Complex Numbers

This presentation contains some useful methods for solving problems involving complex numbers. Before going through the presentation make sure that you're familiar with the basic concepts, such as:

- Complex numbers arithmetic in Cartesian form.
- Argand diagram.
- Conversion between Cartesian and polar forms.
- Finding complex solutions to polynomials, including the use of Conjugate Root Theorem.
- Solving simultaneous equations with complex coefficients.
- De Moivre's Theorem.
- Multiplying and dividing complex numbers in polar form.
- Complex roots of unity and roots of complex numbers.

Some question on the exam papers are worded so that they seem as complex number question, but in fact can be done using vectors alone.

Question 1

Consider complex numbers: $z_1 = 2 + i$, $z_2 = -1 + 3i$ and $z_3 = 4 + 4i$. Show that the triangle in the complex plane with vertices in points represented by z_1, z_2 and z_3 is a right triangle.

Question 1 - solution

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Therefore $\angle BAC = 90^\circ$, what was to be proven.

Question 2

Points A, B and C on the complex plane represent the complex numbers $-1 - i$, 4 and $5 + 4i$ respectively. If $ABCD$ is a parallelogram:

a) find the complex number represented by point D, and

b) find the acute angle between the diagonals of this parallelogram.

Question 2 - solution

a) We have $\overrightarrow{OA} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$.

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b) We need to find the angle between the vectors $\overrightarrow{AC}$ and $\overrightarrow{DB}$.

$$\alpha = \arccos \frac{\overrightarrow{AC} \cdot \overrightarrow{DB}}{|\overrightarrow{AC}||\overrightarrow{DB}|} = \arccos \frac{9}{\sqrt{61}\sqrt{25}} = 76.7^\circ$$

Recall that given a polynomial:

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_1 x + a_0$$

with roots $x_1, x_2, x_3, ...x_n$. We have:

$$x_1 \cdot x_2 \cdot x_3 \cdot ... \cdot x_n = (-1)^n \frac{a_0}{a_n}$$

and

$$x_1 + x_2 + x_3 + ... + x_n = -\frac{a_{n-1}}{a_n}$$

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These equations are of course also true if we work with complex roots of polynomials.

Question 3

Find the fifth roots of i and find their product.

Question 3 - solution

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So we have:

$$z_0 = cis\left(\frac{\pi}{10}\right) \qquad z_1 = cis\left(\frac{\pi}{2}\right) \qquad z_2 = cis\left(\frac{9\pi}{10}\right)$$

$$z_3 = cis\left(\frac{13\pi}{10}\right) \qquad z_4 = cis\left(\frac{17\pi}{10}\right)$$

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In our case $n = 5$, $a_0 = -i$ and $a_n = 1$.

So we get:

$$z_0 \cdot z_1 \cdot z_2 \cdot z_3 \cdot z_4 = (-1)^5 \cdot \frac{-i}{1} = i$$

Question 4

Solve the equation $x^6 - 2x^3 + 4 = 0$ and hence show that:

$$\cos\frac{\pi}{9} + \cos\frac{7\pi}{9} + \cos\frac{13\pi}{9} = 0$$

Question 4 - solution

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$$u_1 = 1 + i\sqrt{3} \qquad\qquad u_2 = 1 - i\sqrt{3}$$

$$x^3 = 1 + i\sqrt{3} \qquad \textit{or} \qquad x^3 = 1 - i\sqrt{3}$$

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We will first solve $x^3 = 1 + i\sqrt{3} = 2cis\frac{\pi}{3}$.

We get: $x_k = \sqrt[3]{2}cis\left(\frac{\frac{\pi}{3} + 2\pi k}{3}\right)$ for $k = 0, 1, 2$.

Question 4 - solution

So we have:

$x_0 = \sqrt[3]{2}cis(\frac{\pi}{9})$ $\quad$ $x_1 = \sqrt[3]{2}cis(\frac{7\pi}{9})$ $\quad$ $x_2 = \sqrt[3]{2}cis(\frac{13\pi}{9})$

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Note that these do not come in conjugate pairs. The original polynomial has degree 6, so there should be 3 more solutions, and these will be the conjugates of the above (so there is no need to solve $x^3 = 1 - i\sqrt{3}$)

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$$x_4 = \sqrt[3]{2}cis(-\frac{\pi}{9}) \qquad x_5 = \sqrt[3]{2}cis(-\frac{7\pi}{9}) \qquad x_6 = \sqrt[3]{2}cis(-\frac{13\pi}{9})$$

Question 4 - solution

The sum of the roots of the polynomial is 0. Consider the real part of this sum (of course it is also 0). Using the fact that cos is an even function, we get:

$$Re(\sum_i x_i) = 2\sqrt[3]{2}\cos\left(\frac{\pi}{9}\right) + 2\sqrt[3]{2}\cos\left(\frac{7\pi}{9}\right) + 2\sqrt[3]{2}\cos\left(\frac{13\pi}{9}\right) = 0$$

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which gives:

$$\cos\left(\frac{\pi}{9}\right) + \cos\left(\frac{7\pi}{9}\right) + \cos\left(\frac{13\pi}{9}\right) = 0$$

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Of course you may be asked to prove identities for any trigonometric function, similar approaches are then used.

Question 5

Prove that

$$\cos^6 \theta = \frac{1}{32}\cos 6\theta + \frac{3}{16}\cos 4\theta + \frac{15}{32}\cos 2\theta + \frac{5}{16}$$

Question 5 - solution

Let $z = cis\theta$. Then

$$(2\cos\theta)^6 = \left(z + \frac{1}{z}\right)^6 =$$

$$= z^6 + 6z^4 + 15z^2 + 20 + \frac{15}{z^2} + \frac{6}{z^4} + \frac{1}{z^6} =$$

$$= \left(z^6 + \frac{1}{z^6}\right) + 6\left(z^4 + \frac{1}{z^4}\right) + 15\left(z^2 + \frac{1}{z^2}\right) + 20 =$$

$$= 2\cos 6\theta + 6 \cdot 2\cos 4\theta + 15 \cdot 2\cos 2\theta + 20$$

Question 5 - solution

We get that:

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Dividing both sides by 64 we get the desired result: Prove that

$$\cos^6\theta = \frac{1}{32}\cos 6\theta + \frac{3}{16}\cos 4\theta + \frac{15}{32}\cos 2\theta + \frac{5}{16}$$

Question 6

Express $\sin 5\theta$ in terms of $\sin\theta$.

Question 6 - solution

$$\sin 5\theta = Im(cis^5\theta) =$$
$$= Im(\cos^5\theta + 5i\cos^4\theta\sin\theta + 10i^2\cos^3\theta\sin^2\theta +$$
$$+ 10i^3\cos^2\theta\sin^3\theta + 5i^4\cos\theta\sin^\theta + i^5\sin^5\theta) =$$
$$= 5\cos^4\theta\sin\theta - 10\cos^2\sin^3\theta + \sin^5\theta$$

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$$= 5\cos^4\theta\sin\theta - 10\cos^2\sin^3\theta + \sin^5\theta$$

Now we use the identity $\cos^2\theta = 1 - \sin^2\theta$ to replace cosines with sines and we get:

$$\sin 5\theta = 5(1-\sin^2\theta)^2\theta\sin\theta - 10(1-\sin^2\theta)\sin^3\theta + \sin^5\theta$$
$$= 16\sin^5\theta - 20\sin^3\theta + \sin\theta$$

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The method rests on the following identities (also used for proving trig identites):

$$cis(\theta) + cis(-\theta) = 2\cos\theta$$

$$cis(\theta) - cis(-\theta) = 2i\sin\theta$$

Question 7

Express in terms of θ the modulus and argument of

$$z = i + \cos\theta - i\sin\theta$$

for $\theta \in [-\frac{3\pi}{2}, \frac{\pi}{2}]$.

Question 7 - solution

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Now we will factor out the complex number whose argument is the average of the two arguments (in our case $\frac{\pi}{4} - \frac{\theta}{2}$) and whose modulus is the same as that of the added numbers (in our case 1).

Question 7 - solution

We get:

$$i + \cos\theta - i\sin\theta = cis\left(\frac{\pi}{2}\right) + cis(-\theta) =$$
$$= cis\left(\frac{\pi}{4} - \frac{\theta}{2}\right)\left(cis\left(\frac{\pi}{4} + \frac{\theta}{2}\right) + cis\left(-\frac{\pi}{4} - \frac{\theta}{2}\right)\right) =$$
$$= cis\left(\frac{\pi}{4} - \frac{\theta}{2}\right)2\cos\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$$

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$$= cis\left(\frac{\pi}{4} - \frac{\theta}{2}\right) 2\cos\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$$

So our complex number $z = 2\cos\left(\frac{\pi}{4} + \frac{\theta}{2}\right) cis\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$. Now we can just read off the modulus and argument:

$$|z| = 2\cos\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$$

$$arg(z) = \frac{\pi}{4} - \frac{\theta}{2}$$

Question 8

Express in terms of θ the modulus and argument of

$$z = i - \cos\theta - i\sin\theta$$

for $\theta \in [\frac{\pi}{2}, \frac{5\pi}{2}]$

Question 8 - solution

We will use a similar approach.

$$i - \cos\theta - i\sin\theta = cis\left(\frac{\pi}{2}\right) - cis(\theta) =$$

$$= cis\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\left(cis\left(\frac{\pi}{4} - \frac{\theta}{2}\right) - cis\left(-\frac{\pi}{4} + \frac{\theta}{2}\right)\right) =$$

$$= cis\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\left(-2i\sin\left(-\frac{\pi}{4} + \frac{\theta}{2}\right)\right)$$

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\begin{aligned}
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&= cis\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\left(-2i\sin\left(-\frac{\pi}{4} + \frac{\theta}{2}\right)\right)
\end{aligned}
$$

Now we need to convert $-i$ into the polar form:

$$
cis\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\left(-2i\sin\left(-\frac{\pi}{4}+\frac{\theta}{2}\right)\right) = 2\sin\left(-\frac{\pi}{4}+\frac{\theta}{2}\right)cis\left(-\frac{\pi}{2}\right)cis\left(\frac{\pi}{4}+\frac{\theta}{2}\right)
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Finally we get:

$$z = 2\sin\left(-\frac{\pi}{4}+\frac{\theta}{2}\right) cis\left(-\frac{\pi}{2}\right) cis\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = 2\sin\left(-\frac{\pi}{4}+\frac{\theta}{2}\right) cis\left(-\frac{\pi}{4}+\frac{\theta}{2}\right)$$

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So:

$$|z| = 2\sin\left(-\frac{\pi}{4} + \frac{\theta}{2}\right)$$

$$arg(z) = -\frac{\pi}{4} + \frac{\theta}{2}$$

Questions 7, 8 - final remarks

In these kinds of questions you need to be very careful about the signs.

As an example consider a complex number $z = \cos(\frac{3\pi}{4})cis(\frac{\pi}{4})$. What is its argument?

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You may think that it is $\frac{\pi}{4}$, but this is not the case since $\cos(\frac{3\pi}{4})$ is negative, so we in fact have:

$$z = \cos\left(\frac{3\pi}{4}\right) cis\left(\frac{\pi}{4}\right) = -\cos\left(\frac{-\pi}{4}\right) cis\left(\frac{\pi}{4}\right) =$$

$$= cis(\pi)\cos\left(\frac{-\pi}{4}\right) cis\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) cis\left(\frac{5\pi}{4}\right)$$

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So $arg(z) = \dfrac{5\pi}{4}$.