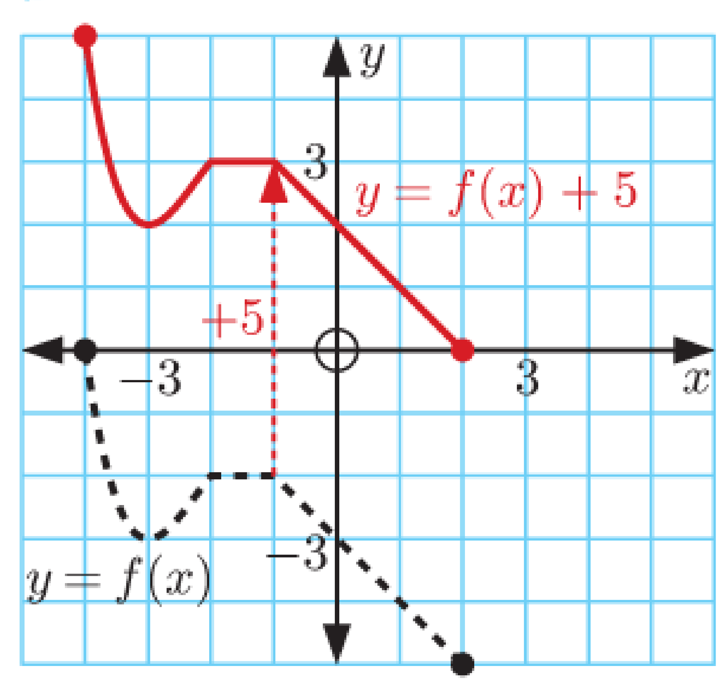
18 **a** $a = 2,\ b = -1$
b Domain is $\{x \mid x \neq 2\}$, Range is $\{y \mid y \neq -1\}$

19 **a** $\dfrac{3x}{x-2}$ **b** $\dfrac{2x+1}{x-1}$

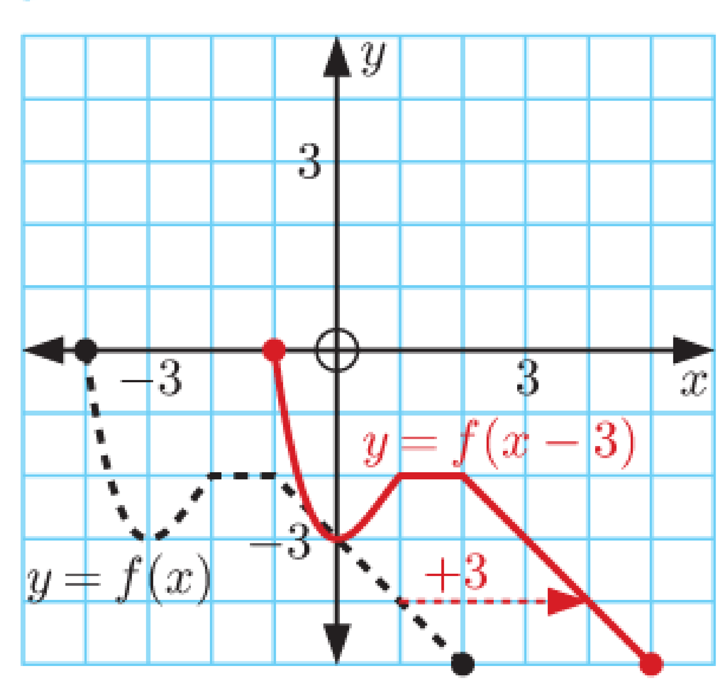
20 **a** $f^{-1}(x) = \sqrt{4 - \sqrt{x + 13}}$
Domain is $\{x \mid -13 \leqslant x \leqslant 3\}$,
Range is $\{y \mid 0 \leqslant y \leqslant 2\}$
b $g^{-1}(x) = \sqrt{4 + \sqrt{x + 13}}$
Domain is $\{x \mid x \geqslant -13\}$, Range is $\{y \mid y \geqslant 2\}$
c $h^{-1}(x) = -\sqrt{4 - \sqrt{x + 13}}$
Domain is $\{x \mid -13 \leqslant x \leqslant 3\}$,
Range is $\{y \mid -2 \leqslant y \leqslant 0\}$
d $j^{-1}(x) = -\sqrt{4 + \sqrt{x + 13}}$
Domain is $\{x \mid x \geqslant -13\}$, Range is $\{y \mid y \leqslant -2\}$

EXERCISE 16A

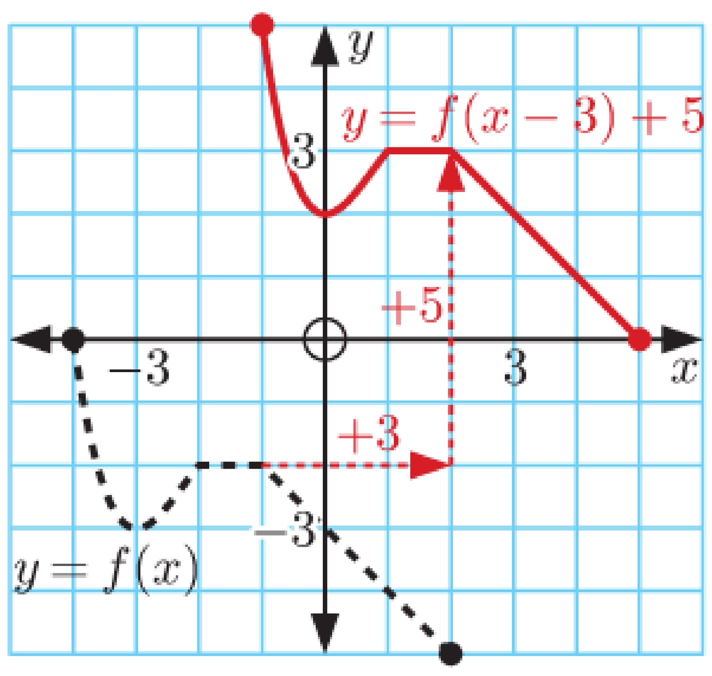
1 **a**



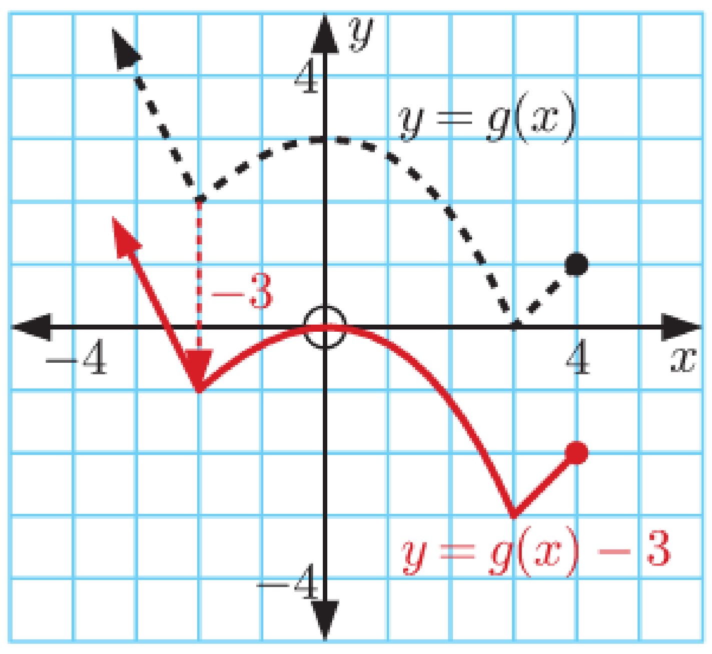
b



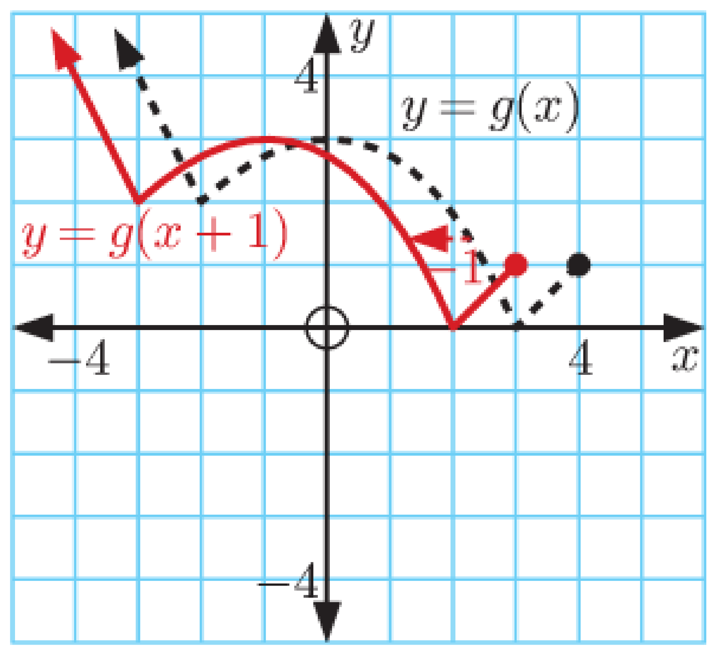
c



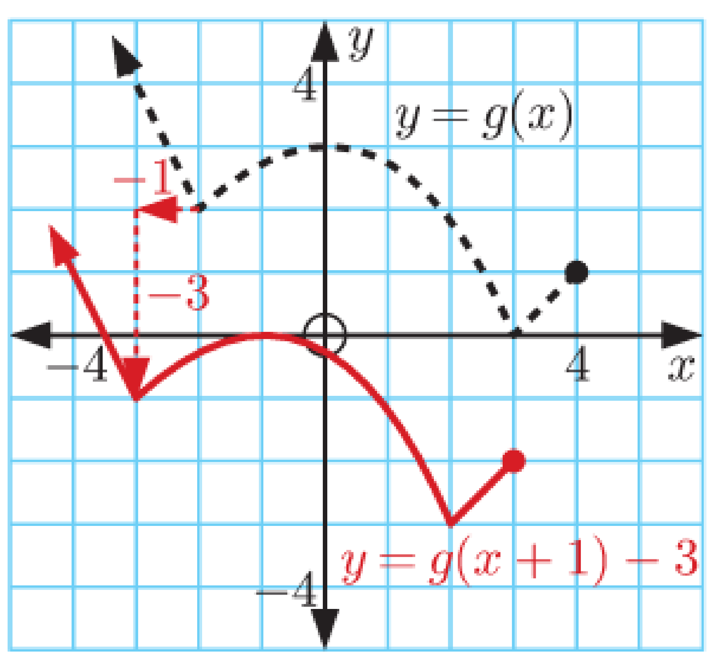
2 **a**



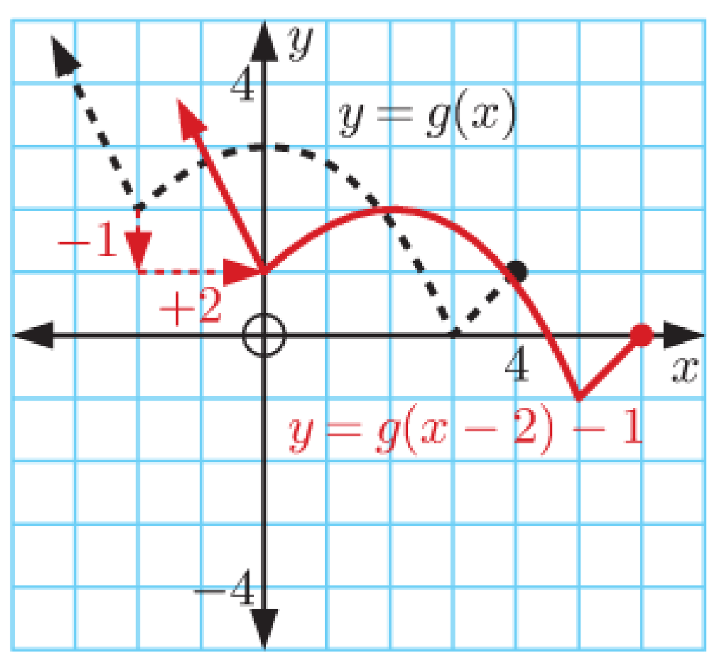
b



c



d

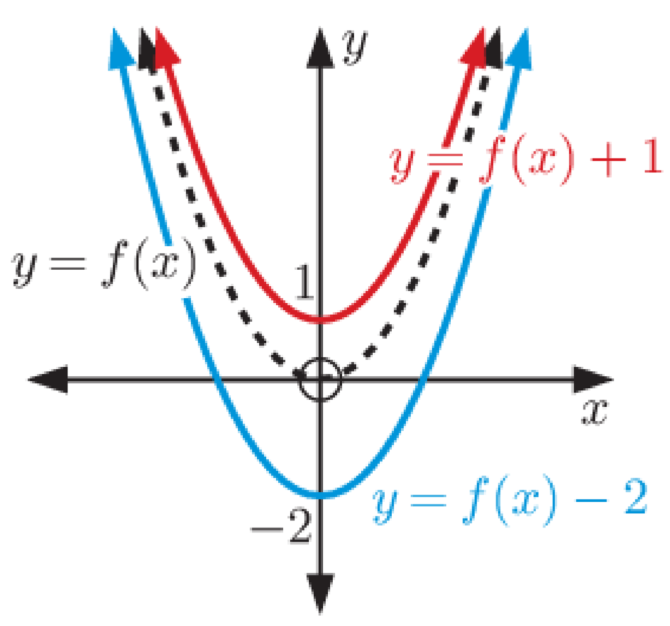


3 **a** $g(x) = f(x-4)$ **b** $g(x) = f(x+1)+3$

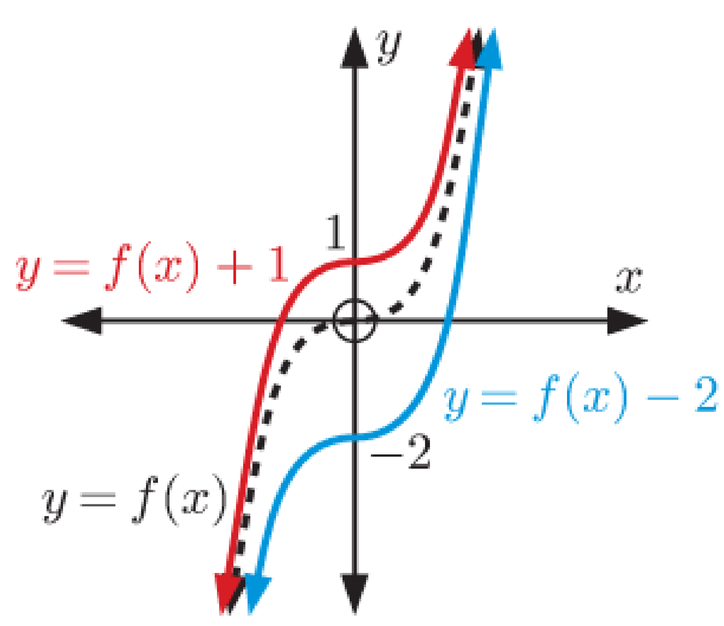
4 **a** $g(x) = 2x-1$ **b** $g(x) = 3x+2$

c $g(x) = -x^2+5x-4$ **d** $g(x) = x^2-6x+4$

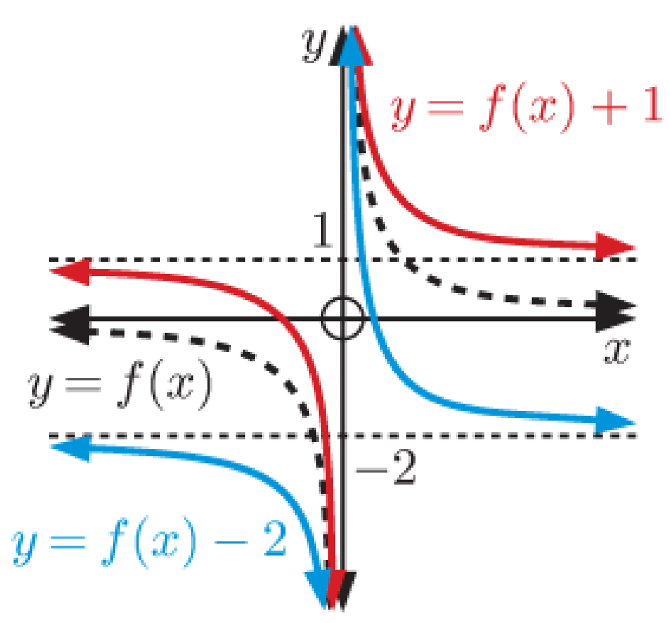
5 **a**



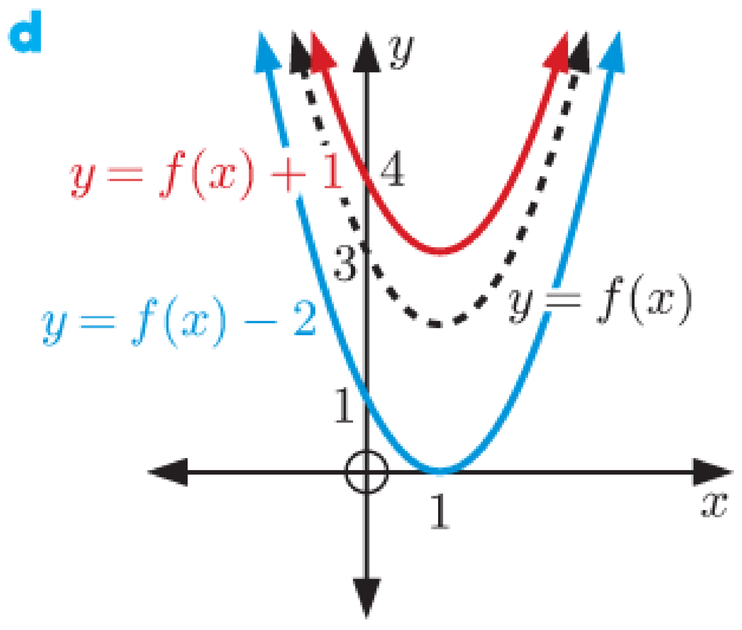
b



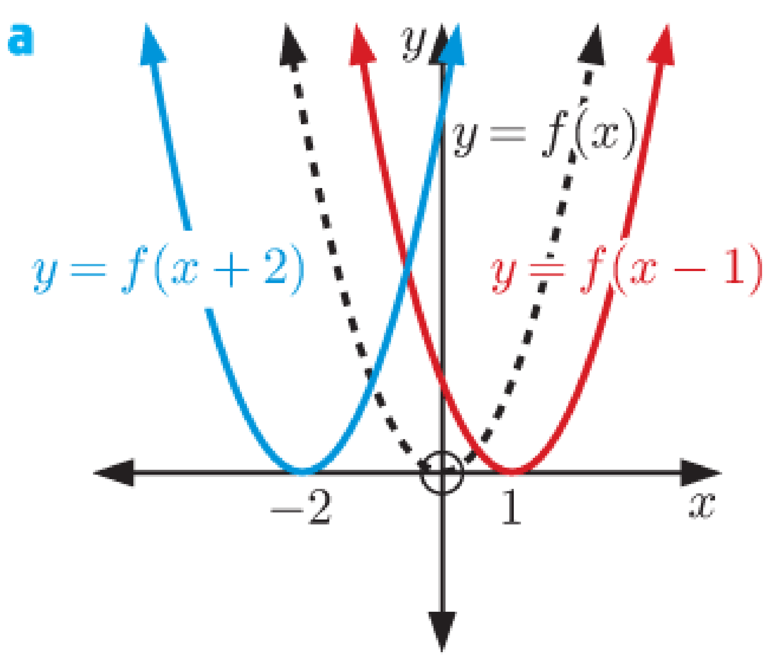
c



d



6 **a**



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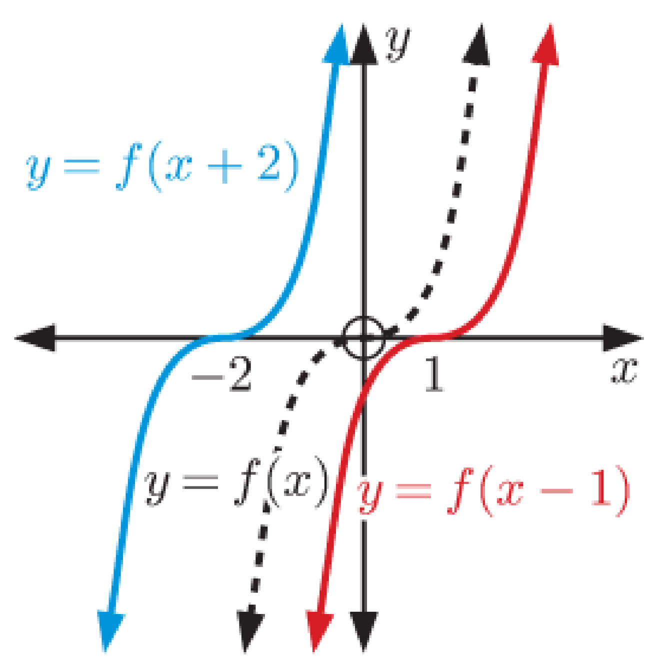
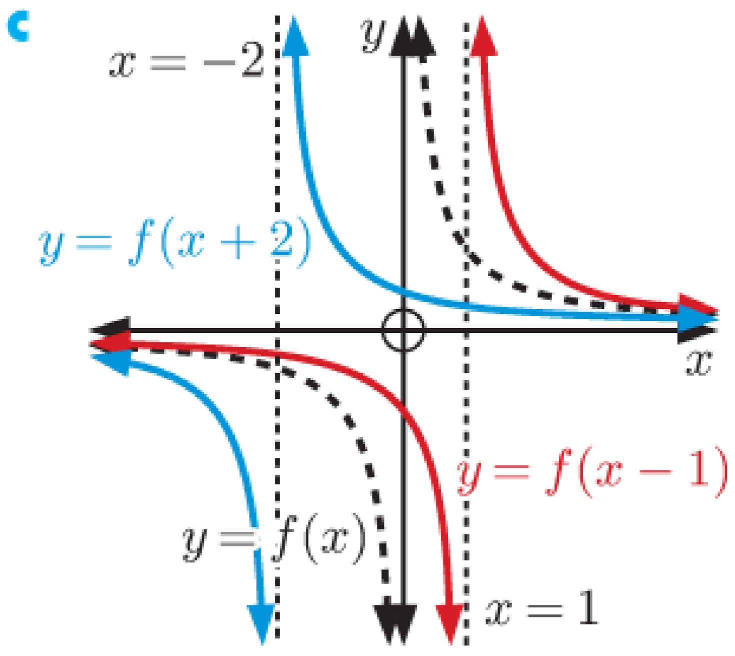
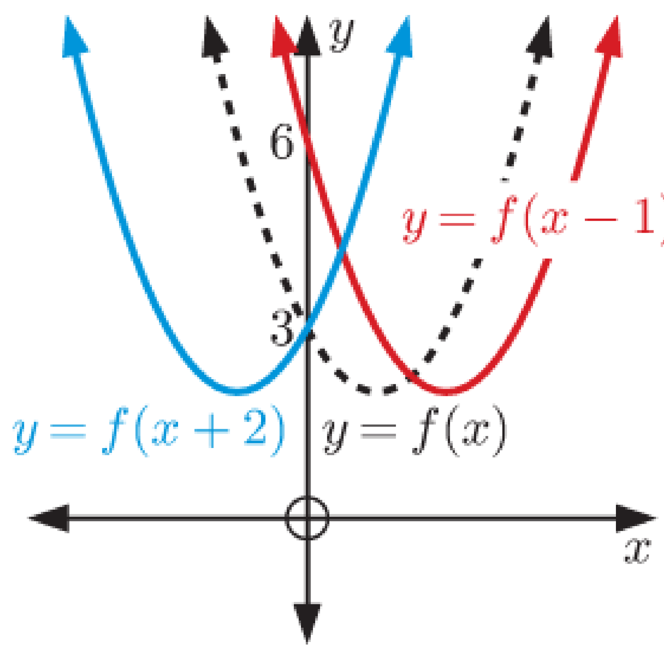


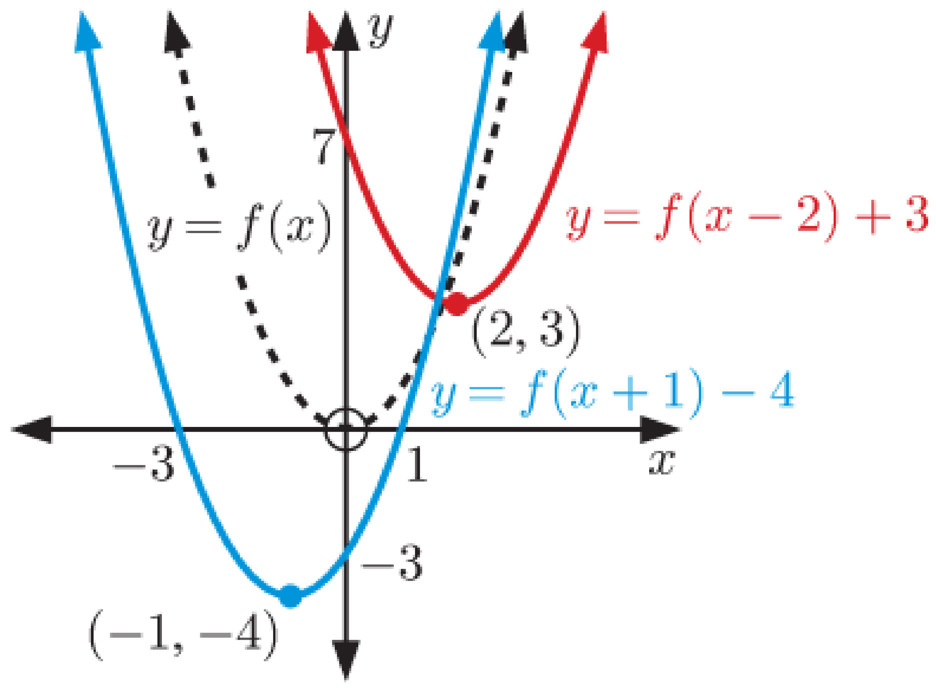
c



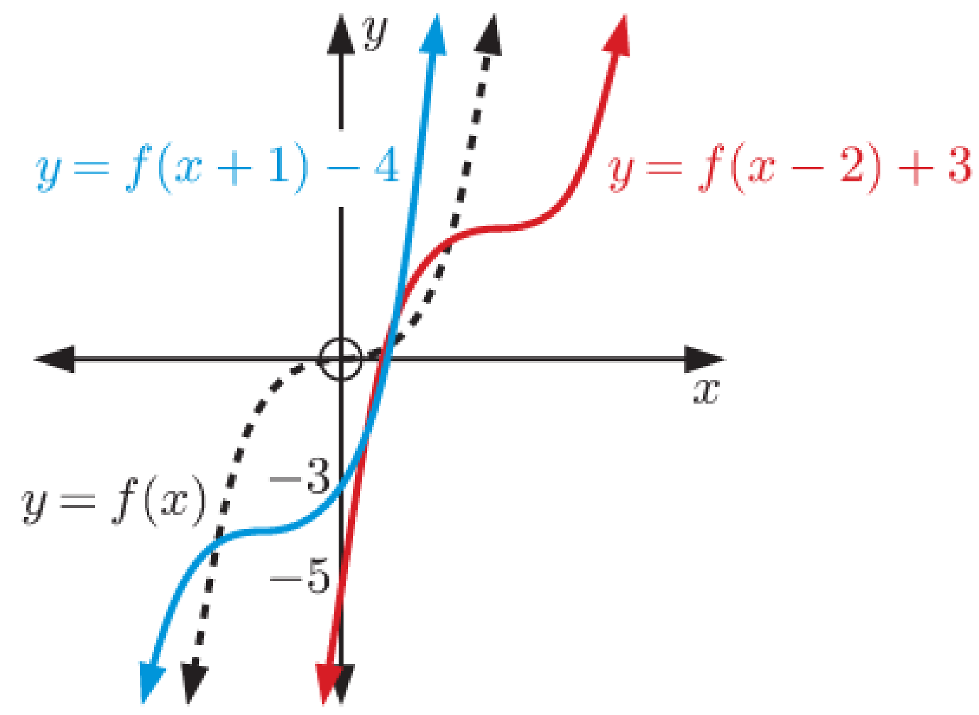
d



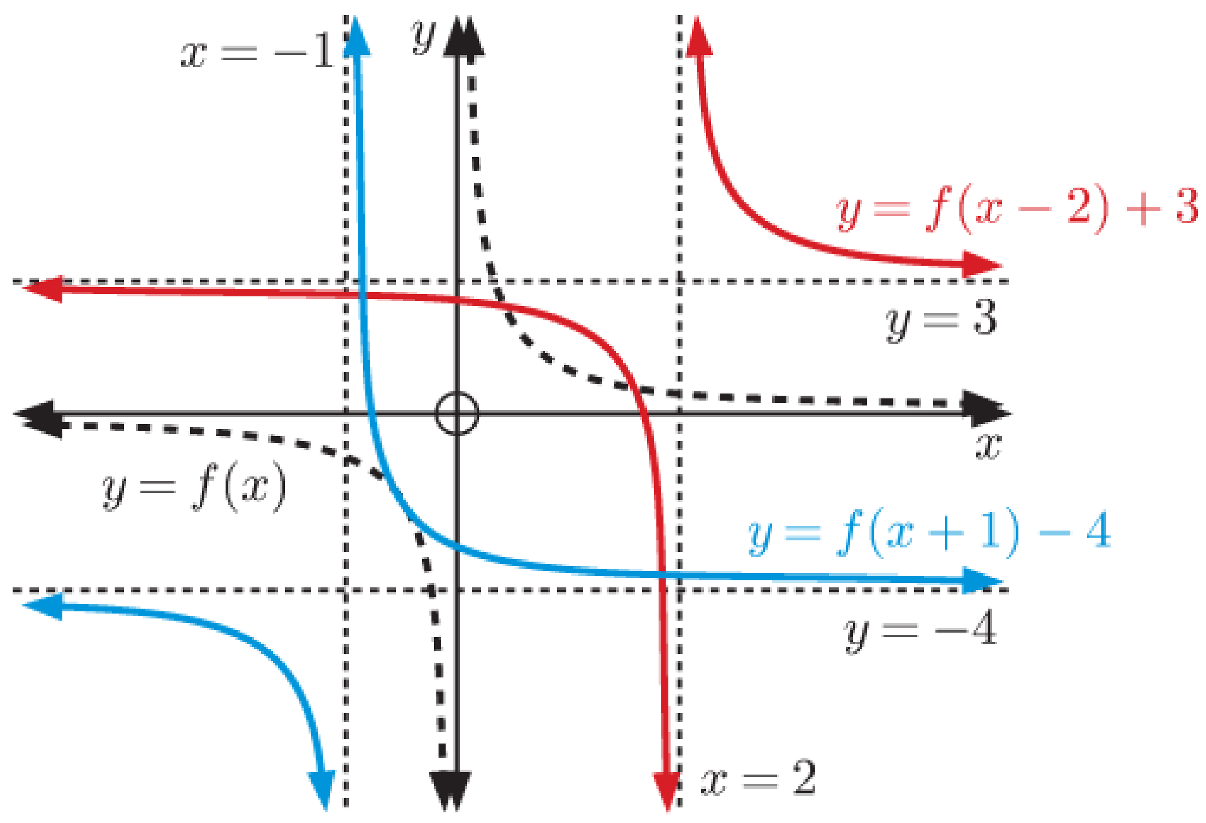
7 **a**



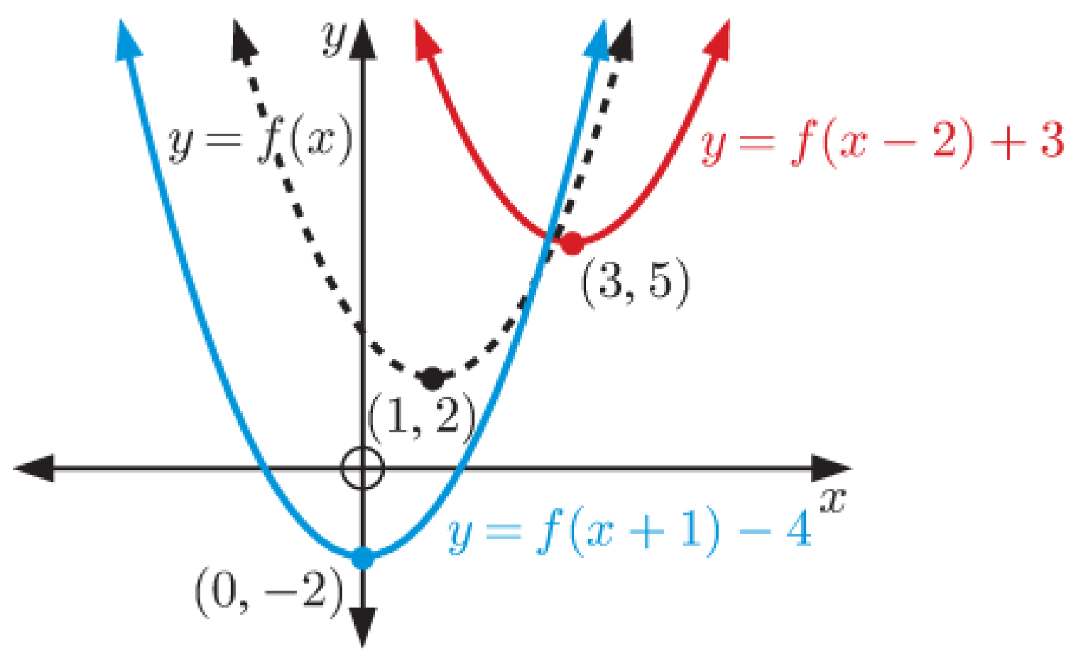
b



c



d



8 $(1, -9)$

9 **a** y-intercept is -1 **b** x-intercepts are -2 and 5

c inconclusive

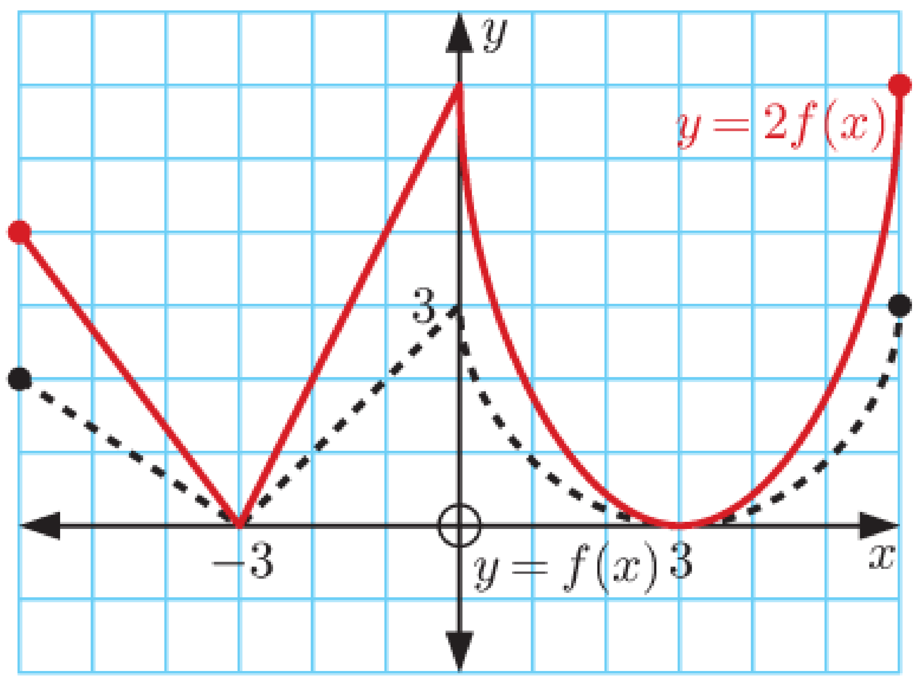
10 $g(x) = x^2 - 8x + 12$ **11** $g(x) = \dfrac{7x+15}{x+2}$

12 **a** **i** $(3, 2)$ **ii** $(0, 11)$ **iii** $(5, 6)$

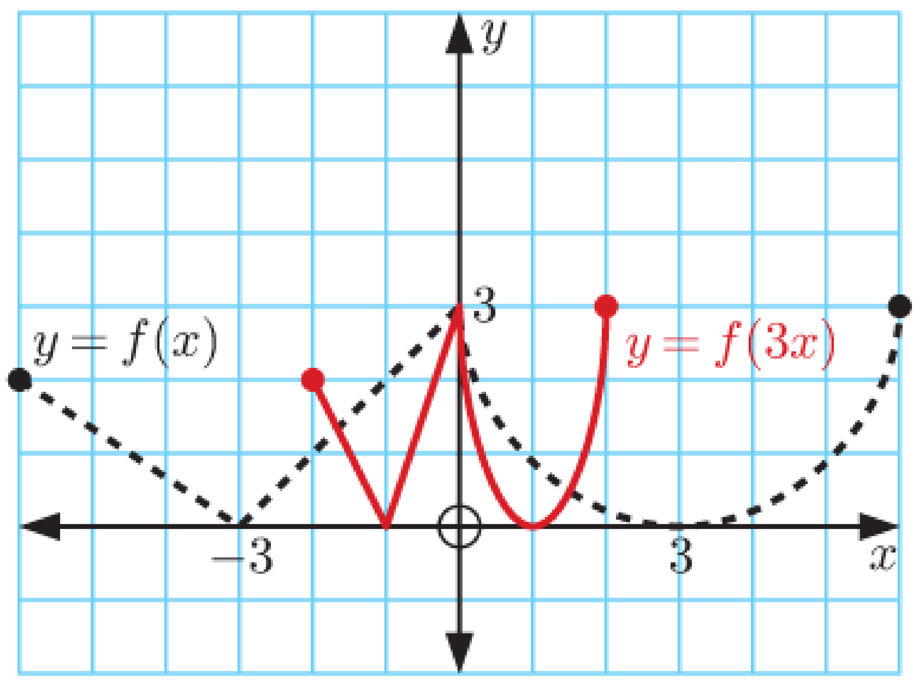
b **i** $(-2, 4)$ **ii** $(-5, 25)$ **iii** $\left(-1\frac{1}{2}, 2\frac{1}{4}\right)$

EXERCISE 16B

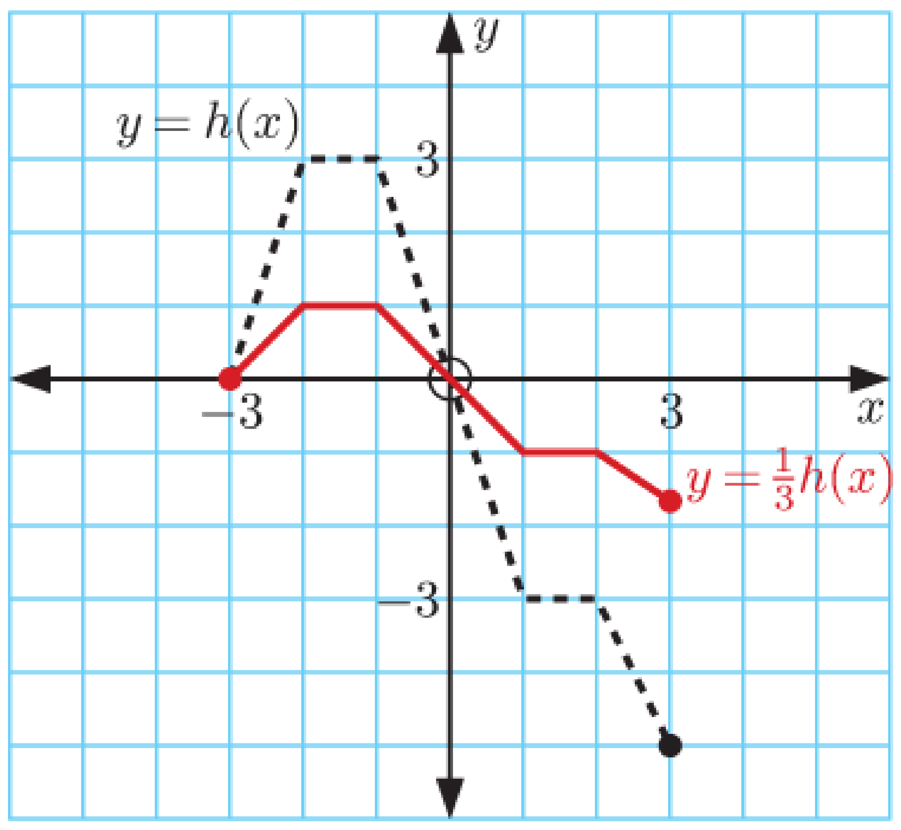
1 **a**



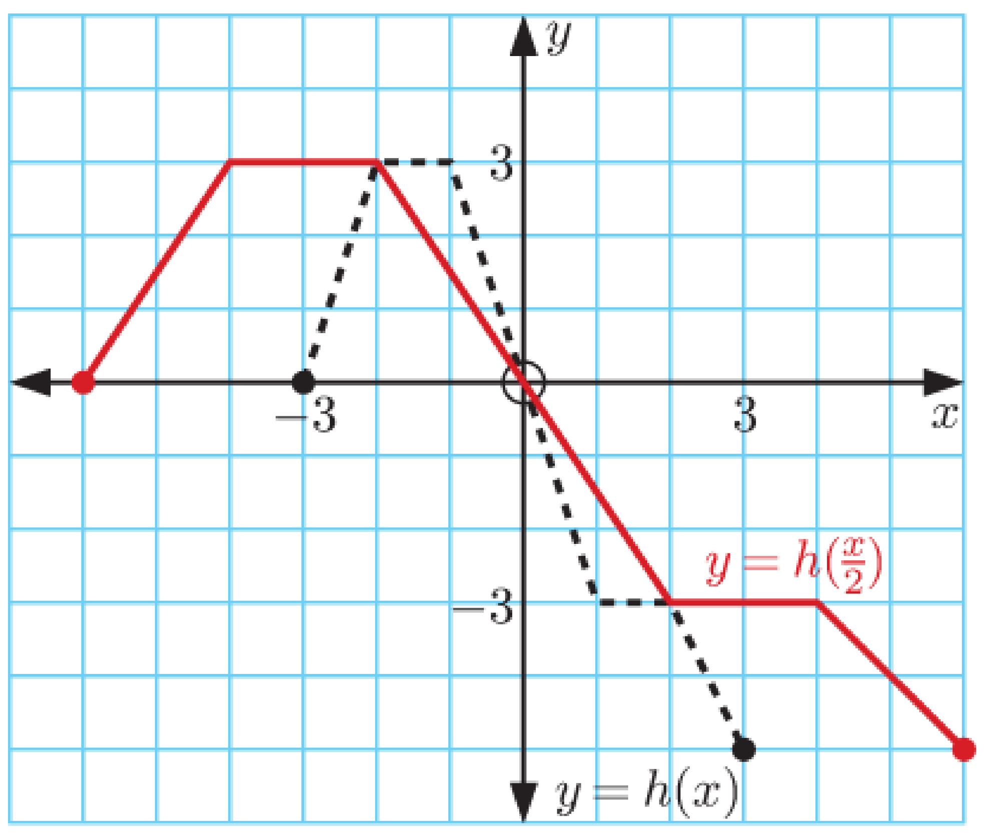
b



2 **a**

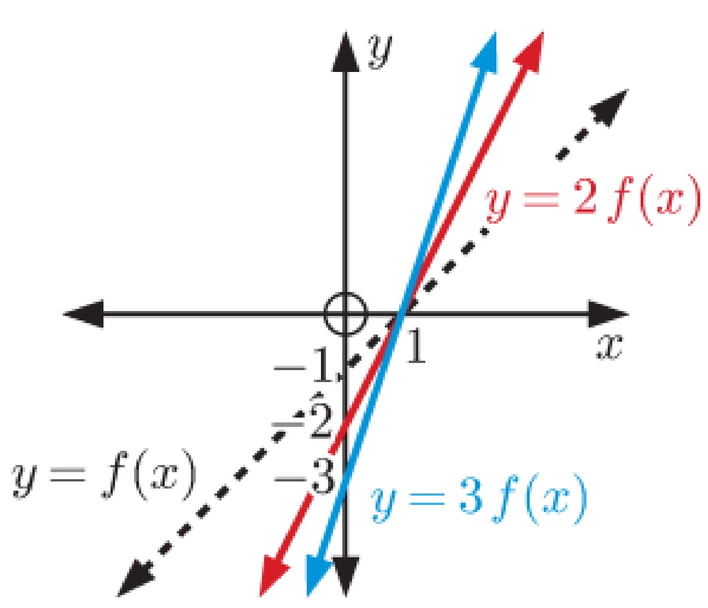


b

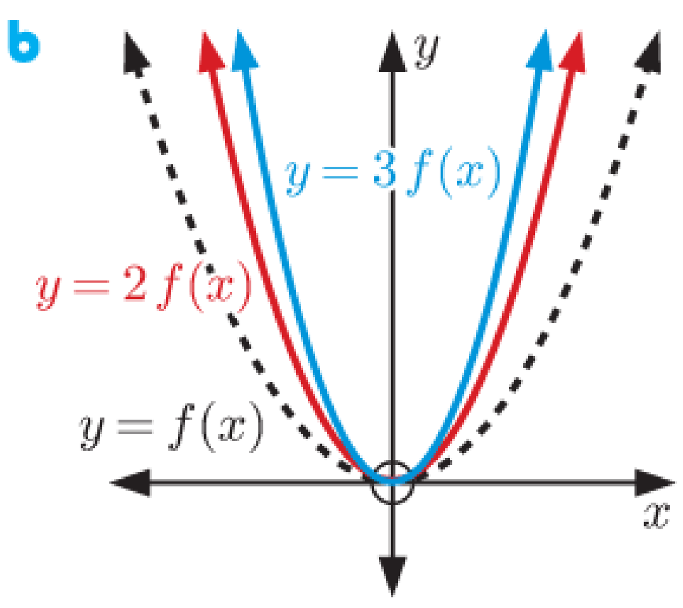


3 **a** $g(x) = 2f(x)$ **b** $g(x) = f\left(\dfrac{x}{3}\right)$ **4** *cm*

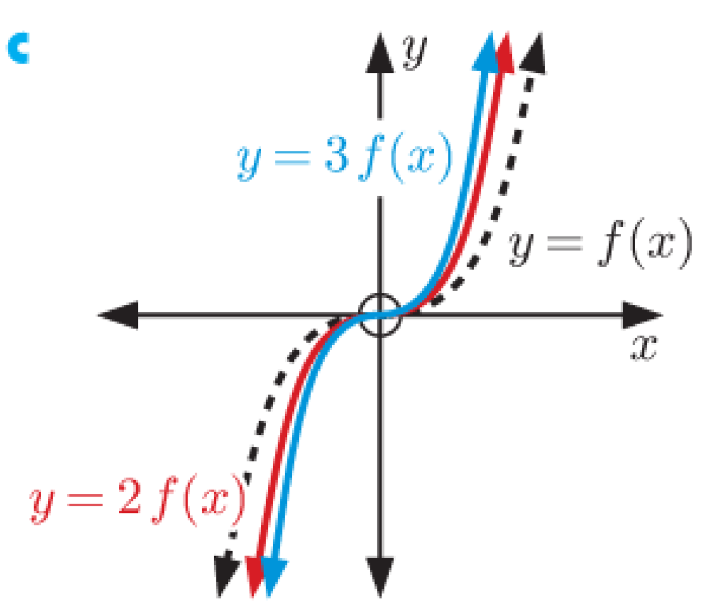
5 **a**



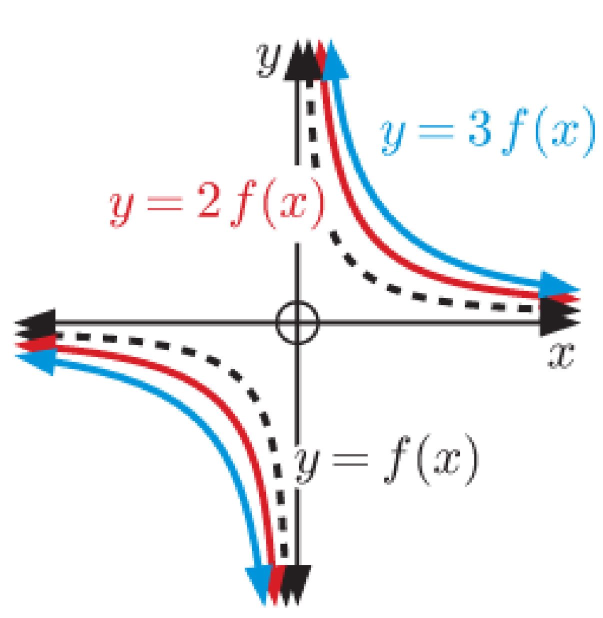
b



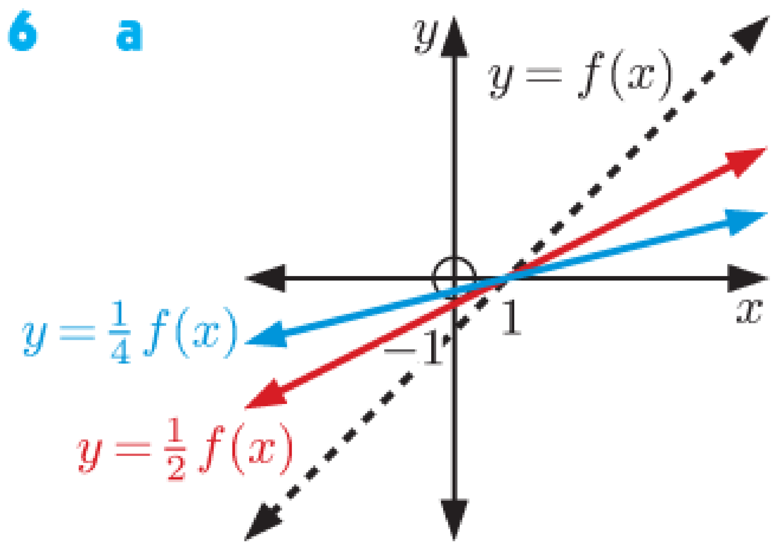
c



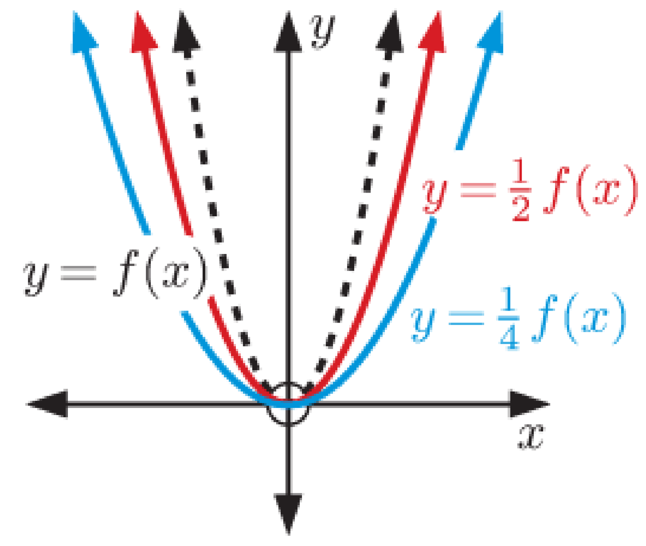
d



6 **a**



b



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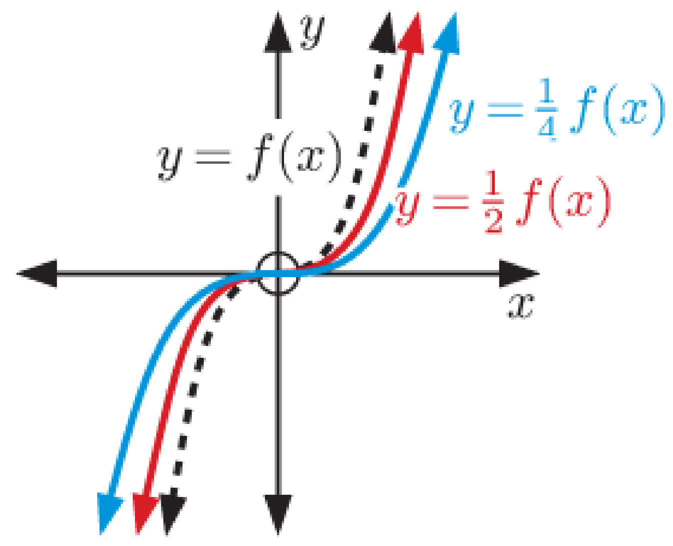
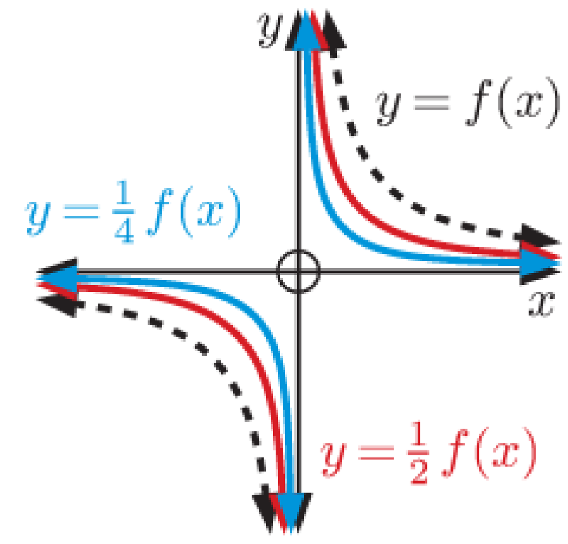
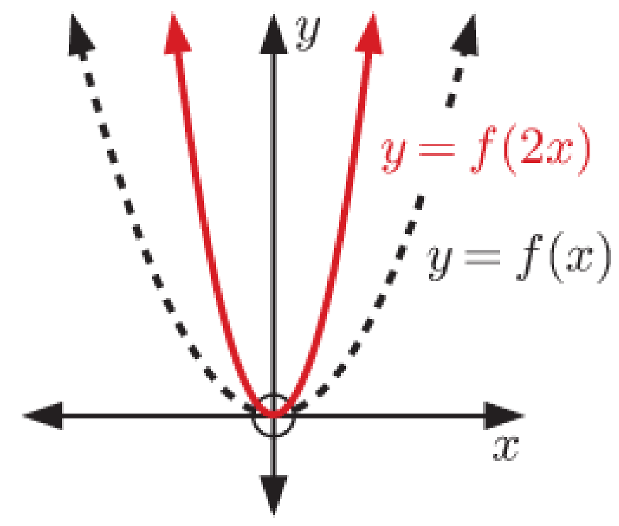


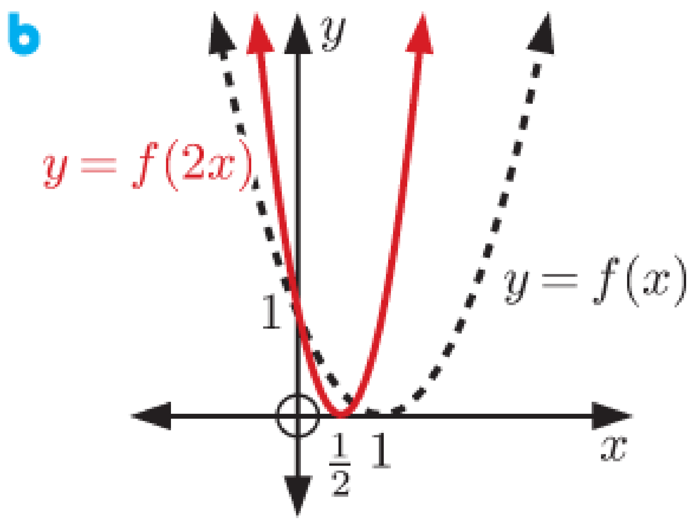
d



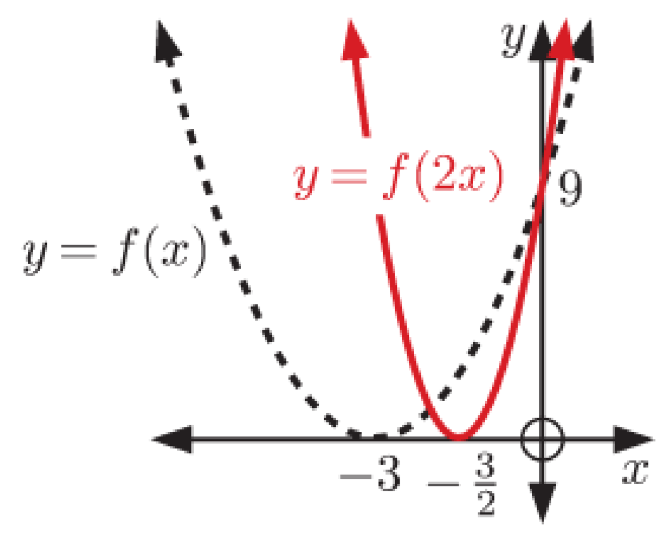
7 **a**



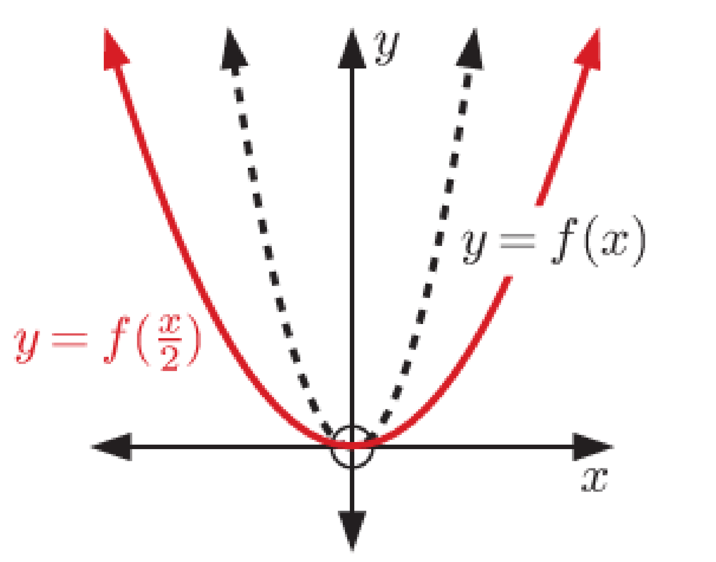
b



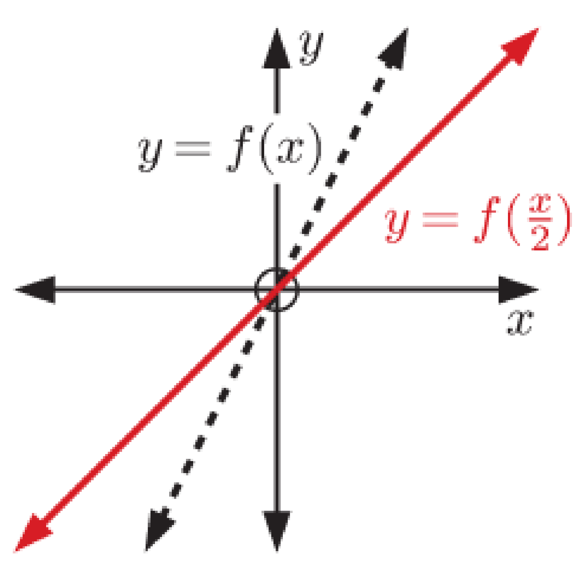
c



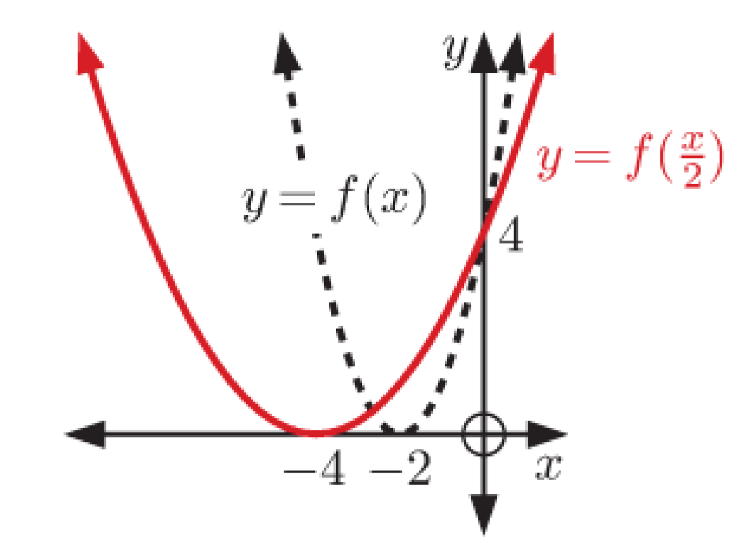
8 **a**



b



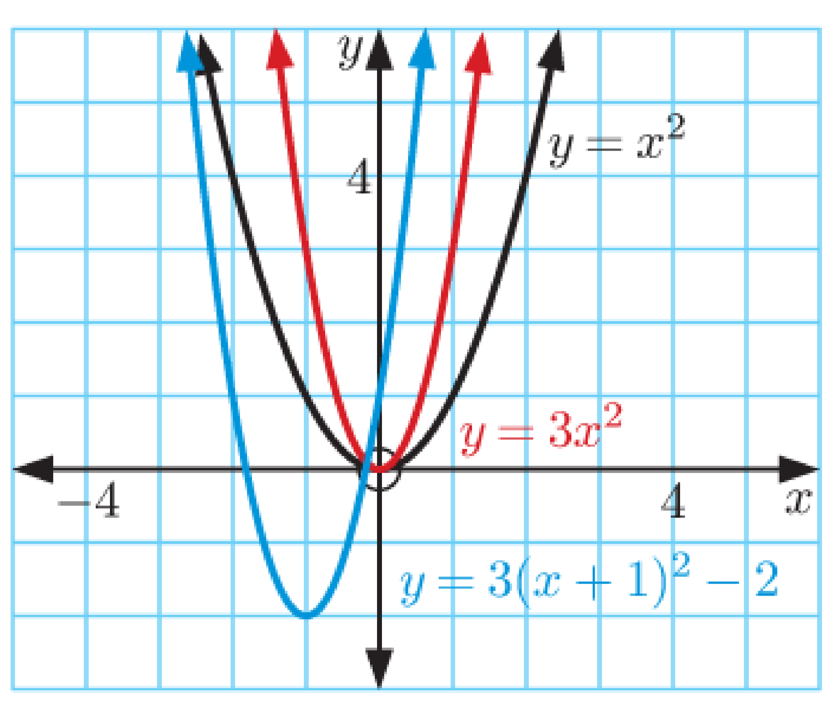
c



9 **a** (2, 25) **b** (−25, −15)

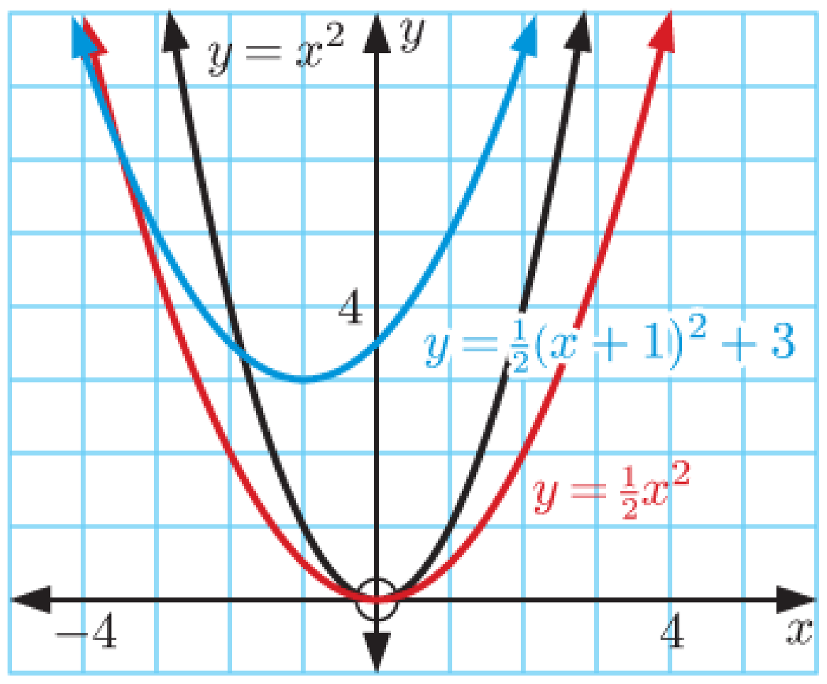
10 **a** $g(x) = 2x^2 + 4$ **b** $g(x) = 5 - x$
c $g(x) = \frac{1}{4}x^3 + 2x^2 - \frac{1}{2}$ **d** $g(x) = 8x^2 + 2x - 3$

11



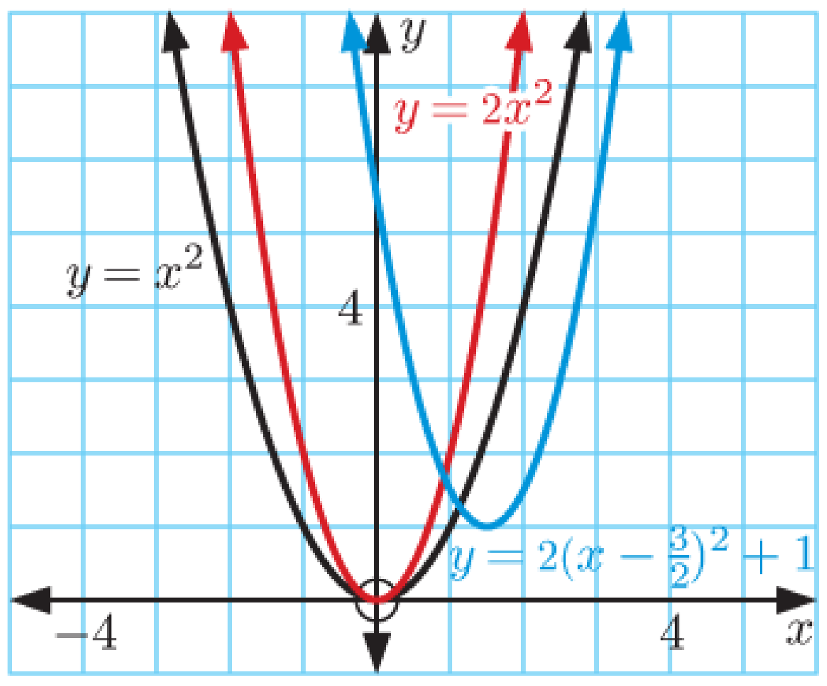
$y = x^2$ is transformed to $y = 3(x+1)^2 - 2$ by vertically stretching with scale factor 3 and then translating through $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$.

12



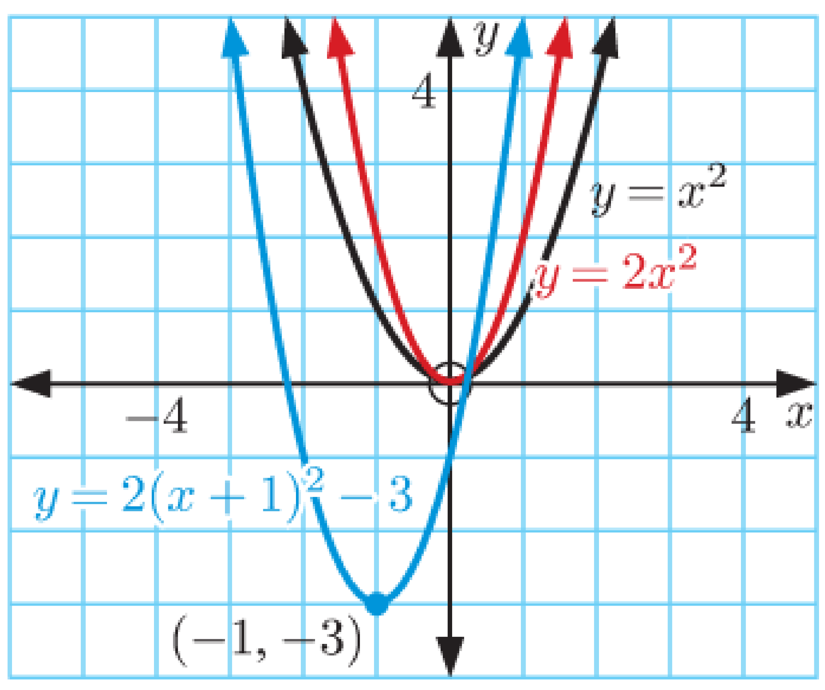
$y=x^2$ is transformed to $y=\frac{1}{2}(x+1)^2+3$ by vertically stretching with scale factor $\frac{1}{2}$ and then translating through $\begin{pmatrix}-1\\3\end{pmatrix}$.

13



$y=x^2$ is transformed to $y=2(x-\frac{3}{2})^2+1$ by vertically stretching with scale factor 2 and then translating through $\begin{pmatrix}\frac{3}{2}\\1\end{pmatrix}$.

14



$y=x^2$ is transformed to $y=2(x+1)^2-3$ by vertically stretching with scale factor 2 and then translating through $\begin{pmatrix}-1\\-3\end{pmatrix}$.

15 **a** Horizontally stretching with scale factor $\frac{1}{2}$, then vertically stretching with scale factor 3.

b **i** $(\frac{3}{2}, -15)$ **ii** $(\frac{1}{2}, 6)$ **iii** $(-1, 3)$

c **i** $(4, \frac{1}{3})$ **ii** $(-6, \frac{2}{3})$ **iii** $(-14, 1)$

16 $a=5$, $b=\sqrt{10}$

17 **a** $y=\dfrac{1}{2x}$ **b** $y=\dfrac{3}{x}$ **c** $y=\dfrac{1}{x+3}$

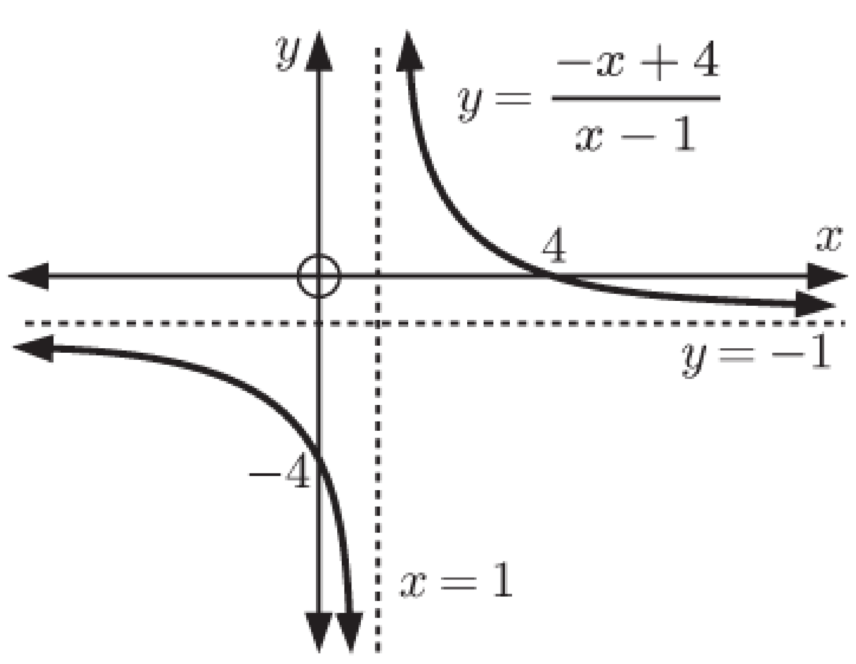
d $y=4+\dfrac{1}{x}=\dfrac{4x+1}{x}$

18 **a** $g(x)=\dfrac{3}{x-1}-1=\dfrac{-x+4}{x-1}$

b vertical asymptote $x=1$, horizontal asymptote $y=-1$

c Domain is $\{x \mid x \neq 1\}$, Range is $\{y \mid y \neq -1\}$

d

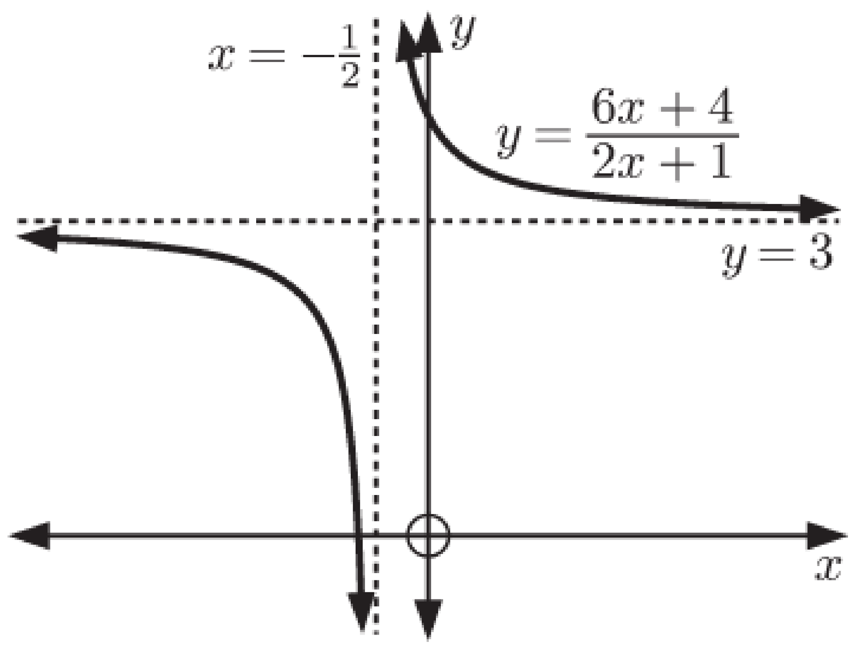


19 **a** $g(x)=\dfrac{6x+4}{2x+1}$

b vertical asymptote $x=-\frac{1}{2}$, horizontal asymptote $y=3$

c Domain is $\{x \mid x \neq -\frac{1}{2}\}$, Range is $\{y \mid y \neq 3\}$

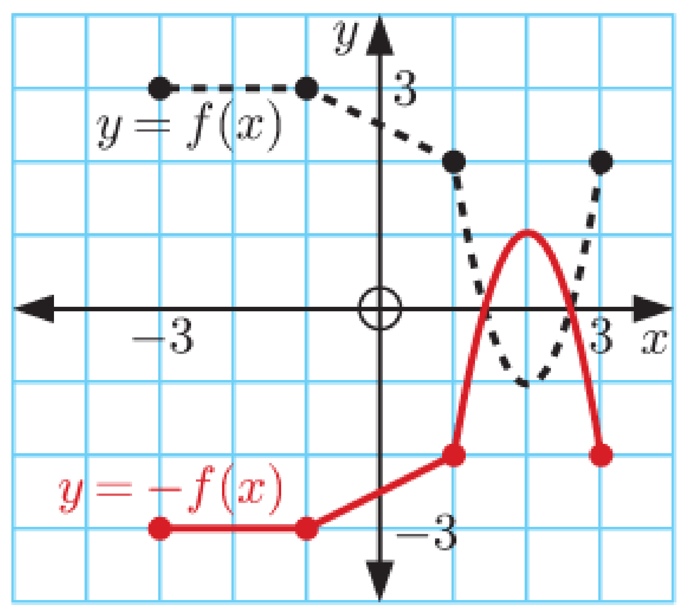
d



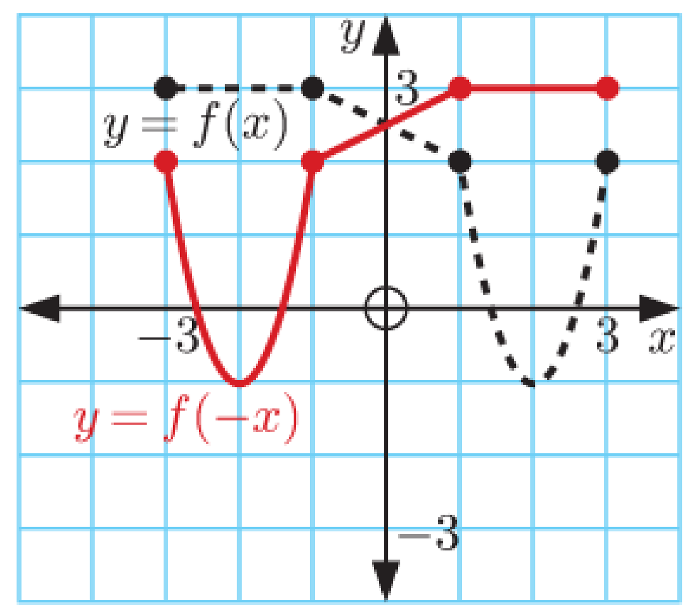
20
- A vertical stretch with scale factor 4 followed by a translation through $\begin{pmatrix}3\\33\end{pmatrix}$, *or*
- a translation through $\begin{pmatrix}3\\8\frac{1}{4}\end{pmatrix}$ followed by a vertical stretch with scale factor 4.

EXERCISE 16C

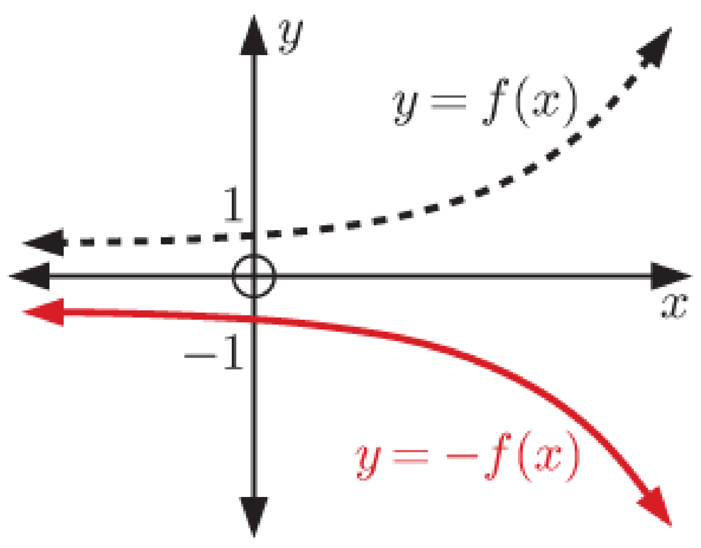
1 **a**



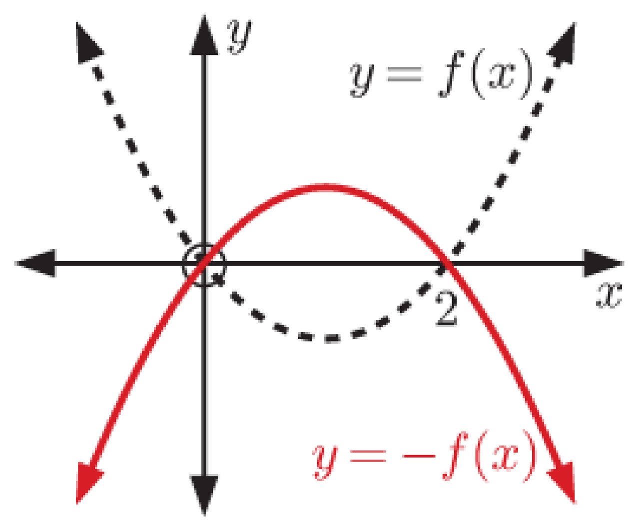
b



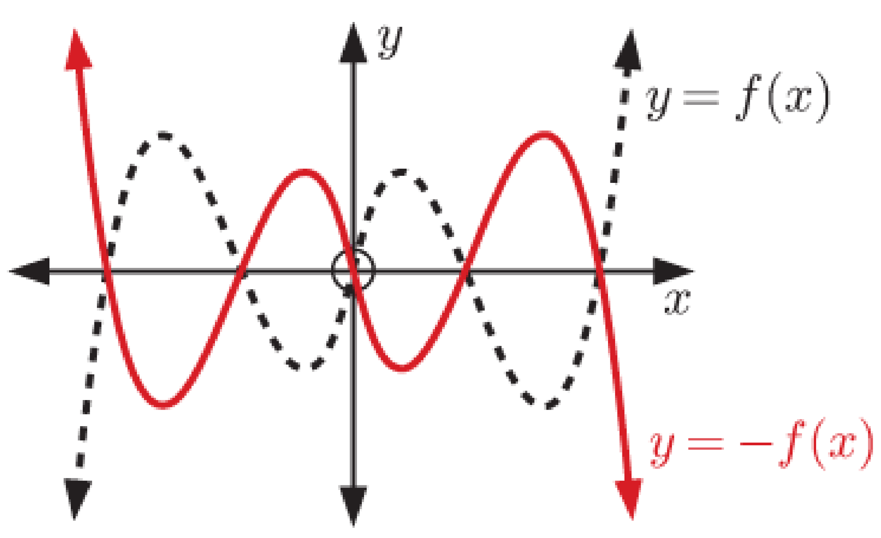
2 **a**



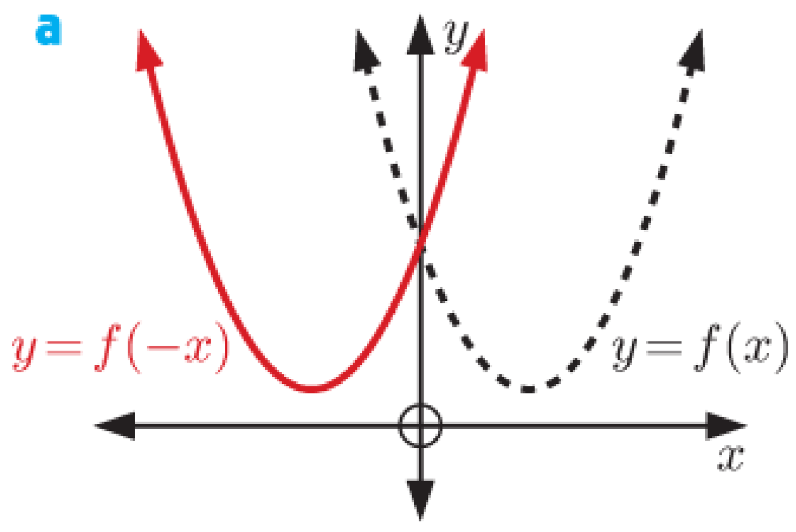
b



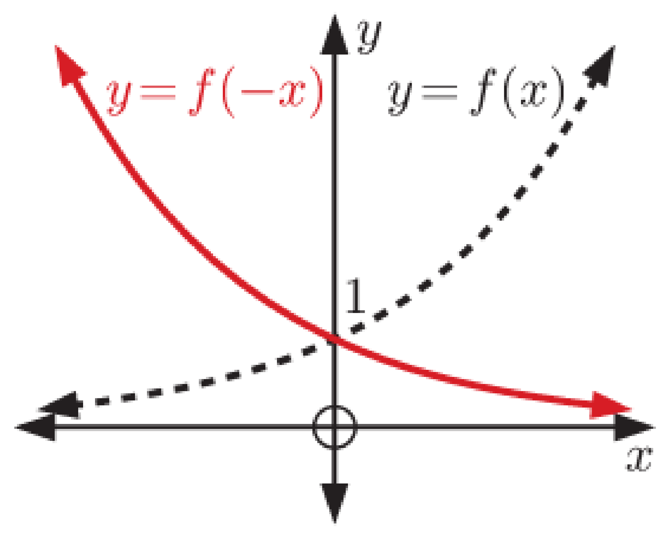
c



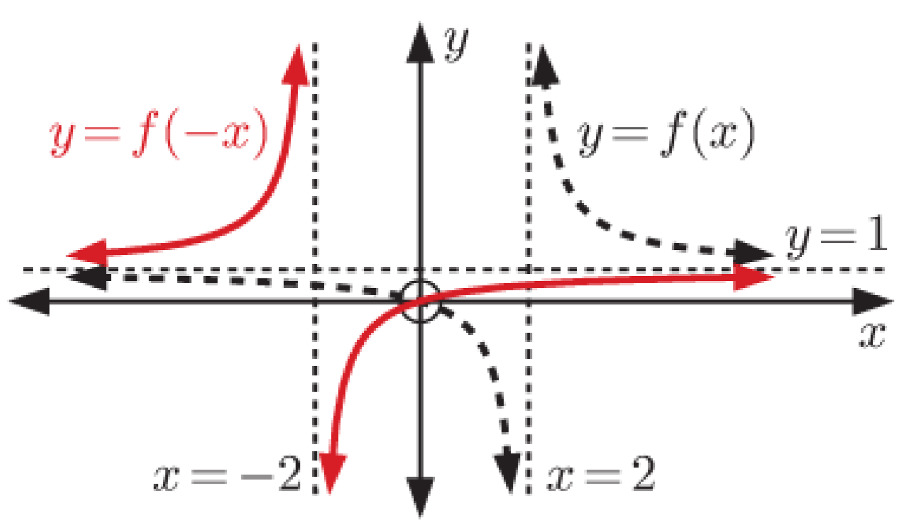
3 **a**



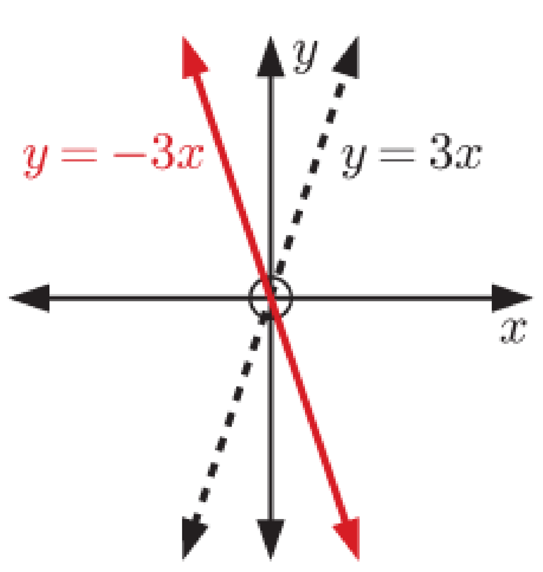
b



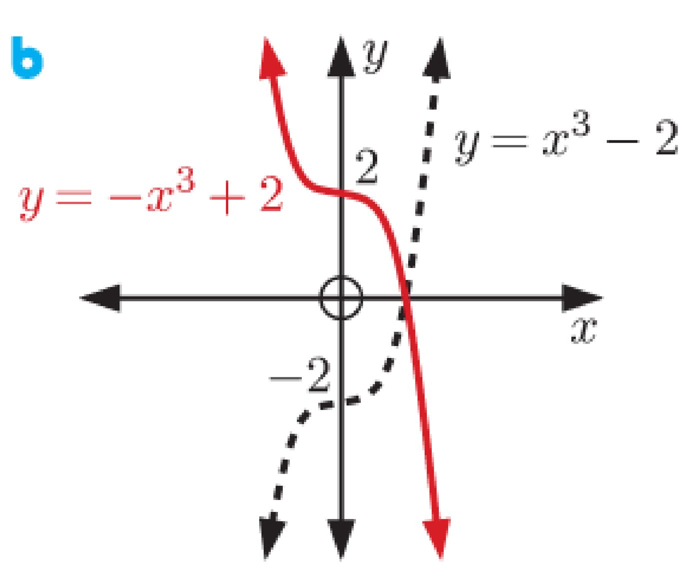
c



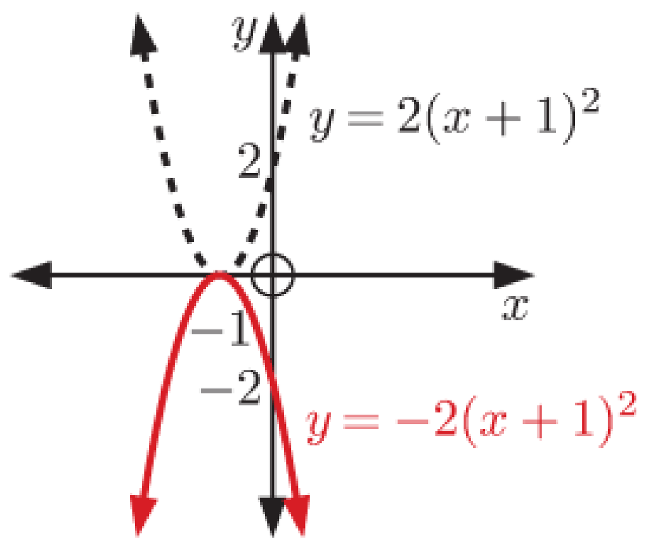
4 **a**



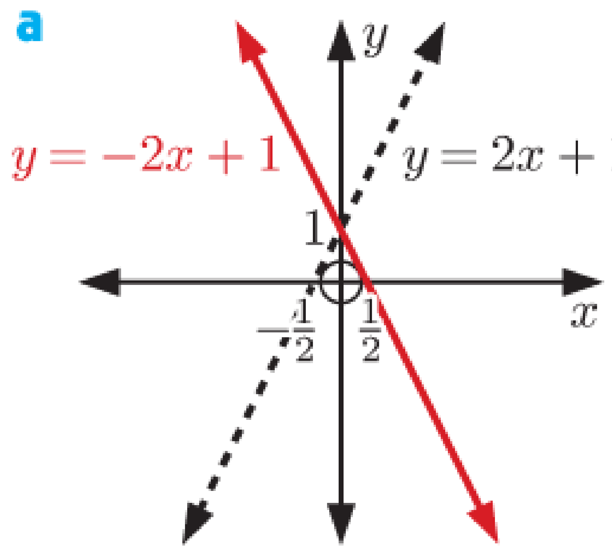
b



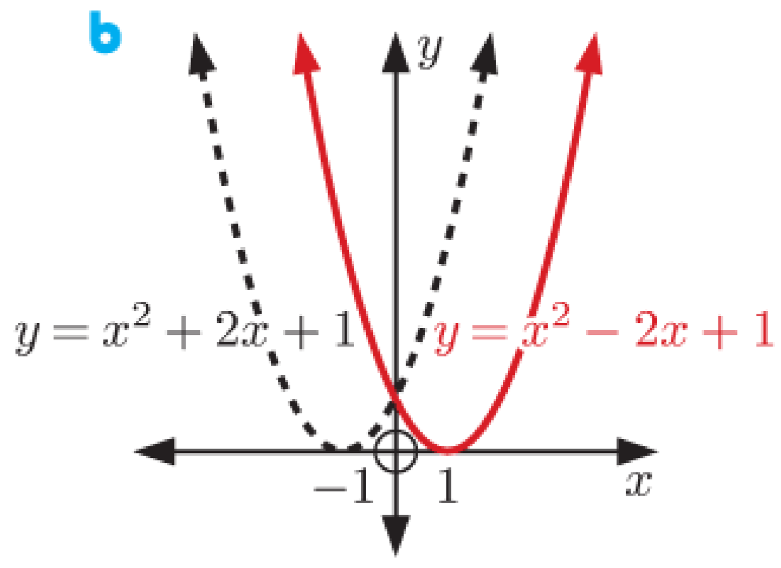
c



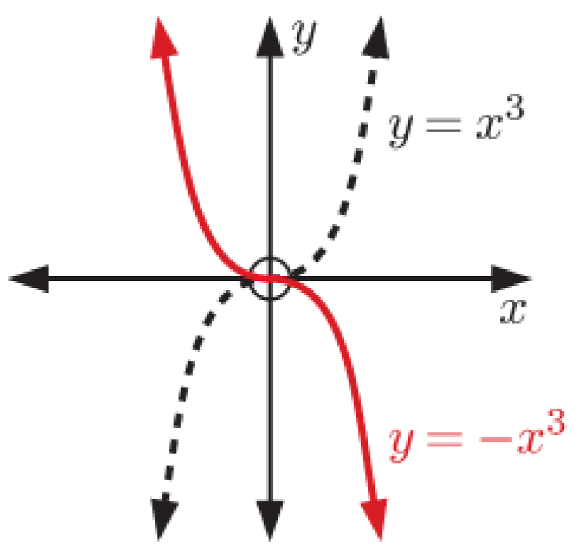
5 **a**



b



c



6 **a** $g(x) = -5x - 7$ **b** $g(x) = 2^{-x}$

c $g(x) = -2x^2 - 1$

d $g(x) = x^4 + 2x^3 - 3x^2 - 5x - 7$

7 **a** **i** $(3, 0)$ **ii** $(2, 1)$ **iii** $(-3, -2)$

b **i** $(7, 1)$ **ii** $(-5, 0)$ **iii** $(-3, 2)$

8 **a** **i** $(-2, -1)$ **ii** $(0, 3)$ **iii** $(1, 2)$

b **i** $(-5, -4)$ **ii** $(0, 3)$ **iii** $(-2, 3)$

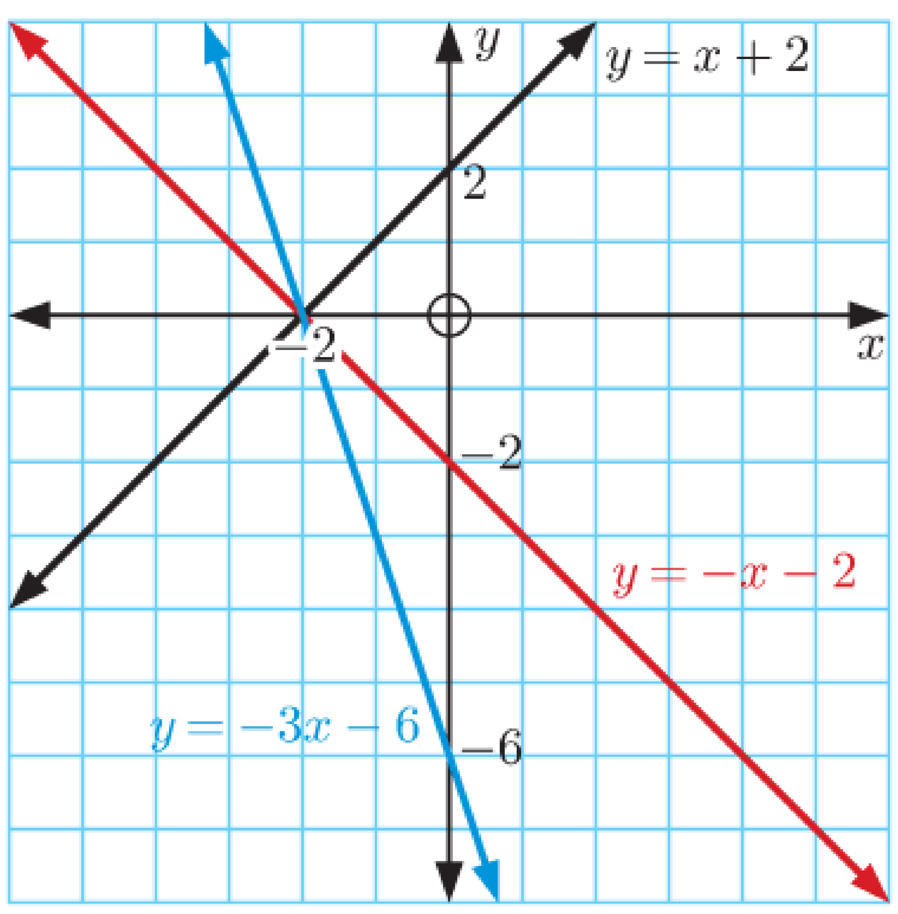
9 **a** A reflection in the y-axis and a reflection in the x-axis.

b $(-3, 7)$ **c** $(5, 1)$

10 **a** A reflection in the x-axis.

b A vertical stretch with scale factor 3.

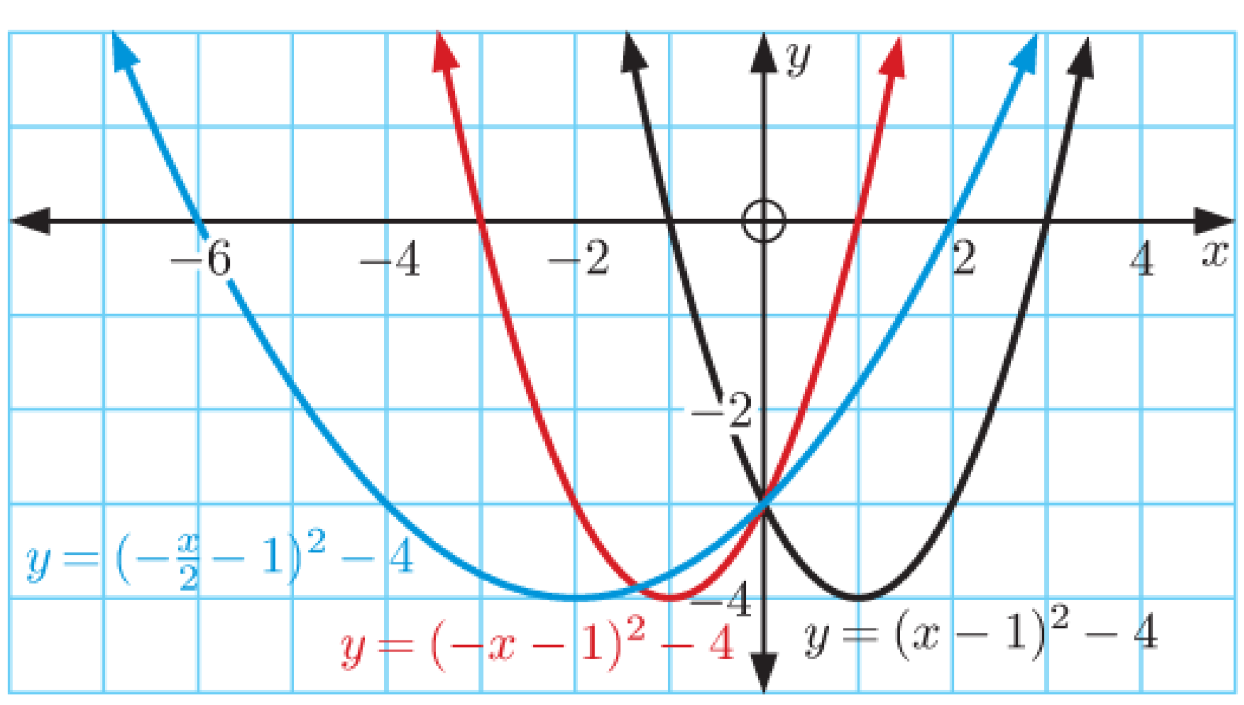
c



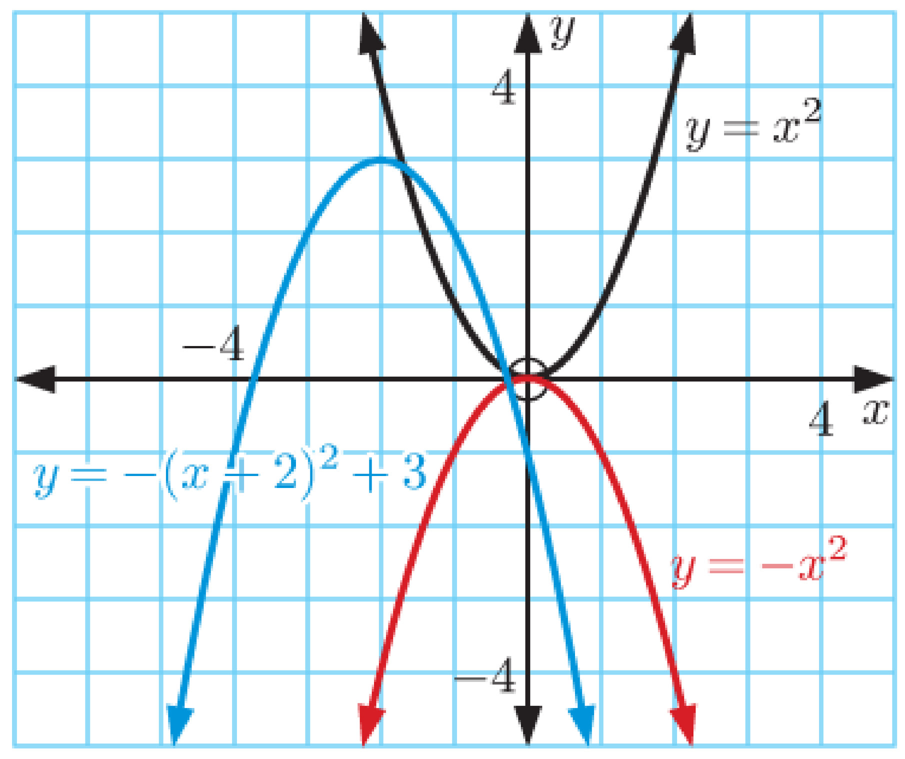
11 **a** A reflection in the y-axis.

b A horizontal stretch with scale factor 2.

c

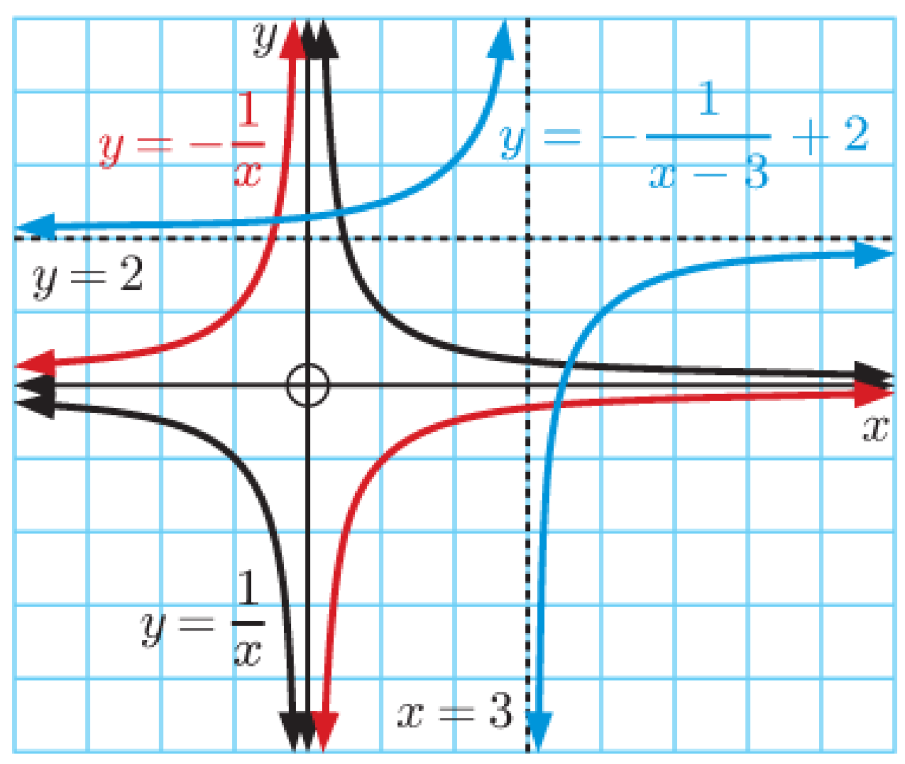


12



$y = x^2$ is transformed to $y = -(x+2)^2 + 3$ by reflecting in the x-axis and then translating through $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$.

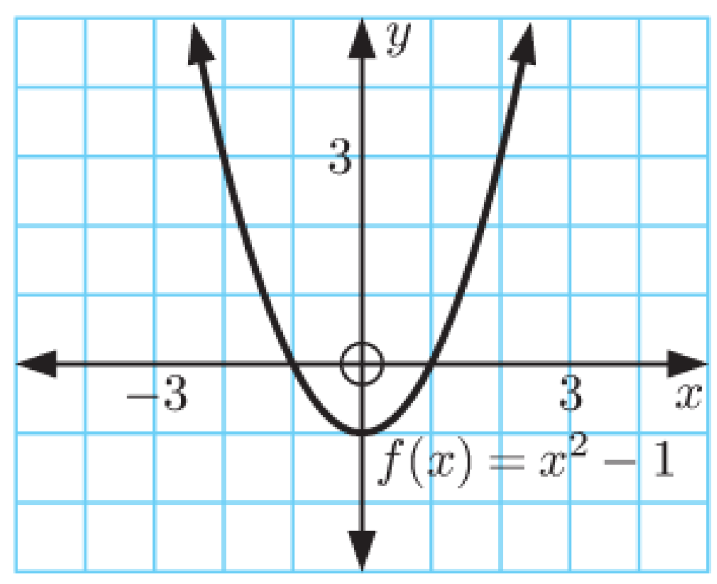
13



$y = \dfrac{1}{x}$ is transformed to $y = -\dfrac{1}{x-3} + 2$ by reflecting in the x-axis and then translating through $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

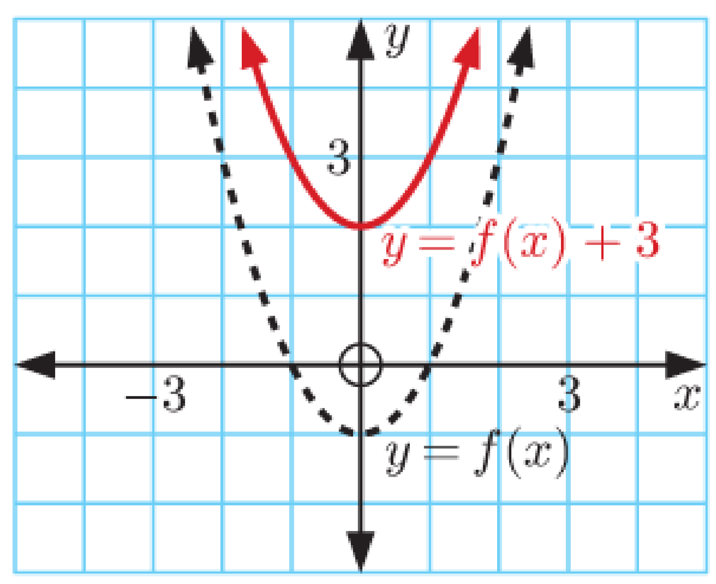
EXERCISE 16D

1 **a**



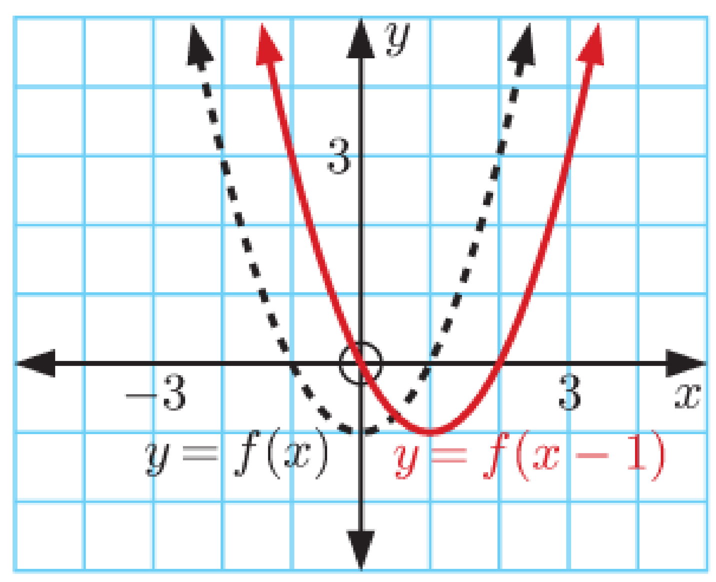
x-intercepts are ± 1, y-intercept is -1

b **i**



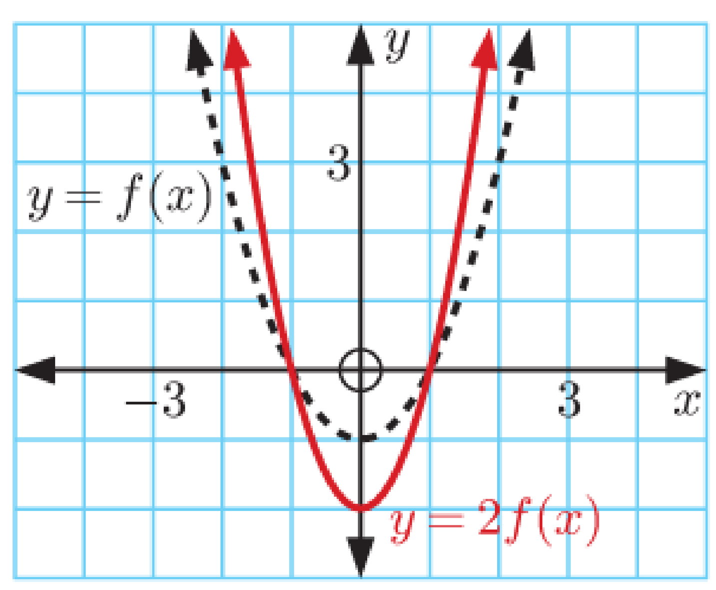
$y = f(x)$ has been translated 3 units upwards.

ii



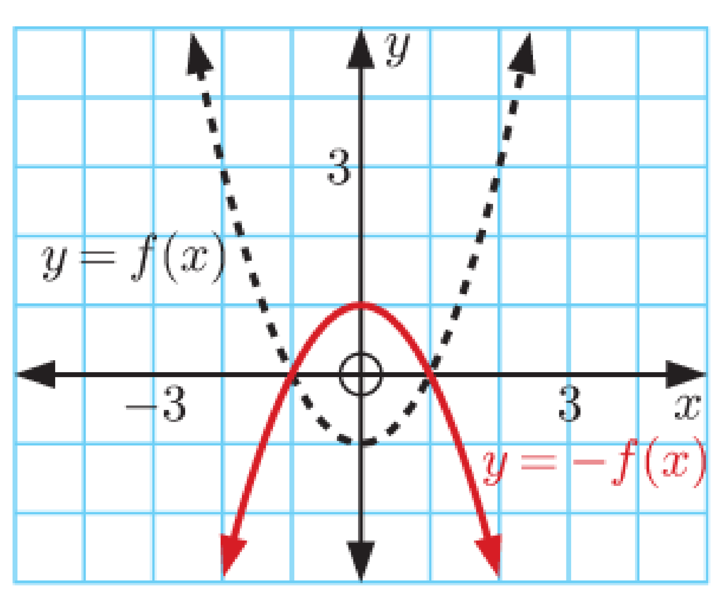
$y = f(x)$ has been translated 1 unit to the right.

iii



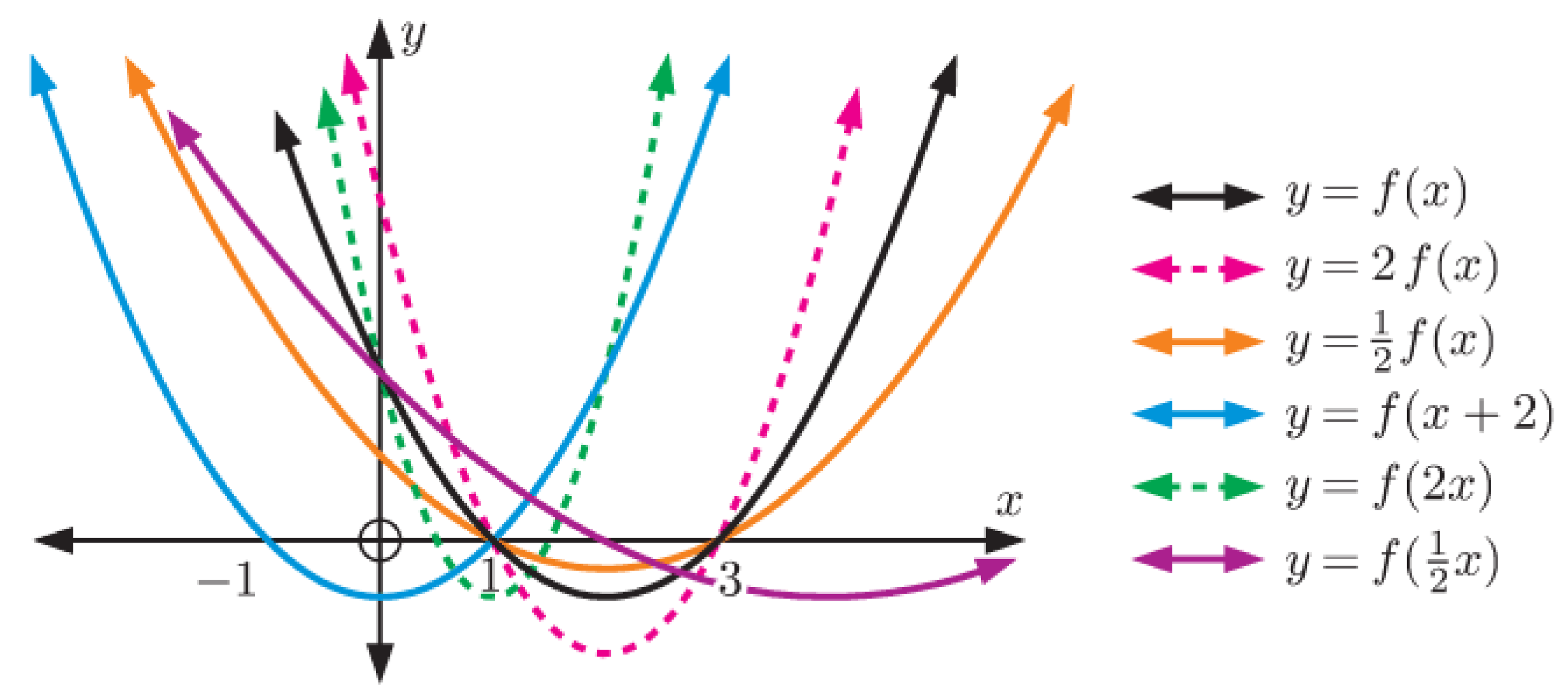
$y = f(x)$ has been vertically stretched with scale factor 2.

iv



$y = f(x)$ has been reflected in the x-axis.

2

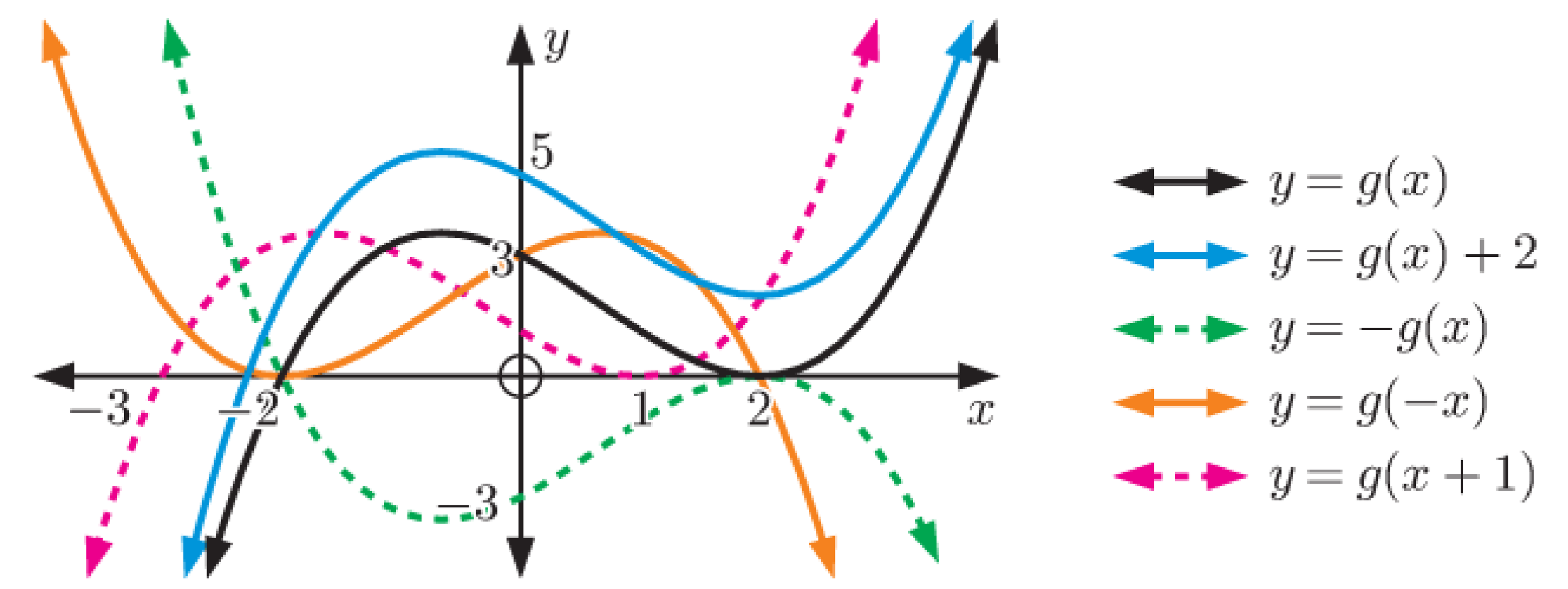


3 **a** **i** A vertical translation through $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$.
ii $g(x) = f(x) - 2$

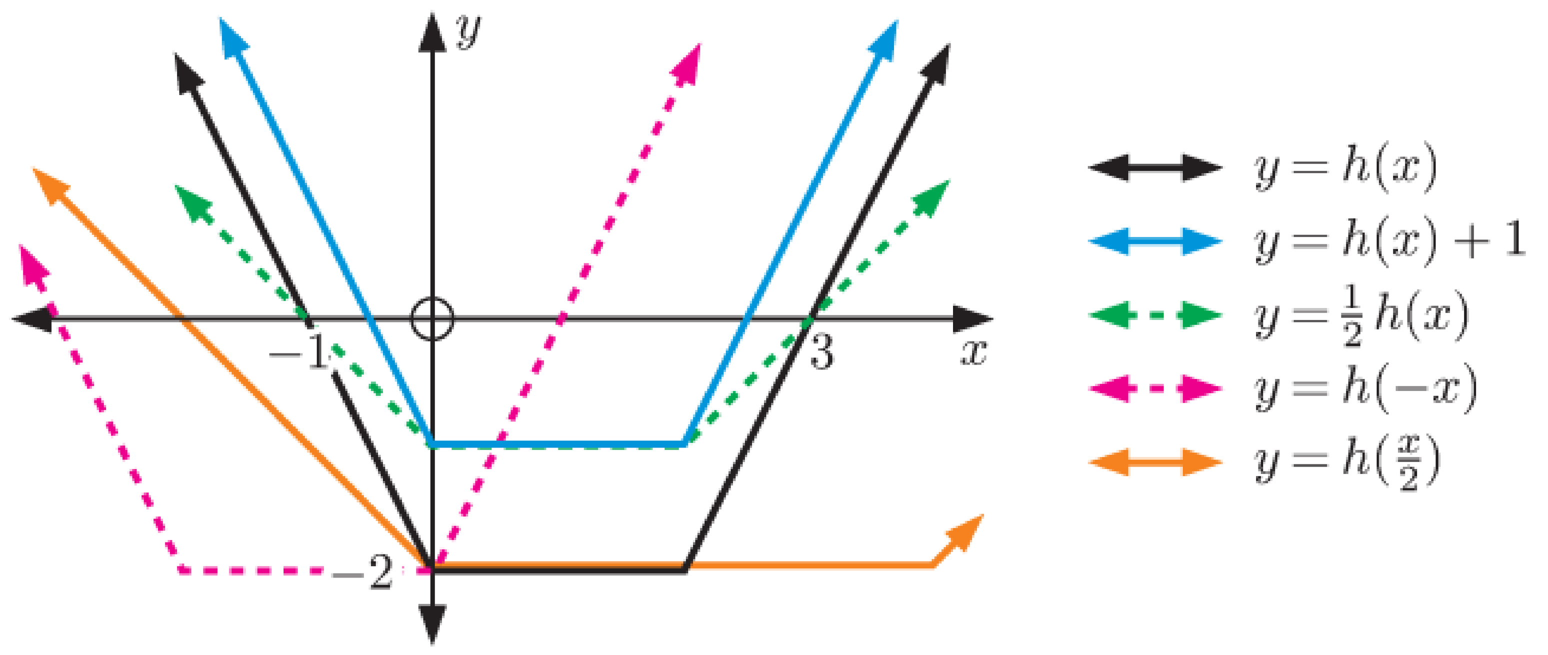
b **i** A vertical stretch with scale factor $\frac{1}{2}$.
ii $g(x) = \frac{1}{2}f(x)$

c **i** A reflection in the y-axis. **ii** $g(x) = f(-x)$

4

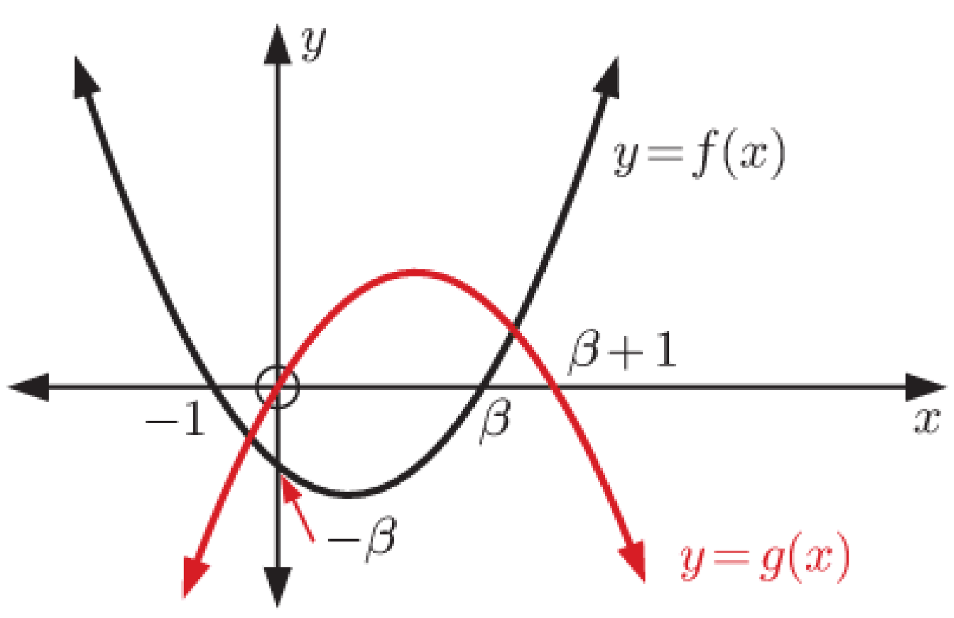


5



6 **a** x-intercepts are -1 and β, y-intercept is $-\beta$

b, c



7 **a** $f(-x-4) - 1$ **b** $f(-x+4) - 1$
c $\frac{1}{2}f(x+2) + \frac{1}{2}$ **d** $\frac{1}{2}f(x+2) + 1$
e $f(\frac{1}{4}x - 3) - 5$ **f** $f\left(\dfrac{x-3}{4}\right) - 5$

8 **a** A reflection in the x-axis, then a translation through $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

b A horizontal stretch with scale factor 2, then a translation through $\begin{pmatrix} 0 \\ -7 \end{pmatrix}$.

c A translation through $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, then a horizontal stretch with scale factor $\frac{1}{3}$.

d A vertical stretch with scale factor 2, a translation through $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, then a horizontal stretch with scale factor 4.

e A vertical stretch with scale factor 2, a horizontal stretch with scale factor $\frac{1}{3}$, then a translation through $\begin{pmatrix} 1 \\ 5 \end{pmatrix}$.

f A reflection in the x-axis, a vertical stretch with scale factor 4, a horizontal stretch with scale factor 2, then a translation through $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$.

9 **a** Domain is $\{x \mid x \geqslant -3\}$, Range is $\{y \mid -3 \leqslant y < 4\}$
b Domain is $\{x \mid x \geqslant \frac{1}{3}\}$, Range is $\{y \mid -10 < y \leqslant 4\}$
c Domain is $\{x \mid x \geqslant 3\}$, Range is $\{y \mid \frac{10}{3} \leqslant y < \frac{17}{3}\}$

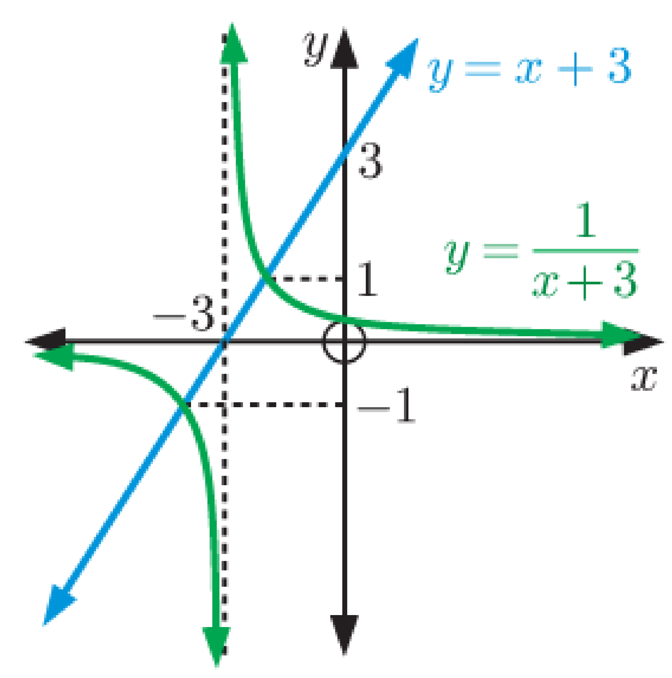
10 **a** $5\sqrt{2-x} + 15$
Domain is $\{x \mid x \leqslant 2\}$, Range is $\{y \mid y \geqslant 15\}$
b $5\sqrt{2-x} + 3$
Domain is $\{x \mid x \leqslant 2\}$, Range is $\{y \mid y \geqslant 3\}$
c $5\sqrt{-x-2} + 3$
Domain is $\{x \mid x \leqslant -2\}$, Range is $\{y \mid y \geqslant 3\}$

11 **a** The vertical stretch has scale factor $|a|$. The reflection in the x-axis occurs if $a < 0$. Each point is then moved h units right and k units up.
b The function has shape ∪ if $a > 0$ and ∩ if $a < 0$.
The function has vertex (h, k), and y-intercept $ah^2 + k$.

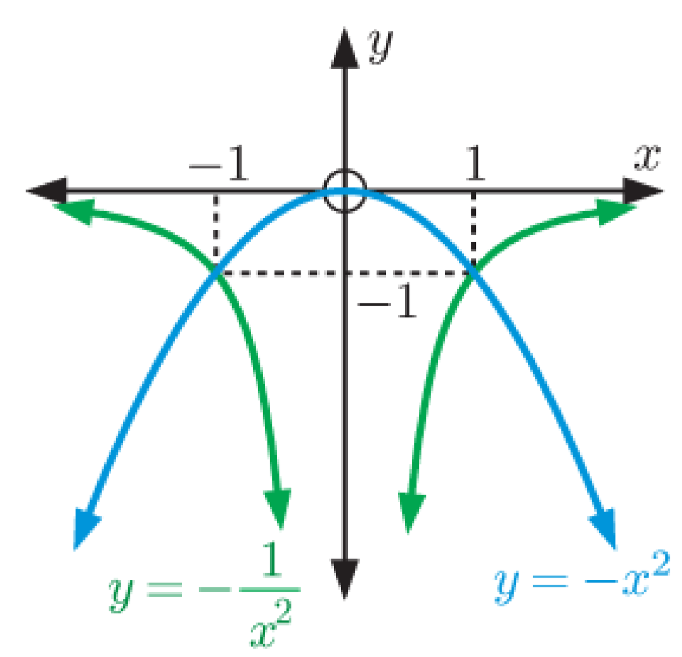
12 **a** $5 + \dfrac{-4}{2x+3}$
b A reflection in the x-axis, a vertical stretch with scale factor 4, a translation through $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$, then a horizontal stretch with scale factor $\frac{1}{2}$.

EXERCISE 16E

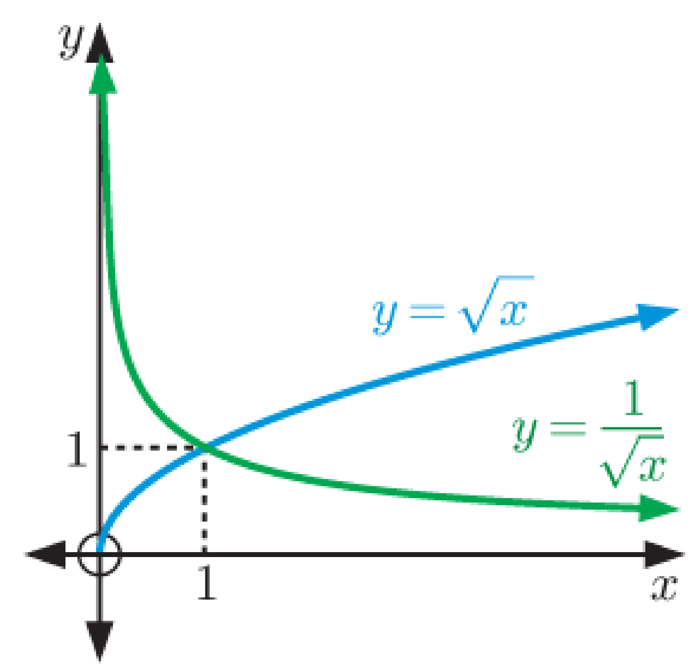
1 **a**



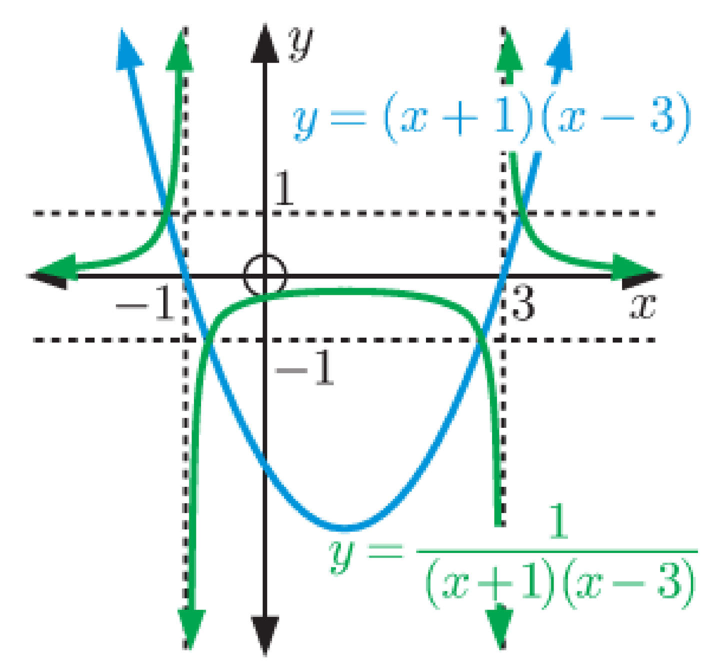
b



c

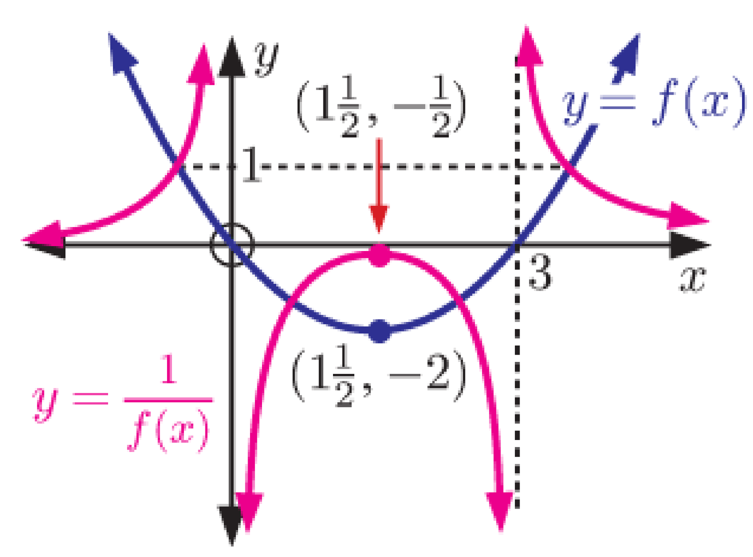


d

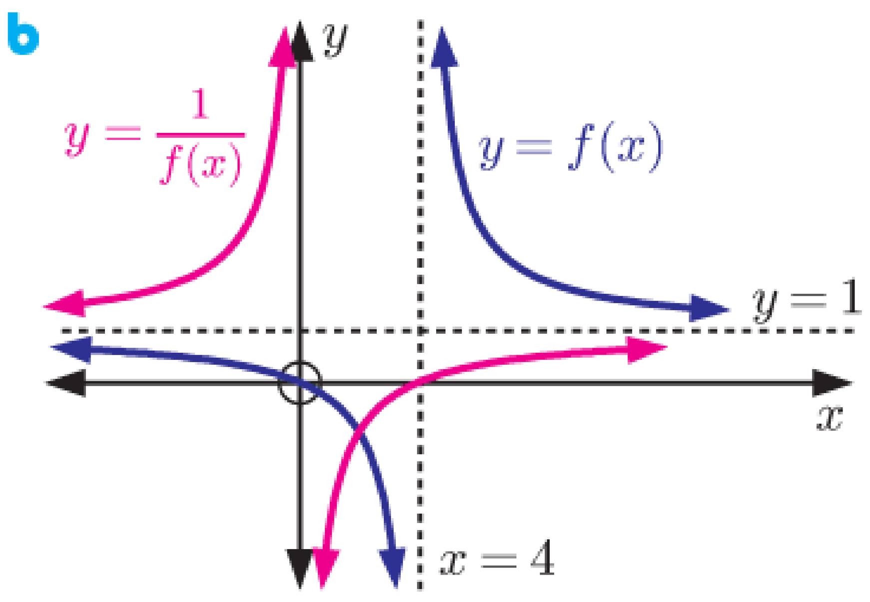


2 **a** invariant points are $(-2, 1)$ and $(-4, -1)$
b invariant points are $(-1, -1)$ and $(1, -1)$
c invariant point is $(1, 1)$
d invariant points are $(\approx -1.24, 1)$, $(\approx -0.732, -1)$, $(\approx 2.73, -1)$, and $(\approx 3.24, 1)$

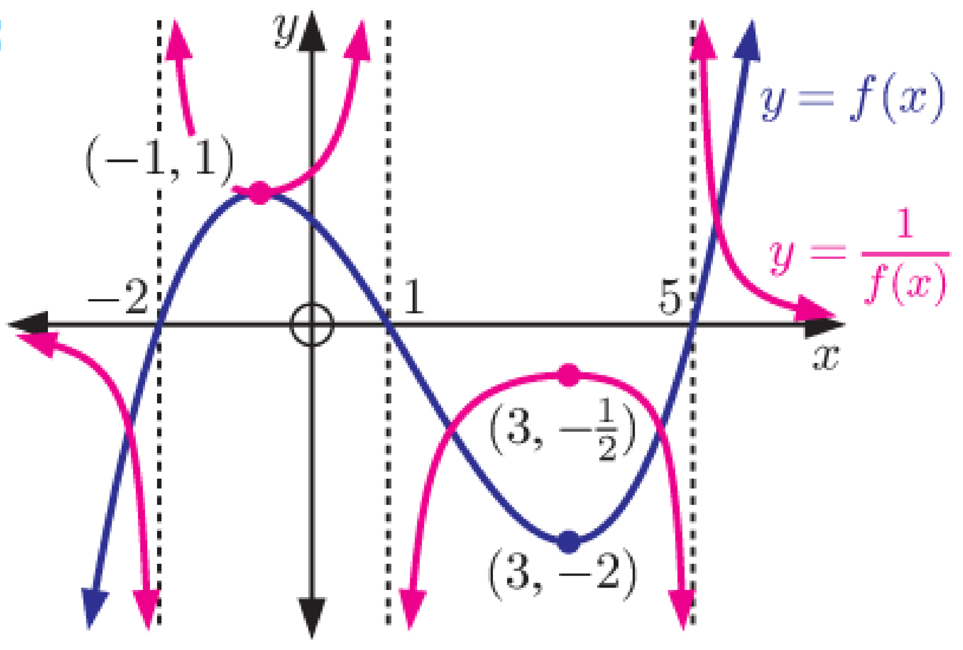
3 **a**



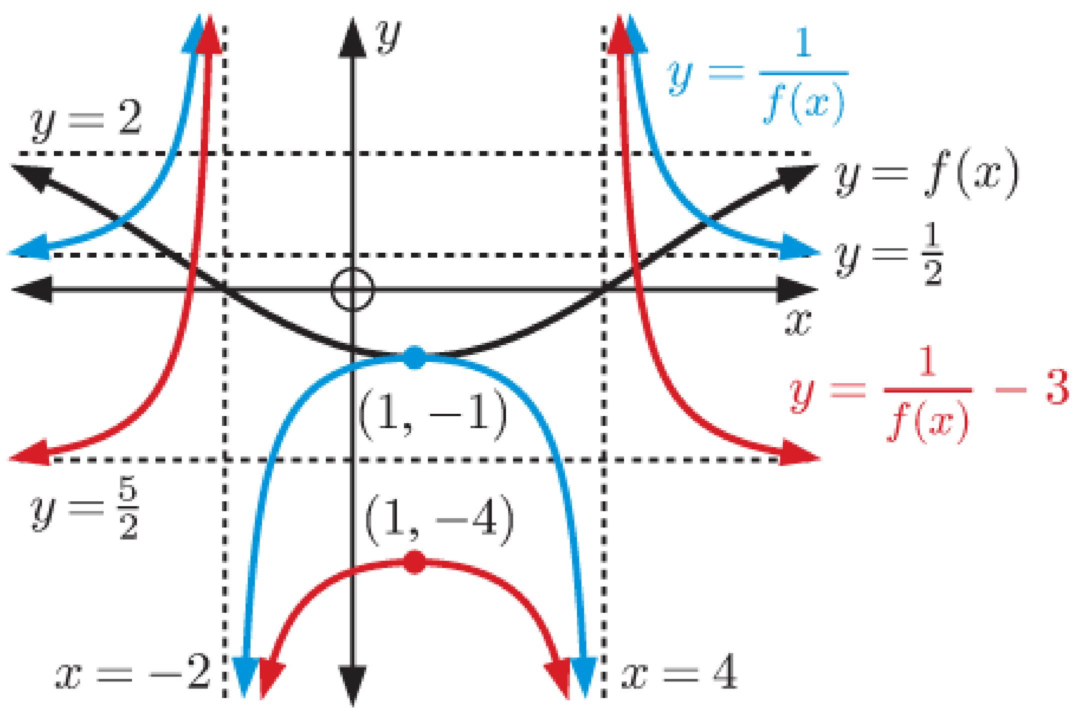
b



c

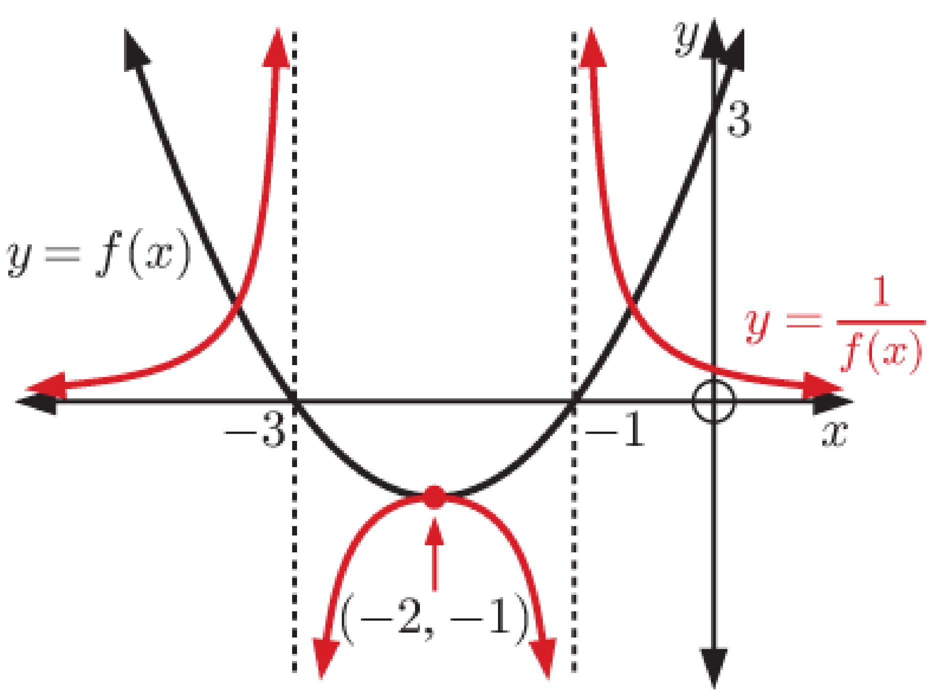


4



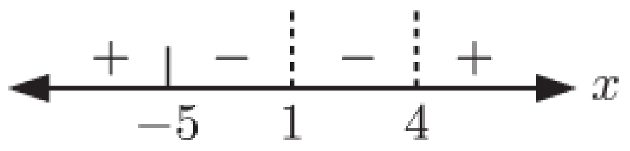
5 **a** x-intercepts -3 and -1, y-intercept 3, vertex $(-2, -1)$

b



c $x = -\frac{9}{2}$ or $\frac{1}{2}$

6

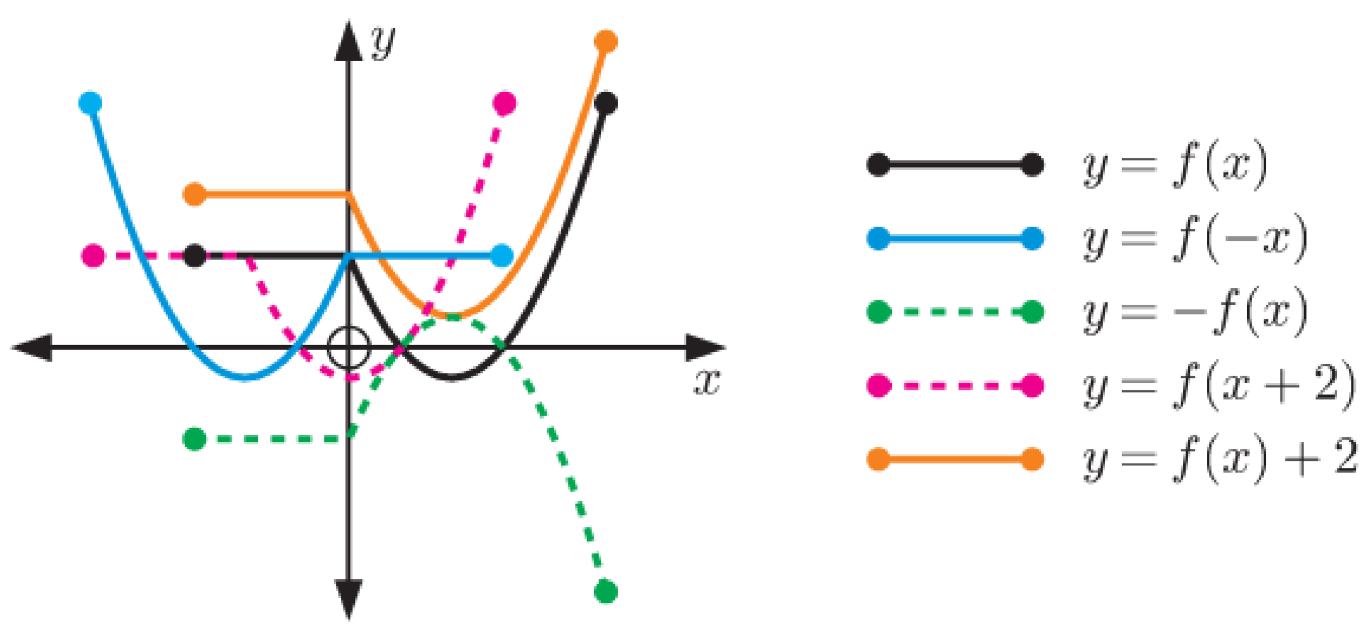


7 **a** Domain is $\{x \mid -1 \leqslant x \leqslant 6\}$, Range is $\{y \mid \frac{1}{5} < y \leqslant \frac{1}{2}\}$

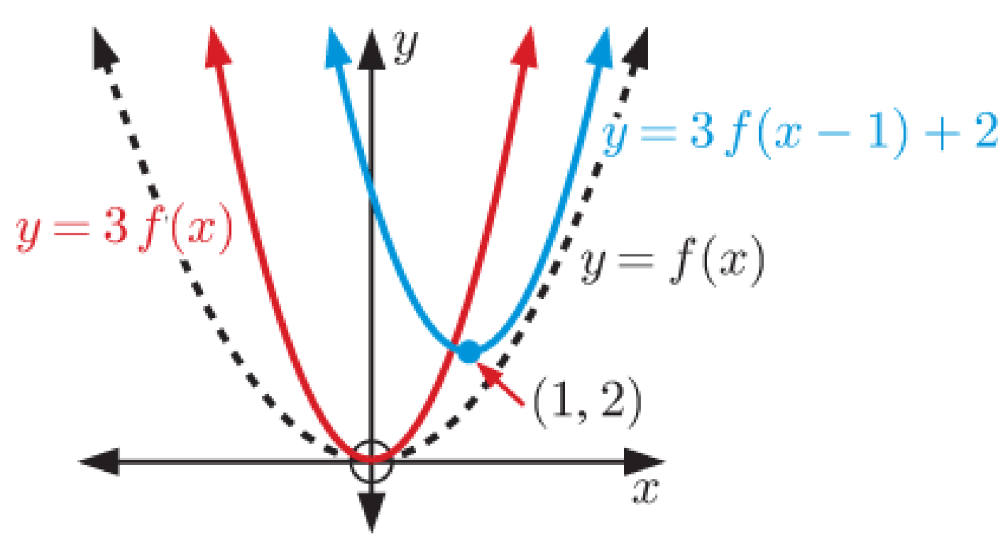
b Range is $\{y \mid y \leqslant -\frac{1}{3}$ or $y \geqslant \frac{1}{3}\}$, cannot say about the domain.

REVIEW SET 16A

1



2

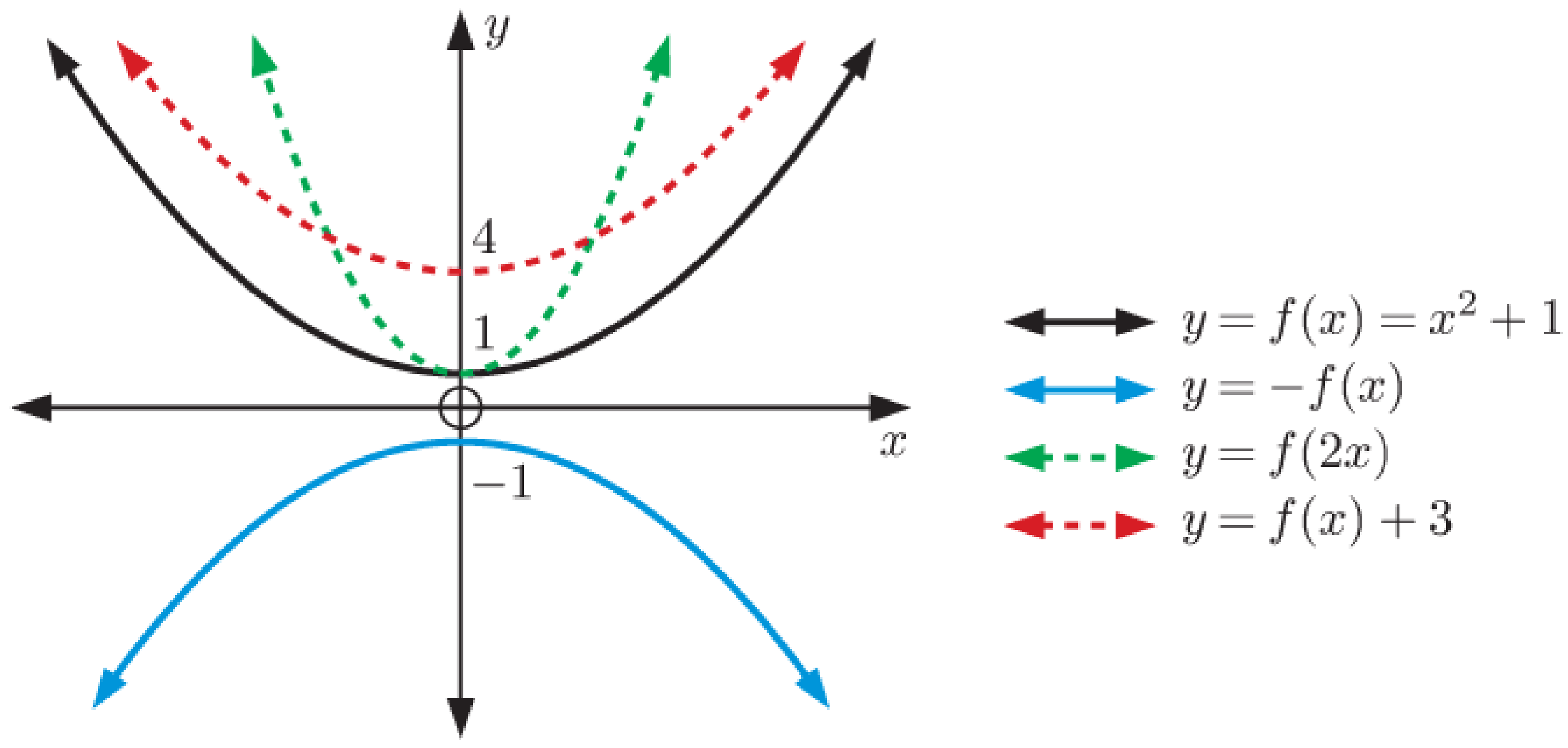


3 **a** $g(x) = 4x - 10$ **b** $g(x) = 5x^2 + 30$

c $g(x) = -3x - 5$ **d** $g(x) = \frac{2}{9}x^2 - \frac{1}{3}x + 4$

e $g(x) = -x^3$

4



5 $g(x)$ is the result of transforming $f(x)$ 3 units to the left and 4 units down.

$\therefore$ domain of $g(x)$ is $\{x \mid -5 \leqslant x \leqslant 0\}$

range of $g(x)$ is $\{y \mid -5 \leqslant y \leqslant 3\}$.

6 **a** $g(x) = (x-1)^2 + 8$

b **i** $\{y \mid y \geqslant 4\}$ **ii** $\{y \mid y \geqslant 8\}$

8 $g(x) = 3x^2 + 5x + 9$

9 **a** $-f(x+2) + 3$ **b** $2f(x-4) - 2$

10 **a** $(0, 4)$ **b** $(0, 6)$ **c** $(\frac{1}{2}, 3)$

11 **a** x-intercepts -9 and -3

b x-intercepts -5 and 1, y-intercept -9

c x-intercepts -10 and 2, y-intercept -3

d x-intercepts -5 and 1, y-intercept 3

12 **a** $g(x) = \dfrac{2x-3}{x-1}$

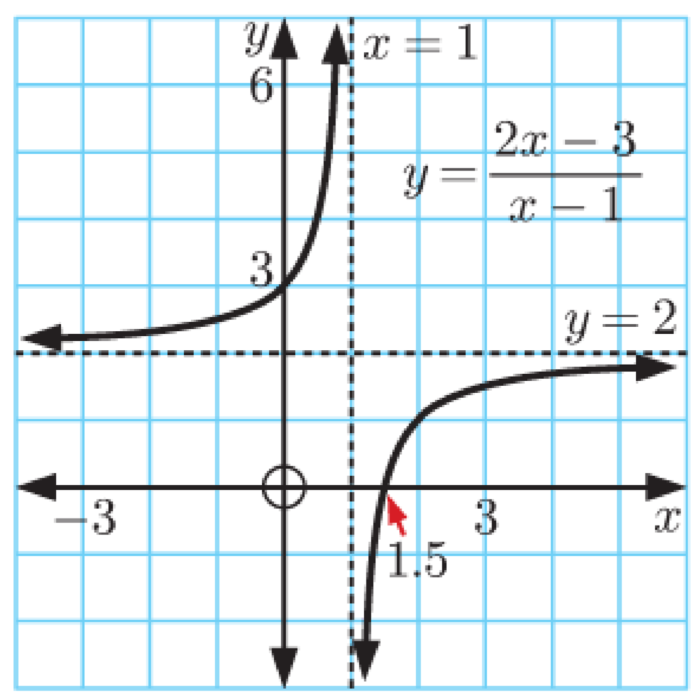
b vertical asymptote $x = 1$,

horizontal asymptote $y = 2$

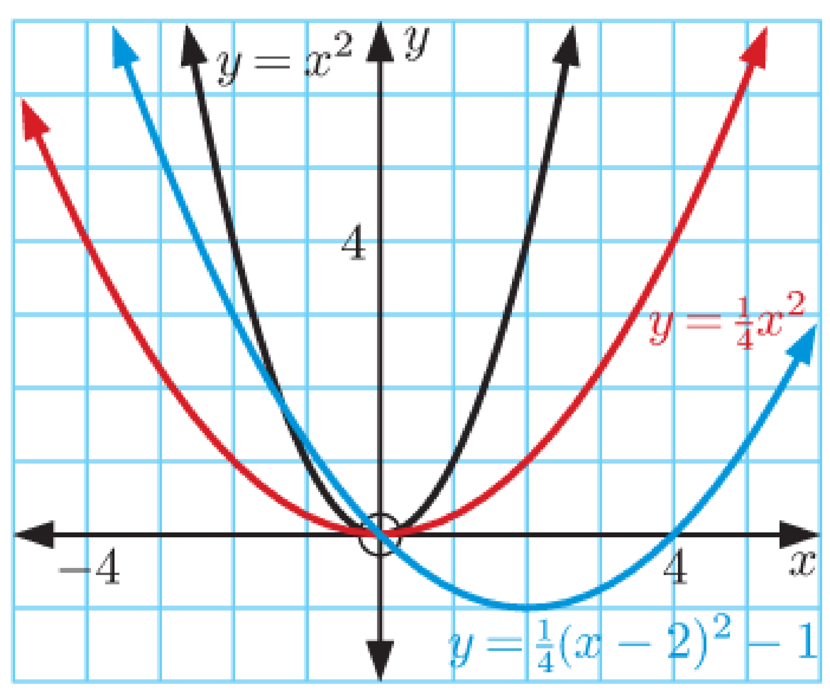
c Domain is $\{x \mid x \neq 1\}$

Range is $\{y \mid y \neq 2\}$

d

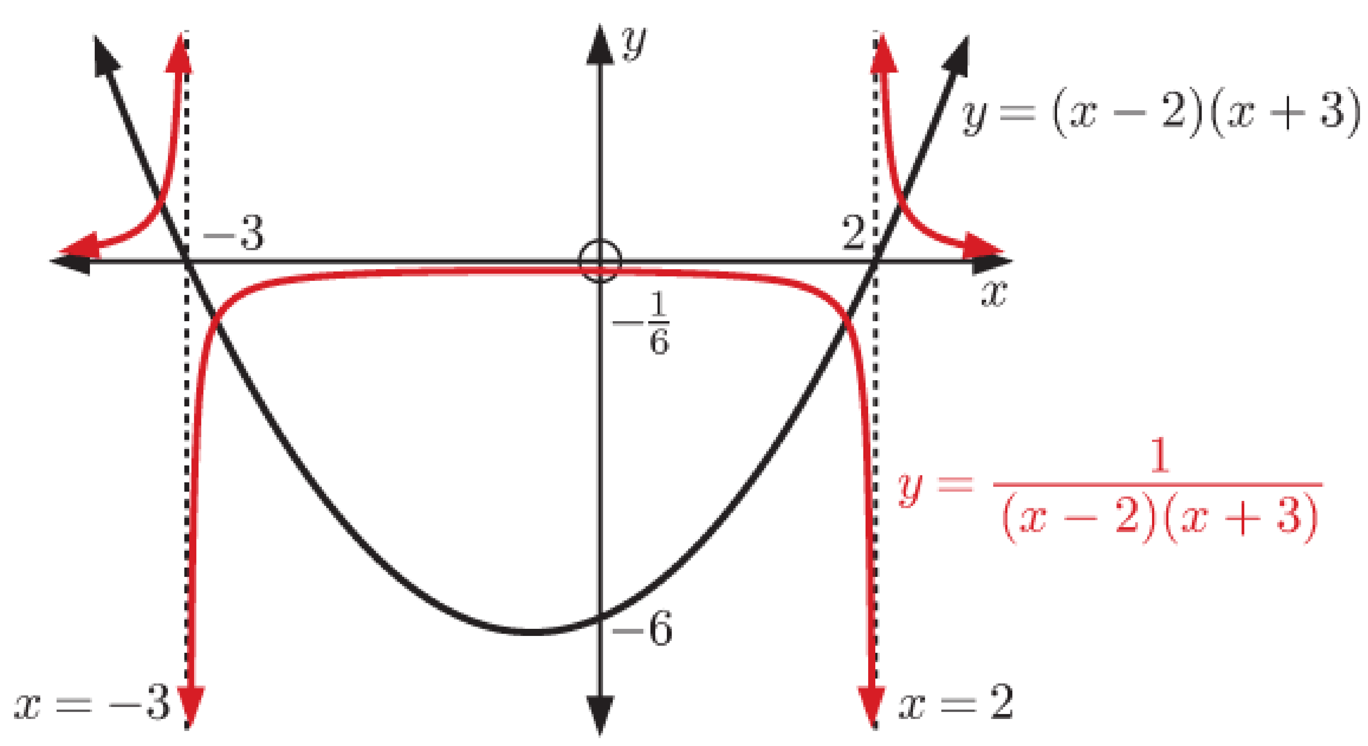


13



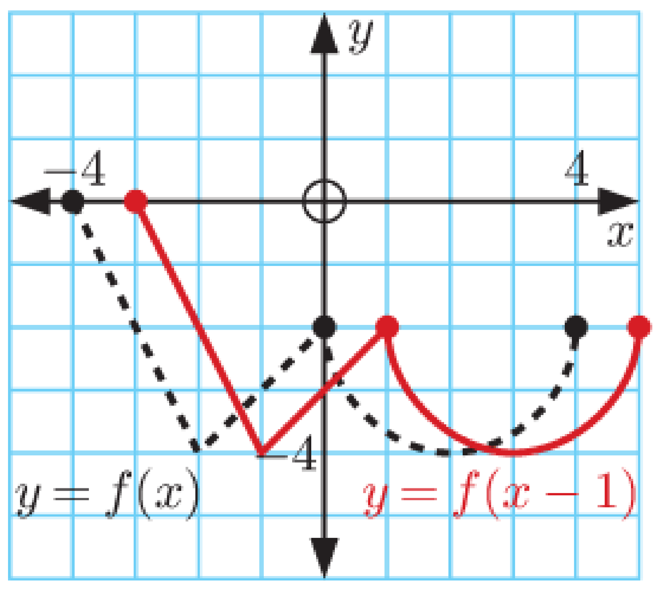
$y = x^2$ is transformed to $y = \frac{1}{4}(x-2)^2 - 1$ by vertically stretching with scale factor $\frac{1}{4}$ and then translating through $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

14

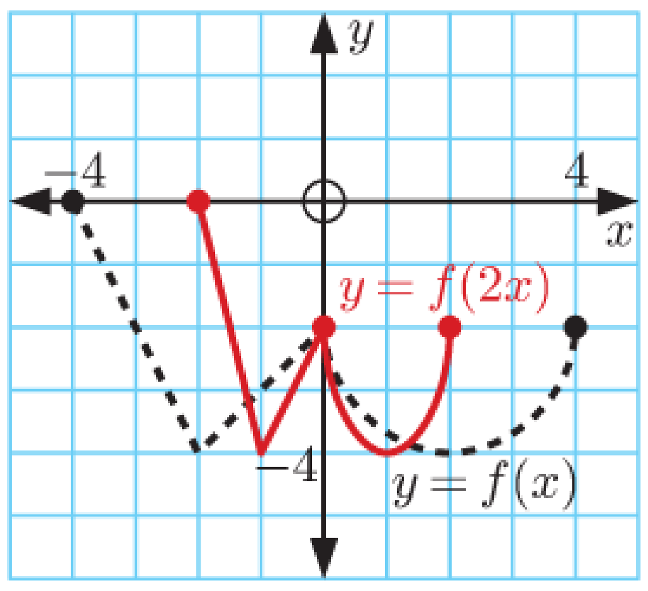


REVIEW SET 16B

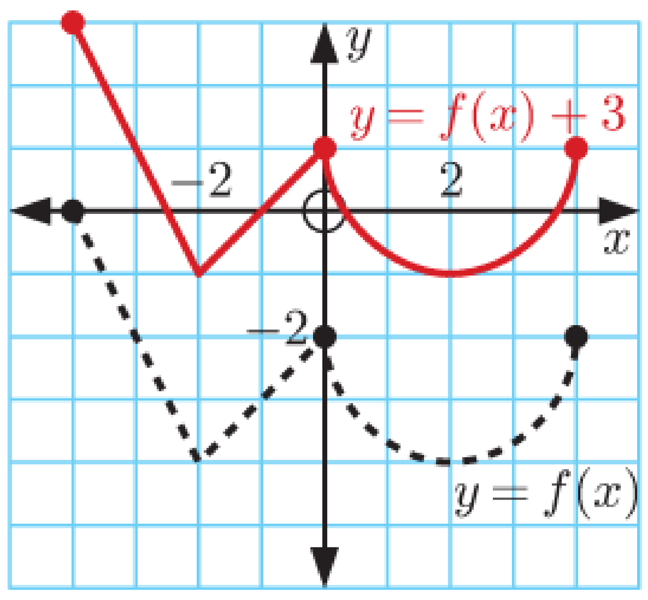
1 **a**



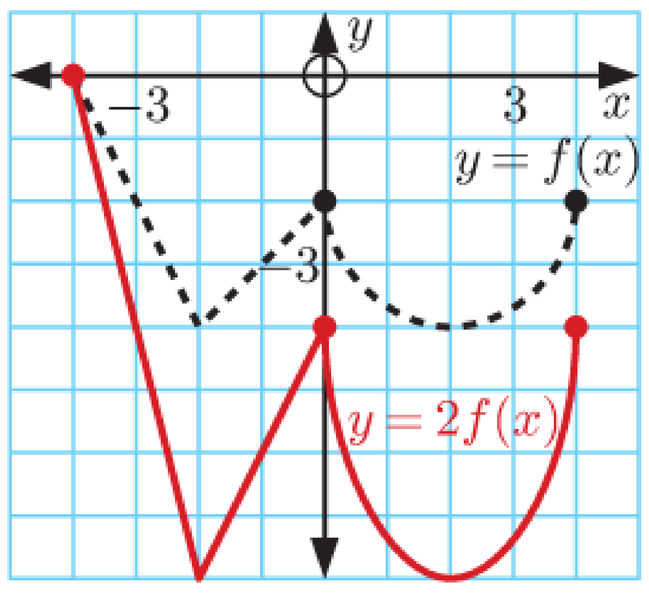
b



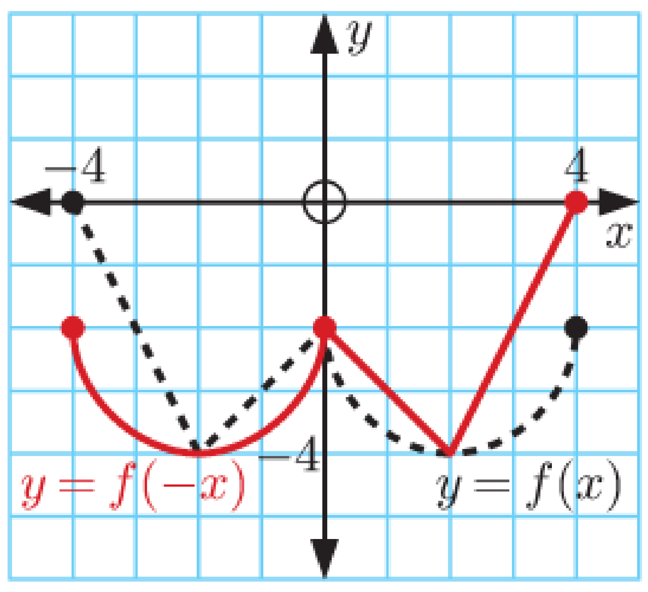
c



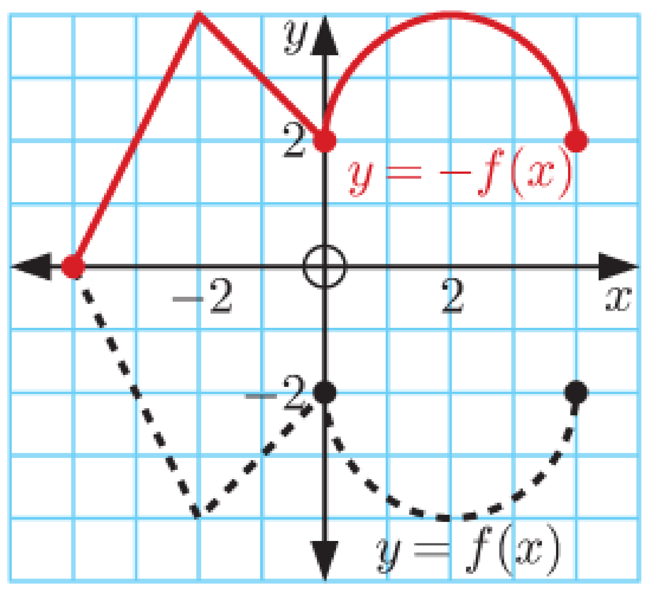
d



e



f

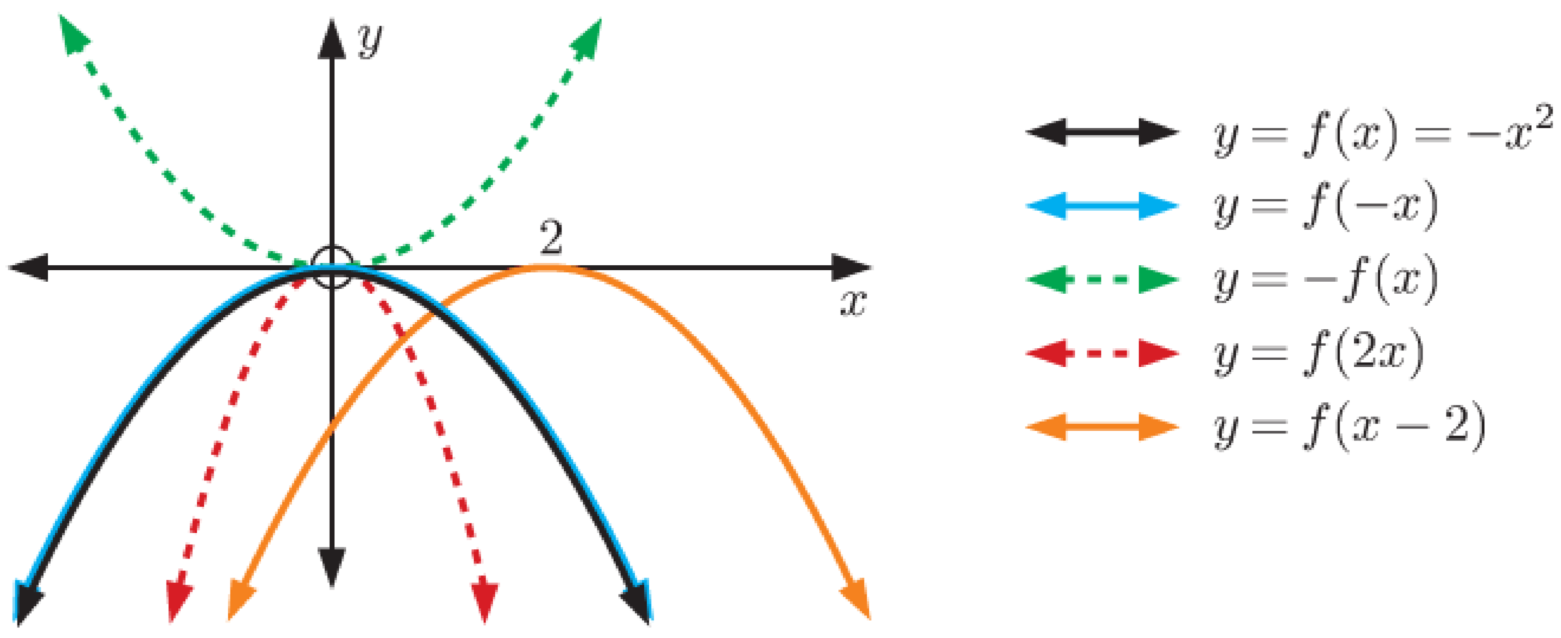


2 **a** $g(x) = 3x - x^2$ **b** $g(x) = 16 - x$

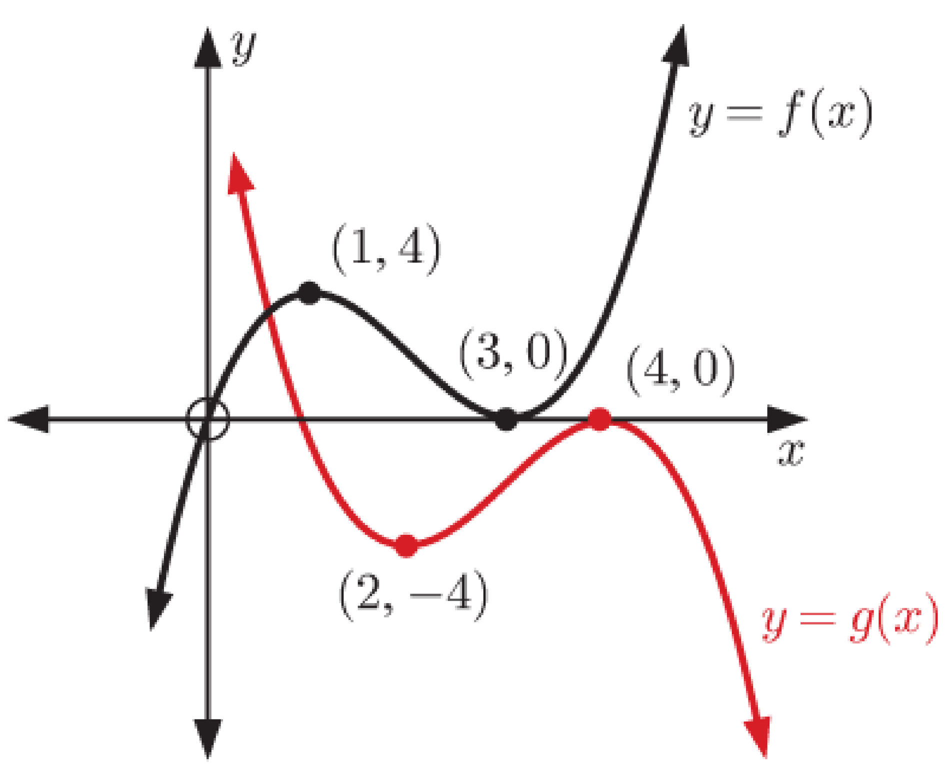
c $g(x) = \frac{1}{12}x + 2$

3 $g(x) = -x^2 - 6x - 7$

4



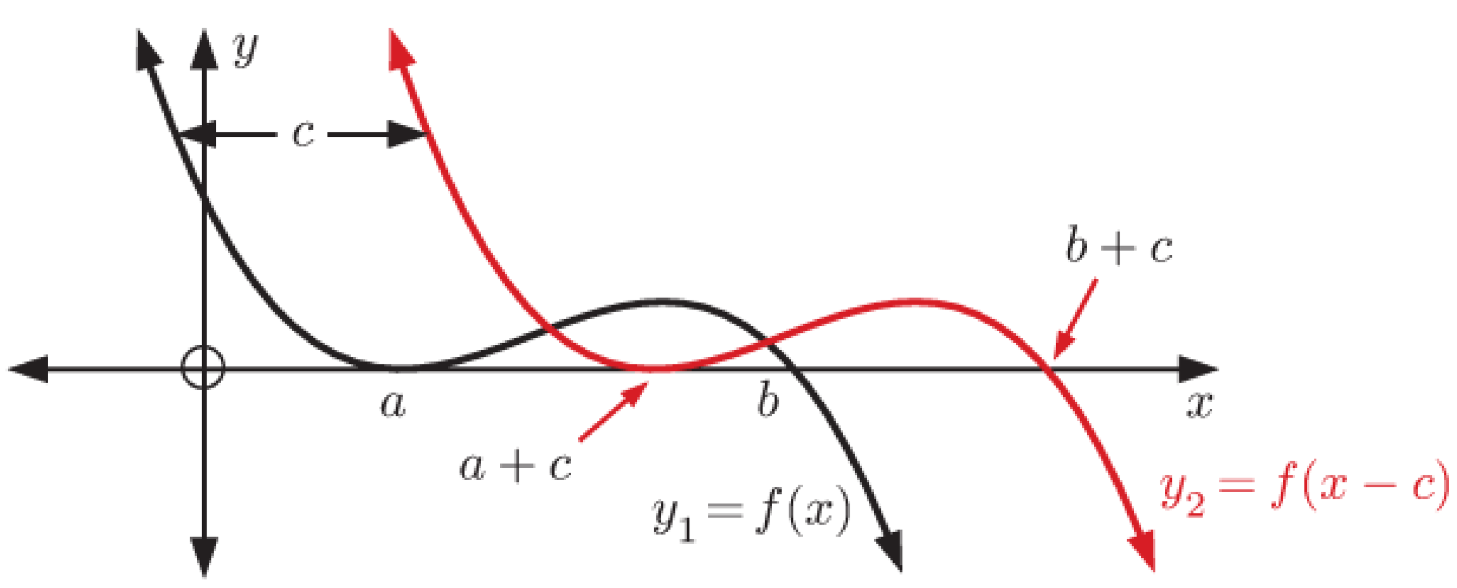
5 **a**



b $(2, -4)$ and $(4, 0)$

6 $y = -2x^2 + 5x - 3$

7



8 A reflection in the x-axis, then a translation through $\begin{pmatrix} \frac{5}{2} \\ -\frac{7}{2} \end{pmatrix}$.

9 $(1, 6)$

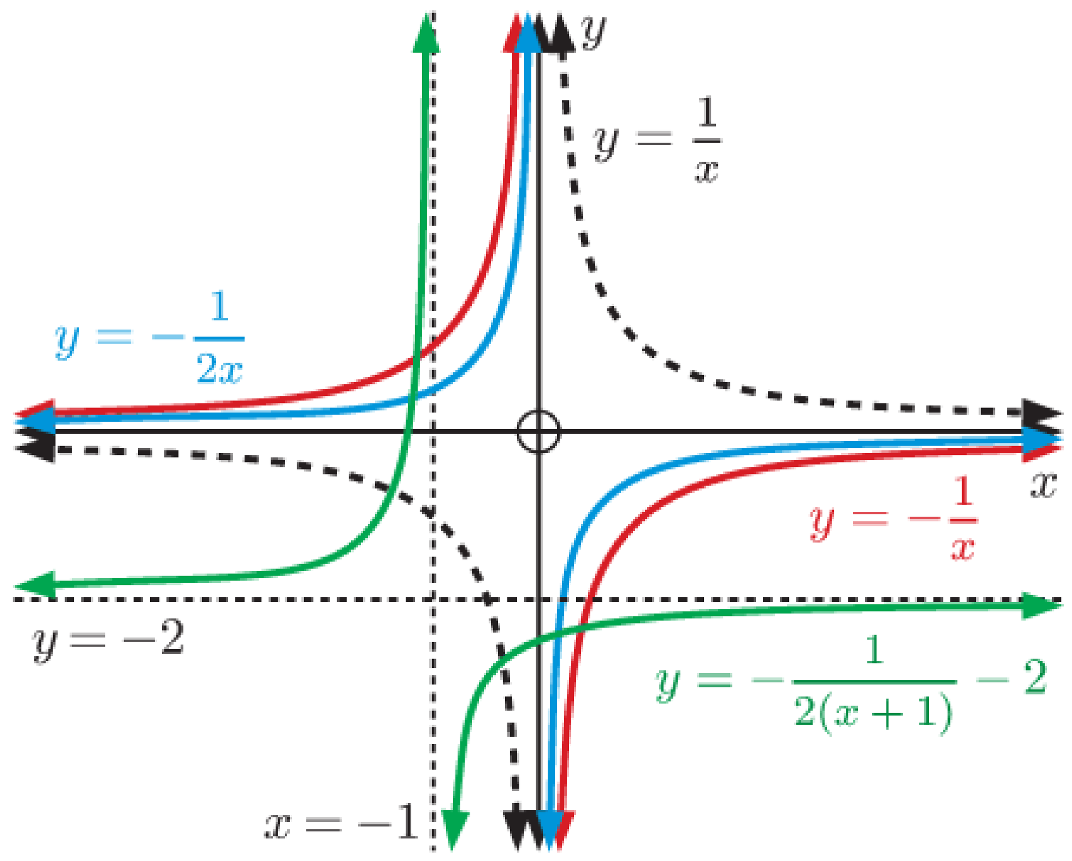
10 **a** A vertical stretch with scale factor 2, then a translation through $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

b A reflection in the x-axis, a horizontal stretch with scale factor $\frac{3}{2}$, then a translation through $\begin{pmatrix} 0 \\ -6 \end{pmatrix}$.

c A vertical stretch with scale factor $\frac{1}{3}$, a reflection in the y-axis, then a translation through $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

11 $b = 8$, $c = -20$

12 **a**

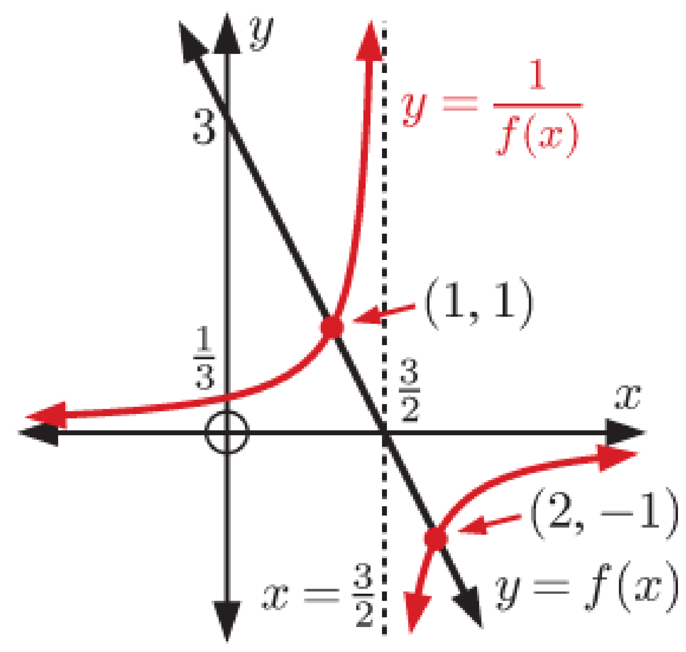


b A reflection in the x-axis, a vertical stretch with scale factor $\frac{1}{2}$, then a translation through $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$.

c $y = \dfrac{-4x - 5}{2x + 2}$

Domain is $\{x \mid x \neq -1\}$, Range is $\{y \mid y \neq -2\}$

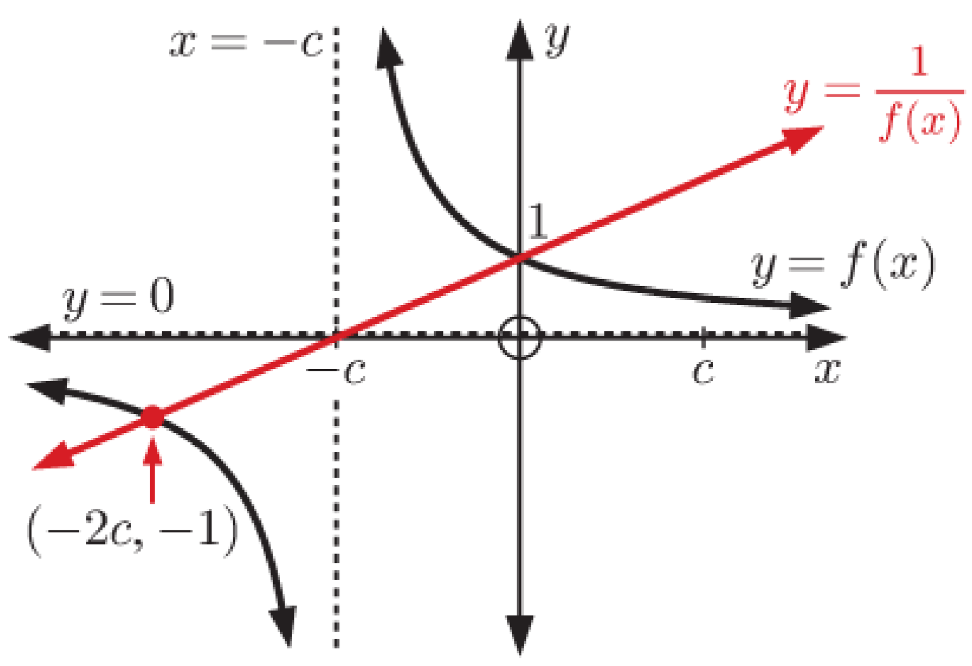
13 **a, d**



b $(1, 1)$ and $(2, -1)$

c y-intercept $\frac{1}{3}$ and vertical asymptote $x = \frac{3}{2}$

14



EXERCISE 17A

1 **a** periodic **b** periodic **c** periodic **d** not periodic
e periodic **f** periodic **g** not periodic **h** not periodic

2 **a**