

Mixed examination practice 1

Short questions

1. Seven athletes take part in the 100 m final of the Olympic games. In how many ways can three medals be awarded? [4 marks]
2. In how many ways can five different letters be put into five different envelopes? [5 marks]
3. In how many ways can ten cartoon characters stand in a queue if Mickey, Bugs Bunny and Jerry must occupy the first three places in some order? [5 marks]
4. How many three digit numbers contain no zeros? [6 marks]
5. A committee of four children is chosen from eight children. The two oldest children cannot both be chosen. Find the number of ways the committee may be chosen. [6 marks]
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6. Solve the equation $(n+1)! = 30(n-1)!$ for $n \in \mathbb{N}$.
(Remember: \mathbb{N} is the set of natural (whole non-negative) numbers.) [5 marks]
7. How many permutations of the word 'CAROUSEL' start and end in a consonant? [5 marks]
8. Solve the equation $\binom{n}{2} = 105$. [6 marks]
9. A group of 15 students contains seven boys and eight girls. In how many ways can a committee of five be selected if it must contain at least one boy? [6 marks]
10. Abigail, Bahar, Chris, Dasha, Eustace and Franz are sitting next to each other in six seats in a cinema. Bahar and Eustace cannot sit next to each other. In how many different ways can they permute themselves? [6 marks]
11. A committee of five is to be selected from a group of 12 children. The two youngest cannot both be on the committee. In how many ways can the committee be selected? [6 marks]
12. A car registration number consists of three different letters followed by five digits chosen from 1–9 (the digits can be repeated). How many different registration numbers can be made? [6 marks]
13. A van has eight seats, two at the front, a row of three in the middle and a row of three at the back. If only 5 out of a group of 8 people can drive, in how many different ways can they be arranged in the car? [6 marks]
14. Ten people are to travel in one car (taking four people) and one van (taking six people). Only two of the people can drive. In how many ways can they be allocated to the two vehicles? (The permutation of the passengers in each vehicle is not important.) [7 marks]

Long questions

- Five girls, Anya, Beth, Carol, Dasha and Elena, stand in a line. How many possible permutations are there in which
 - Anya is at one end of the line?
 - Anya is not at either end?
 - Anya is at the left of the line or Elena is on the right, or both? [9 marks]
- In how many ways can five different sweets be split amongst two people if
 - each person must have at least one sweet?
 - one person can take all of the sweets?
 - one of the sweets is split into two to be shared, and each person gets two of the remaining sweets? [9 marks]
- In a doctor's waiting room, there are 14 seats in a row. Eight people are waiting to be seen.
 - In how many ways can they be seated?
 - Three of the people are all in the same family and they want to sit together. How many ways can this happen?
 - The family no longer have to sit together, but there is someone with a very bad cough who must sit at least one seat away from anyone else. How many ways can this happen? [8 marks]
- Explain why the number of ways of arranging the letters RRDD, given that all the R's and all the D's are indistinguishable is $\binom{4}{2}$.
 - How many ways are there of arranging n R's and n D's?
 - A miner is digging a tunnel on a four by four grid. He starts in the top left box and wants to get to the gold in the bottom right box. He can only tunnel directly right or directly down one box at a time. How many different routes can he take?
 - What will be the general formula for the number of routes when digging on an n by m grid? [10 marks]

5. 12 people need to be split up into teams for a quiz.

- (a) Show that the number of ways of splitting them into two groups of the same size is $\frac{1}{2} \binom{12}{6}$.
- (b) How many ways are there of splitting them into two groups of any size (but there must be at least one person in each group)?
- (c) How many ways are there of splitting them into three groups of four people? [9 marks]



6. (a) How many different ways are there to select a group of three from a class of 31 people?
- (b) In another class there are 1540 ways of selecting a group of three people. How many people are there in the class?
- (c) In another class the teacher noted that the number of ways to select a group of size three is 100 times larger than the number of people in the class. How many people are in the class? [9 marks]

3. (a) (i) 14 (ii) 27
(b) (i) 21 (ii) 17

4. 5005

5. (a) 35 (b) 15

6. 15 380 937

7. 36960

8. 31 500

9. $\binom{140}{12} \binom{128}{10} \binom{118}{10} = 1.62 \times 10^{45}$ (3SF)

10. (a) 43 680
(b) 65 520

11. (a) 35 (b) 35
(c) 31 (d) 33

12. (a) 126
(b) 120

13. 105

14. (a) 120
(b) 210

15. 24

16. $\binom{45}{15} \binom{30}{15} = 5.35 \times 10^{19}$ (3SF)

Exercise 1E

1. 560

2. 600

3. (a) 120
(b) 1320

4. (a) 4920
(b) 4800

5. 19557

6. 270200

7. 65 559

8. (a) 11082
(b) 48387

9. 696

Exercise 1F

1. (a) (i) 6 (ii) 5
(b) (i) 56 (ii) 110
(c) (i) 720 (ii) 1320

2. (i) 5040 (ii) 5040

3. (i) 60 (ii) 210

4. (a) (i) $n=7$ (ii) $n=10$
(b) (i) $n=11$ (ii) $n=14$

5. 7.75×10^{10} (3SF)

6. 255 024

7. 504

8. 336

9. 3 276 000

11. 186

12. 84

13. 4624

14. $n=3$

Exercise 1G

1. $13! \times 2 = 1.25 \times 10^{10}$

2. 2 488 320

3. 30 240

4. 150×10^{14}

5. (a) 32 432 400
(b) 45 360

6. (a) 17280
(b) 5760
(c) 43200
(d) 2880

Mixed examination practice 1

Short questions

1. 210
2. 120
3. 30 240
4. 729
5. 55
6. $n=5$
7. 8640
8. $n=15$
9. 2947
10. 480

11. 672
 12. 921 164 400
 13. 25 200
 14. 112

Long questions

1. (a) 48
 (b) 72
 (c) 42
2. (a) 20
 (b) 22
 (c) 30
3. (a) 121 080 960
 (b) 3 991 680
 (c) 27 941 760
4. (a) We select 2 out of 4 places to put R's in.
 (b) $\binom{2n}{n}$
 (c) 20
 (d) $\binom{n+m-2}{n-1}$
5. (b) 2047
 (c) 5775
6. (a) 4495
 (b) 22
 (c) 26

Chapter 2

Exercise 2A

1. (a) (i) 6^7 (ii) 5^8
 (b) (i) a^8 (ii) x^9
 (c) (i) 7^{-3} (ii) 5^5
 (d) (i) x^2 (ii) x^5
 (e) (i) g^{-12} (ii) k^{-8}
2. (a) (i) 6^1 (ii) 5^{-2}
 (b) (i) a^{-2} (ii) x^3
 (c) (i) 5^9 (ii) 7^{15}
 (d) (i) x^6 (ii) x^{-11}
- (e) (i) 2^2 (ii) 3^{-14}
 (f) (i) g^6 (ii) k^{-8}
3. (a) (i) 2^{12} (ii) 3^{14}
 (b) (i) 5^{-4} (ii) 7^{-6}
 (c) (i) 11^2 (ii) 13^{15}
 (d) (i) 2^{17} (ii) 3^3
 (e) (i) 6^{12} (ii) 3^6
4. (a) (i) 2^{10} (ii) 3^{14}
 (b) (i) 2^9 (ii) 2^{20}
 (c) (i) 2^{13} (ii) 3^4
 (d) (i) 2^9 (ii) 3^{11}
 (e) (i) 2^{-6} (ii) 3^{-6}
 (f) (i) 2^2 (ii) 3^{10}
5. (a) (i) $8x^6$ (ii) $9x^8$
 (b) (i) $2x^6$ (ii) $3x^8$
 (c) (i) $9a^{10}$ (ii) 16
 (d) (i) $\frac{1}{2x}$ (ii) $\frac{y^2}{9}$
 (e) (i) $\frac{2}{x}$ (ii) $3y^2$
 (f) (i) $\frac{5x^2y^4}{9}$ (ii) $\frac{ab^5}{8}$
 (g) (i) $\frac{p^3}{2q^2}$ (ii) $\frac{2^73^{10}}{x^7}$
6. (a) (i) x^3 (ii) x^{12}
 (b) (i) $2x^5$ (ii) $\frac{1}{2x^4}$
 (c) (i) $\frac{4}{3x^3}$ (ii) $\frac{y^{12}}{x^6}$
7. (a) (i) $\frac{5}{3}$ (ii) $-\frac{3}{2}$
 (b) (i) $-\frac{1}{2}$ (ii) $-\frac{3}{4}$
 (c) (i) 4 (ii) 2
 (d) (i) 4 (ii) 0
 (e) (i) 4 (ii) 11
 (f) (i) 3 (ii) 3
8. 5×10^{-4}
9. 8cm
10. (a) $k = \frac{1}{3}$
 (b) $A = 16 \text{ cm}^2$
11. $2^{350} = (2^7)^{50} = (128)^{50}$
 $5^{150} = (5^3)^{50} = (125)^{50}$
12. $b = 1, a = \frac{3}{2}$