1. A vector equation of a line is $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + t\begin{pmatrix} -2 \\ 3 \end{pmatrix}$, $t \in \mathbb{R}$.

Find the equation of this line in the form $ax + by = c$, where a, b, and $c \in \mathbb{Z}$.

(Total 6 marks)

2. A vector equation for the line L is $\boldsymbol{r} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + t\begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

Which of the following are also vector equations for the same line L?

A. $\boldsymbol{r} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

B. $\boldsymbol{r} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + t\begin{pmatrix} 6 \\ 2 \end{pmatrix}$.

C. $\boldsymbol{r} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

D. $\boldsymbol{r} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} + t\begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

(Total 6 marks)

3. Calculate the acute angle between the lines with equations

$\boldsymbol{r} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + s\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

(Total 6 marks)

4. The vector equations of two lines are given below.

$$\boldsymbol{r}_1 = \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ -2 \end{pmatrix}, \quad \boldsymbol{r}_2 = \begin{pmatrix} -2 \\ 2 \end{pmatrix} + t\begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

The lines intersect at the point P. Find the position vector of P.

(Total 6 marks)

5. Car 1 moves in a straight line, starting at point A (0, 12). Its position p seconds after it starts is given by $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 12 \end{pmatrix} + p\begin{pmatrix} 5 \\ -3 \end{pmatrix}$.

(a) Find the position vector of the car after 2 seconds.

(2)

Car 2 moves in a straight line starting at point B (14, 0). Its position q seconds after it starts is given by $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 14 \\ 0 \end{pmatrix} + q\begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

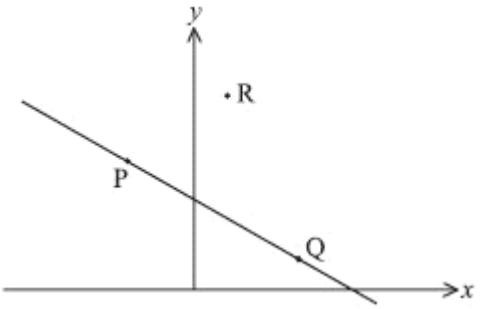
Cars 1 and 2 collide at point P.

(b) (i) Find the value of p and the value of q when the collision occurs.

(ii) Find the coordinates of P.

(6)
(Total 8 marks)

6. The points P(−2, 4), Q (3, 1) and R (1, 6) are shown in the diagram below.



(a) Find the vector $\overrightarrow{PQ}$.

(b) Find a vector equation for the line through R parallel to the line (PQ).

(Total 6 marks)

7. The line L passes through A (0, 3) and B (1, 0). The origin is at O. The point R $(x, 3 - 3x)$ is on L, and (OR) is perpendicular to L.

(a) Write down the vectors $\overrightarrow{AB}$ and $\overrightarrow{OR}$.

(b) Use the scalar product to find the coordinates of R.

(Total 6 marks)

8. In this question, a unit vector represents a displacement of 1 metre.

A miniature car moves in a straight line, starting at the point (2, 0). After t seconds, its position, (x, y), is given by the vector equation

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0.7 \\ 1 \end{pmatrix}$$

(a) How far from the point (0, 0) is the car after 2 seconds?

(2)

(b) Find the speed of the car.

(2)

(c) Obtain the equation of the car's path in the form $ax + by = c$.

(2)

Another miniature vehicle, a motorcycle, starts at the point (0, 2), and travels in a straight line with constant speed. The equation of its path is

$$y = 0.6x + 2, \quad x \geq 0.$$

Eventually, the two miniature vehicles collide.

(d) Find the coordinates of the collision point.

(3)

(e) If the motorcycle left point (0, 2) at the same moment the car left point (2, 0), find the speed of the motorcycle.

(5)

(Total 14 marks)