Name:

Mathematics Analysis & Approaches
Higher level
Paper 1

October 6, 2020 (morning)

2 hours

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the space provided.
- Section B: answer all questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the Mathematics Analysis & Approaches formula booklet is required for this paper.
- The maximum mark for this examination paper is [**110 marks**].

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working

Section A

Answer **all** questions in the space provided.

1. [Maximum mark: 6]

The 2nd, 5th and 11th terms of an arithmetic sequence form the first three terms of a geometric sequence. If the first term of the arithmetic sequence is 4, determine the possible values of the common difference.

2. [Maximum mark: 6]

Consider a function $f(x)$ such that $f'(x) = ae^{2x}$, where a is a positive constant. Find $f(x)$ given that $f(\ln 3) = 26$ and $f(0) = 2$.

3. [Maximum mark: 6]

Consider a function $f(x)$ whose domain is all real numbers. The graph of $y = f(x)$ is symmetric in the y-axis. It is given that $\int_0^3 f(x)\ dx = 5$ and $\int_3^4 f(x)\ dx = -1$.

(a) Write down: [4]

(i) $\int_0^4 f(x)\ dx$

(ii) $\int_{-3}^3 f(x)\ dx$

(iii) $\int_0^3 4f(x)\ dx$

(iv) $\int_1^4 f(x-1)\ dx$

(b) Calculate $\int_0^3 f(x) + x\ dx$ [2]

4. [Maximum mark: 4]

Solve the equation:

$$16^x - 10 \times 4^x + 16 = 0$$

5. [Maximum mark: 5]

Consider the equation

$$2x^3 - 17x^2 + ax + b = 0$$

where $a, b \in \mathbb{R}$.

One solution to this equation is $x = 3 - 2i$.

(a) Determine the other two solutions. [2]

(b) Find the values of a and b. [3]

6. [Maximum mark: 5]

Solve the equation:

$$2\cos^2\theta = 11\sin\theta + 7$$

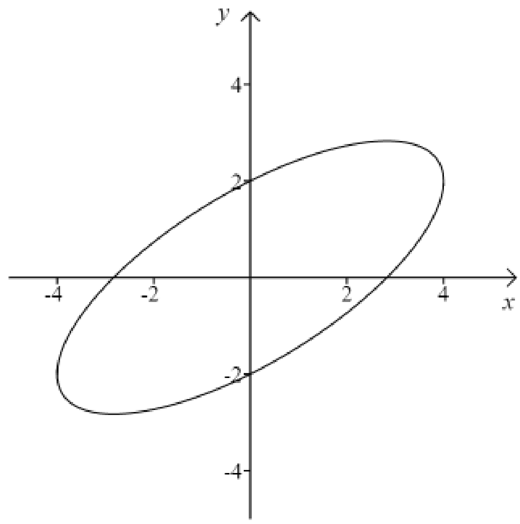
for $-2\pi \leqslant \theta \leqslant 2\pi$.

7. [Maximum mark: 6]

Consider two events A and B such that $P(A) = \frac{5}{12}$, $P(B') = \frac{2}{3}$ and $P(A|B') = 2P(A|B)$. Find $P(B|A)$.

8. [Maximum mark: 9]

The graph below shows the curve described by the equation $x^2 - 2xy + 2y^2 = 8$.



(a) Use implicit differentiation to find $\dfrac{dy}{dx}$. [3]

(b) Determine the exact coordinates of the points on the curve where the tangent is: [6]

(i) horizontal,

(ii) vertical.

9. [Maximum mark: 6]

Use l'Hopital's rule to eveluate

$$\lim_{x \to 0} \frac{x - \sin x}{x^3}$$

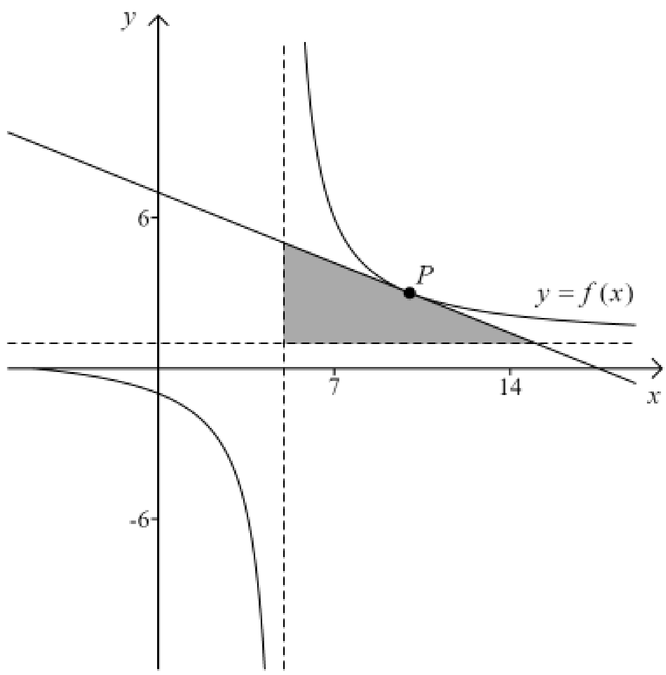
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 19]

Let $f(x) = \dfrac{x+5}{x+a}$, where $a \in \mathbb{Z}$.

The diagram below shows the graph of $y = f(x)$ and the tangent line to graph at point P which has x-coordinate equal to 10. The region between the tangent line and the two asymptotes has been shaded.



(a) Write down the equation of the: [2]

(i) horizontal asymptote,

(ii) vertical asymptote.

(b) Find $f'(x)$ [3]

(c) Write down the expression of the gradient of the tangent line in terms of a. [1]

(d) Show that the equation for the tangent line is: [3]

$$(5-a)x+(10+a)^2y-200-5a=0$$

(e) Determine the expression (in simplified form), in terms of a, for: [6]

(i) the x-coordinate of the point where the tangent intersects the horizontal asymptote,

(ii) the y-coordinate of the point where the tangent intersects the vertical asymptote.

(f) If the area of the shaded region is 20, calculate the value of a. [4]

11. [Maximum mark: 18]

Consider the function $f(x)=-(x-a)(x-3a)$, where a is a positive constant.

(a) Find the coordinates of the vertex of of the graph $y=f(x)$. [2]

(b) Sketch the graph of $y=f(x)$. [2]

(c) Find the expression for x in terms of y in both cases when $x \leqslant 2a$ and when $x \geqslant 2a$. [5]

(d) The region enclosed by the graph of $y=f(x)$ and the x-axis has been rotated through 2π around the y-axis. The resulting solid has an area of 18π. Find the value of a. [9]

12. [Maximum mark: 20]

Consider the function $f(x) = \dfrac{x^2 - x - 6}{x + 6}$.

(a) Find the coordinates of the: [3]

(i) the x-intercepts,

(ii) the y-intercept.

(b) Write down the equation of the vertical asymptote. [1]

(c) Find the equation of the oblique asymptote. [2]

(d) Find in simplified form: [4]

(i) $\dfrac{dy}{dx}$,

(ii) $\dfrac{d^2y}{dx^2}$.

(e) Hence find the coordinates of any maximum or minimum points. In each case justify why each point is a maximum or a minimum. [6]

(f) Explain why the graph has no points of inflexion. [1]

(g) Sketch the graph of $y = f(x)$ including all of the above information. [3]