1. Consider the plane with equation $4x - 2y - z = 1$ and the line given by the parametric equations

$$x = 3 - 2\lambda$$
$$y = (2k - 1) + \lambda$$
$$z = -1 + k\lambda.$$

Given that the line is perpendicular to the plane, find

(a) the value of k;

(4)

(b) the coordinates of the point of intersection of the line and the plane.

(4)
(Total 8 marks)

2. The points A(1, 2, 1), B(–3, 1, 4), C(5, –1, 2) and D(5, 3, 7) are the vertices of a tetrahedron.

(a) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(2)

(b) Find the Cartesian equation of the plane Π that contains the face ABC.

(4)

(c) Find the vector equation of the line that passes through D and is perpendicular to Π. Hence, or otherwise, calculate the shortest distance to D from Π.

(5)

(d) (i) Calculate the area of the triangle ABC.

(ii) Calculate the volume of the tetrahedron ABCD.

(4)

(e) Determine which of the vertices B or D is closer to its opposite face.

(4)
(Total 19 marks)

3. The equations of three planes, are given by

$$\begin{aligned} ax + 2y + z &= 3 \\ -x + (a + 1)y + 3z &= 1 \\ -2x + y + (a + 2)z &= k \end{aligned}$$

where $a \in \mathbb{R}$.

(a) Given that $a = 0$, show that the three planes intersect at a point.

(3)

(b) Find the value of a such that the three planes do **not** meet at a point.

(5)

(c) Given a such that the three planes do **not** meet at a point, find the value of k such that the planes meet in one line and find an equation of this line in the form

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \lambda \begin{pmatrix} l \\ m \\ n \end{pmatrix}.$$

(6)
(Total 14 marks)

4. The points P(–1, 2, –3), Q(–2, 1, 0), R(0, 5, 1) and S form a parallelogram, where S is diagonally opposite Q.

(a) Find the coordinates of S.

(2)

(b) The vector product $\overrightarrow{PQ} \times \overrightarrow{PS} = \begin{pmatrix} -13 \\ 7 \\ m \end{pmatrix}$. Find the value of m.

(2)

(c) Hence calculate the area of parallelogram PQRS.

(2)

(d) Find the Cartesian equation of the plane, Π_1, containing the parallelogram PQRS.

(3)

(e) Write down the vector equation of the line through the origin (0, 0, 0) that is perpendicular to the plane Π_1.

(1)

(f) Hence find the point on the plane that is closest to the origin.

(3)

(g) A second plane, Π_2, has equation $x - 2y + z = 3$. Calculate the angle between the two planes.

(4)
(Total 17 marks)

5. (a) Show that the two planes

$$\pi_1 : x + 2y - z = 1$$
$$\pi_2 : x + z = -2$$

are perpendicular.

(3)

(b) Find the equation of the plane π_3 that passes through the origin and is perpendicular to both π_1 and π_2.

(4)
(Total 7 marks)

6. The three planes

$$2x - 2y - z = 3$$
$$4x + 5y - 2z = -3$$
$$3x + 4y - 3z = -7$$

intersect at the point with coordinates (a, b, c).

(a) Find the value of each of a, b and c.

(2)

(b) The equations of three other planes are

$$2x - 4y - 3z = 4$$
$$-x + 3y + 5z = -2$$
$$3x - 5y - z = 6.$$

Find a vector equation of the line of intersection of these three planes.

(4)
(Total 6 marks)

7. (a) Show that a Cartesian equation of the line, l_1, containing points A(1, –1, 2) and B(3, 0, 3) has the form $\frac{x-1}{2}=\frac{y+1}{1}=\frac{z-2}{1}$.

(2)

(b) An equation of a second line, l_2, has the form $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{1}$. Show that the lines l_1 and l_2 intersect, and find the coordinates of their point of intersection.

(5)

(c) Given that direction vectors of l_1 and l_2 are $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$ respectively, determine $\boldsymbol{d}_1 \times \boldsymbol{d}_2$.

(3)

(d) Show that a Cartesian equation of the plane, Π, that contains l_1 and l_2 is $-x - y + 3z = 6$.

(3)

(e) Find a vector equation of the line l_3 which is perpendicular to the plane Π and passes through the point T(3, 1, –4).

(2)

(f) (i) Find the point of intersection of the line l_3 and the plane Π.

(ii) Find the coordinates of T′, the reflection of the point T in the plane Π.

(iii) Hence find the magnitude of the vector $\overrightarrow{\mathrm{TT'}}$.

(7)
(Total 22 marks)