1. A ray of light coming from the point $(-1, 3, 2)$ is travelling in the direction of vector $\begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix}$ and meets the plane $\pi : x + 3y + 2z - 24 = 0$.

Find the angle that the ray of light makes with the plane.

(Total 6 marks)

2. Find the angle between the lines $\frac{x-1}{2} = 1 - y = 2z$ and $x = y = 3z$.

(Total 6 marks)

3. Consider the planes defined by the equations $x + y + 2z = 2$, $2x - y + 3z = 2$ and $5x - y + az = 5$ where a is a real number.

(a) If $a = 4$ find the coordinates of the point of intersection of the three planes.

(2)

(b) (i) Find the value of a for which the planes do not meet at a unique point.

(ii) For this value of a show that the three planes do not have any common point.

(6)

(Total 8 marks)

4. Two lines are defined by

$$l_1 : \boldsymbol{r} = \begin{pmatrix} -3 \\ -4 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \text{ and } l_2 : \frac{x-4}{-3} = \frac{y+7}{4} = -(z+3).$$

(a) Find the coordinates of the point A on l_1 and the point B on l_2 such that $\overrightarrow{AB}$ is perpendicular to both l_1 and l_2.

(13)

(b) Find $|AB|$.

(3)

(c) Find the Cartesian equation of the plane Π that contains l_1 and does not intersect l_2.

(3)

(Total 19 marks)

5. The points A, B, C have position vectors $\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$, $\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$, $3\boldsymbol{i} + \boldsymbol{k}$ respectively and lie in the plane π.

(a) Find

(i) the area of the triangle ABC;

(ii) the shortest distance from C to the line AB;

(iii) the cartesian equation of the plane π.

(14)

The line L passes through the origin and is normal to the plane π, it intersects π at the point D.

(b) Find

(i) the coordinates of the point D;

(ii) the distance of π from the origin.

(6)
(Total 20 marks)

6. Consider the points A(1, −1, 4), B(2, − 2, 5) and O(0, 0, 0).

(a) Calculate the cosine of the angle between $\overrightarrow{OA}$ and $\overrightarrow{AB}$.

(5)

(b) Find a vector equation of the line L_1 which passes through A and B.

(2)

The line L_2 has equation $\boldsymbol{r} = 2\boldsymbol{i} + 4\boldsymbol{j} + 7\boldsymbol{k} + t(2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k})$, where $t \in \mathbb{R}$.

(c) Show that the lines L_1 and L_2 intersect and find the coordinates of their point of intersection.

(7)

(d) Find the Cartesian equation of the plane that contains both the line L_2 and the point A.

(6)
(Total 20 marks)

7. Find the vector equation of the line of intersection of the three planes represented by the following system of equations.

$$\begin{aligned} 2x - 7y + 5z &= 1 \\ 6x + 3y - z &= -1 \\ -14x - 23y + 13z &= 5 \end{aligned}$$

(Total 6 marks)

8. (a) Write the vector equations of the following lines in parametric form.

$$\boldsymbol{r}_1 = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + m\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$$

$$\boldsymbol{r}_2 = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} + n\begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}$$

(2)

(b) Hence show that these two lines intersect and find the point of intersection, A.

(5)

(c) Find the Cartesian equation of the plane Π that contains these two lines.

(4)

(d) Let B be the point of intersection of the plane Π and the line $\boldsymbol{r} = \begin{pmatrix} -8 \\ -3 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ 8 \\ 2 \end{pmatrix}$.

Find the coordinates of B.

(4)

(e) If C is the mid-point of AB, find the vector equation of the line perpendicular to the plane Π and passing through C.

(3)

(Total 18 marks)