Name:

Mathematics Analysis & Approaches
Higher level
Paper 2

October 7, 2020 (morning)

2 hours

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the space provided.
- Section B: answer all questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the Mathematics Analysis & Approaches formula booklet is required for this paper.
- The maximum mark for this examination paper is [**110 marks**].

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working

Section A

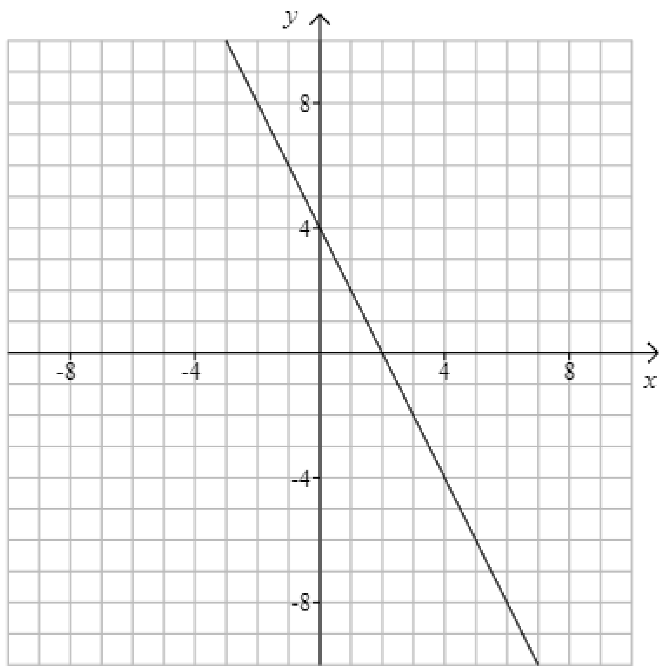
Answer **all** questions in the space provided.

1. [Maximum mark: 4]

Tomasz invests 1000PLN into an account that pays 7% annual interest rate compunded m times per year. At the end of the first year the investment is worth 1072.29PLN to two decimal places. Calculate the value of m, give your answer to the nearest integer.

2. [Maximum mark: 5]

The diagram below shows the graph of $y = f'(x)$.



Given that $f(1) = 5$ find the function $f(x)$.

3. [Maximum mark: 6]

The equation of the tangent to the graph of $y = ax^2 + bx + 3$ at $x = -1$ is $y = -7x + 1$. Find the values of a and b.

4. [Maximum mark: 4]

Describe the transformations that map the parabola $y = 2x^2 - 4x + 5$ onto the parabola $y = -x^2 - 2x - 6$.

5. [Maximum mark: 5]

The polynomial $P(x)$ gives remainder of 1 when divided by $x - 3$ and 5 when divided by $x + 1$. Find the remainder when $P(x)$ is divided by $x^2 - 2x - 3$.

6. [Maximum mark: 5]

The following table shows values of functions f and g and their derivatives for various values of x.

x	1	2	3	4	5
$f(x)$	5	3	1	2	4
$g(x)$	4	5	3	1	2
$f'(x)$	1	3	2	5	4
$g'(x)$	2	1	4	5	3

Let $p(x) = \dfrac{f(x)}{g(x)}$ and $q(x) = (f \circ g)(x)$.

Find:

(a) $p'(1)$, [2]

(b) $q'(2)$. [3]

7. [Maximum mark: 6]

Maria is standing some distance away from a building. The angle of elevation from that point to the top of the building is $20°$. She then moves 10 metres closer to the building and the angle of elevation changes to $30°$. Find the angle of elevation if she moves another 10 metres towards the building.

8. [Maximum mark: 8]

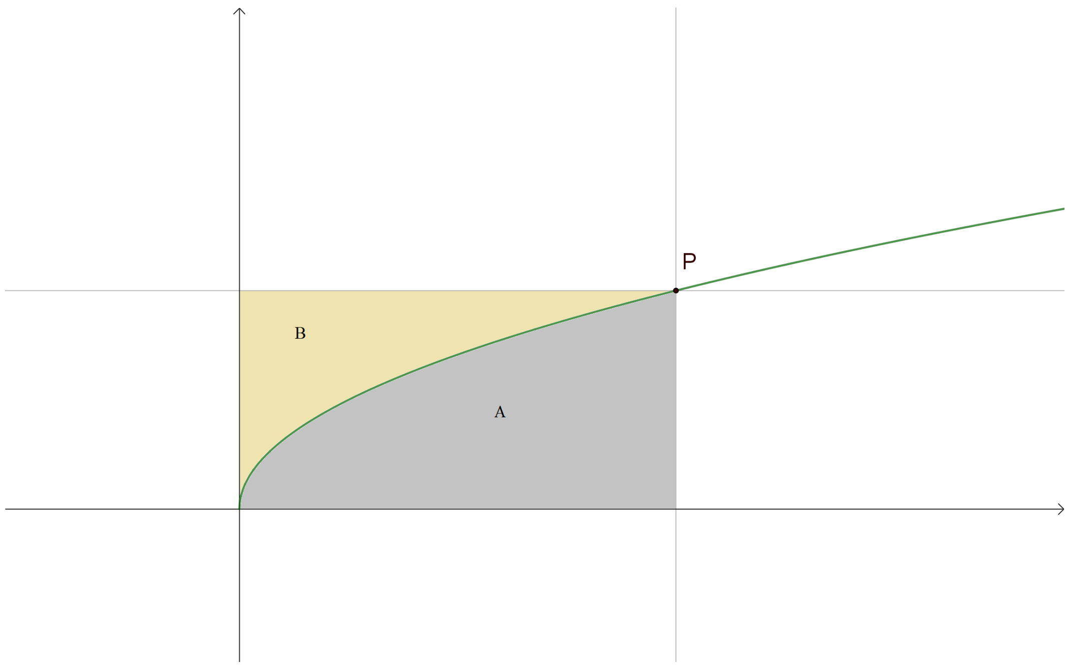
Consider the following polynomial:

$$P(x) = x^5 - 62x^4 + Ax^3 + Bx^2 + Cx - 32768$$

where $A, B, C \in \mathbb{R}$. Given that the five roots of this polynomial form a geometric sequence find all these roots.

9. [Maximum mark: 6]

Consider the curve $y = \sqrt{x}$. Find coordinates of the point P on this curve, so that the volume of solid formed by rotating the region A though 2π around the x-axis is equal to the volume of the solid formed by rotating the region B though 2π around the y-axis.



10. [Maximum mark: 8]

A bag contains 9 balls including a certain number blue balls. Maria and Tomasz play the following game. Maria picks one ball without replacement. Then Tomasz picks two balls without replacement. Tomasz wins if he has more blue balls than Maria. Find the number of blue balls if the probability that Tomasz wins is $\frac{65}{126}$.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

11. [Maximum mark: 17]

A particle moves in a straight line so that its velocity, in metres per second, is given by

$$v(t) = (e^x - 2)(x - 1) - 1$$

.

with $t \geqslant 0$. Initially the particle is at the origin.

(a) Find the initial velocity and acceleration of the particle. [3]

(b) Sketch the graph of $v(t)$ for $0 \leqslant t \leqslant 2.5$. [2]

(c) Find the time at which the particle changes direction. [2]

The particle comes back to the origin at time $t = a$ $(a > 0)$.

(d) Find the value of a. [4]

(e) Find the total distance travelled by the particle in the first a seconds. [3]

(f) Find the highest speed of the particle in the first a seconds. Justify your answer. [3]

12. [Maximum mark: 16]

Three cities A, B and C are connected with roads in such a way that the road AC is 2km long, the road BC is 3km long and they intersect at C at the angle of $50°$. A person walks from B to C with a constant speed of 4km/h.

(a) Find the rate at which the distance from that person to A is changing when she is midway between B and C. [5]

(b) Let P be the point where the person is at a given time. Find the rate at which the angle APC is changing when she is 1km away from C. [5]

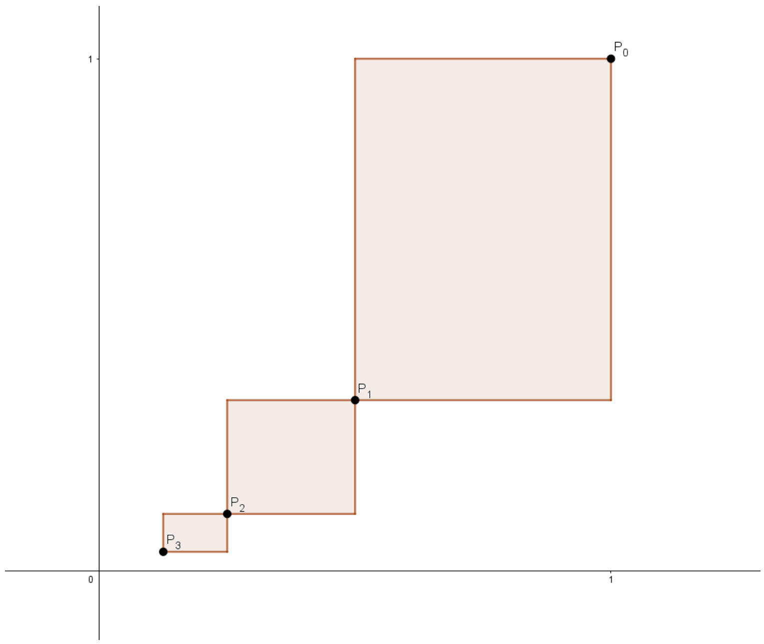
20 minutes after the first person began to walk, another person starts to walk from A to C with a constant speed of 3km/h.

(c) Find the rate at which the distance between them is changing when the second person began to walk. [6]

13. [Maximum mark: 20]

Consider a sequence of points $P_0(1,1),\ P_1(\frac{1}{2},\frac{1}{3}),\ P_2(\frac{1}{4},\frac{1}{9}),...,P_n(\frac{1}{2^n},\frac{1}{3^n}),....$

An infinite sequence of rectangles $R_1, R_2, R_3, ...$ is created, so thtat each rectangle R_n has sides parallel to x and y axes and two opposite vertices at points P_{n-1} and P_n - see diagram below:



(a) Calculate the area of rectangle R_1. [2]

(b) Show that the area of the n-th rectangle R_n is equal to $\dfrac{2}{6^n}$. [4]

(c) Calculate the total area of all the rectangles. [3]

An infinite sequence of circles is circumsribed on these rectangles, so that circle O_n circumsribes rectangle R_n.

(d) Find the area of O_1. [3]

(e) Find the total area of all these circles. [8]