13 $3\sin x - 5\cos x \approx \sqrt{34}\cos(x + 3.68)$

14 **a** $2\sin x + \sqrt{3}\cos x \approx \sqrt{7}\sin(x + 0.714)$

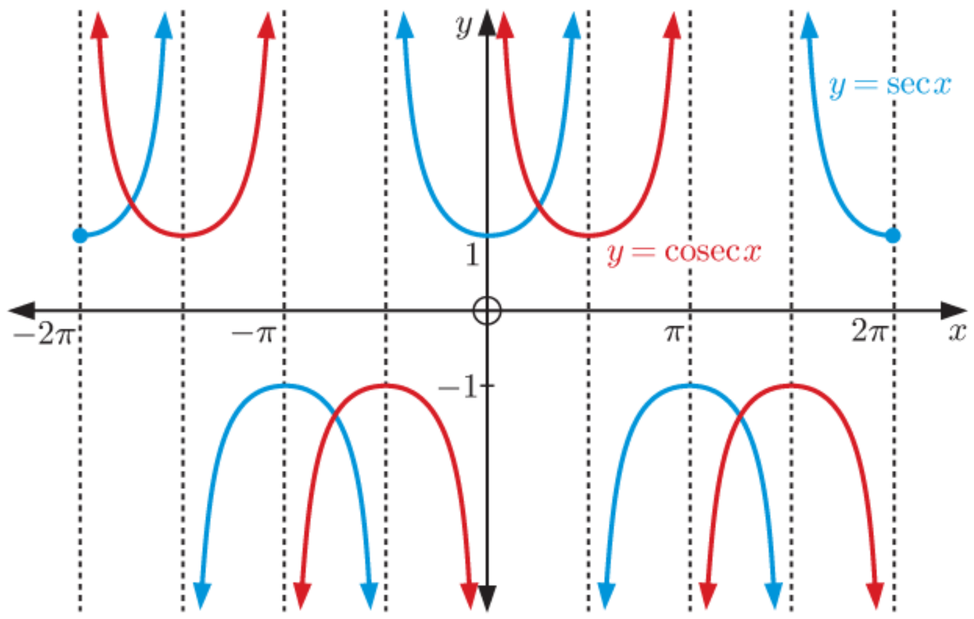
b **i** $A = \sqrt{7}$ **ii** $b \approx 2.43$

15 $\frac{\pi}{4}$

REVIEW SET 1B

1 $\sin x = -\frac{2\sqrt{2}}{3}$, $\tan x = 2\sqrt{2}$, $\operatorname{cosec} x = -\frac{3}{2\sqrt{2}}$, $\sec x = -3$, $\cot x = \frac{1}{2\sqrt{2}}$

2 **a**



b translation $\frac{\pi}{2}$ units right

3 $x = -\frac{5\pi}{6}$ or $\frac{\pi}{6}$ **4** **a** $x = \frac{\sqrt{3}}{2}$ **b** $x = 2 + \frac{1}{\sqrt{3}}$

5 **a** $\sec x$ **b** $\sin x$ **c** $\cos x$

6 **a** $\cos\theta$ **b** $-\sin\theta$ **c** $5\cos^2\theta$ **d** $-\cos\theta$
e $\operatorname{cosec}\theta$ **f** $\sin 2\theta$

7 **a** $\frac{120}{169}$ **b** $\frac{119}{169}$ **c** $\frac{120}{119}$

10 **a** $x = -\frac{2\pi}{3}, -\frac{\pi}{2}, -\frac{\pi}{3}$, or $\frac{\pi}{2}$ **b** $\theta = \frac{\pi}{3}$

11 $\sin\left(\theta + \frac{\pi}{6}\right) = \frac{3\sqrt{3}-\sqrt{7}}{8}$ **12** $\tan\theta = \frac{9}{19}$

13 $3\sin x + 4\cos x \approx 5\sin(x + 0.927)$ **14** 1.5 m

15 **b** $y = 2\sec 2x$ has range $\{y \mid y \leqslant -2 \text{ or } y \geqslant 2\}$

$\therefore \quad \frac{1}{1+\sqrt{2}\sin x} + \frac{1}{1-\sqrt{2}\sin x} = 1$ has no solutions.

EXERCISE 2A

1 **a** $2^{\frac{1}{5}}$ **b** $2^{-\frac{1}{5}}$ **c** $2^{\frac{3}{2}}$ **d** $2^{\frac{5}{2}}$ **e** $2^{-\frac{1}{3}}$
f $2^{\frac{4}{3}}$ **g** $2^{\frac{3}{2}}$ **h** $2^{\frac{3}{2}}$ **i** $2^{-\frac{4}{3}}$ **j** $2^{-\frac{3}{2}}$

2 **a** $3^{\frac{1}{3}}$ **b** $3^{-\frac{1}{3}}$ **c** $3^{\frac{1}{4}}$ **d** $3^{\frac{3}{2}}$ **e** $3^{-\frac{5}{2}}$

3 **a** $7^{\frac{1}{3}}$ **b** $3^{\frac{3}{4}}$ **c** $2^{\frac{4}{5}}$ **d** $2^{\frac{5}{3}}$ **e** $7^{\frac{2}{7}}$
f $7^{-\frac{1}{3}}$ **g** $3^{-\frac{3}{4}}$ **h** $2^{-\frac{4}{5}}$ **i** $2^{-\frac{5}{3}}$ **j** $7^{-\frac{2}{7}}$

4 **a** $x^{\frac{1}{2}}$ **b** $x^{\frac{3}{2}}$ **c** $x^{-\frac{1}{2}}$ **d** $x^{\frac{5}{2}}$ **e** $x^{-\frac{3}{2}}$

5 **a** ≈ 2.28 **b** ≈ 0.435 **c** ≈ 1.68 **d** ≈ 1.93
e ≈ 0.523

6 **a** $\sqrt[3]{5}$ **b** $\frac{1}{\sqrt{3}}$ **c** $9\sqrt{3}$ **d** $m\sqrt{m}$ **e** $x^3\sqrt{x}$

7 **a** 8 **b** 32 **c** 8 **d** 125 **e** 4
f $\frac{1}{2}$ **g** $\frac{1}{27}$ **h** $\frac{1}{16}$ **i** $\frac{1}{81}$ **j** $\frac{1}{25}$

EXERCISE 2B

1 **a** 1 **b** x **c** $x^{\frac{1}{2}}$ or $\sqrt{x}$

2 **a** $x^5 + 2x^4 + x^2$ **b** $2^{2x} + 2^x$ **c** $x + 1$
d $7^{2x} + 2(7^x)$ **e** $2(3^x) - 1$ **f** $x^2 + 2x + 3$
g $1 + 5(2^{-x})$ **h** $5^x + 1$ **i** $x^{\frac{3}{2}} + x^{\frac{1}{2}} + 1$
j $3^{2x} + 5(3^x) + 1$ **k** $2x^{\frac{3}{2}} - x^{\frac{1}{2}} + 5$ **l** $2^{3x} - 3(2^{2x}) - 1$

3 **a** $2^{2x} + 2^{x+1} - 3$ **b** $3^{2x} + 7(3^x) + 10$
c $5^{2x} - 6(5^x) + 8$ **d** $2^{2x} + 6(2^x) + 9$
e $3^{2x} - 2(3^x) + 1$ **f** $4^{2x} + 14(4^x) + 49$
g $x - 4$ **h** $4^x - 9$ **i** $x - \frac{1}{x}$ **j** $x^2 + 4 + \frac{4}{x^2}$
k $7^{2x} - 2 + 7^{-2x}$ **l** $25 - 10(2^{-x}) + 2^{-2x}$

4 **a** $5^x(5^x + 1)$ **b** $10(3^n)$ **c** $7^n(1 + 7^{2n})$
d $5(5^n - 1)$ **e** $6(6^{n+1} - 1)$ **f** $16(4^n - 1)$
g $2^n(2^n - 8)$ **h** $\frac{5}{2}(2^n)$ **i** $\frac{9}{2}(2^{2n})$

5 **a** $(3^x + 2)(3^x - 2)$ **b** $(2^x + 5)(2^x - 5)$
c $(4 + 3^x)(4 - 3^x)$ **d** $(5 + 2^x)(5 - 2^x)$
e $(3^x + 2^x)(3^x - 2^x)$ **f** $(2^x + 3)^2$
g $(3^x + 5)^2$ **h** $(2^x - 7)^2$ **i** $(5^x - 2)^2$

6 **a** $(2^x + 1)(2^x - 2)$ **b** $(3^x + 3)(3^x - 2)$
c $(2^x - 3)(2^x - 4)$ **d** $(2^x + 3)(2^x + 6)$
e $(2^x + 4)(2^x - 5)$ **f** $(3^x + 2)(3^x + 7)$
g $(3^x + 5)(3^x - 1)$ **h** $(5^x + 2)(5^x - 1)$
i $(7^x - 4)(7^x - 3)$

7 **a** 2^n **b** 10^a **c** 3^b **d** $\frac{1}{5^n}$ **e** 5^x
f $(\frac{3}{4})^a$ **g** $(\frac{8}{3})^k$ **h** 5 **i** 5^n

8 **a** $3^m + 1$ **b** $1 + 6^n$ **c** $4^n + 2^n$ **d** $4^x - 1$
e 6^n **f** 5^n **g** 4 **h** $2^n - 1$ **i** $\frac{1}{2}$

9 **a** $n\,2^{n+1}$ **b** -3^{n-1}

EXERCISE 2C

1 **a** $x = 5$ **b** $x = 2$ **c** $x = 4$ **d** $x = 0$
e $x = -1$ **f** $x = \frac{1}{2}$ **g** $x = -3$ **h** $x = 2$
i $x = -3$ **j** $x = -4$ **k** $x = 2$ **l** $x = \frac{3}{4}$

2 **a** $x = \frac{5}{3}$ **b** $x = -\frac{3}{2}$ **c** $x = -\frac{3}{2}$ **d** $x = -\frac{1}{2}$
e $x = -\frac{2}{3}$ **f** $x = -\frac{5}{4}$ **g** $x = \frac{3}{2}$ **h** $x = \frac{5}{2}$
i $x = \frac{1}{8}$ **j** $x = \frac{9}{2}$ **k** $x = -4$ **l** $x = -\frac{7}{2}$
m $x = 0$ **n** $x = \frac{7}{2}$ **o** $x = -\frac{2}{3}$ **p** $x = -6$

3 **a** $x = \frac{1}{7}$ **b** no solution **c** $x = \frac{5}{2}$
d $x = \frac{1}{3}$ **e** $x = -\frac{1}{4}$ **f** $x = -1$ or 3

4 **a** $x = 3$ **b** $x = 2$ **c** $x = -1$ **d** $x = 2$
e $x = -2$ **f** $x = -2$

5 **a** $x = 1$ or 2 **b** $x = 1$ **c** $x = 1$ or 2
d $x = 1$ **e** $x = 2$ **f** $x = 0$
g $x = 1$ **h** $x = 1$ or -1 **i** $x = 2$
j $x = -2$ or 1 **k** $x = 2$ **l** $x = \frac{1}{2}$

6 $x = \frac{15}{7}$, $y = \frac{10}{7}$

EXERCISE 2D

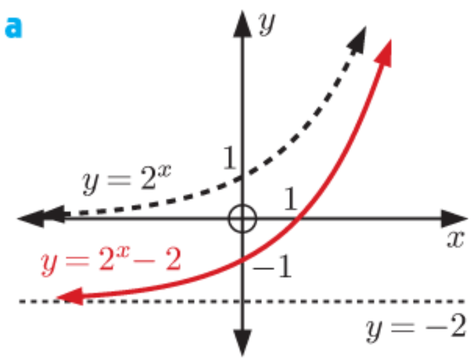
1 **a** **i** ≈ 1.4 **ii** ≈ 1.7 **iii** ≈ 2.8 **iv** ≈ 0.4

b **i** $x \approx 1.6$ **ii** $x \approx -0.7$

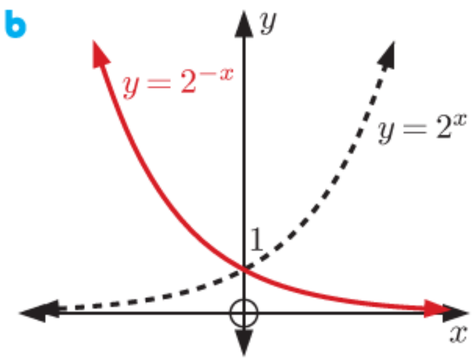
c $y = 2^x$ has a horizontal asymptote of $y = 0$.

2 **a** **C** **b** **B** **c** **E** **d** **A** **e** **D**

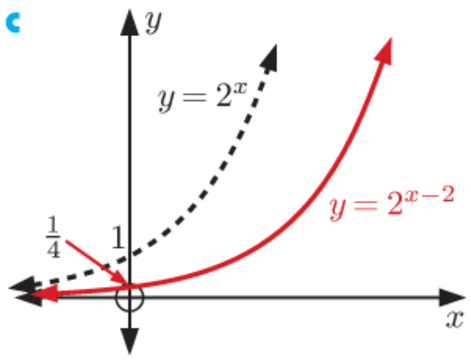
3 **a**



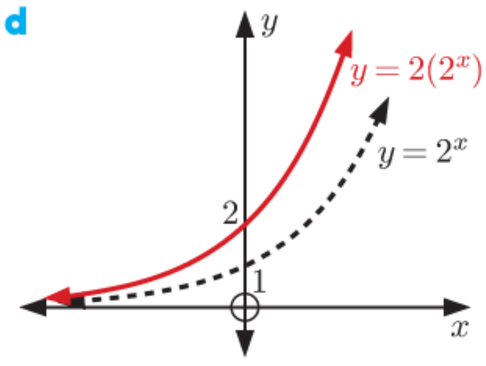
b



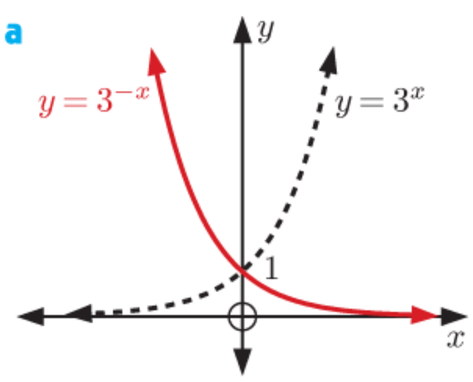
c



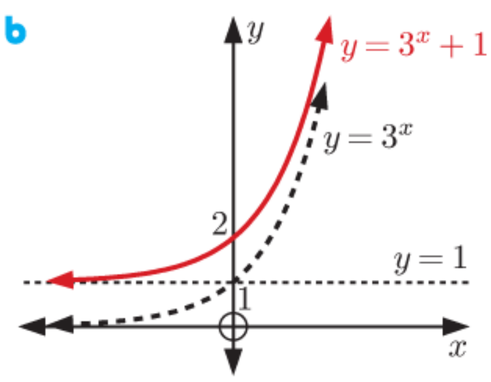
d



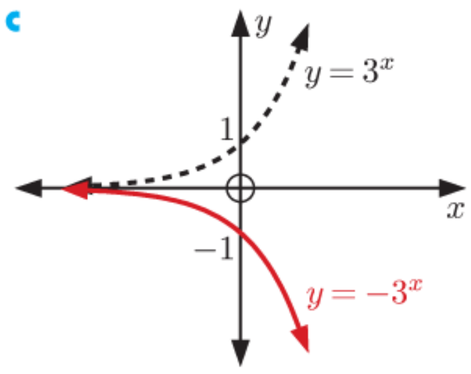
4 **a**



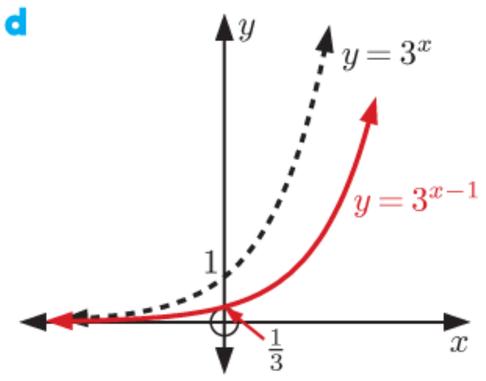
b



c



d

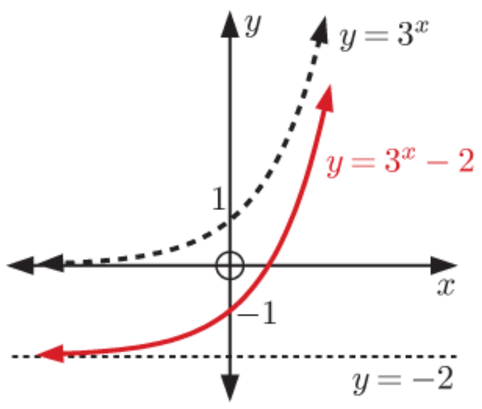


5 **a** $y = 0$ **b** $y = -1$ **c** $y = 3$ **d** $y = 2$
e $y = 0$ **f** $y = -4$

6 **a** **i** -1 **ii** 7
iii $-\frac{17}{9} = -1\frac{8}{9}$

b $y = -2$

c

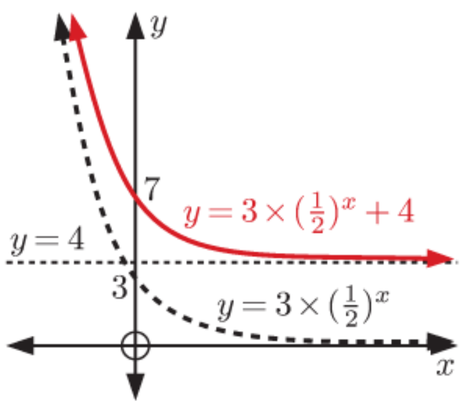


d Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y > -2\}$

7 **a** **i** 7
ii $\frac{19}{4} = 4\frac{3}{4}$
iii 16

b $y = 4$

c

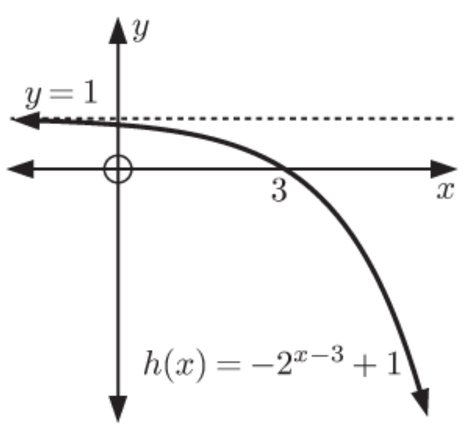


d Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y > 4\}$

8 **a** **i** $\frac{7}{8}$ **ii** 0
iii -7

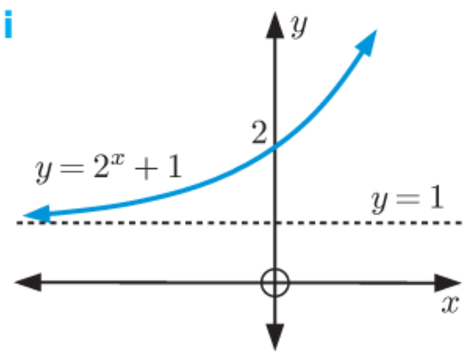
b $y = 1$

c



d Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y < 1\}$

9 **a** **i**



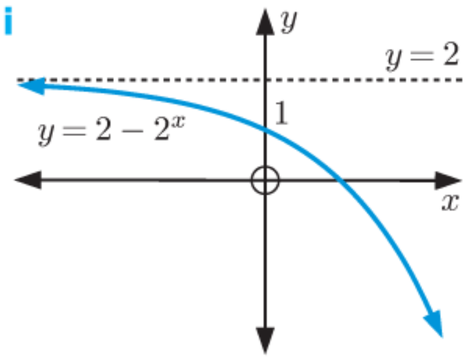
ii Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y > 1\}$

iii $y \approx 3.67$

iv as $x \to \infty$, $y \to \infty$
as $x \to -\infty$, $y \to 1^+$

v $y = 1$

b **i**



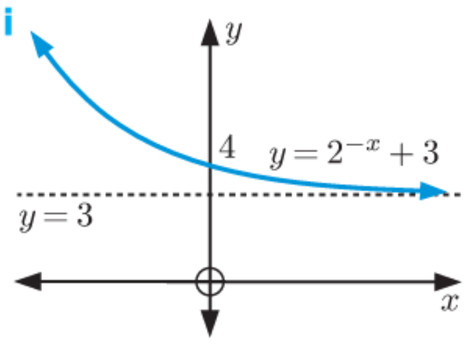
ii Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y < 2\}$

iii $y \approx -0.665$

iv as $x \to \infty$, $y \to -\infty$
as $x \to -\infty$, $y \to 2^-$

v $y = 2$

c **i**



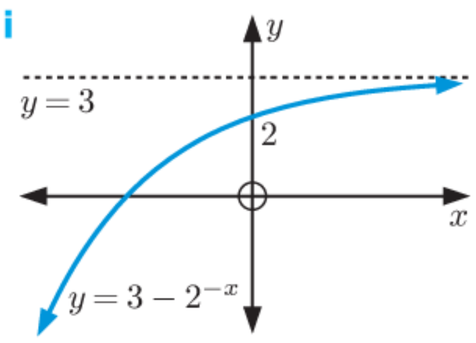
ii Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y > 3\}$

iii $y \approx 3.38$

iv as $x \to \infty$, $y \to 3^+$
as $x \to -\infty$, $y \to \infty$

v $y = 3$

d **i**



ii Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y < 3\}$

iii $y \approx 2.62$

iv as $x \to \infty$, $y \to 3^-$
as $x \to -\infty$, $y \to -\infty$

v $y = 3$

10 **a** $a = 5,\ b = -10$ **b** $y = 310$

11 **a** P$(0, 2.5)$ **b** $a = 1.5$ **c** $y = 3.5$

12 **a** Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid y \geqslant 2\}$
b Domain is $\{x \mid x \neq 0\}$, Range is $\{y \mid y > 0,\ y < -1\}$
c Domain is $\{x \mid x \geqslant 1\}$, Range is $\{y \mid y \geqslant 0\}$

13 **a** $(f \circ g)(x) = 3^{\sqrt{x}} - 9$
Domain is $\{x \mid x \geqslant 0\}$, Range is $\{y \mid y \geqslant -8\}$

b $(g \circ f)(x) = \sqrt{3^x - 9}$
Domain is $\{x \mid x \geqslant 2\}$, Range is $\{y \mid y \geqslant 0\}$

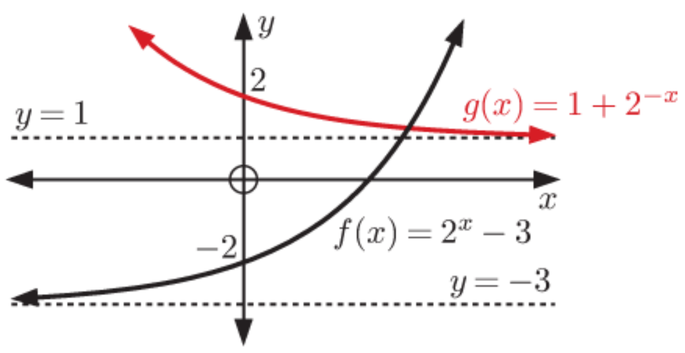
c **i** $x = 4$ **ii** $x = 3$

14 **a** **i** $f(x)$: $y = -3$, $g(x)$: $y = 1$

ii $f(x)$: Range is $\{y \mid y > -3\}$
$g(x)$: Range is $\{y \mid y > 1\}$

iii $f(x)$: y-intercept -2, $g(x)$: y-intercept 2

b

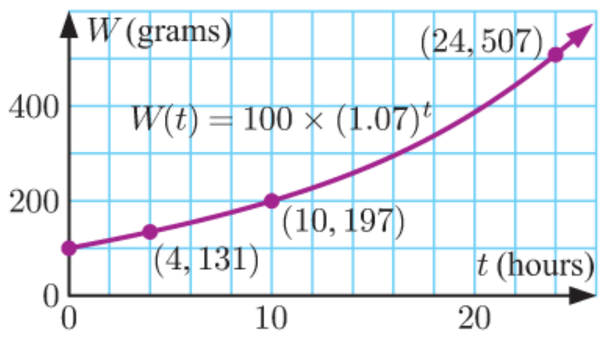


c $-1 + \sqrt{5}$

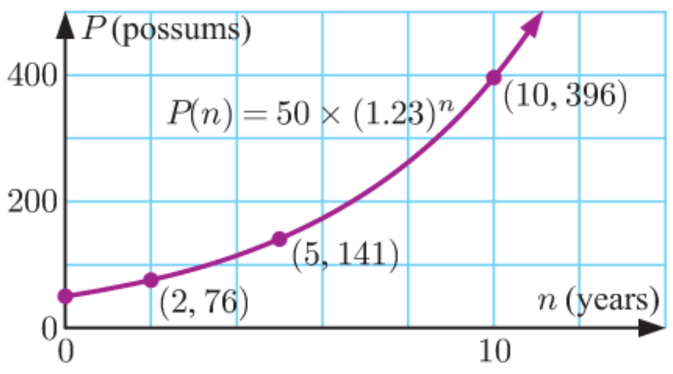
15 **a** $x \approx 3.46$ **b** $x \approx 2.46$ **c** $x \approx 1.16$
d $x \approx -0.738$ **e** $x \approx 1.85$ **f** $x \approx 0.0959$
g $x \approx 6.03$ **h** $x \approx 50.0$ **i** $x \approx 31.0$

EXERCISE 2E.1

1 **a** 100 grams
b **i** ≈ 131 g **ii** ≈ 197 g **iii** ≈ 507 g
c



2 **a** $P_0 = 50$
b **i** ≈ 76 possums **ii** ≈ 141 possums
iii ≈ 396 possums
c



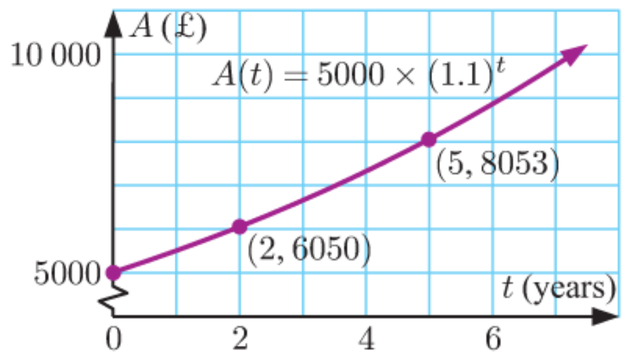
d ≈ 11 years **e** ≈ 11.1 years

3 **a** 4 people **b** ≈ 393 people **c** ≈ 19.9 days

4 **a** $B_0 = 200$
b $a = 1.1$, the bear population is increasing by 10% every year.
c ≈ 1350 bears **d** $\approx 159\%$ increase **e** ≈ 24.2 years

5 **a** **i** V_0 **ii** $2V_0$ **b** 100%
c $\approx 183\%$ increase, it is the percentage increase at 50°C compared with 20°C.

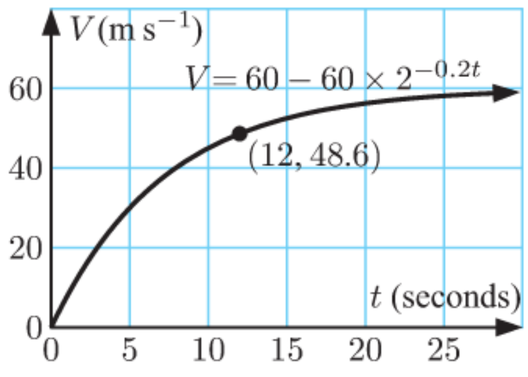
6 **a** $A(t) = 5000 \times (1.1)^t$ **b** **i** £6050 **ii** £8052.55
c



d ≈ 4.93 years

7 **a** $a = 1.08$, the expected value of the house is increasing by 8% per year.
$k = 375\,000$, the original value of the house was \$375 000.
b ≈ 4.98 years

8 **a** When $t = 0$, $V = c - 60 = 0$
$\therefore \quad c = 60$
b $k = -\frac{1}{5} = -0.2$
c $\approx 48.6 \text{ m s}^{-1}$ **d**

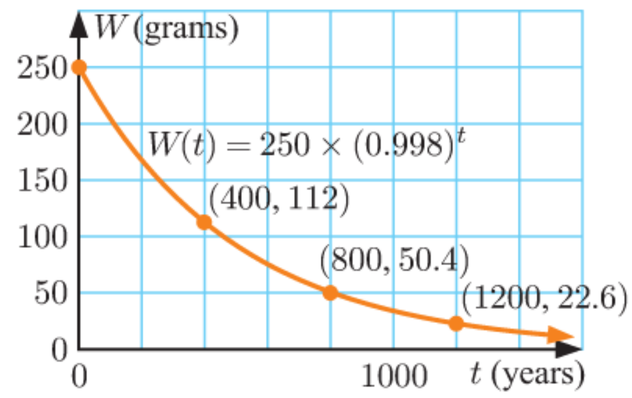


e The parachutist accelerates rapidly until he approaches his terminal velocity of 60 m s^{-1}.

9 ≈ 2.27 hours

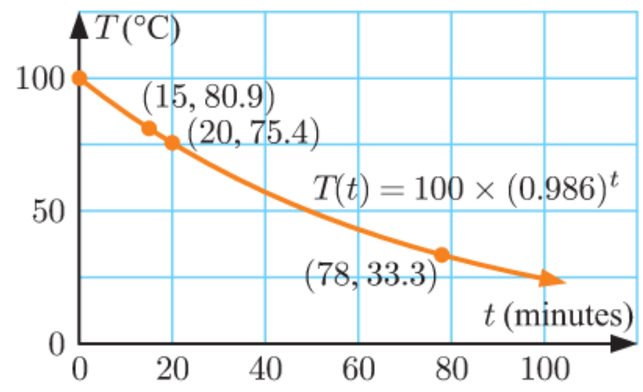
EXERCISE 2E.2

1 **a** 250 g **b** **i** ≈ 112 g **ii** ≈ 50.4 g **iii** ≈ 22.6 g
c

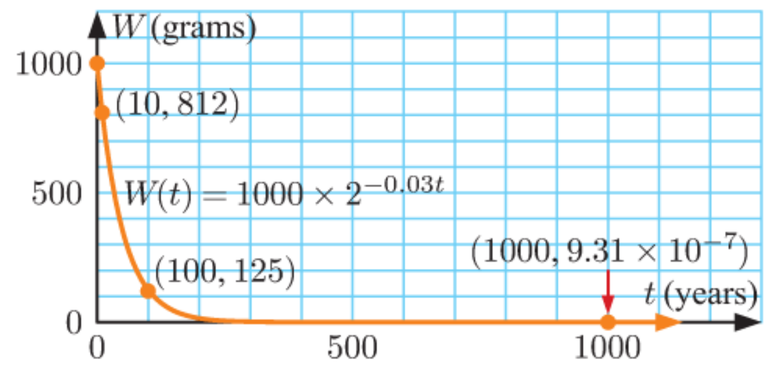


d ≈ 346 years

2 **a** 100°C
b **i** ≈ 80.9°C **ii** ≈ 75.4°C **iii** ≈ 33.3°C
c

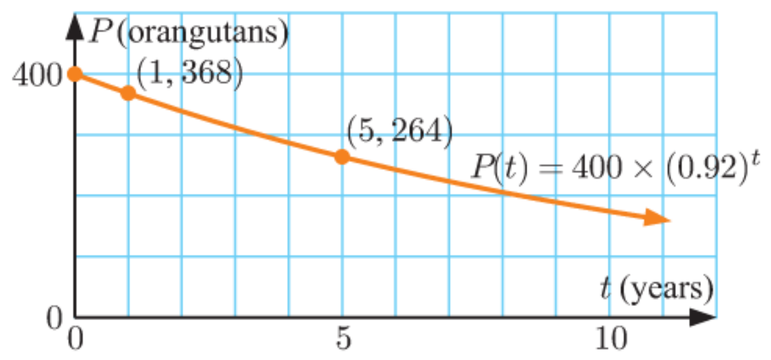


3 **a** 1000 g
b **i** ≈ 812 g **ii** 125 g **iii** $\approx 9.31 \times 10^{-7}$ g
c



d ≈ 221 years **e** $1000(1 - 2^{-0.03t})$ grams

4 **a** $P(t) = 400 \times (0.92)^t$
b **i** 368 orangutans **ii** ≈ 264 orangutans
c

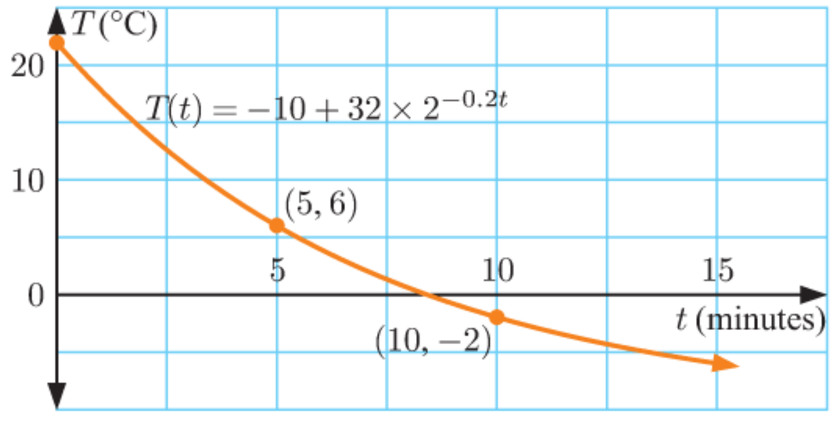


d ≈ 8.31 years, or ≈ 8 years 114 days

5 **a** $L_0 = 10$ units **b** ≈ 2.77 units **c** ≈ 17.9 m
d between ≈ 23.5 m and ≈ 44.9 m

6 **a** \$24 000 **b** $r = 0.85$ **c** 7 years

7 **a** **i** 22°C **ii** 6°C **iii** −2°C
b



c ≈ 8.39 min or ≈ 8 min 23 s
d No, as $32 \times 2^{-0.2t} > 0$ for any value of t.

8 **a** W_0 **b** $\approx 12.9\%$ **c** 45 000 years

9 **a** $A(t) = 150 \times (1.48)^{\frac{t}{3}}$, $B(t) = 400 \times (0.8)^t + 100$
b **i** $t \approx 4.16$ years **ii** $t \approx 3.45$ years **iii** $t \approx 1.69$ years

10 **a** The initial weight of the isotope is 10 mg.

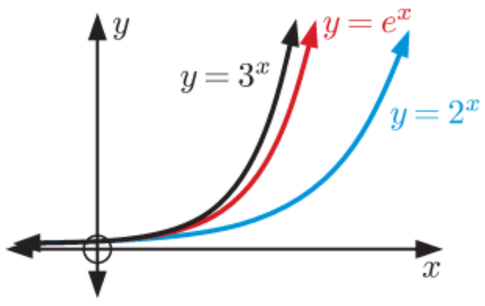
b $a \approx 0.7937$; each day the isotope's weight is decreasing by $\approx 20.63\%$.

c ≈ 6.30 mg **d** **i** ≈ 5.21 days **ii** ≈ 9.00 days

11 ≈ 33.2 minutes or ≈ 33 minutes 13.2 seconds

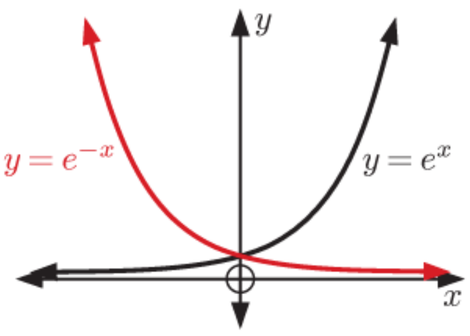
EXERCISE 2F

1



The graph of $y = e^x$ lies between $y = 2^x$ and $y = 3^x$.

2



One is the other reflected in the y-axis.

3 p

4 **a** $e^x > 0$ for all x

b **i** $y \approx 4.12 \times 10^{-9}$ **ii** $y \approx 9.70 \times 10^8$

5 **a** ≈ 7.39 **b** ≈ 20.1 **c** ≈ 2.01 **d** ≈ 1.65 **e** ≈ 0.368

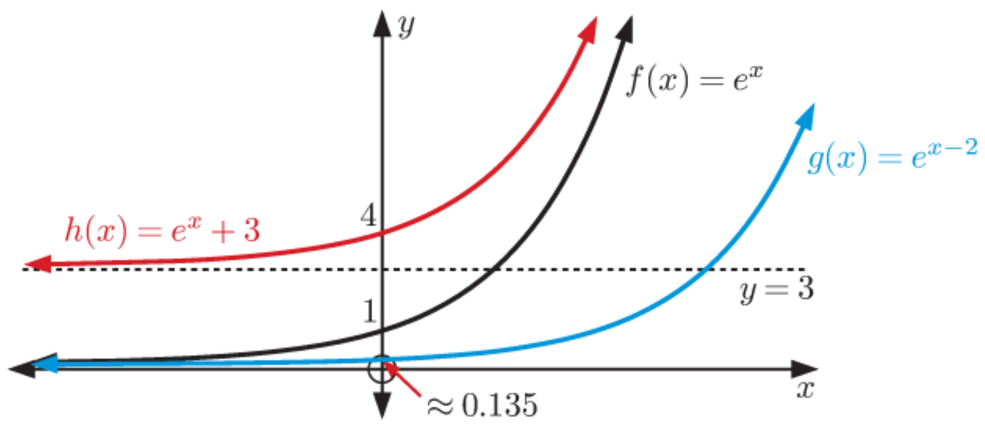
6 **a** $e^{\frac{1}{2}}$ **b** $e^{-\frac{1}{2}}$ **c** e^{-2} **d** $e^{\frac{3}{2}}$

7 **a** ≈ 10.074 **b** $\approx 0.099\,261$ **c** ≈ 125.09
d $\approx 0.007\,994\,5$ **e** ≈ 41.914 **f** ≈ 42.429
g ≈ 3540.3 **h** $\approx 0.006\,342\,4$

8 **a** $e^{2x} + 2e^x + 1$ **b** $1 - e^{2x}$ **c** $1 - 3e^x$

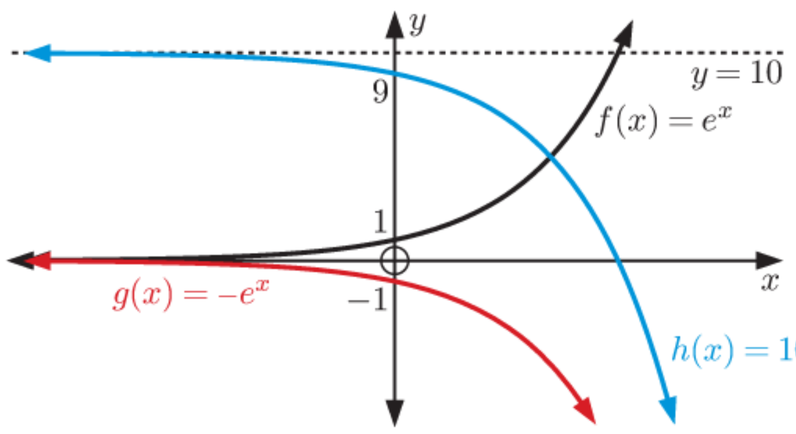
9 **a** $e^x(e^x + 1)$ **b** $(e^x + 4)(e^x - 4)$ **c** $(e^x - 6)(e^x - 2)$

10 **a**



b Domain of f, g, and h is $\{x \mid x \in \mathbb{R}\}$
Range of f is $\{y \mid y > 0\}$, Range of g is $\{y \mid y > 0\}$,
Range of h is $\{y \mid y > 3\}$

11 **a**



b Domain of f, g, and h is $\{x \mid x \in \mathbb{R}\}$
Range of f is $\{y \mid y > 0\}$, Range of g is $\{y \mid y < 0\}$,
Range of h is $\{y \mid y < 10\}$

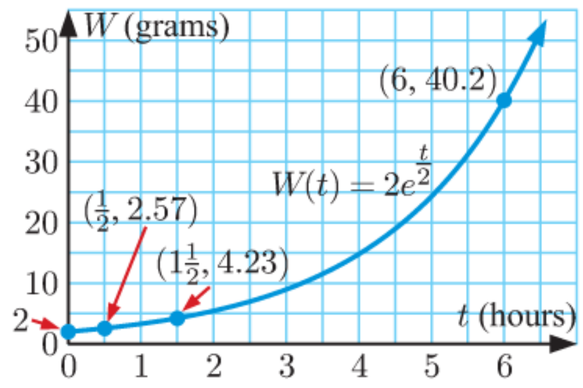
c For f: as $x \to \infty$, $y \to \infty$
as $x \to -\infty$, $y \to 0^+$
For g: as $x \to \infty$, $y \to -\infty$
as $x \to -\infty$, $y \to 0^-$
For h: as $x \to \infty$, $y \to -\infty$
as $x \to -\infty$, $y \to 10^-$

12 **a** $(f \circ g)(x) = e^{\frac{1}{x}} - 1$
Domain is $\{x \mid x \neq 0\}$
Range is $\{y \mid -1 < y < 0 \text{ or } y > 0\}$

b $(g \circ f)(x) = \dfrac{1}{e^x - 1}$
Domain is $\{x \mid x \neq 0\}$
Range is $\{y \mid y < -1 \text{ or } y > 0\}$

13 **a** **i** 2 g **ii** ≈ 2.57 g **iii** ≈ 4.23 g **iv** ≈ 40.2 g

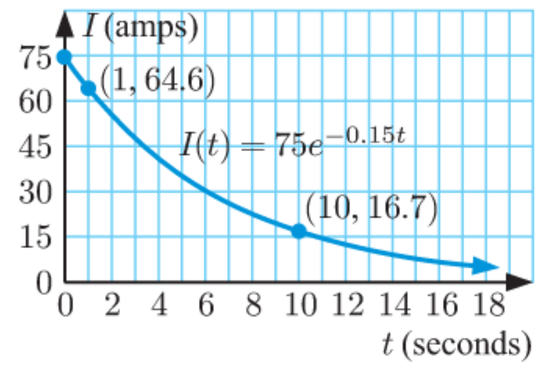
b



14 **a** $x = \frac{1}{2}$ **b** $x = -4$ **c** $x = 0$

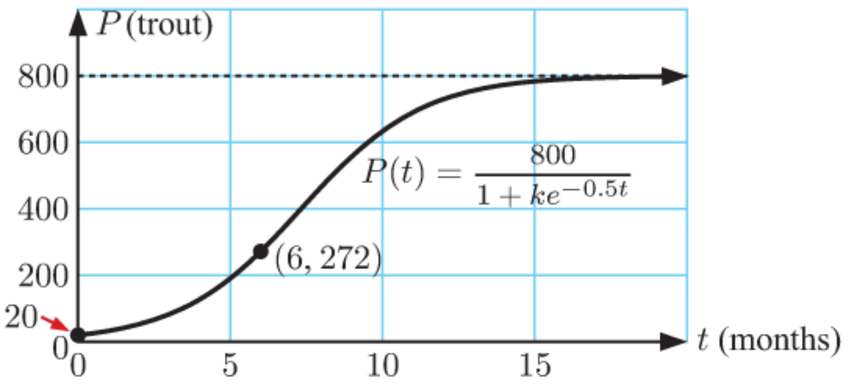
15 **a** **i** ≈ 64.6 amps **ii** ≈ 16.7 amps
c ≈ 28.8 seconds

b



16 **a** $k = 39$ **b** ≈ 272 trout

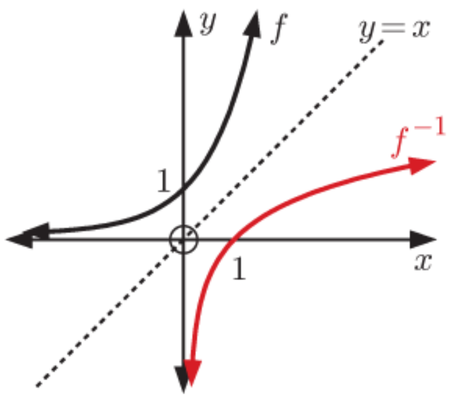
c



d As t increases, the population approaches a limiting value of 800 trout.

e ≈ 9.52 months

17 **a** $f^{-1}(x) = \log_e x$



b Domain of f^{-1} is $\{x \mid x > 0\}$
Range of f^{-1} is $\{y \mid y \in \mathbb{R}\}$

18 $e^1 \approx \sum_{i=0}^{19} \dfrac{1}{k!}\, 1^k \approx 2.718\,281\,828$

REVIEW SET 2A

1 **a** 4 **b** $\frac{1}{9}$ **c** $\frac{1}{3}$

2 **a** $x = -2$ **b** $x = \frac{3}{4}$ **c** $x = -\frac{1}{4}$

3 **a** $1 + e^{2x}$ **b** $2^{2x} + 10(2^x) + 25$ **c** $x - 49$

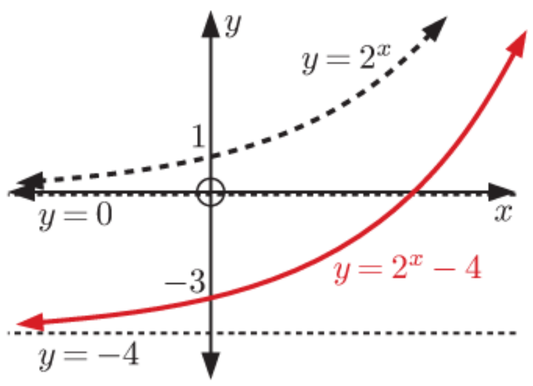
4 **a** $x = 5$ **b** $x = -\frac{5}{2}$ **c** $x = 3$ **5** $k = \frac{3}{2}$

6 **a** **i** ≈ 2.2 **ii** ≈ 0.6

b **i** $x \approx 1.45$ **ii** $x \approx -0.6$ **iii** $x \approx 1.1$

7 **a** 3 **b** 24 **c** $\frac{3}{4}$

8

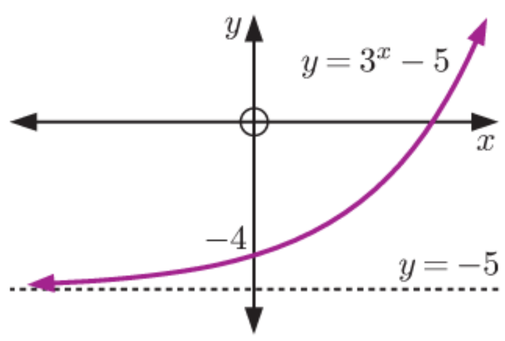


9 **a**

x	-2	-1	0	1	2
y	$-4\frac{8}{9}$	$-4\frac{2}{3}$	-4	-2	4

b as $x \to \infty$, $y \to \infty$
as $x \to -\infty$, $y \to -5^+$

c



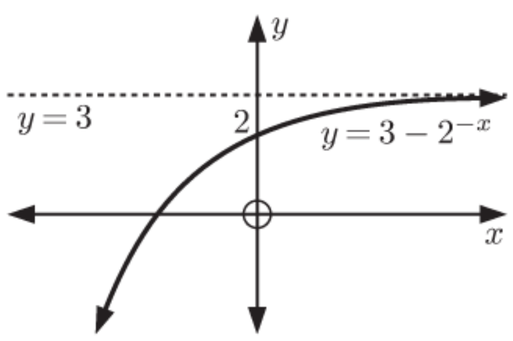
d $y = -5$

10 **a**

x	-2	-1	0	1	2
y	-1	1	2	$2\frac{1}{2}$	$2\frac{3}{4}$

b as $x \to \infty$, $y \to 3^-$
as $x \to -\infty$, $y \to -\infty$

c



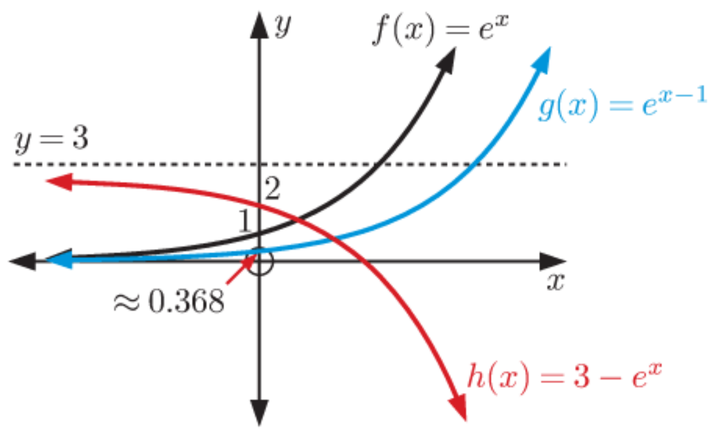
d $y = 3$

11 **a** $(f \circ g)(x) = 2^{3-x^2}$
Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid 0 < y \leqslant 8\}$

b $(g \circ f)(x) = 3 - 2^{2x} = 3 - 4^x$
Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid y < 3\}$

c **i** $x = \pm\sqrt{2}$ **ii** $x = 2$

12 **a**



b For $f(x)$: domain is $\{x \mid x \in \mathbb{R}\}$, range is $\{y \mid y > 0\}$
For $g(x)$: domain is $\{x \mid x \in \mathbb{R}\}$, range is $\{y \mid y > 0\}$
For $h(x)$: domain is $\{x \mid x \in \mathbb{R}\}$, range is $\{y \mid y < 3\}$

c For $f(x)$: as $x \to \infty$, $f(x) \to \infty$
as $x \to -\infty$, $f(x) \to 0^+$
For $g(x)$: as $x \to \infty$, $g(x) \to \infty$
as $x \to -\infty$, $g(x) \to 0^+$
For $h(x)$: as $x \to \infty$, $h(x) \to -\infty$
as $x \to -\infty$, $h(x) \to 3^-$

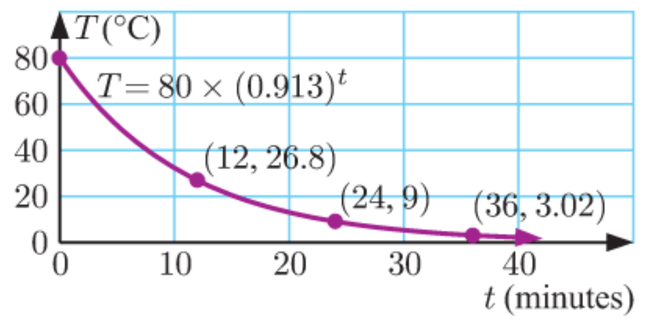
13 about every ≈ 7.92 days

14 **a** 80°C

b **i** ≈ 26.8°C
ii ≈ 9.00°C
iii ≈ 3.02°C

d ≈ 12.8 min

c



REVIEW SET 2B

1 **a** ≈ 3.95 **b** ≈ 0.517 **c** ≈ 3.16

2 **a** $9 - 6e^x + e^{2x}$ **b** $x - 2 - x^{-1}$ **c** $2^x + 1$

3 **a** $8(3^x)$ **b** $(2^x - 4)(2^x + 3)$ **c** $(e^x + 5)(e^x - 3)$

4 **a** $x = -4$ **b** $x = 0$ or 2 **c** $x = -1$ or -2

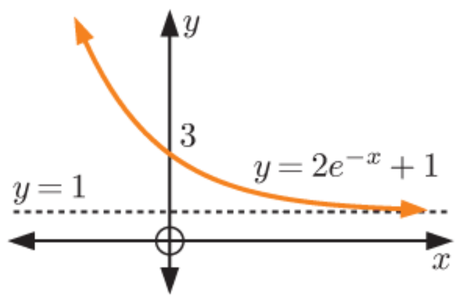
5 **a** $\frac{1}{\sqrt{2}} + 1 \approx 1.71$ **b** $a = -1$

6 **a**

x	-2	-1	0	1	2
y	15.8	6.44	3	1.74	1.27

b as $x \to \infty$, $y \to 1^+$
as $x \to -\infty$, $y \to \infty$

c



d $y = 1$

7 **a** clock: £525, vase: £428

b clock: $V(t) = 500 \times (1.05)^t$
vase: $V(t) = 400 \times (1.07)^t$

c clock $\approx$ £1039.46, vase $\approx$ £1103.61 $\therefore$ the vase

d $500 \times (1.05)^t = 400 \times (1.07)^t$ and solve for t;
$t \approx 11.8$ years

8 Domain is $\{x \mid x \geqslant -1\}$, Range is $\{y \mid y \geqslant 1\}$

9 **a** $f(x)$: y-intercept 2, $g(x)$: y-intercept -1

b $-\frac{5}{2} + \frac{1}{2}\sqrt{5}$ units

10 **a** **i** 81 **ii** $\frac{1}{3}$ **b** $k = 9$

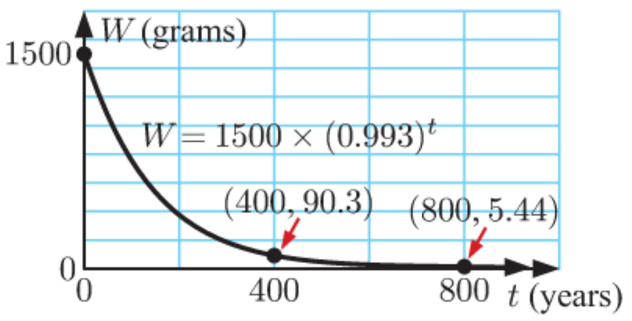
11 **a** y^2 **b** y^{-1} **c** $\frac{1}{\sqrt{y}}$ or $y^{-\frac{1}{2}}$

12 **a** 1500 g

b **i** ≈ 90.3 g
ii ≈ 5.44 g

d ≈ 386 years

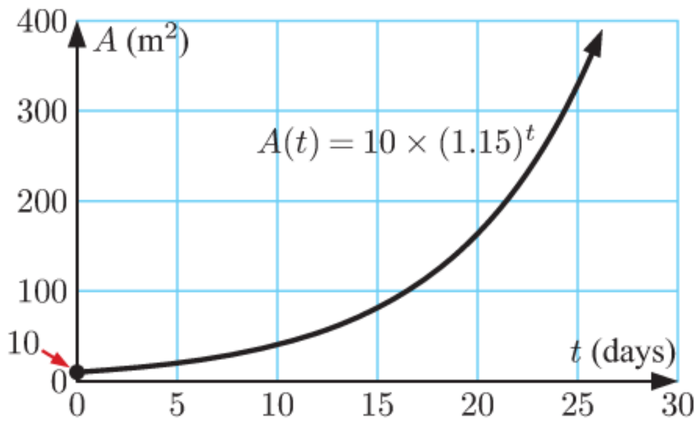
c



13 **a** $A(t) = 10 \times (1.15)^t$

b **i** 13.225 m^2 **ii** ≈ 20.1 m^2

c



d ≈ 24.3 days

EXERCISE 3A

1 **a** 4 **b** -3 **c** 1 **d** 0 **e** $\frac{1}{2}$ **f** $\frac{1}{3}$
g $-\frac{1}{4}$ **h** $1\frac{1}{2}$ **i** $\frac{2}{3}$ **j** $1\frac{1}{2}$ **k** $1\frac{1}{3}$ **l** $3\frac{1}{2}$

2 **a** n **b** $a+2$ **c** $1-m$ **d** $a-b$

3 **a** $100 < 237 < 1000$
$\therefore \log 100 < \log 237 < \log 1000$
$\therefore 2 < \log 237 < 3$ **b** ≈ 2.37

4 **a** $-1 < \log(0.6) < 0$ **b** ≈ -0.22

5 **a** ≈ 1.88 **b** ≈ 2.06 **c** ≈ 0.48 **d** ≈ 2.92
e ≈ -0.40 **f** ≈ 3.51 **g** ≈ -2.10 **h** does not exist

6 **a** $x > 1$ **b** $x = 1$ **c** $0 < x < 1$ **d** $x \leqslant 0$

7 **a** $\approx 10^{0.7782}$ **b** $\approx 10^{1.7782}$ **c** $\approx 10^{3.7782}$
d $\approx 10^{-0.2218}$ **e** $\approx 10^{-2.2218}$ **f** $\approx 10^{1.1761}$
g $\approx 10^{3.1761}$ **h** $\approx 10^{0.1761}$ **i** $\approx 10^{-0.8239}$
j $\approx 10^{-3.8239}$

8 **a** **i** ≈ 0.477 **ii** ≈ 2.477
b $\log 300 = \log(3 \times 10^2) = \log(10^{\log 3} \times 10^2) = \ldots.$

9 **a** **i** ≈ 0.699 **ii** ≈ -1.301
b $\log(0.05) = \log(5 \times 10^{-2}) = \log(10^{\log 5} \times 10^{-2}) = \ldots.$

10 **a** $x = 100$ **b** $x = 10$ **c** $x = 1$
d $x = \frac{1}{10}$ **e** $x = \sqrt{10}$ **f** $x = \frac{1}{\sqrt{10}}$
g $x = 10\,000$ **h** $x = 0.000\,01$ **i** $x \approx 6.84$
j $x \approx 140$ **k** $x \approx 0.0419$ **l** $x \approx 0.000\,631$

EXERCISE 3B

1 **a** $10^2 = 100$ **b** $10^4 = 10\,000$ **c** $10^{-1} = 0.1$
d $10^{\frac{1}{2}} = \sqrt{10}$ **e** $2^3 = 8$ **f** $3^2 = 9$
g $2^{-2} = \frac{1}{4}$ **h** $3^{1.5} = \sqrt{27}$ **i** $5^{-\frac{1}{2}} = \frac{1}{\sqrt{5}}$

2 **a** $\log_4 64 = 3$ **b** $\log_5 25 = 2$
c $\log_7 49 = 2$ **d** $\log_2 64 = 6$
e $\log_2\left(\frac{1}{8}\right) = -3$ **f** $\log_{10}(0.01) = -2$
g $\log_2\left(\frac{1}{2}\right) = -1$ **h** $\log_3\left(\frac{1}{27}\right) = -3$

3 **a** 5 **b** -2 **c** $\frac{1}{2}$ **d** 2 **e** 6 **f** 7
g 2 **h** 3 **i** -3 **j** $\frac{1}{2}$ **k** 2 **l** $\frac{1}{2}$
m 5 **n** $\frac{1}{3}$ **o** $\frac{1}{3}$ **p** $\frac{3}{2}$ **q** 0 **r** 1
s -1 **t** $\frac{3}{4}$ **u** $-\frac{1}{2}$ **v** $\frac{5}{2}$ **w** $-\frac{3}{2}$ **x** $-\frac{3}{4}$

4 **a** 2 **b** -1 **c** $\frac{1}{2}$ **d** 3 **e** $\frac{1}{4}$ **f** $\frac{3}{2}$
g -2 **h** $-\frac{1}{2}$ **i** $\frac{5}{2}$

5 **a** $x = 8$ **b** $x = 2$ **c** $x = \frac{1}{125}$ **d** $x = 3$
e $x = 14$ **f** $x = \sqrt{3}$

6 $\log_b a = \frac{1}{x}$ **7** $x = \frac{2^{2y}+1}{5}$

EXERCISE 3C

1 **a** $\log 16$ **b** $\log 20$ **c** $\log 8$ **d** $\log\left(\frac{p}{m}\right)$
e 1 **f** $\log 2$ **g** 3 **h** 2
i $\log 24$ **j** 1 **k** 0 **l** $\log 28$

2 **a** $\log 700$ **b** $\log\left(\frac{2}{5}\right)$ **c** $\log_2 6$
d $\log_3\left(\frac{5}{9}\right)$ **e** $\log 200$ **f** $\log(0.005)$
g $\log(10^t \times w)$ **h** $\log_m\left(\frac{40}{m^2}\right)$ **i** $\log_5\left(\frac{5}{2}\right)$

3 **a** $\log 96$ **b** $\log 72$ **c** $\log 8$ **d** $\log_3\left(\frac{25}{8}\right)$
e 1 **f** $\log\left(\frac{1}{2}\right)$ **g** $\log 20$ **h** $\log 25$ **i** $\log_n\left(\frac{n^2}{10}\right)$

4 **a** 2 **b** $\frac{3}{2}$ **c** 3 **d** $\frac{1}{2}$ **e** -2 **f** $-\frac{3}{2}$

5 For example, for **a**, $\log 9 = \log(3^2) = 2\log 3$.

7 **a** $p+q$ **b** $2q+r$ **c** $2p+3q$ **d** $r+\frac{1}{2}q-p$
e $r-5p$ **f** $p-2q$

8 **a** $x+z$ **b** $z+2y$ **c** $x+z-y$ **d** $2x+\frac{1}{2}y$
e $3y-\frac{1}{2}z$ **f** $2z+\frac{1}{2}y-3x$

9 **a** 0.86 **b** 2.15 **c** 1.075 **10** $x = \frac{2}{a^2-1}$

11 **a** $\log 384$ **b** $4+\log_2 45$ **12** $\log\left(\frac{x^{40}}{y^{45}}\right)$

13 $\log \sqrt[3]{3}$ **14** **Hint:** Subtract $2xy$ from both sides.

EXERCISE 3D

1 **a** 2 **b** 4 **c** $\frac{3}{2}$ **d** 0 **e** -1
f $\frac{1}{3}$ **g** -2 **h** $-\frac{1}{2}$

2 **a** 3 **b** 9 **c** $\frac{1}{5}$ **d** $\frac{1}{4}$ **e** a
f $1+a$ **g** $a+b$ **h** ab

3 **a** ≈ 2.485 **b** ≈ 4.220 **c** ≈ 0.336
d ≈ -0.357 **e** ≈ 6.215

4 x does not exist such that $e^x = -2$ or 0 since $e^x > 0$ for all $x \in \mathbb{R}$.

5 **a** $\approx e^{1.7918}$ **b** $\approx e^{4.0943}$ **c** $\approx e^{8.6995}$
d $\approx e^{-0.5108}$ **e** $\approx e^{-5.1160}$ **f** $\approx e^{2.7081}$
g $\approx e^{7.3132}$ **h** $\approx e^{0.4055}$ **i** $\approx e^{-1.8971}$
j $\approx e^{-8.8049}$

6 **a** $x \approx 20.1$ **b** $x \approx 2.72$ **c** $x = 1$
d $x \approx 0.368$ **e** $x \approx 0.006\,74$ **f** $x \approx 2.30$
g $x \approx 8.54$ **h** $x \approx 0.0370$

7 **a** **i** x **ii** x **b** They are inverses of each other.

8 **a** $\ln 45$ **b** $\ln 5$ **c** $\ln 4$ **d** $\ln 24$
e $\ln 1 = 0$ **f** $\ln 30$ **g** $\ln(4e)$ **h** $\ln\left(\frac{6}{e}\right)$
i $\ln 20$ **j** $\ln(4e^2)$ **k** $\ln\left(\frac{20}{e^2}\right)$ **l** $\ln 1 = 0$

9 **a** $\ln 972$ **b** $\ln 200$ **c** $\ln 1 = 0$ **d** $\ln 16$
e $\ln 6$ **f** $\ln\left(\frac{1}{3}\right)$ **g** $\ln\left(\frac{1}{2}\right)$ **h** $\ln 2$
i $\ln 16$ **j** $\ln(16e^2)$ **k** $\ln\left(\frac{3}{e}\right)$ **l** $\ln\left(\frac{\sqrt{e}}{8}\right)$

10 For example, for **a**, $\ln 27 = \ln(3^3) = 3\ln 3$.

11 $x = e^4,\ y = \frac{1}{e^2}$

EXERCISE 3E

1 **a** $\log y = x\log 2$ **b** $\log y = \log 20 + 3\log b$
c $\log M = \log a + 4\log d$ **d** $\log T = \log 5 + \frac{1}{2}\log d$
e $\log R = \log b + \frac{1}{2}\log l$ **f** $\log Q = \log a - n\log b$
g $\log y = \log a + x\log b$ **h** $\log F = \log 20 - \frac{1}{2}\log n$
i $\log L = \log a + \log b - \log c$
j $\log N = \frac{1}{2}\log a - \frac{1}{2}\log b$
k $\log S = \log 200 + t\log 2$ **l** $\log y = m\log a - n\log b$

2 **a** $D = 2e$ **b** $F = \frac{5}{t}$ **c** $P = \sqrt{x}$ **d** $M = b^2c$
e $B = \frac{m^3}{n^2}$ **f** $N = \frac{1}{\sqrt[3]{p}}$ **g** $P = 10x^3$ **h** $Q = \frac{a^2}{x}$

3 **a** $D = ex$ **b** $F = \frac{e^2}{p}$ **c** $P = \sqrt{x}$
d $M = e^3y^2$ **e** $B = \frac{t^3}{e}$ **f** $N = \frac{1}{\sqrt[3]{g}}$
g $Q \approx 8.66x^3$ **h** $D \approx 0.518n^{0.4}$

4 **a** $\log_2 y = \log_2 3 + x$ **b** $x = \log_2\left(\frac{y}{3}\right)$
c **i** $x = 0$ **ii** $x = 2$ **iii** $x \approx 3.32$

5 **a** $x = 9$ **b** $x = 2$ or 4 **c** $x = 25\sqrt{5}$ **d** $x = 200$
e $x = 5$ **f** $x = 3$ **g** $x = 3$ **h** $x = -2$

6 **a** $x = 2,\ y = 4$ **b** $x = 2,\ y = 8$

7 **a** $2^x = 7$
b $\log 2^x = \log 7$
$\therefore\ x \log 2 = \log 7$
$\therefore\ x = \log_2 7 = \frac{\log 7}{\log 2} \approx 2.81$

8 **a** Taking the logarithm in base a of both sides, $x = \log_a b$.
b $\log a^x = \log b$
c Using **b**, $x \log a = \log b$
$\therefore\ x = \frac{\log b}{\log a}$
and using part **a**, $x = \log_a b = \frac{\log b}{\log a}$

EXERCISE 3F

1 **a** ≈ 1.77 **b** ≈ 5.32 **c** ≈ 3.23 **d** ≈ -10.3
e ≈ -2.46 **f** ≈ 5.42

2 2

4 **a** $x = 16$ **b** $x = \frac{1}{5}$ **c** $x = \sqrt[3]{5}$ **d** $x = 64$
e $x = 2$ or 4

5 $\frac{8}{x}$ **6** $2m + 3$

7 **a** We get $\log_9 2 \neq 0$. **b** $x = 81$ **c** $x = a^4$
d **Hint:** First show that $\log_a x = k^{\frac{k}{k-1}}$.

EXERCISE 3G

1 **a** $3^3 = 27,\ 3^4 = 81,\ \therefore$ if $3^x = 40$, then $3 < x < 4$
b $x = \frac{\log 40}{\log 3}$ **c** $x \approx 3.36$

2 **a** **i** $x = \frac{1}{\log 2}$ **ii** $x \approx 3.32$
b **i** $x = \frac{\log 20}{\log 3}$ **ii** $x \approx 2.73$
c **i** $x = \frac{\log 50}{\log 4}$ **ii** $x \approx 2.82$
d **i** $x = 4$ **ii** $x = 4$
e **i** $x = -\frac{1}{\log\left(\frac{3}{4}\right)}$ **ii** $x \approx 8.00$
f **i** $x = \log(0.000\,015)$ **ii** $x \approx -4.82$

3 **a** $x \approx 2.29$ **b** $x \approx 5.13$ **c** $x \approx 0.194$

4 **a** $x = \ln 10$ **b** $x = \ln 1000$ **c** $x = \ln(0.15)$
d $x = 2 \ln 5$ **e** $x = \frac{1}{2} \ln 18$ **f** $x = 0$

5 **a** $x = \frac{\log 25}{\log 2}$ **b** $x = \frac{\log\left(\frac{20}{7}\right)}{\log 1.5}$ **c** $x = \frac{\log(0.6)}{\log(0.8)}$
d $x = -\frac{\log(0.03)}{\log 2}$ **e** $x = \frac{10 \log\left(\frac{10}{3}\right)}{\log 5}$ **f** $x = 4 \ln 8$

6 **a** $x = \frac{\log 3}{\log 5}$ **b** $x = -\frac{\log 8}{\log 3}$ **c** $x = -1$

7 $x = -\frac{2 \ln 2}{\ln 3}$ **8** $x = \frac{3 \ln 2}{\ln 5}$

9 **a** $x = \ln 2$ **b** $x = 0$ **c** $x = \ln 2$ or $\ln 3$ **d** $x = 0$
e $x = \ln 4$ **f** $x = \ln\left(\frac{3+\sqrt{5}}{2}\right)$ or $\ln\left(\frac{3-\sqrt{5}}{2}\right)$

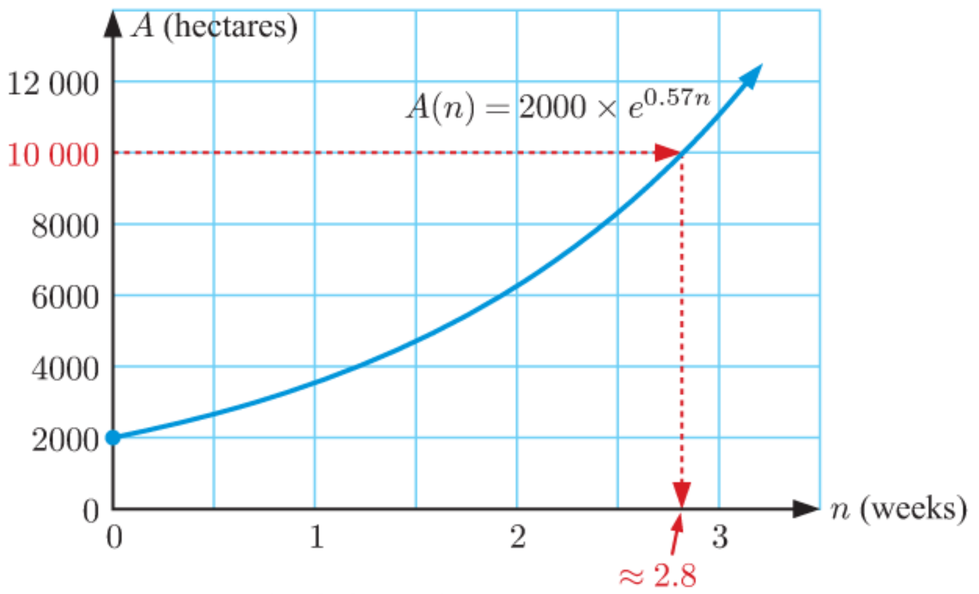
10 **a** $(\ln 3,\ 3)$ **b** $(\ln 2,\ 5)$ **c** $(0,\ 2)$ and $(\ln 5,\ -2)$

11 **a** $(-\ln 2,\ 2)$ **b** $\ln\left(\frac{11}{2}\right)$ units

12 **a** ≈ 2.37 years **b** ≈ 8.36 years

13 **a** ≈ 3.90 hours **b** ≈ 15.5 hours

14 **a**, **b** see graph below



$\therefore$ approximately 2.8 weeks.

15 In ≈ 5.86 years or ≈ 5 years 10 months. **16** 9 years

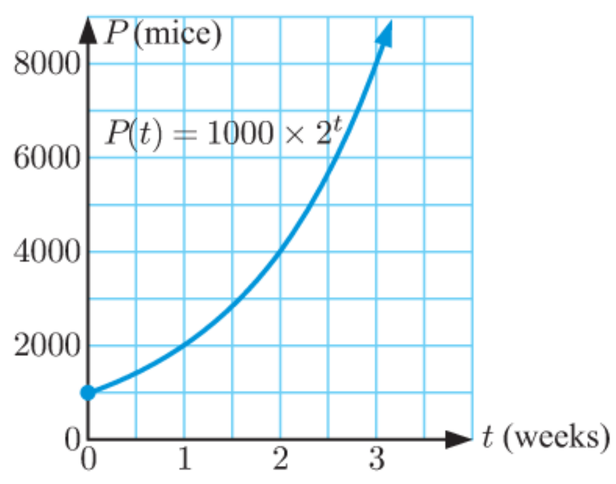
17 **a** $\frac{8.4\%}{12} = 0.7\% = 0.007,\quad r = 1 + 0.007 = 1.007$
b after 74 months

18 **a** ≈ 17.3 years **b** ≈ 92.2 years **c** ≈ 115 years

19 **Hint:** $0.1 \times I_0 = I_0 \times 2^{-0.02t}$
$\therefore\ 0.1 = 2^{-0.02t}$ and solve for t using logarithms.

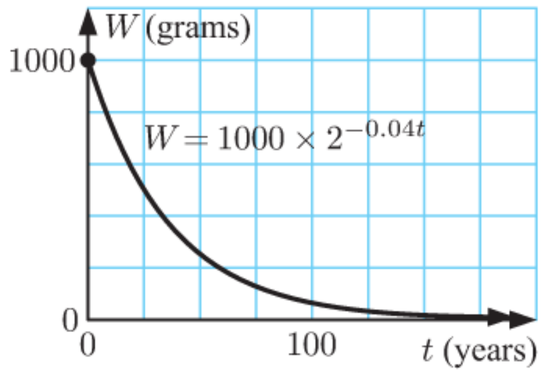
20 **a** **Hint:** Set $V = 40$, solve for t.
b $t = -5 \ln\left(1 - \frac{V}{50}\right)$ s

21 **a**



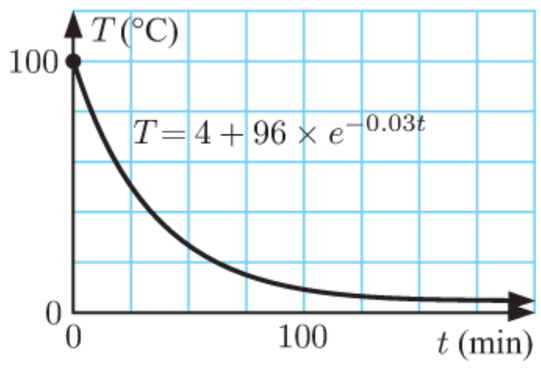
b ≈ 4.32 weeks
c $t = \frac{\log P - 3}{\log 2}$

22 **a**



b $t = \frac{3 - \log W}{0.04 \log 2}$
c **i** $t \approx 141$ years
ii $t \approx 498$ years

23 **a**



b $t = \dfrac{\ln 96 - \ln(T-4)}{0.03}$

c **i** ≈ 50.7 minutes **ii** ≈ 152 minutes

24 **a** decreasing **b** **i** $3900\ \text{m s}^{-1}$ **ii** $\approx 2600\ \text{m s}^{-1}$

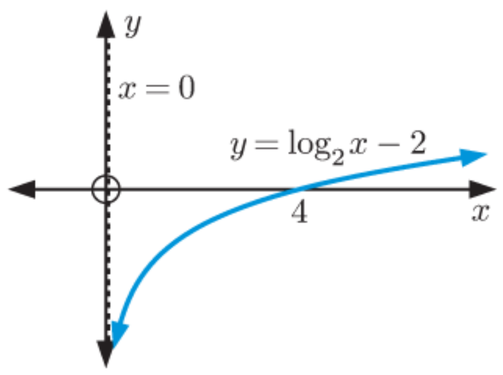
c ≈ 11.8 s

EXERCISE 3H

1 **a** **i** Domain is $\{x \mid x > 0\}$, Range is $\{y \mid y \in \mathbb{R}\}$

ii vertical asymptote is $x = 0$, x-intercept 4, no y-intercept

iii

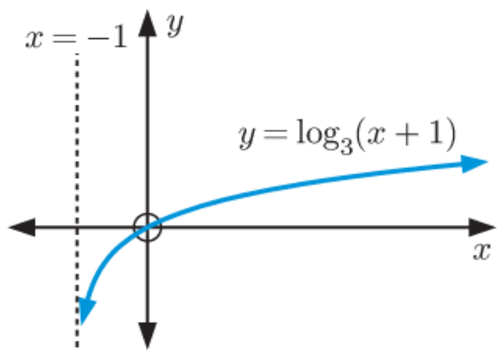


iv $x = 2$ **v** $f^{-1}(x) = 2^{x+2}$

b **i** Domain is $\{x \mid x > -1\}$, Range is $\{y \mid y \in \mathbb{R}\}$

ii vertical asymptote is $x = -1$, x and y-intercepts 0

iii

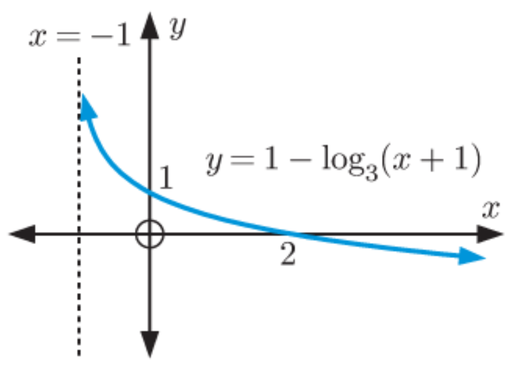


iv $x = -\frac{2}{3}$ **v** $f^{-1}(x) = 3^x - 1$

c **i** Domain is $\{x \mid x > -1\}$, Range is $\{y \mid y \in \mathbb{R}\}$

ii vertical asymptote is $x = -1$, x-intercept 2, y-intercept 1

iii

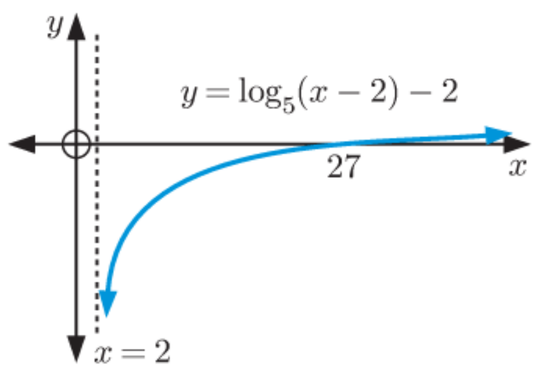


iv $x = 8$ **v** $f^{-1}(x) = 3^{1-x} - 1$

d **i** Domain is $\{x \mid x > 2\}$, Range is $\{y \mid y \in \mathbb{R}\}$

ii vertical asymptote is $x = 2$, x-intercept 27, no y-intercept

iii

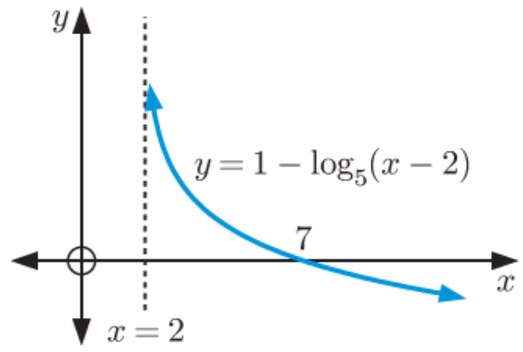


iv $x = 7$ **v** $f^{-1}(x) = 5^{x+2} + 2$

e **i** Domain is $\{x \mid x > 2\}$, Range is $\{y \mid y \in \mathbb{R}\}$

ii vertical asymptote is $x = 2$, x-intercept 7, no y-intercept

iii

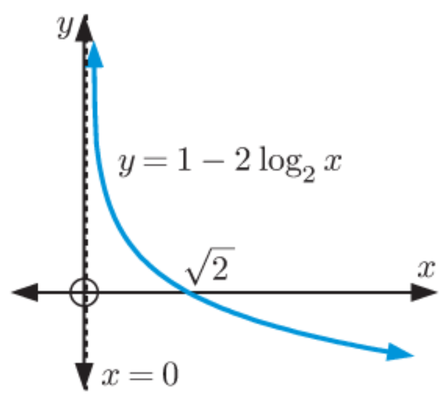


iv $x = 27$ **v** $f^{-1}(x) = 5^{1-x} + 2$

f **i** Domain is $\{x \mid x > 0\}$, Range is $\{y \mid y \in \mathbb{R}\}$

ii vertical asymptote is $x = 0$, x-intercept $\sqrt{2}$, no y-intercept

iii



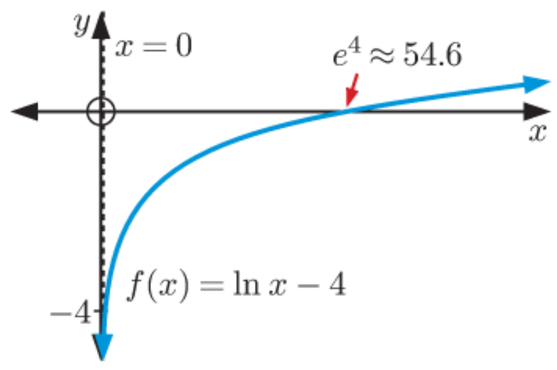
iv $x = 2$ **v** $f^{-1}(x) = 2^{\frac{1-x}{2}}$

2 **a** **i** A translation through $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$.

ii Domain is $\{x \mid x > 0\}$, Range is $\{y \mid y \in \mathbb{R}\}$

iii vertical asymptote is $x = 0$, x-intercept e^4, no y-intercept

iv



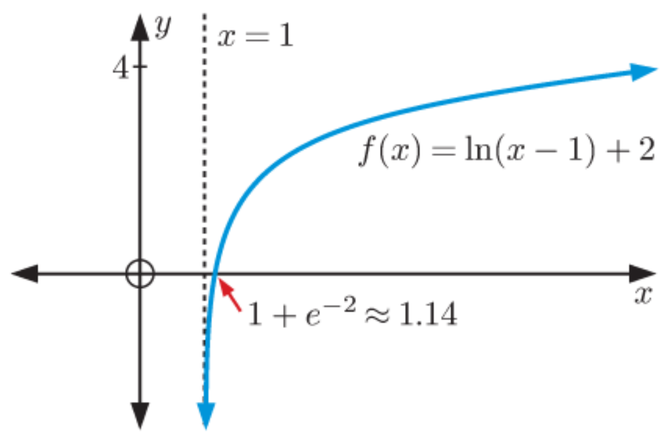
v $f^{-1}(x) = e^{x+4}$

b **i** A translation through $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

ii Domain is $\{x \mid x > 1\}$, Range is $\{y \mid y \in \mathbb{R}\}$

iii vertical asymptote is $x = 1$, x-intercept $1 + e^{-2}$, no y-intercept

iv



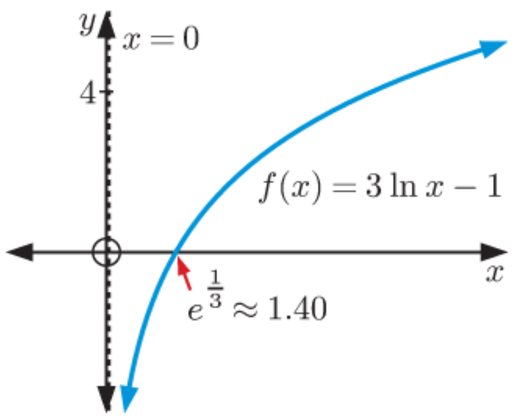
v $f^{-1}(x) = e^{x-2} + 1$

c **i** A vertical stretch with scale factor 3, then a translation through $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$.

ii Domain is $\{x \mid x > 0\}$, Range is $\{y \mid y \in \mathbb{R}\}$

iii vertical asymptote is $x = 0$, x-intercept $e^{\frac{1}{3}}$, no y-intercept

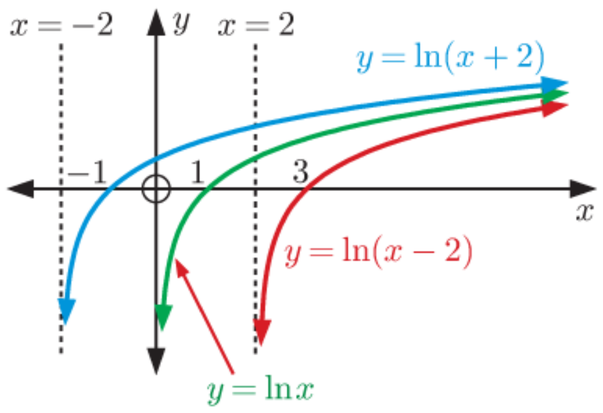
iv



v $f^{-1}(x) = e^{\frac{x+1}{3}}$

3 **a** A is $y = \ln x$ as its x-intercept is 1. B is $y = \ln(x-2)$.

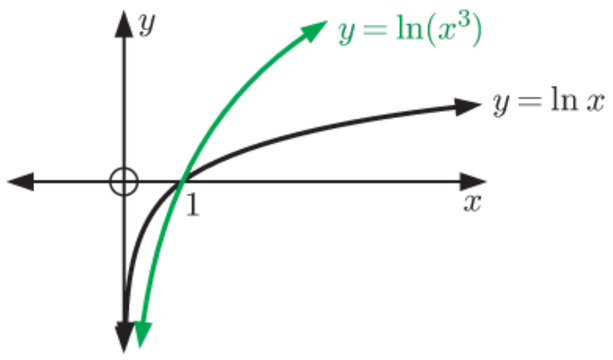
b



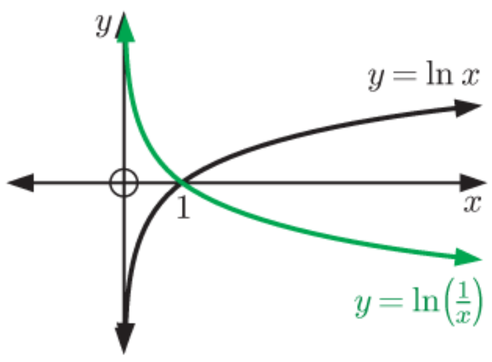
c $y = \ln x$ has vertical asymptote $x = 0$
$y = \ln(x-2)$ has vertical asymptote $x = 2$
$y = \ln(x+2)$ has vertical asymptote $x = -2$

4 $y = \ln(x^2) = 2\ln x$, so she is correct.
This is because the y-values are twice as large for $y = \ln(x^2)$ as they are for $y = \ln x$.

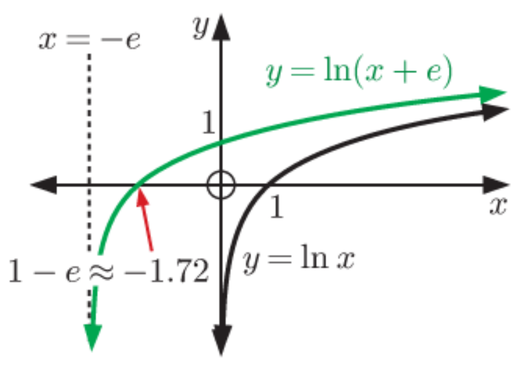
5 **a**



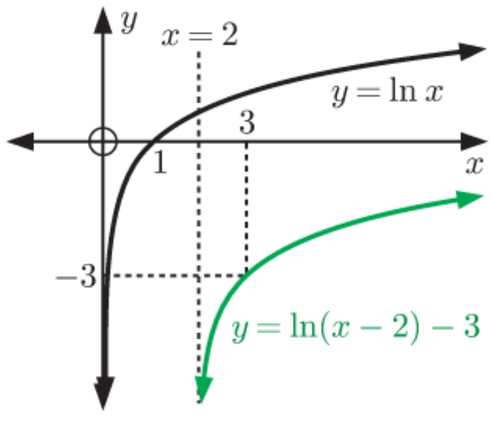
b



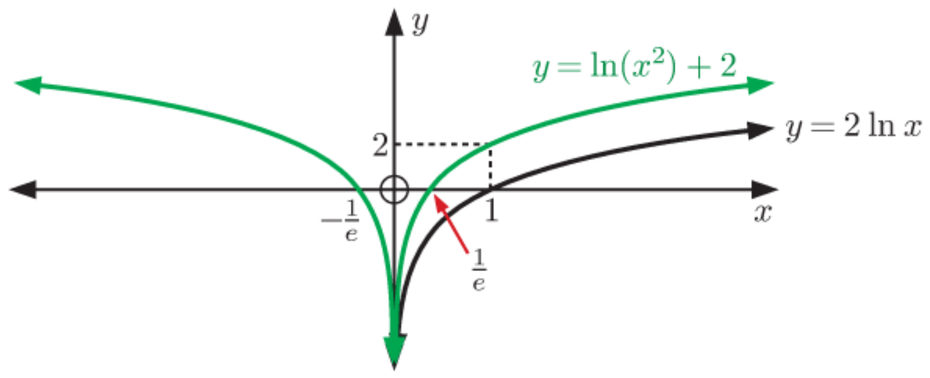
c



d



e



6 A vertical stretch with scale factor $\log_5 2$.

7 **a** **A** is $y = \log_2(x+2)$ since it is increasing.
B is $y = 3 - \log_2(3x+1)$ since it is decreasing.

b **A**: x-intercept -1, y-intercept 1,
vertical asymptote $x = -2$
B: x-intercept $\frac{7}{3}$, y-intercept 3,
vertical asymptote $x = -\frac{1}{3}$

c $(\frac{2}{3},\ 3 - \log_2 3)$

8 **a** $f^{-1}(x) = \log_3 x$
Domain is $\{x \mid x > 0\}$, Range is $\{y \mid y \in \mathbb{R}\}$

b $f^{-1}(x) = \log_2 x - 1$
Domain is $\{x \mid x > 0\}$, Range is $\{y \mid y \in \mathbb{R}\}$

c $f^{-1}(x) = \frac{1}{2}\ln x$
Domain is $\{x \mid x > 0\}$, Range is $\{y \mid y \in \mathbb{R}\}$

d $f^{-1}(x) = \log_5(x+3)$
Domain is $\{x \mid x > -3\}$, Range is $\{y \mid y \in \mathbb{R}\}$

9 **a** $b^2 x$ **b** $2\ln b + x$ **c** $x = \dfrac{2\ln b}{b^2 - 1}$

10 **a** $\dfrac{x}{e}$, Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid y \in \mathbb{R}\}$

b $x - 1$, Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid y \in \mathbb{R}\}$

11 **a** Domain is $\{x \mid x < \frac{1}{3}\}$, Range is $\{y \mid y \in \mathbb{R}\}$

b **i** $x = -\frac{17}{3}$ **ii** $x = \frac{1}{3} - \frac{1}{3\sqrt{2}}$

c $f^{-1}(x) = \dfrac{1 - 2^x}{3}$
Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid y < \frac{1}{3}\}$

12 $f^{-1}(x) = \frac{1}{2}\ln x$

a $(f^{-1} \circ g)(x) = \frac{1}{2}\ln(2x-1)$

b $(g \circ f)^{-1}(x) = \frac{1}{2}\ln\left(\dfrac{x+1}{2}\right)$

13 **a** $k = 3$ **b** x-intercepts $1 \pm 2\sqrt{2}$, y-intercept $\log_3 8$

c $g^{-1}(x) = 1 - \sqrt{9 - 3^x}$
Domain is $\{x \mid x \leqslant 2\}$, Range is $\{y \mid -2 < y \leqslant 1\}$

REVIEW SET 3A

1 **a** $\frac{1}{2}$ **b** $-\frac{1}{3}$ **c** $a + b + 1$

2 **a** 3 **b** 8 **c** -2 **d** $\frac{1}{2}$ **e** 0
f $\frac{1}{4}$ **g** -1 **h** $\frac{1}{2}$, $k > 0$, $k \neq 1$

3 **a** ≈ 1.431 **b** ≈ -0.237 **c** ≈ 2.602 **d** ≈ 3.689

4 $x = 2 - 3^{2y}$

5 **a** $\ln 144$ **b** $\ln\left(\frac{3}{2}\right)$ **c** $\ln\left(\dfrac{25}{e}\right)$ **d** $\ln 3$

6 **a** $\log 144$ **b** $\log_2\left(\frac{16}{9}\right)$ **c** $\log_4 80$

7 **a** $2A + 2B$ **b** $A + 3B$ **c** $3A + \frac{1}{2}B$
d $\frac{1}{2}(A + B)$ **e** $4B - 2A$ **f** $3A - 2B$

8 **a** $\log M = \log a + n \log b$ **b** $\log T = \log 5 - \frac{1}{2}\log l$
c $\log G = 2\log a + \log b - \log c$

9 **a** $x \approx 5.19$ **b** $x \approx 4.29$ **c** $x \approx -0.839$

10 **a** $x = \ln 3$ **b** $x = \ln 3$ or $\ln 4$

11 **a** $P = TQ^{1.5}$ **b** $M = \dfrac{e^{1.2}}{\sqrt{N}}$

12 **a** $x = 0$ or $\ln\left(\frac{2}{3}\right)$ **b** $x = e^2$

13 **a** $x = \frac{1}{8}$ **b** $x \approx 82.7$ **c** $x \approx 0.0316$

14 **a** **i** $x = \dfrac{\log 50}{\log 2}$ **ii** $x \approx 5.64$

b **i** $x = \dfrac{\log 4}{\log 7}$ **ii** $x \approx 0.71$

c **i** $x = \dfrac{-2}{\log(0.6)}$ **ii** $x \approx 9.02$

15 $\log_a\left(\frac{1}{b}\right) = -x$ **16** $4\log_3 5$

17 **Hint:** $2^{4x} - 5 \times 2^{3x} = 0$

18 **a** $x = e^5$ **b** $x = e^{-\frac{2}{3}}$ **c** $x = \ln 400$

d $x = \dfrac{\ln 11 - 1}{2}$ **e** $x = 2\ln 30$

19 **b** $\frac{1}{3}\ln\left(\frac{7}{2}\right)$ **20** $x = 2,\ y = \frac{1}{8}$ or $x = 64,\ y = 4$

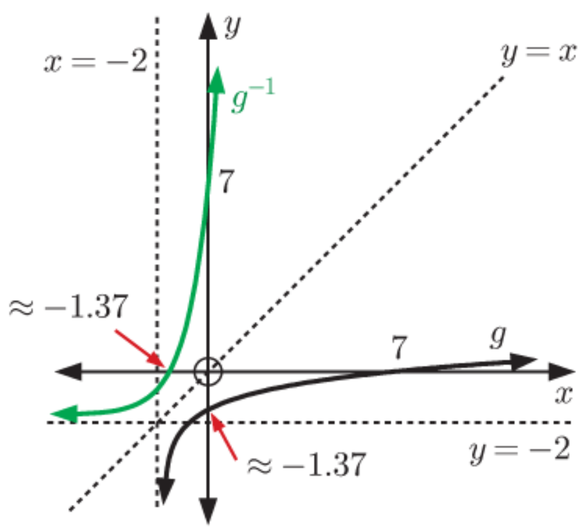
21 **a** A translation through $\begin{pmatrix} -2 \\ -2 \end{pmatrix}$.

b Domain is $\{x \mid x > -2\}$, Range is $\{y \mid y \in \mathbb{R}\}$

c vertical asymptote is $x = -2$, x-intercept is 7, y-intercept is ≈ -1.37

d $g^{-1}(x) = 3^{x+2} - 2$

e



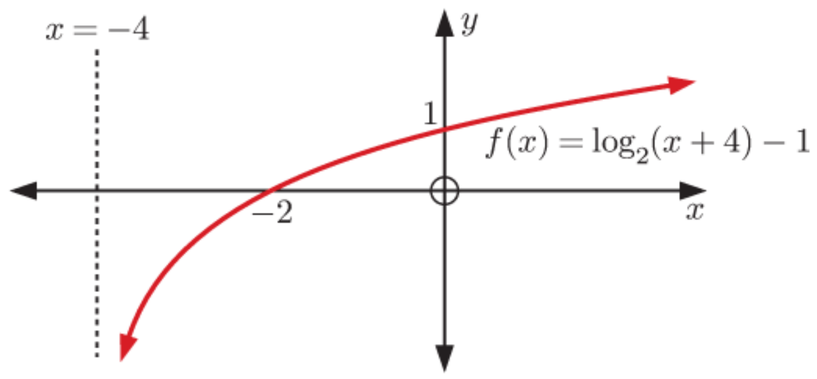
22 **a** ≈ 13.9 weeks **b** ≈ 41.6 weeks **c** ≈ 138 weeks

23 **a** ≈ 4.96 years or ≈ 4 years $11\frac{1}{2}$ months **b** $\approx 74.9\%$

24 **a** **i** Domain is $\{x \mid x > -4\}$, Range is $\{y \mid y \in \mathbb{R}\}$

ii vertical asymptote is $x = -4$, x-intercept -2, y-intercept 1

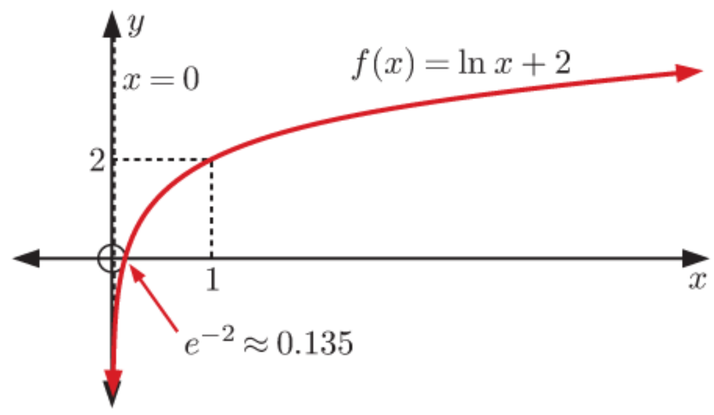
iii



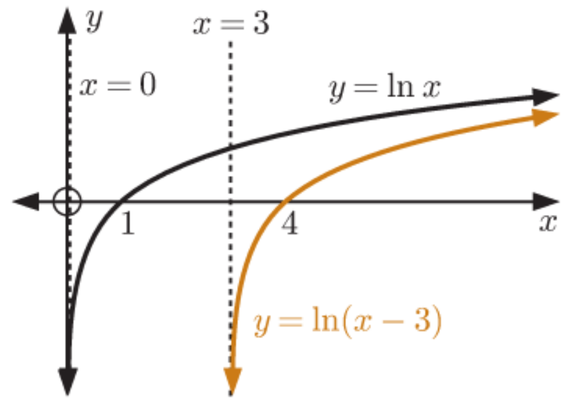
b **i** Domain is $\{x \mid x > 0\}$, Range is $\{y \mid y \in \mathbb{R}\}$

ii vertical asymptote is $x = 0$, x-intercept e^{-2}, no y-intercept

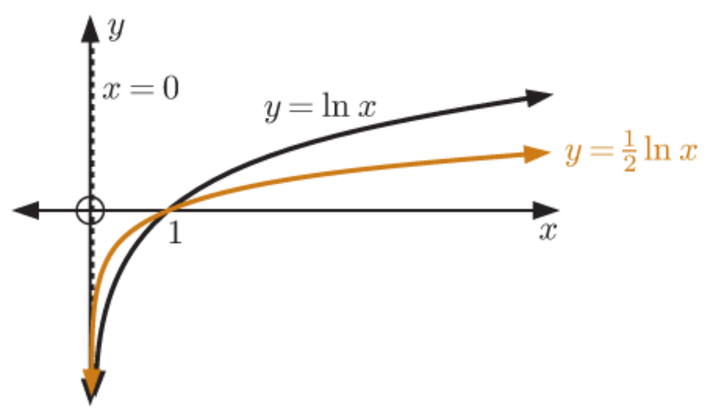
iii



25 **a**



b



26 **a** 9 **b** $\ln 5$

REVIEW SET 3B

1 **a** $\frac{3}{2}$ **b** $\frac{2}{3}$ **c** $a + b$

2 **a** 7 **b** -3 **c** $-\frac{1}{2}$

3 **a** $\approx 10^{1.5051}$ **b** $\approx 10^{-2.8861}$ **c** $\approx 10^{-4.0475}$

4 **a** $\frac{3}{2}$ **b** -3 **c** $2x$ **d** $1 - x$

5 **a** $\frac{2}{3}$ **b** $\frac{6}{5}$ **c** 8

6 **a** x^4 **b** 5 **c** $\frac{1}{2}$ **d** $3x$ **e** $-x$ **f** $\log x$

7 **a** $\approx e^{2.9957}$ **b** $\approx e^{8.0064}$ **c** $\approx e^{-2.5903}$

8 **a** **i** $x = \dfrac{\log 7}{\log 5}$ **ii** $x \approx 1.21$

b **i** $x = -\dfrac{1}{\log 2}$ **ii** $x \approx -3.32$

9 **a** $\ln 3$ **b** $\ln 4$ **c** $\ln 125$

10 **a** $x = \dfrac{\ln 70}{2}$ **b** $x = \dfrac{\log\left(\frac{11}{3}\right)}{\log 1.3}$ **c** $x = \dfrac{10\log\left(\frac{16}{5}\right)}{3\log 2}$

11 $x = 1$

13 **a** $\log P = \log 3 + x\log b$ **b** $\log m = 3\log n - 2\log p$

14 **Hint:** Use the change of base rule.

15 **a** $x = 2^{\frac{18}{13}}$ **b** $x = 10^{\frac{3}{11}}$

16 **a** $T = \dfrac{x^2}{y}$ **b** $K = n\sqrt{t}$

17 **a** $5\ln 2$ **b** $3\ln 5$ **c** $6\ln 3$

18

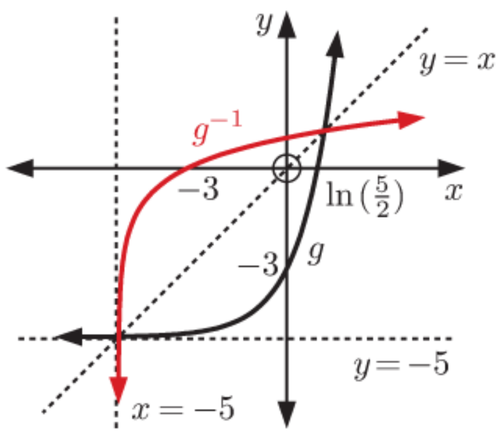
	$y = \log_2 x$	$y = \ln(x + 5)$
Domain	$x > 0$	$x > -5$
Range	$y \in \mathbb{R}$	$y \in \mathbb{R}$

19 **a** $(2^x + 4)(2^x - 5)$ **b** $x = \dfrac{\log 5}{\log 2}$

c **i** $x = \dfrac{1}{p}$ **ii** $x = \dfrac{1}{3p+1}$

20 **a** $g^{-1}(x) = \ln\left(\dfrac{x+5}{2}\right)$

b



c Domain of g is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid y > -5\}$
Domain of g^{-1} is $\{x \mid x > -5\}$, Range is $\{y \mid y \in \mathbb{R}\}$

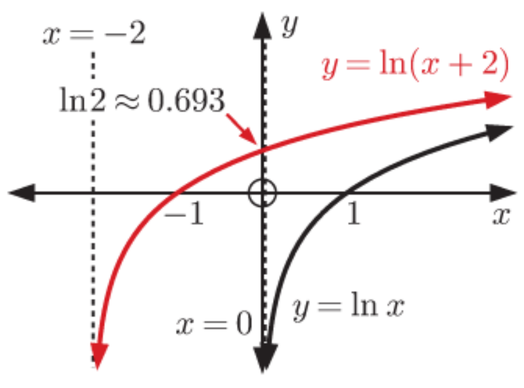
d g has horizontal asymptote $y = -5$,
x-intercept is $\ln\left(\frac{5}{2}\right) \approx 0.916$, y-intercept is -3
g^{-1} has vertical asymptote $x = -5$,
x-intercept is -3, y-intercept is ≈ 0.916

21 **Hint:** Set $T = 40$, and solve for t.

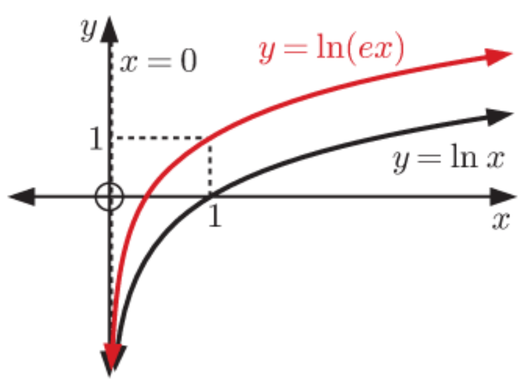
22 **a** 2500 g **b** $\approx$ 3288 years **c** $\approx 42.3\%$

23 **a** $x = \dfrac{2\log 9}{\log 5}$ **b** $x = \ln 30$ **c** $x = \dfrac{1 - \ln 2}{3}$

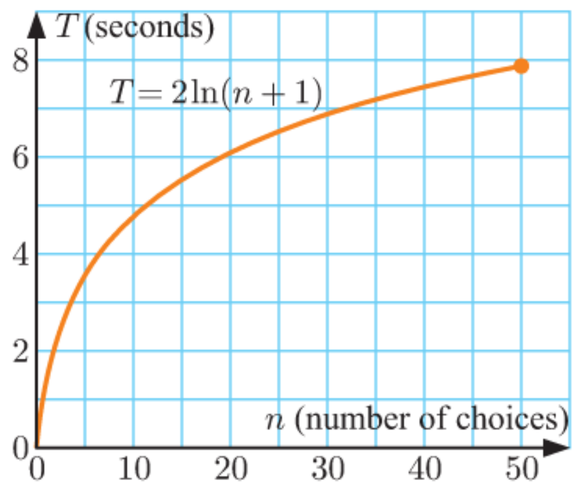
24 **a**



b



25 **a**



b **i** $\approx$ 3.58 seconds **ii** $\approx$ 5.55 seconds

c $\approx$ 1.34 seconds longer

26 **a** **i** $f^{-1}(x) = 3^{x+2}$
Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid y > 0\}$

ii $(f \circ g)(x) = \log_3(3 - \sqrt{x}) - 2$
Domain is $\{x \mid 0 \leqslant x < 9\}$, Range is $\{y \mid y \leqslant -1\}$

iii $(g \circ f)(x) = 3 - \sqrt{\log_3 x - 2}$
Domain is $\{x \mid x \geqslant 9\}$, Range is $\{y \mid y \leqslant 3\}$

b **i** $x = 4$ **ii** $x = 3^{11} = 177\,147$

c $(g \circ f)^{-1}(x) = 3^{x^2 - 6x + 11}$, $x \leqslant 3$
Domain is $\{x \mid x \leqslant 3\}$, Range is $\{y \mid y \geqslant 9\}$

EXERCISE 4A

1

z	$\mathcal{Re}(z)$	$\mathcal{Im}(z)$	z^*
$3 + 2i$	3	2	$3 - 2i$
$5 - i$	5	-1	$5 + i$
3	3	0	3
0	0	0	0
$-3 + 4i$	-3	4	$-3 - 4i$
$-7 - 2i$	-7	-2	$-7 + 2i$
$-11i$	0	-11	$11i$
$i\sqrt{3}$	0	$\sqrt{3}$	$-i\sqrt{3}$

2 **a** $3i$ **b** $8i$ **c** $\frac{1}{2}i$ **d** $i\sqrt{5}$ **e** $i\sqrt{8}$

3 **a** $x = \pm 5$ **b** $x = \pm 5i$ **c** $x = \pm\sqrt{5}$
d $x = \pm i\sqrt{5}$ **e** $x = \pm\frac{3}{2}$ **f** $x = \pm\frac{3}{2}i$

4 **a** $x = 5 \pm 2i$ **b** $x = -3 \pm 4i$ **c** $x = -7 \pm i$
d $x = \frac{3}{2} \pm \frac{1}{2}i$ **e** $x = \sqrt{3} \pm i$ **f** $x = \frac{1}{4} \pm i\frac{\sqrt{7}}{4}$

EXERCISE 4B

1 **a** $(x+3)(x-3)$ **b** $(x+3i)(x-3i)$
c $(x+\sqrt{7})(x-\sqrt{7})$ **d** $(x+i\sqrt{7})(x-i\sqrt{7})$
e $(2x+1)(2x-1)$ **f** $(2x+i)(2x-i)$
g $(\sqrt{2}x+3)(\sqrt{2}x-3)$ **h** $(\sqrt{2}x+3i)(\sqrt{2}x-3i)$

2 $(a+bi)(a-bi) = a^2 - b^2i^2 = a^2 + b^2$ {since $i^2 = -1$}

3 **a** $x = \pm 4i$ **b** $x = \pm 6i$ **c** $x = \pm i\sqrt{5}$
d $x = \pm\frac{1}{3}i$ **e** $x = \pm\frac{5}{2}i$ **f** $x = \pm i\frac{\sqrt{2}}{\sqrt{3}}$

4 **a** $x(x+1)(x-1)$ **b** $x(x+i)(x-i)$
c $(x+1)(x-1)(x+i)(x-i)$
d $(x+2)(x-2)(x+2i)(x-2i)$

5 **a** $x = 0$ or ± 2 **b** $x = 0$ or $\pm 2i$
c $x = 0$ or $\pm\sqrt{3}$ **d** $x = 0$ or $\pm i\sqrt{3}$
e $x = \pm 1$ or $\pm i$ **f** $x = \pm 3$ or $\pm 3i$

6 **a** $x = \pm i\sqrt{3}$ or ± 1 **b** $x = \pm\sqrt{3}$ or $\pm i\sqrt{2}$
c $x = \pm 3i$ or ± 2 **d** $x = \pm i\sqrt{7}$ or $\pm i\sqrt{2}$
e $x = \pm 1$ **f** $x = \pm i$

EXERCISE 4C

1 **a** $7 - i$ **b** $10 - 4i$ **c** $-1 + 2i$ **d** $3 - 3i$
e $4 - 7i$ **f** $12 + i$ **g** $3 + 4i$ **h** $21 - 20i$

2 **a** $-3 + 7i$ **b** $2i$ **c** $-2 + 2i$ **d** $-1 + i$
e $-5 - 12i$ **f** $-5 + i$ **g** $-6 - 4i$ **h** $-1 - 5i$

3 $(a + \cancel{bi}) + (a - \cancel{bi}) = 2a$ which is real

4 **a** $i^0 = 1$, $i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$, $i^5 = i$,
$i^6 = -1$, $i^7 = -i$, $i^8 = 1$, $i^9 = i$, $i^{-1} = -i$,
$i^{-2} = -1$, $i^{-3} = i$, $i^{-4} = 1$, $i^{-5} = -i$

b $i^{4n+3} = -i$

5 $(1+i)^4 = -4$, $(1+i)^{101} = -2^{50}(1+i)$

6 **a** -6 **b** No, $\sqrt{-4} \times \sqrt{-9} = -6 \neq \sqrt{36}$.

7 **a** $-\frac{1}{10} - \frac{7}{10}i$ **b** $-\frac{1}{5} + \frac{2}{5}i$ **c** $\frac{7}{5} + \frac{1}{5}i$ **d** $\frac{3}{25} + \frac{4}{25}i$

8 **a** $-\frac{2}{5} + \frac{1}{5}i$ **b** $-\frac{1}{13} + \frac{8}{13}i$ **c** $-\frac{2}{5} + \frac{3}{5}i$
d $\dfrac{2}{x^2+1}$

9 **a** -2 **b** -4 **c** 3 **d** 0

10 $z = 65 - 72i$ **11** $z = i\sqrt{2}$