1. The line L is given by the parametric equations $x = 1 - \lambda$, $y = 2 - 3\lambda$, $z = 2$.
Find the coordinates of the point on L that is nearest to the origin.

(Total 6 marks)

2. (a) Show that the following system of equations will have a unique solution when $a \neq -1$.

$$\begin{aligned} x + 3y - z &= 0 \\ 3x + 5y - z &= 0 \\ x - 5y + (2 - a)z &= 9 - a^2 \end{aligned}$$

(5)

(b) State the solution in terms of a.

(6)

(c) Hence, solve

$$\begin{aligned} x + 3y - \quad z &= 0 \\ 3x + 5y - z &= 0 \\ x - 5y + z &= 8 \end{aligned}$$

(2)

(Total 13 marks)

3. Consider the points A(1, 2, 1), B(0, –1, 2), C(1, 0, 2) and D(2, –1, –6).

(a) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

(2)

(b) Calculate $\overrightarrow{AB} \times \overrightarrow{BC}$.

(2)

(c) Hence, or otherwise find the area of triangle ABC.

(3)

(d) Find the Cartesian equation of the plane P containing the points A, B and C.

(3)

(e) Find a set of parametric equations for the line *L* through the point D and perpendicular to the plane *P*.

(3)

(f) Find the point of intersection E, of the line *L* and the plane *P*.

(4)

(g) Find the distance from the point D to the plane *P*.

(2)

(h) Find a unit vector that is perpendicular to the plane *P*.

(2)

(i) The point F is a reflection of D in the plane *P*. Find the coordinates of F.

(4)
(Total 25 marks)

4. (a) Show that lines $\frac{x-2}{1}=\frac{y-2}{3}=\frac{z-3}{1}$ and $\frac{x-2}{1}=\frac{y-3}{4}=\frac{z-4}{2}$ intersect and find the coordinates of P, the point of intersection.

(8)

(b) Find the Cartesian equation of the plane Π that contains the two lines.

(6)

(c) The point Q(3, 4, 3) lies on Π. The line *L* passes through the midpoint of [PQ]. Point S is on *L* such that $\left|\overrightarrow{PS}\right|=\left|\overrightarrow{QS}\right| = 3$, and the triangle PQS is normal to the plane Π. Given that there are two possible positions for S, find their coordinates.

(15)
(Total 29 marks)