ANSWERS

EXERCISE 1A

1 **a** $m = 3,\ c = 7$ **b** $m = -2,\ c = -5$
c $m = \frac{2}{3},\ c = -\frac{1}{3}$ **d** $m = \frac{7}{9},\ c = \frac{2}{9}$
e $m = \frac{1}{3},\ c = -\frac{1}{2}$ **f** $m = -\frac{5}{8},\ c = \frac{3}{8}$

2 **a** $y = 3x - 11$ **b** $y = -2x - 1$ **c** $y = \frac{1}{4}x - 4$
d $y = -\frac{3}{4}x + 4$

3 **a** The gradient is -10 which means that the balance in the account decreases by \$10 each year.
The y-intercept is 90 which means that the initial balance was \$90.
b $y = -10x + 90$ **c** 9 years

4 **a** $-\frac{23}{910}$ **b** $y = -\frac{23}{910}x + 46$

5 **a** 150 metres
b The height of the helicopter above sea level increases by 120 metres each minute after taking off.
c 390 metres **d** 4 minutes 10 seconds

6 **a** $4x + y = 6$ **b** $5x - y = 3$ **c** $3x + 4y = 5$
d $3x - 5y = 1$

7 **a** $y = -5x + 2$ **b** $y = -\frac{3}{7}x - \frac{2}{7}$ **c** $y = 2x - 6$
d $y = \frac{3}{13}x + \frac{4}{13}$

8 $ax + by = d$ can be written as $y = -\frac{a}{b}x + \frac{d}{b}$ which has the form $y = mx + c$. $\therefore\ m = -\frac{a}{b}$

9 **a** $4x + y = 6$ **b** $x - 2y = 13$ **c** $5x + 3y = 8$
d $7x - 6y = 17$

10 **a** $y = 2x + 5$ **b** $y = -x + 9$ **c** $y = \frac{7}{5}x - \frac{11}{5}$
d $y = -\frac{5}{6}x + \frac{19}{6}$

11 **a** $2x - y = -2$ **b** $3x + 10y = 8$ **c** $8x + 5y = -13$

12 **a** $y = \frac{3}{4}x - \frac{5}{4}$ **b** $-\frac{5}{4}$

13 **a** $y = 3x + 1$ **b** $2x - y = 7$ **c** $y = \frac{1}{2}x + \frac{11}{2}$
d $2x - y = -3$

14 *Line 1*: $y = \frac{2}{3}x + \frac{1}{3}$, *Line 2*: $y = -\frac{3}{2}x - 4$

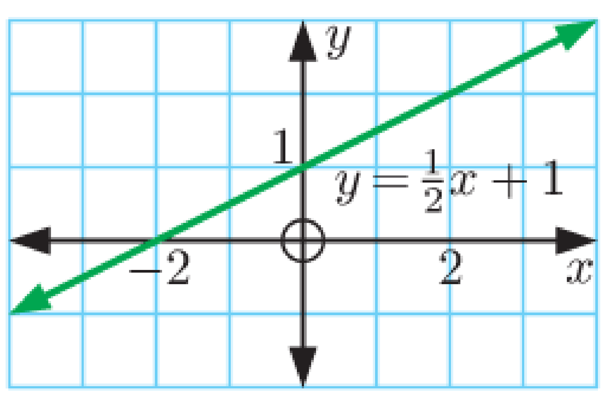
15 **a** yes **b** no **c** yes **d** yes

16 **a** $c = 7$ **b** $m = 11$ **c** $t = 8$

17 **a** $k = -3$ **b** $k = -51$ **c** $k = -12$

18 **a** $x - y + 2 = 0$ **b** -2 **19** 126 units^2

EXERCISE 1B

1 **a**


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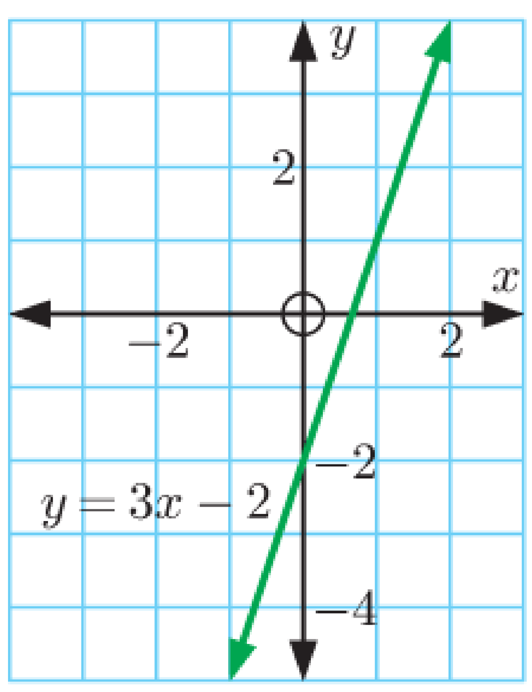
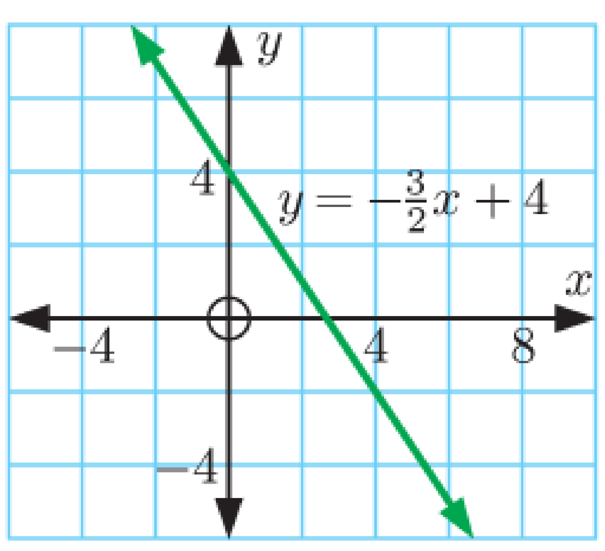


c


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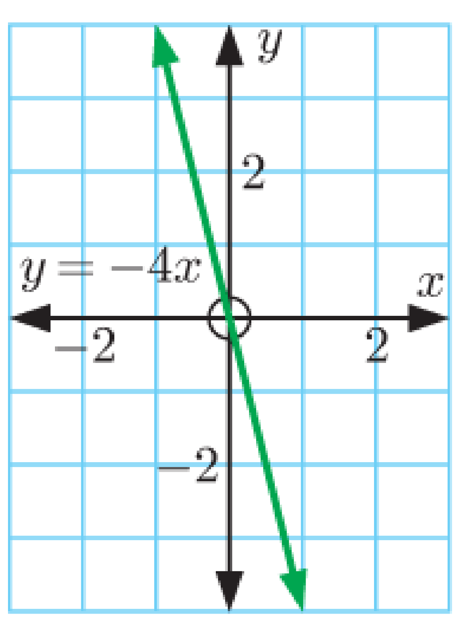

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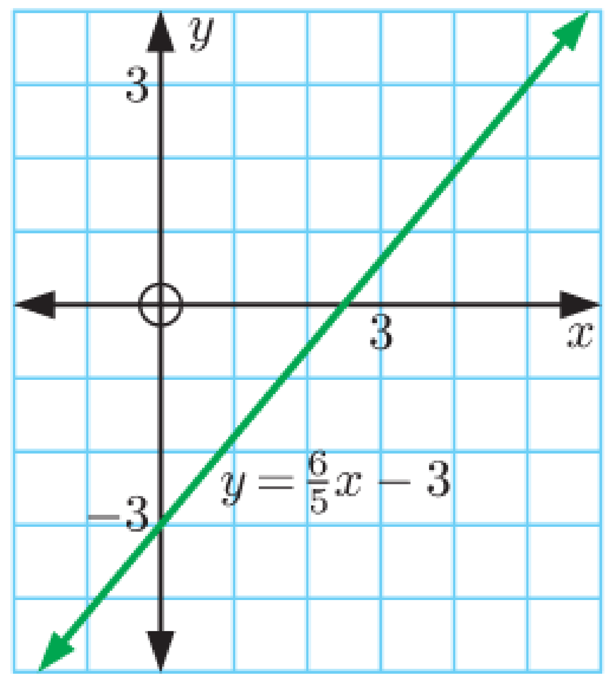

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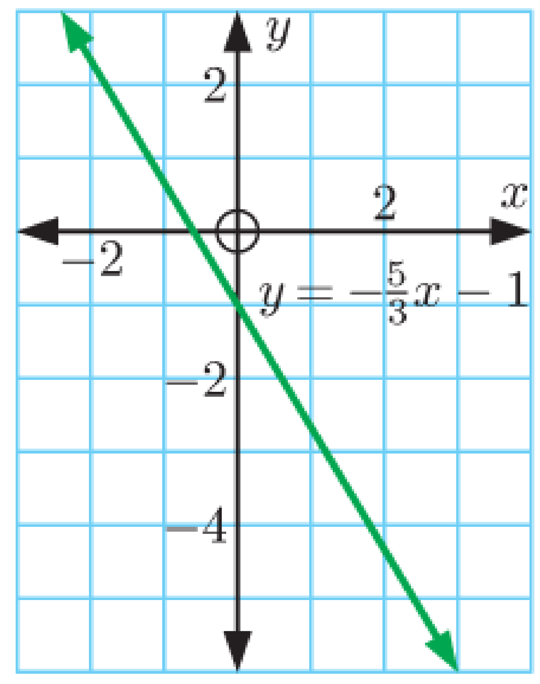
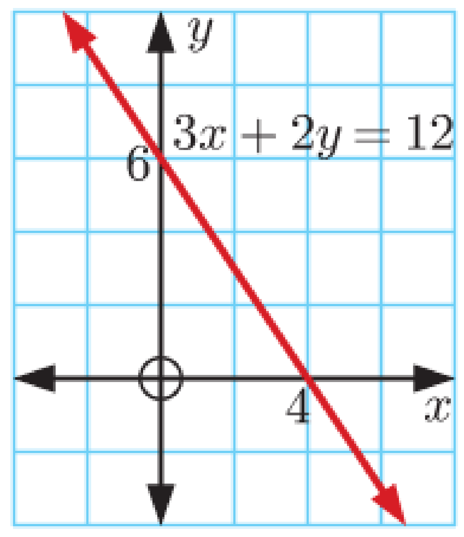


2 **a**


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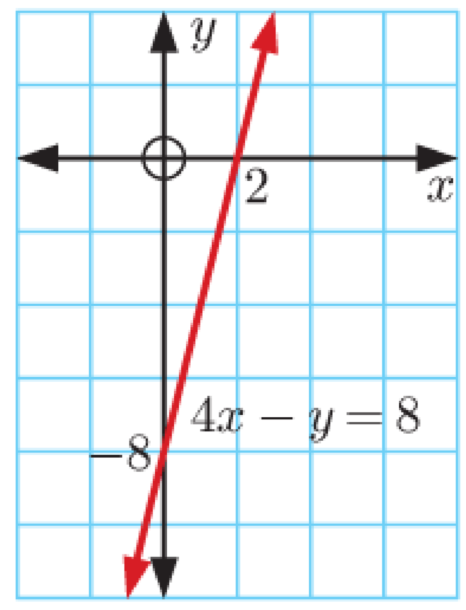

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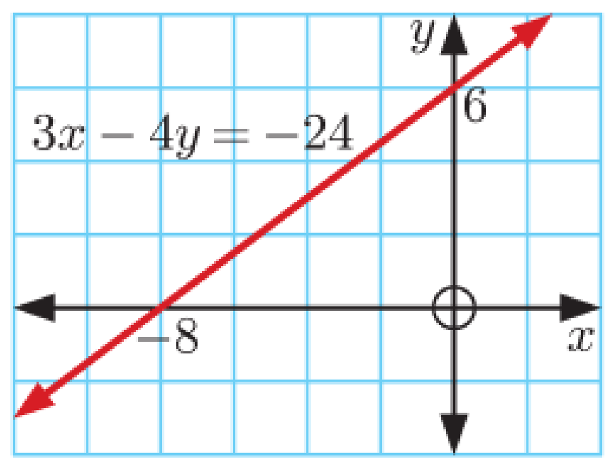

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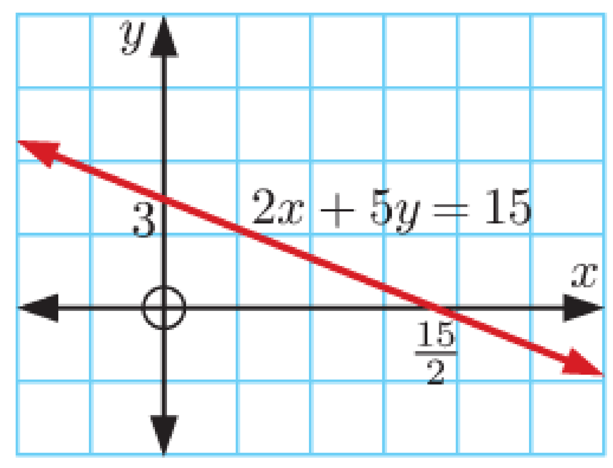

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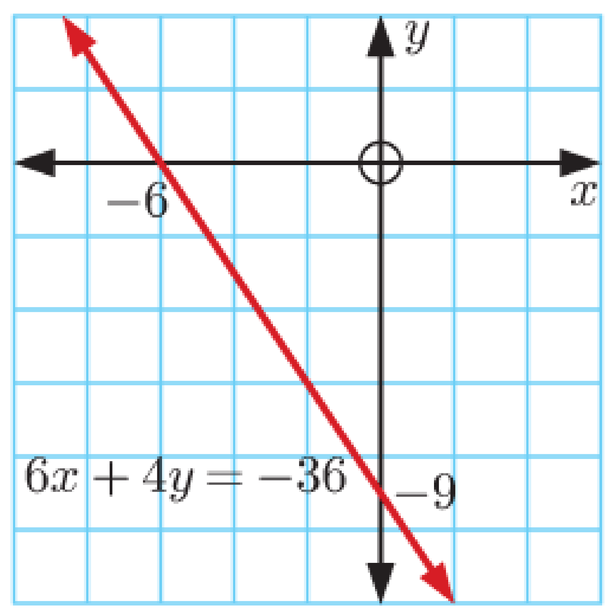

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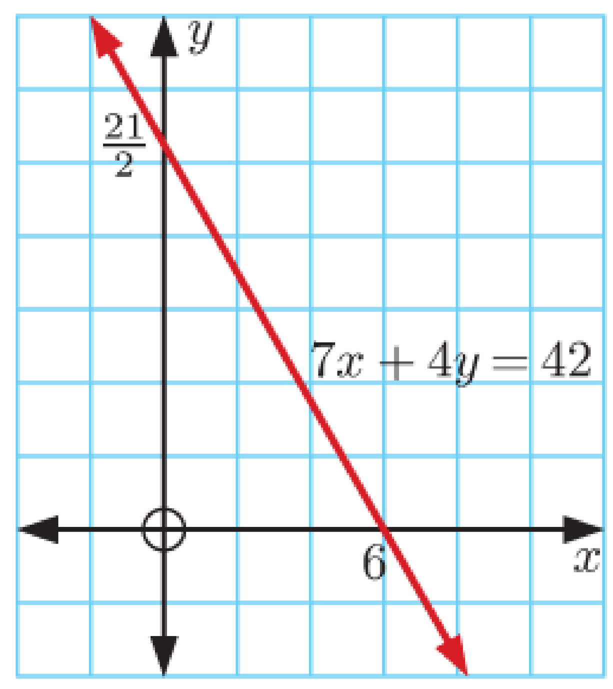
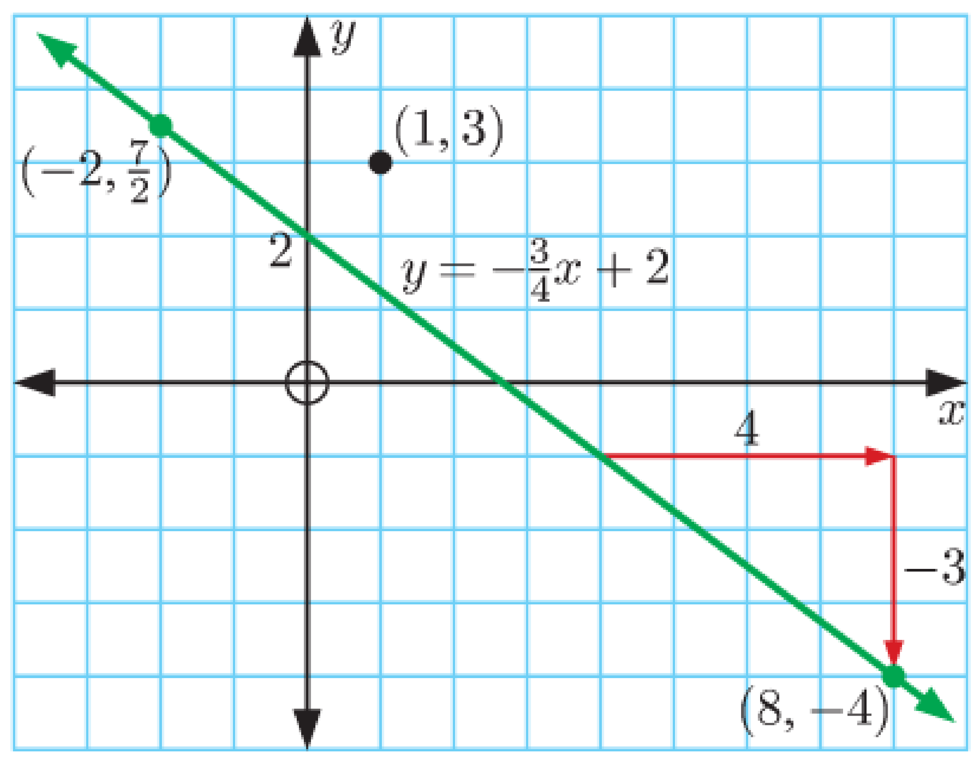
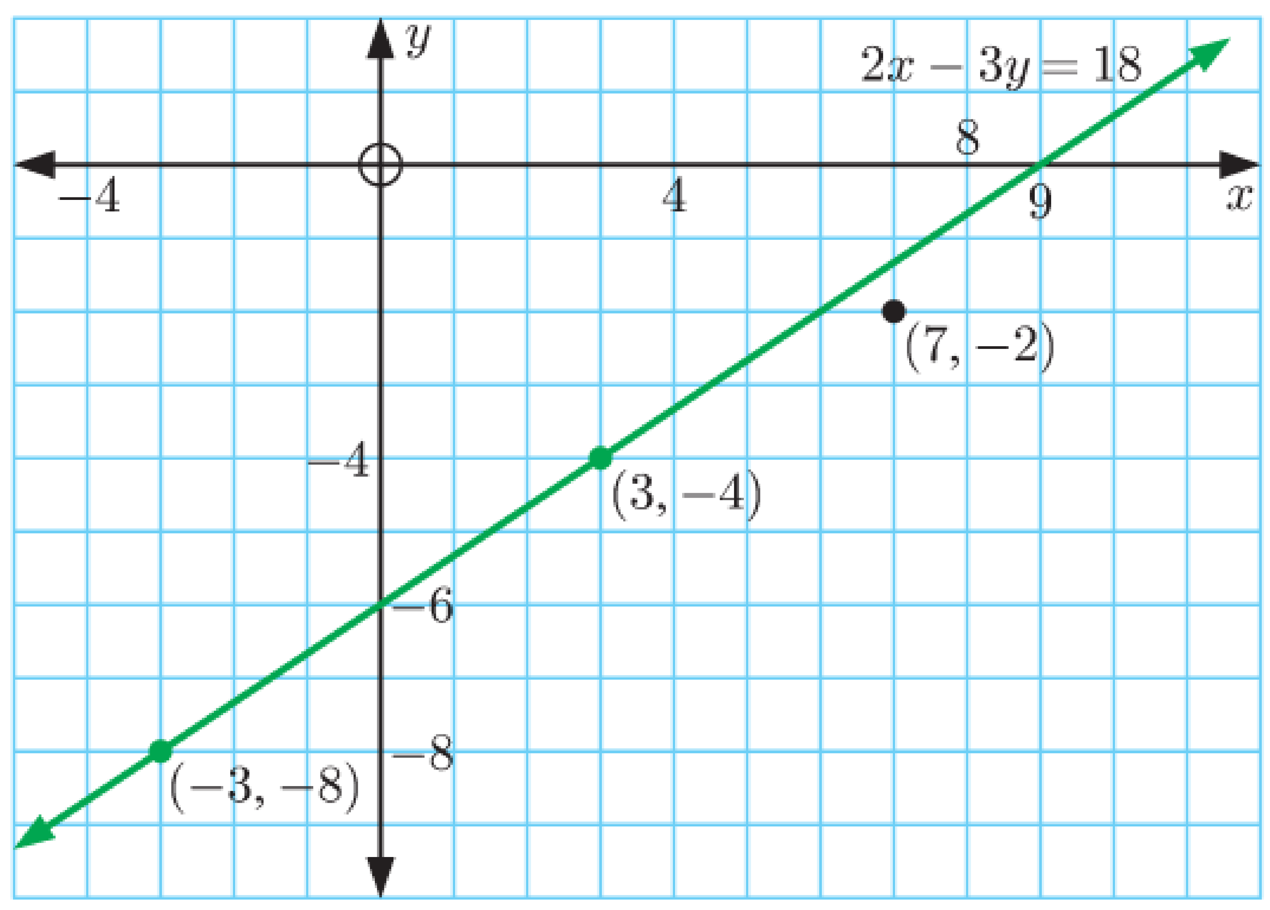


3 **a** $m = -\frac{3}{4},\ c = 2$ **b** **i** yes **ii** no **iii** yes
c


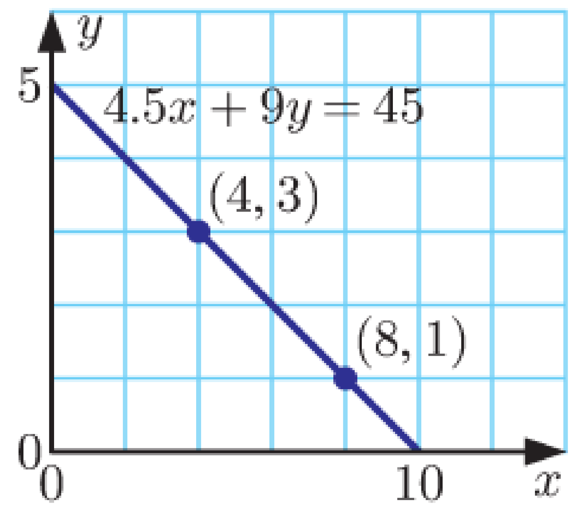
4 **a** x-intercept 9, y-intercept -6
b **i** yes **ii** no **c** $c = -8$
d


5 **a** x serves of nigiri at \$4.50 each and y serves of sashimi at \$9 each adds up to a total of \$45. $\therefore$ $4.5x + 9y = 45$

b 3 serves of sashimi

c 8 serves of nigiri

d



EXERCISE 1C

1 **a** $(4, 4)$ **b** 3 **c** $-\frac{1}{3}$ **d** $x + 3y = 16$

2 **a** $2x - y = 3$ **b** $3x - y = 2$ **c** $x - y = 5$ **d** $2x - y = 2$ **e** $x = \frac{9}{2}$ **f** $8x + 6y = 35$

3 **a** $2x - 3y = 2$ **b** $2(1) - 3(0) = 2$ ✓ **c** $\text{PR} = \text{QR} = \sqrt{26}$ units

4 **a** $\text{AB} = \text{BC} = \text{CD} = \text{AD} = \sqrt{29}$ units $\therefore$ ABCD is a rhombus.

b $y = -x$ **c** B: $2 = -(-2)$ ✓ D: $-1 = -(1)$ ✓

5 **a** **i** $\frac{3}{2}$ **ii** $-\frac{2}{3}$ **b** $2x + 3y - 21 = 0$

6 **a** $(-\frac{1}{2}, 1)$

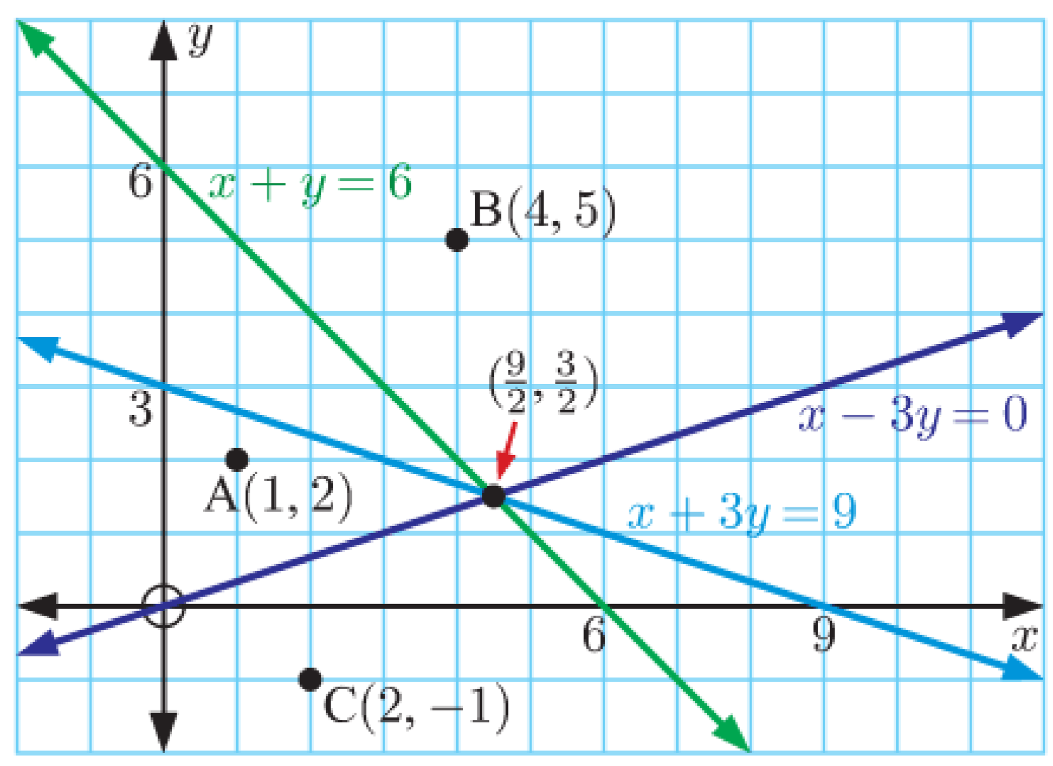
b The perpendicular bisector of the line joining the two hospitals is $10x + 12y = 7$. An ambulance crew should be sent from A to locations below this line, and from B to locations above this line.

7 **a** **Hint:** Start by finding the gradient and midpoint of [AB].

b We can find the perpendicular bisector of any two points $\text{A}(x_1, y_1)$ and $\text{B}(x_2, y_2)$ by substituting in the values of x_1, x_2, y_1, and y_2.

8 **a** **i** $x + y = 6$ **ii** $x - 3y = 0$ **iii** $x + 3y = 9$

b

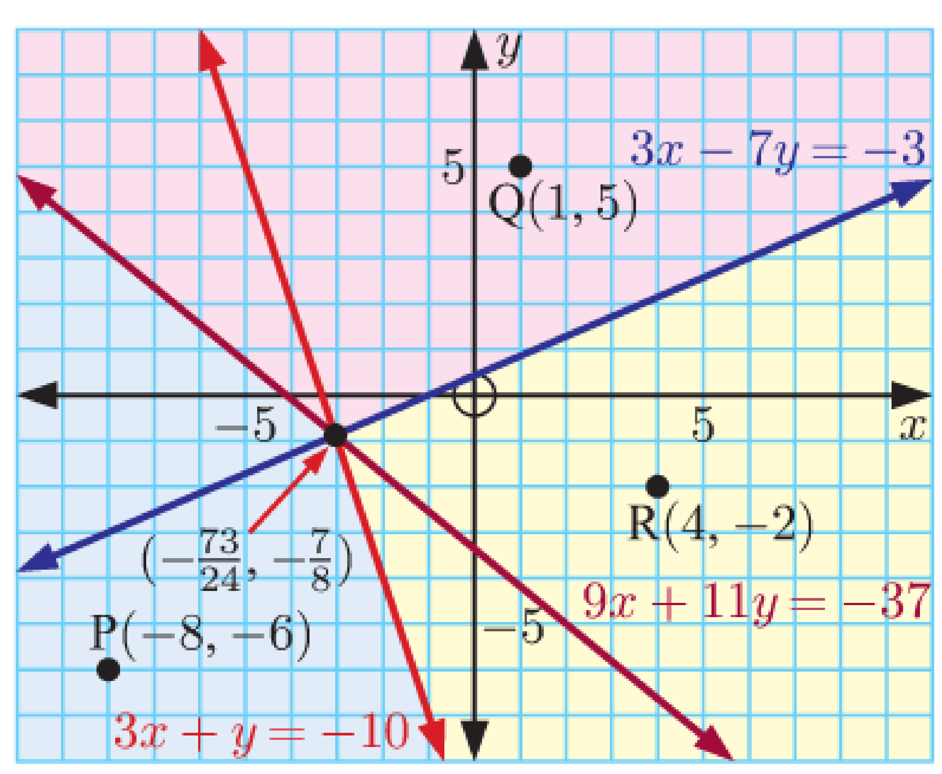


The perpendicular bisectors all intersect at $(\frac{9}{2}, \frac{3}{2})$.
A, B, and C are all equidistant from this point.

c The perpendicular bisectors of each pair of points will meet at a single point. As the three points are equidistant from the point of intersection, a circle centred at the point of intersection that passes through one of them will pass through all of them.

9 **a** **i** $9x + 11y = -37$ **ii** $3x + y = -10$ **iii** $3x - 7y = -3$

b



EXERCISE 1D

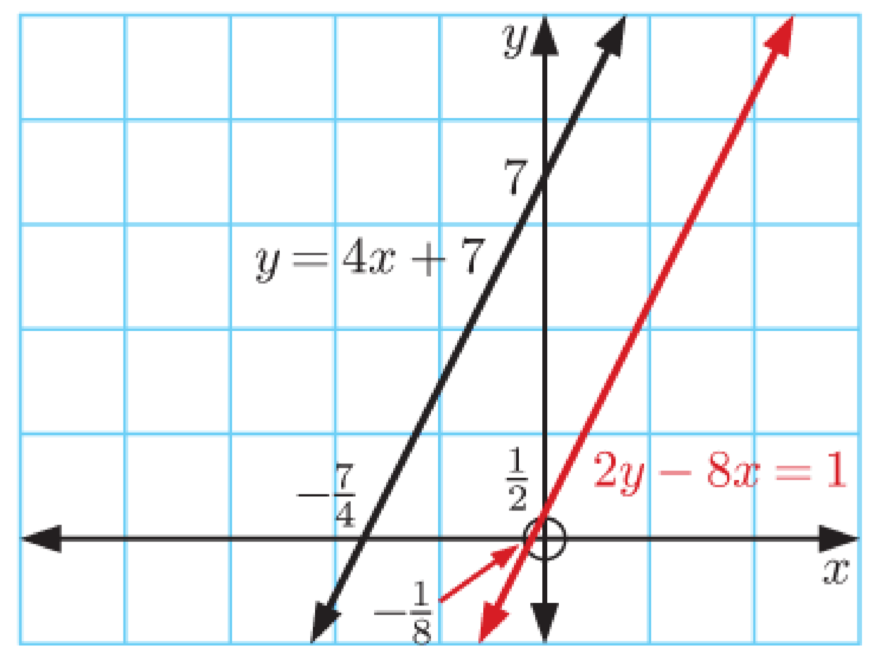
1 **a** $x = -2,\ y = -4$ **b** $x = 1,\ y = -3$ **c** $x = 6,\ y = 7$ **d** $x = 3,\ y = 2$ **e** $x = 6,\ y = 1$ **f** $x = 2,\ y = -12$

2 **a** $x = 3,\ y = 5$ **b** $x = 1,\ y = -1$ **c** $x = -1,\ y = 8$ **d** $x = 5,\ y = 8$ **e** $x = -4,\ y = -\frac{1}{4}$ **f** $x = -1\frac{11}{31},\ y = -\frac{4}{31}$ **g** $x = -3,\ y = 3\frac{1}{2}$ **h** $x = -2\frac{1}{4},\ y = 3$ **i** $x = \frac{1}{4},\ y = \frac{3}{4}$

3 **a** $x = 2,\ y = 1$ **b** $x = 3,\ y = -1$ **c** $x = 3,\ y = 7$ **d** $x = \frac{1}{3},\ y = 4$ **e** $x = \frac{1}{4},\ y = 1\frac{1}{4}$ **f** $x = 5,\ y = -2$ **g** $x = -3,\ y = -4$ **h** $x = -4\frac{1}{2},\ y = -2\frac{1}{2}$ **i** $x = -28\frac{2}{3},\ y = -17\frac{2}{3}$

4 **a** $12\frac{1}{4}$ units^2 **b** $1\frac{4}{25}$ units^2

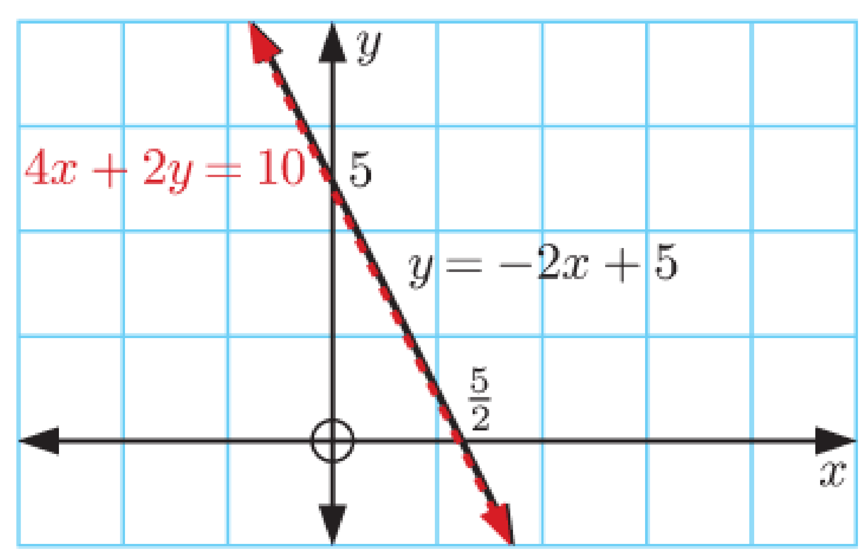
5 **a**



The lines are parallel.

c no solutions

6 **a**



The lines are coincident.

c infinitely many solutions

7 **a** $\frac{3}{2}$ and m

b $m = \frac{3}{2}$, in this case the lines are coincident and hence there are infinitely many solutions.

c $x = 0,\ y = -6$

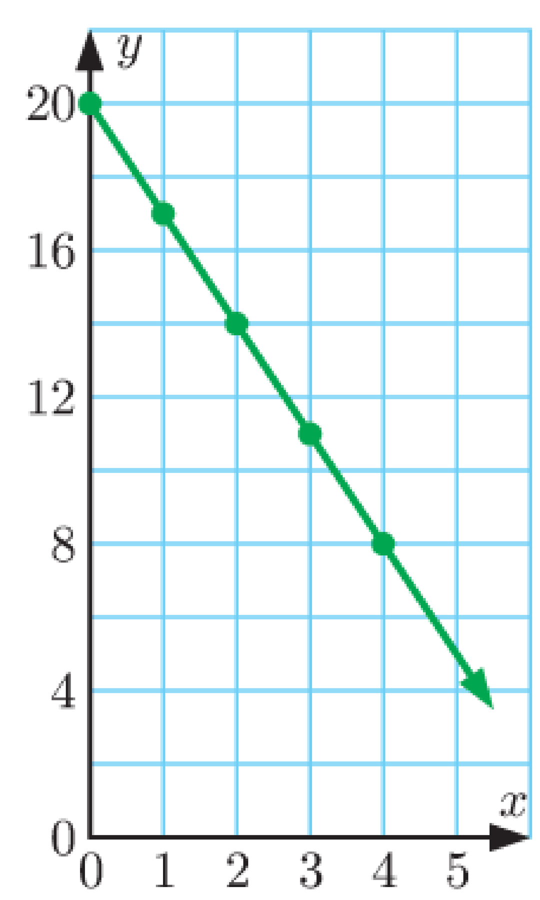
8 **a** $\dfrac{4}{c}$ and $\frac{2}{3}$

b $c = 6$, in this case the lines are parallel and hence there are no solutions.

c $x = \dfrac{18 + 3c}{6 - c},\quad y = \dfrac{24}{6 - c}$

REVIEW SET 1A

1 **a**



b Yes, the variables are linearly related as the points all lie on a straight line.

c gradient is -3, y-intercept is 20

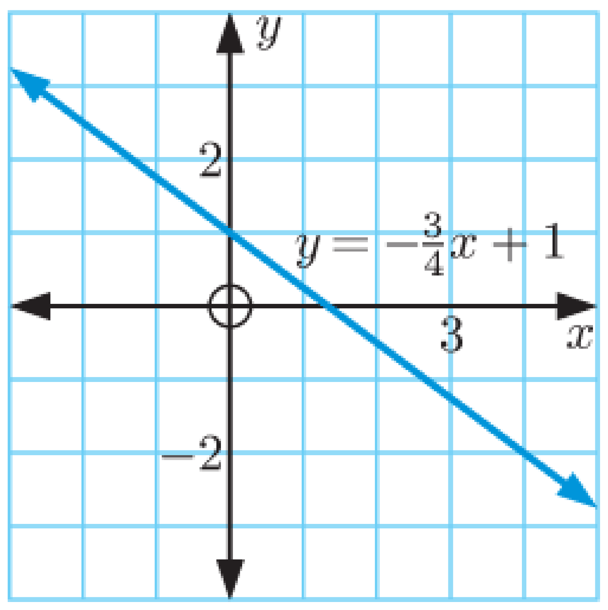
d $y = -3x + 20$

e $y = -1$

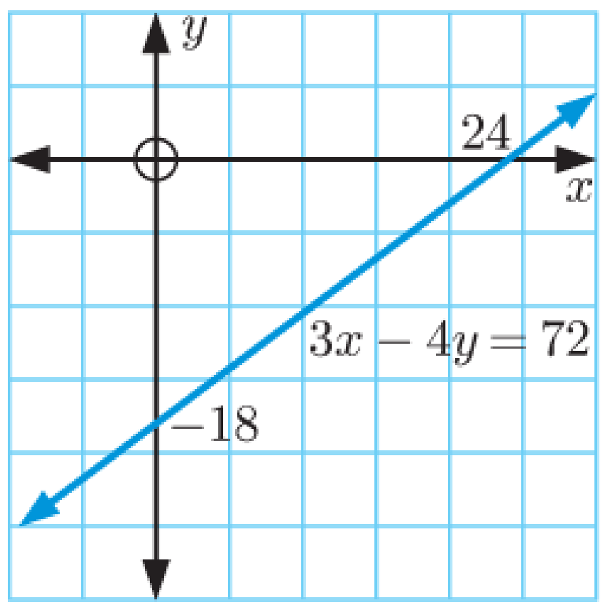
2 **a** $y = -\frac{1}{3}x + 4$ **b** $x + 3y - 12 = 0$

3 **a** $3x - 2y = 12$ **b** 4 **4** **a** yes **b** yes

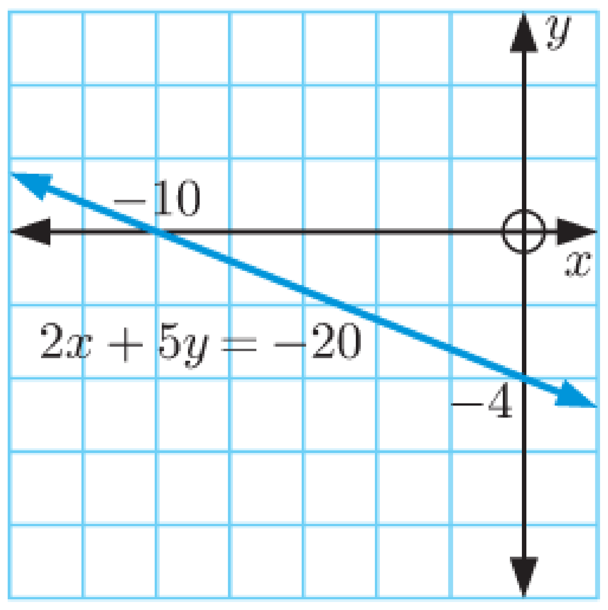
5 **a**



b



c



6 **a** $y = -1$ **b** $3x - 2y = 9$

7 **a** **i** $7x + 5y = -6$ **ii** $5x - 7y = 1$

b ABCD is a square.

8 **a** $x = -2,\ y = -5$ **b** $x = 4,\ y = -2$

9 **a** $x = 1,\ y = 7$ **b** $x = -1,\ y = 2$

10 **a** $x = 3,\ y = -1$ **b** $x = -4,\ y = 3$

11 **a** $-\frac{1}{2}$ and $-\frac{1}{2}$

b **i** $k \neq 4$ **ii** $k = 4$

If $k \neq 4$, the lines are parallel and hence there are no solutions.

If $k = 4$, the lines are coincident and hence there are infinitely many solutions.

REVIEW SET 1B

1 **a** The gradient is 10 which means that the speed increases by 10 m s^{-1} each second.

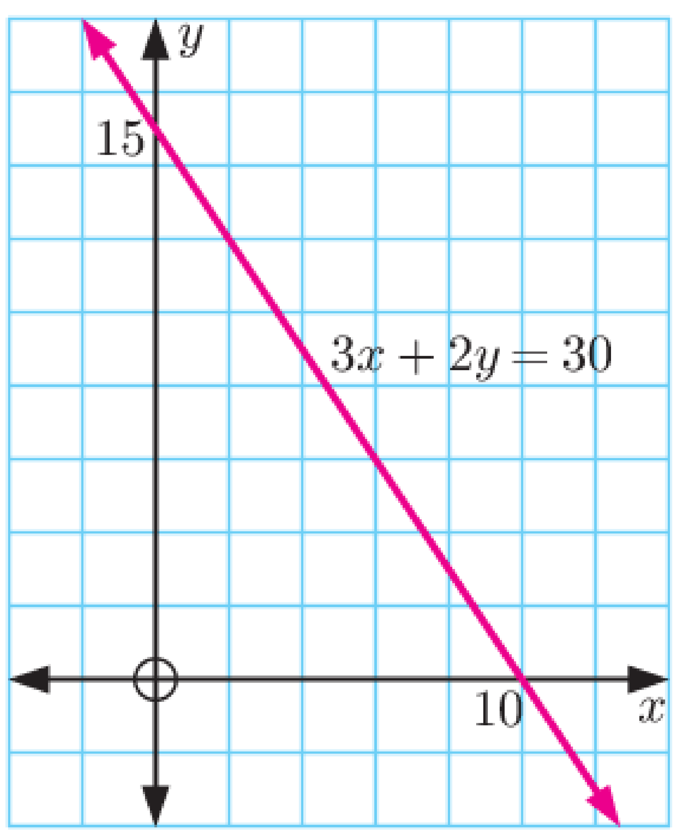
The y-intercept is 5 which means that the initial speed was 5 m s^{-1}.

b $y = 10x + 5$ **c** 85 m s^{-1}

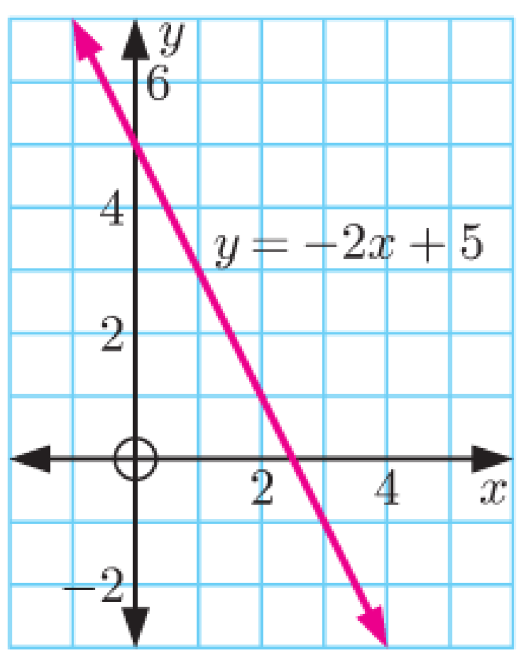
2 **a** $y = 3x + 1$ **b** $5x - 2y = -3$

3 **a** $k = 7$ **b** $k = -11$

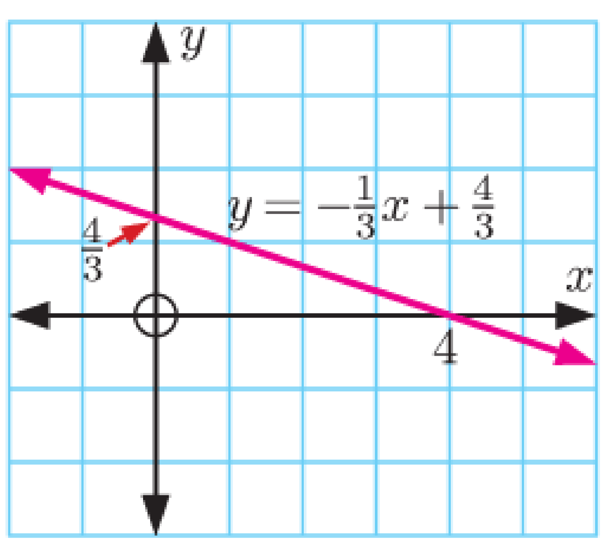
4 **a**



b



c

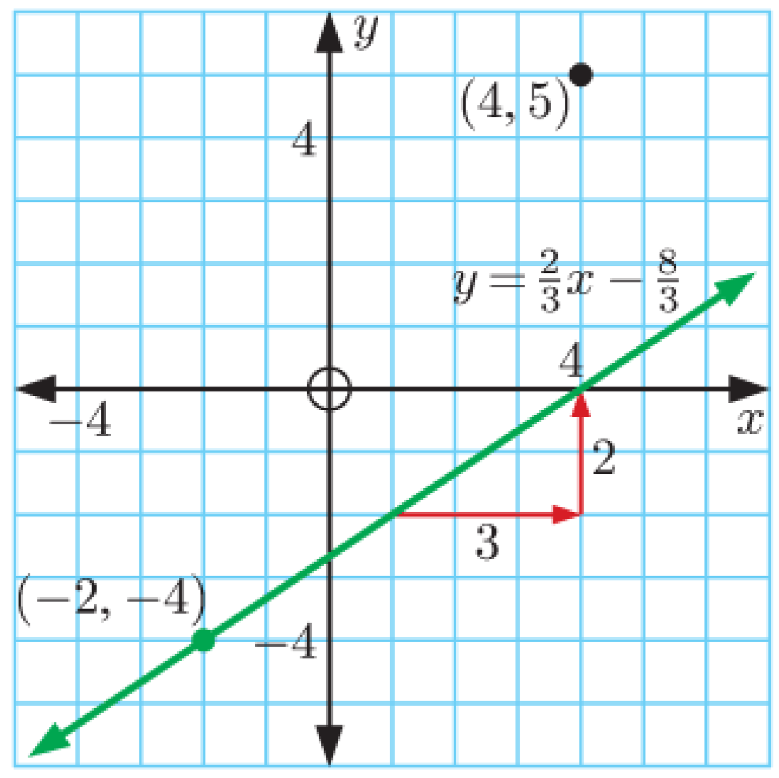


5 **a** $m = \frac{2}{3}$

b **i** yes

ii no

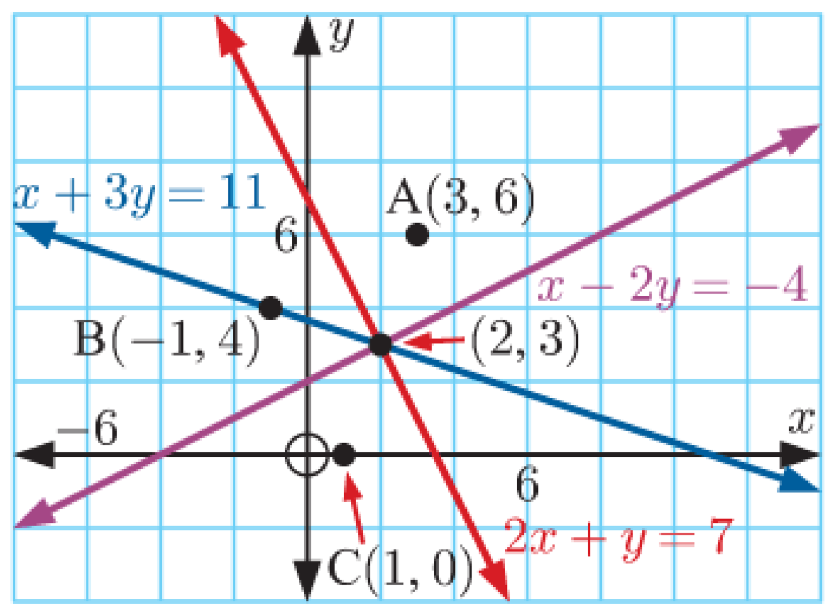
c



6 $108\frac{1}{3}$ units^2 **7** **a** **i** $\frac{1}{5}$ **ii** -5 **b** $5x + y = 22$

8 **a** **i** $2x + y = 7$ **ii** $x + 3y = 11$ **iii** $x - 2y = -4$

b



All three perpendicular bisectors intersect at (2, 3).
A, B, and C are all equidistant from this point.

9 **a** $x = \frac{1}{3},\ y = 4$ **b** $x = -2,\ y = 4$

10 **a** $x = 3,\ y = -\frac{1}{2}$ **b** $x = 1\frac{1}{2},\ y = -3\frac{1}{2}$

11 **a** $-\dfrac{a}{4}$ and $\frac{1}{2}$

b $a = -2$, in this case the lines are parallel and hence there are no solutions.

c $x = \dfrac{2}{a+2},\quad y = \dfrac{a+3}{a+2}$

EXERCISE 2A

1 **a** $A = \{1, 2, 4, 8\},\quad n(A) = 4$

b $A = \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18\},\quad n(A) = 10$

c $A = \{\text{A, R, D, V, K}\},\quad n(A) = 5$

d $A = \{41, 43, 47\},\quad n(A) = 3$

2 **a** finite **b** infinite **c** infinite

3 **a** **i** 6 **ii** 3

b **i** true **ii** false **iii** true **iv** true **v** true

4 **a** subsets of S: $\varnothing, \{1\}, \{2\}, \{1, 2\}$

subsets of T: $\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$

b yes **c** $\frac{1}{2}$

5 Each of the four elements can be *included* or *not included* in a subset.

$\therefore$ the set has $2 \times 2 \times 2 \times 2 = 16$ subsets.

6 $x = 3$

7 If $A \subseteq B$, then all elements of A are in B.

If $B \subseteq A$, then all elements of B are in A.

This is only possible if A and B contain exactly the same elements.

$\therefore\ A = B$.

EXERCISE 2B

1 **a** **i** $A \cap B = \{9\}$

ii $A \cup B = \{5, 6, 7, 8, 9, 10, 11, 12, 13\}$

b **i** $A \cap B = \varnothing$ **ii** $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

c **i** $A \cap B = \{1, 3, 5, 7\}$
ii $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
d **i** $A \cap B = \{5, 8\}$
ii $A \cup B = \{0, 1, 3, 4, 5, 8, 11, 13, 14\}$

2 **a** disjoint **b** not disjoint

3 **a** True, $R \cap S = \varnothing$ tells us that R and S have no elements in common, and hence are disjoint.
b True, every element of $A \cap B$ is an element of A, and every element of $A \cap B$ is an element of B.
c True, if $A \cap B = A \cup B$ then there are no elements that are in only A or only B. $\therefore$ $A = B$.
d False, consider $A = \{1, 2, 3, ...\}$ and $B = \{-1, -2, -3, ...\}$ which are infinite but $A \cap B = \varnothing$ which is finite.

4 Not necessarily, consider $A = \{1, 2\}$, $B = \{3, 4\}$, and $C = \{1, 6\}$. $A \cap B = \varnothing$ and $B \cap C = \varnothing$, but $A \cap C = \{1\}$, so A and C are not disjoint sets.

5 **a** $n(A \cap B) = 0, 1, 2, 3, 4, 5, 6, 7$, or 8
b $n(A \cup B) = 11, 12, 13, 14, 15, 16, 17, 18$, or 19

6 Each element in $A \cup B$ must be in A or B, or both. It is not possible that $n(A \cup B) > n(A) + n(B)$
$\therefore$ $n(A \cup B) \leqslant n(A) + n(B)$

EXERCISE 2C

1 **a** $A' = \{1, 4, 5, 9\}$
b No, since 1 is neither prime nor composite.

2 **a** $A = \{10, 12, 15, 20\}$ **b** $B = \{12, 15, 18\}$
c $A' = \{11, 13, 14, 16, 17, 18, 19\}$
d $B' = \{10, 11, 13, 14, 16, 17, 19, 20\}$
e $A \cap B = \{12, 15\}$ **f** $A \cup B = \{10, 12, 15, 18, 20\}$
g $A' \cap B = \{18\}$
h $A' \cup B = \{11, 12, 13, 14, 15, 16, 17, 18, 19\}$
i $A \cap B' = \{10, 20\}$
j $A \cup B' = \{10, 11, 12, 13, 14, 15, 16, 17, 19, 20\}$
k $A' \cap B' = \{11, 13, 14, 16, 17, 19\}$
l $A' \cup B' = \{10, 11, 13, 14, 16, 17, 18, 19, 20\}$

3 **a** **i** 7 **ii** 3 **iii** 4 **iv** 4 **v** 3
b For any set S within a universal set U, $n(S) + n(S') = n(U)$.

4 **a** 9 **b** 11

5 As $P \subseteq Q$, then all elements of P are in Q.
$\therefore$ if an element is not in Q then it is not in P. $\therefore$ $Q' \subseteq P'$

6 Let $U = \{2, 3, 4, ...\}$ and $P = \{\text{primes}\}$.
$P' = \{\text{composites}\}$ which is an infinite set.
Let $U = \{0, 1, 2, 3, ...\}$ and $P = \{1, 2, 3, ...\}$.
$P' = \{0\}$ which is a finite set.

EXERCISE 2D

1

Number	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$
6	✓	✓	✓	✓
$-\frac{3}{8}$	✗	✗	✓	✓
1.8	✗	✗	✓	✓
$1.\overline{8}$	✗	✗	✓	✓
-17	✗	✓	✓	✓
$\sqrt{64}$	✓	✓	✓	✓
$\frac{\pi}{2}$	✗	✗	✗	✓
$\sqrt{-3}$	✗	✗	✗	✗
$-\sqrt{3}$	✗	✗	✗	✓

2 **a** false **b** true **c** false **d** true
e false **f** true **g** true **h** false

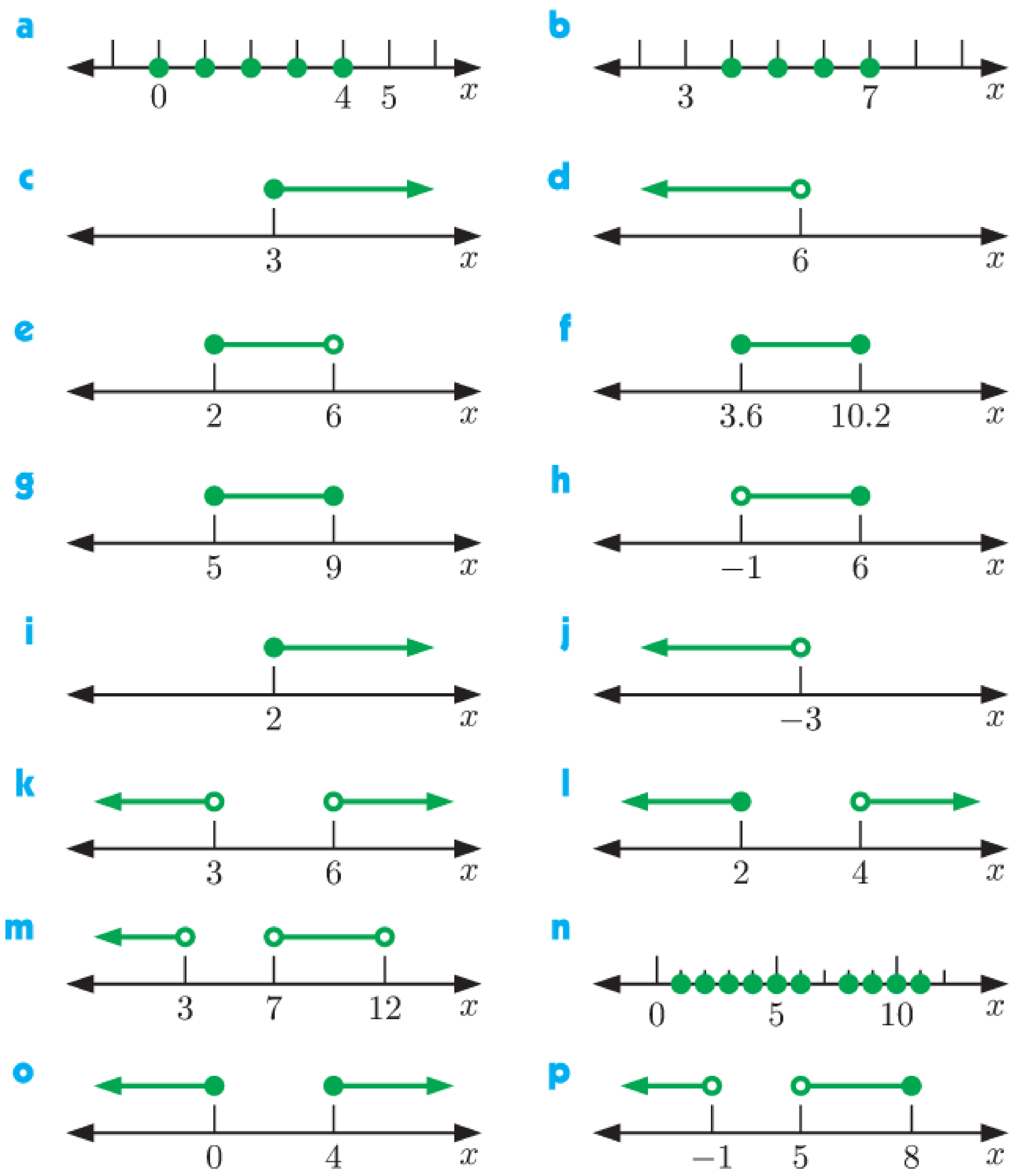
3 **a** true **b** true **c** false **d** true
e false **f** true **g** true **h** false

4 **a** finite **b** infinite **c** infinite **d** infinite

5 $\mathbb{Z}^- \cup \{0\}$

EXERCISE 2E

1 **a** **i** The set of all x such that x is an integer between -1 and 7, including -1 and 7.
ii $A = \{-1, 0, 1, 2, 3, 4, 5, 6, 7\}$ **iii** 9
b **i** The set of all x such that x is a natural number between -2 and 8.
ii $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$ **iii** 8
c **i** The set of all real x such that x is greater than or equal to 0, and less than or equal to 1.
ii not possible
iii A is an infinite set, $n(A)$ is undefined.
d **i** The set of all x such that x is a rational number greater than or equal to 5, and less than or equal to 6.
ii not possible
iii A is an infinite set, $n(A)$ is undefined.

2


3 **a** $\{x \in \mathbb{Z} \mid -100 < x < 100\}$ **b** $\{x \in \mathbb{R} \mid x > 1000\}$
c $\{x \in \mathbb{Q} \mid 2 \leqslant x \leqslant 3\}$

4 **a** $\{x \mid x \geqslant 8\}$ **b** $\{x \mid -1 \leqslant x < 4\}$
c $\{x \in \mathbb{Z} \mid -3 < x < 4\}$
d $\{x \in \mathbb{N} \mid x \leqslant 4\} \cup \{x \in \mathbb{N} \mid x = 6\}$

5 **a** $x \in [-3, 2[$ **b** $x \in [3, \infty[$ **c** $x \in]0, 2[$
d $x \in [1, 4] \cup [6, \infty[$

6 **a** $A \subseteq B$ **b** $A \not\subseteq B$ **c** $A \subseteq B$ **d** $A \subseteq B$
e $A \not\subseteq B$ **f** $A \not\subseteq B$

7 **a** $A = \{2, 3, 4, 5, 6, 7\}$ **b** $A' = \{0, 1, 8\}$
c $B = \{5, 6, 7, 8\}$ **d** $B' = \{0, 1, 2, 3, 4\}$
e $A \cap B = \{5, 6, 7\}$ **f** $A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$

g $A \cap B' = \{2, 3, 4\}$

8 **a** $P = \{1, 2, 4, 7, 14, 28\}$, $Q = \{1, 2, 4, 5, 8, 10, 20, 40\}$

b $P \cap Q = \{1, 2, 4\}$

c $P \cup Q = \{1, 2, 4, 5, 7, 8, 10, 14, 20, 28, 40\}$

d $n(P) + n(Q) - n(P \cap Q) = 6 + 8 - 3 = 11$
$= n(P \cup Q)$

9 **a** $C = \{-4, -3, -2, -1\}$
$D = \{-7, -6, -5, -4, -3, -2, -1\}$

b $C \cap D = \{-4, -3, -2, -1\}$

c $C \cup D = \{-7, -6, -5, -4, -3, -2, -1\}$

d $n(C) + n(D) - n(C \cap D) = 4 + 7 - 4 = 7$
$= n(C \cup D)$

10 **a** $A = \{6, 12, 18, 24, 30\}$, $B = \{1, 2, 3, 5, 6, 10, 15, 30\}$,
$C = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

b **i** $A \cap B = \{6, 30\}$ **ii** $B \cap C = \{2, 3, 5\}$

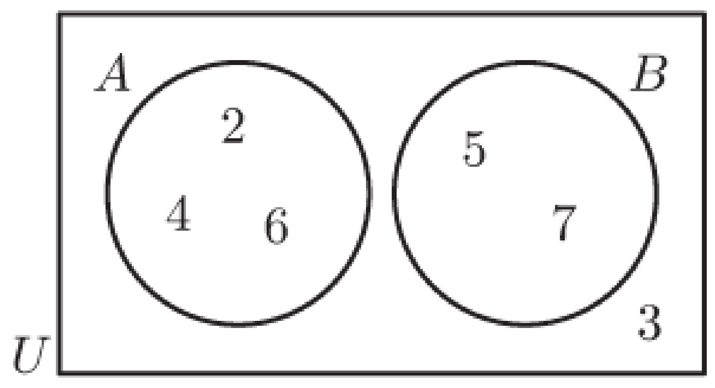
iii $A \cap C = \varnothing$ **iv** $A \cap B \cap C = \varnothing$

v $A \cup B \cup C = \{1, 2, 3, 5, 6, 7, 10, 11, 12, 13, 15, 17, 18, 19, 23, 24, 29, 30\}$

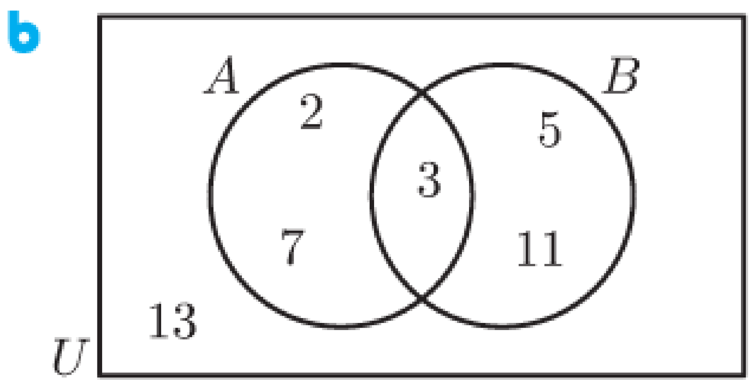
c $n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
$= 5 + 8 + 10 - 2 - 3 - 0 + 0$
$= 18$
$= n(A \cup B \cup C)$

EXERCISE 2F

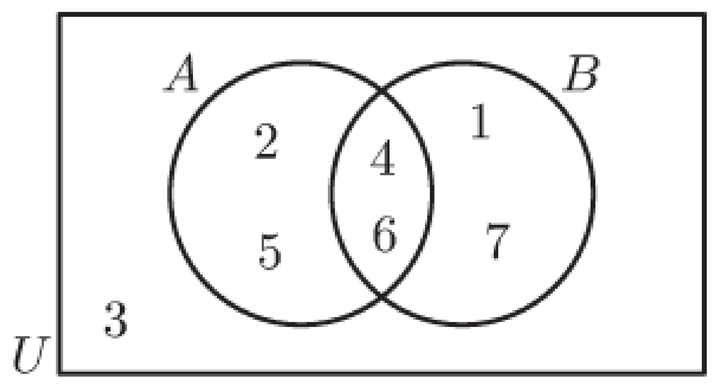
1 **a**



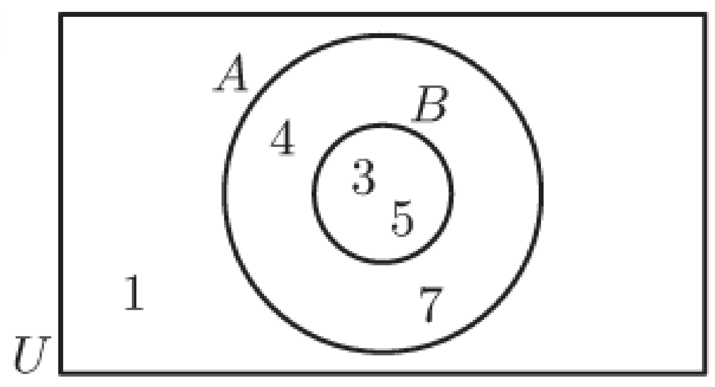
b



c



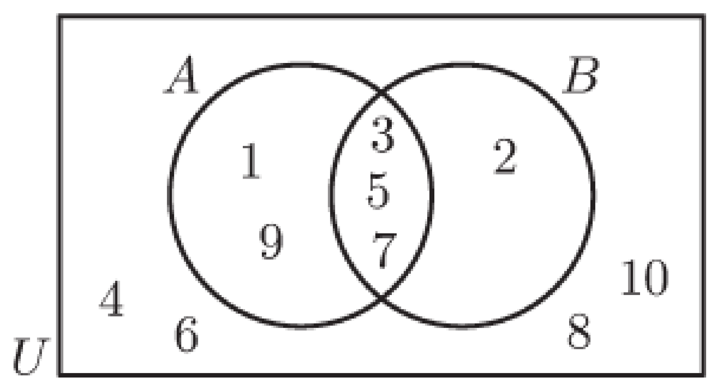
d



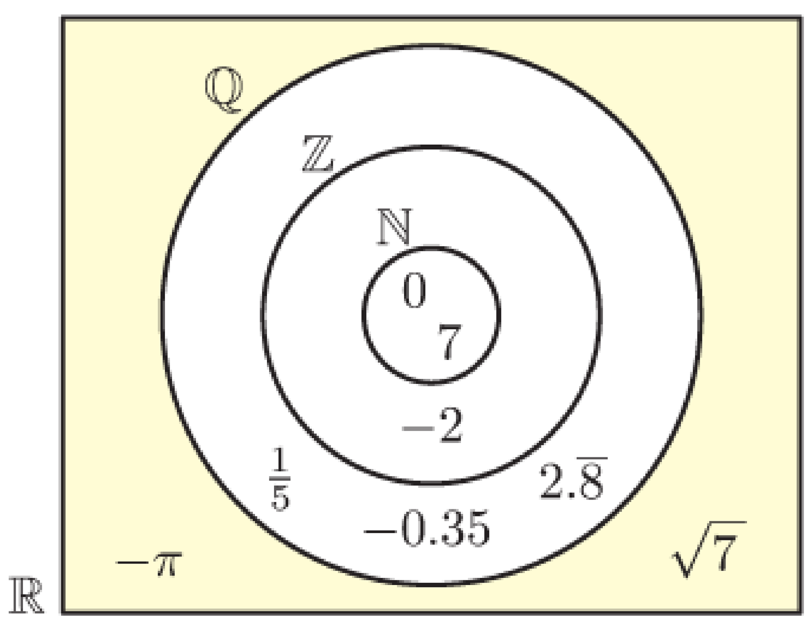
2 **a** $A = \{1, 3, 5, 7, 9\}$
$B = \{2, 3, 5, 7\}$

b $A \cap B = \{3, 5, 7\}$
$A \cup B = \{1, 2, 3, 5, 7, 9\}$

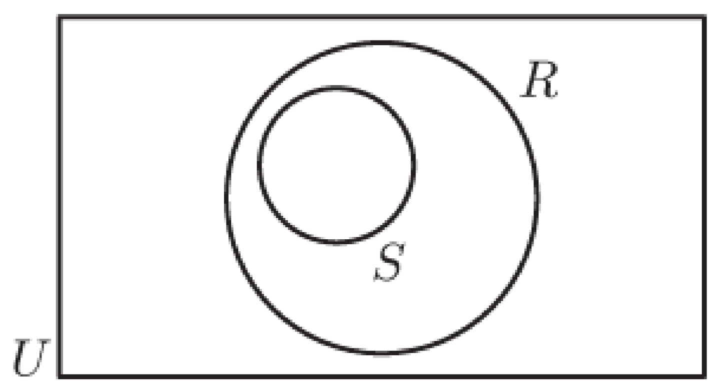
c



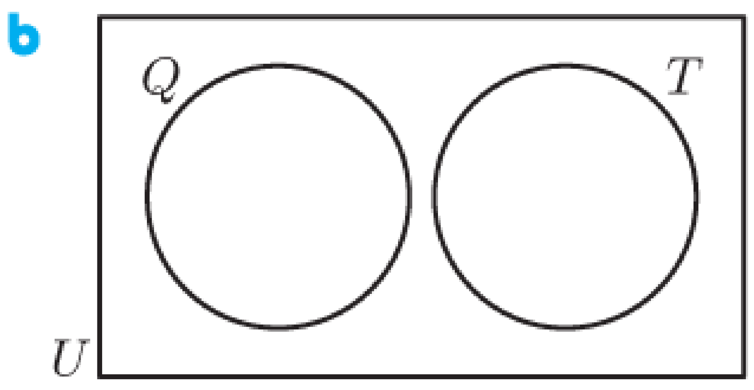
3



4 **a**



b



5 **a** **i** $A = \{a, b, c, d, h, j\}$ **ii** $B = \{a, c, d, e, f, g, k\}$

iii $C = \{a, b, e, f, i, l\}$ **iv** $A \cap B = \{a, c, d\}$

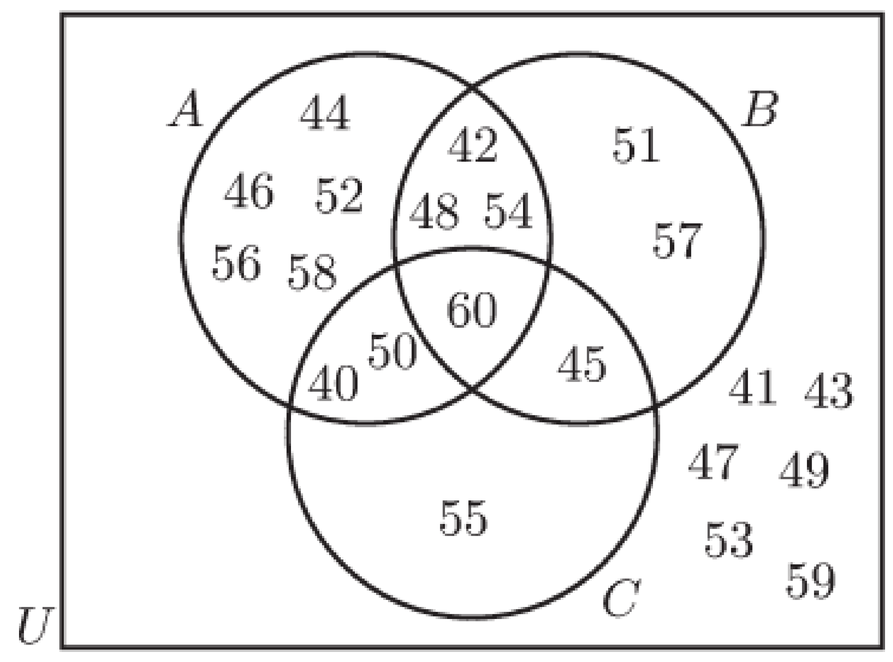
v $A \cup B = \{a, b, c, d, e, f, g, h, j, k\}$

vi $B \cap C = \{a, e, f\}$ **vii** $A \cap B \cap C = \{a\}$

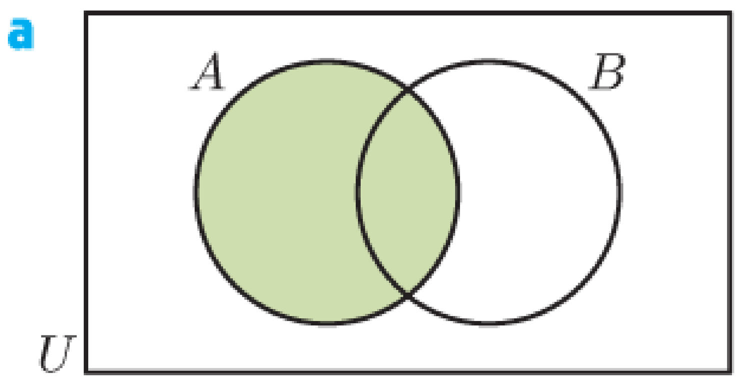
viii $A \cup B \cup C = \{a, b, c, d, e, f, g, h, i, j, k, l\}$

b $n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$
$= 6 + 7 + 6 - 3 - 2 - 3 + 1$
$= 12$
$= n(A \cup B \cup C)$

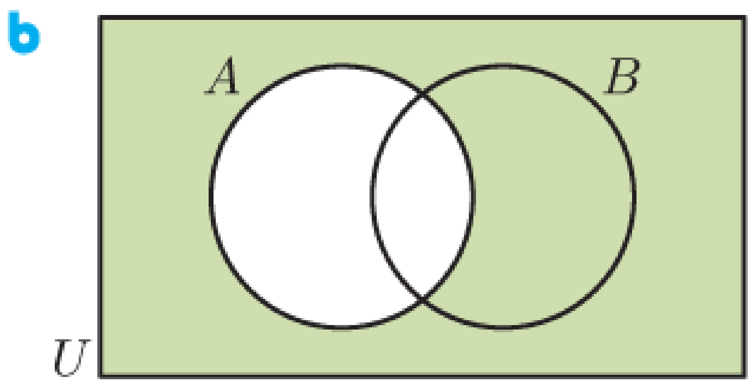
6



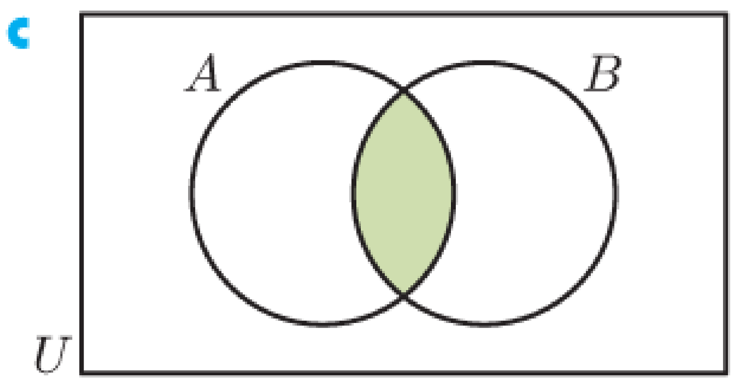
7 **a**



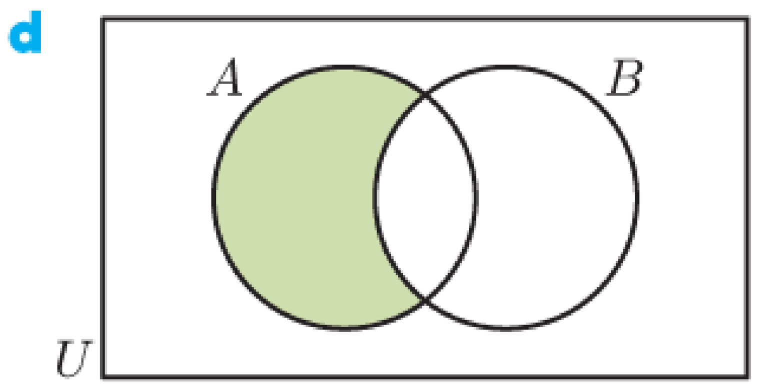
b



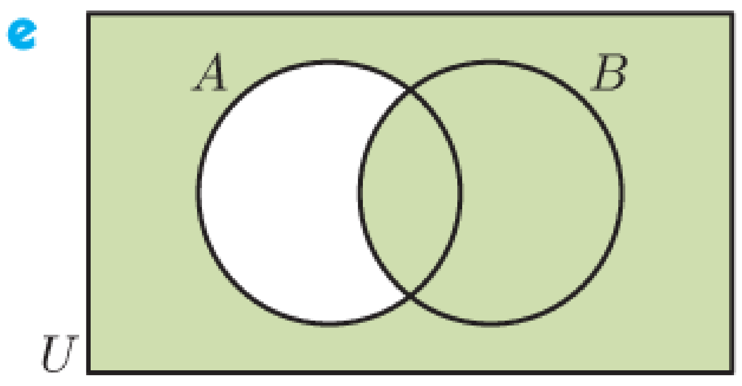
c



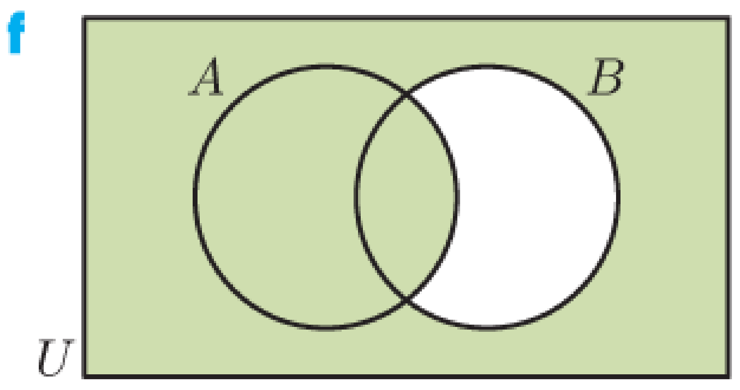
d



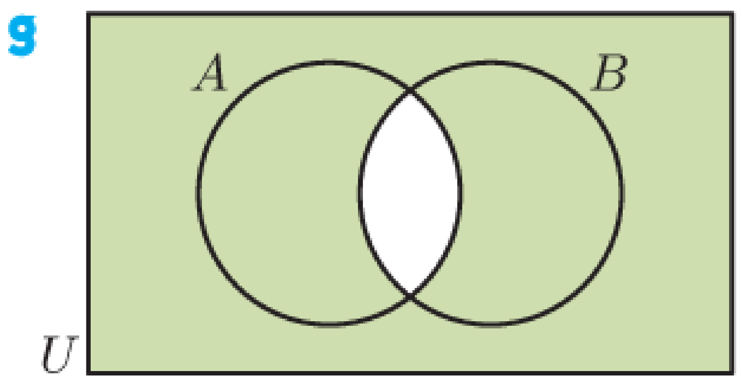
e



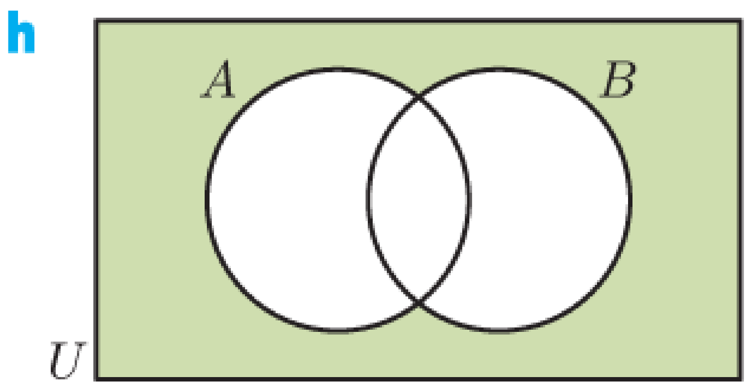
f



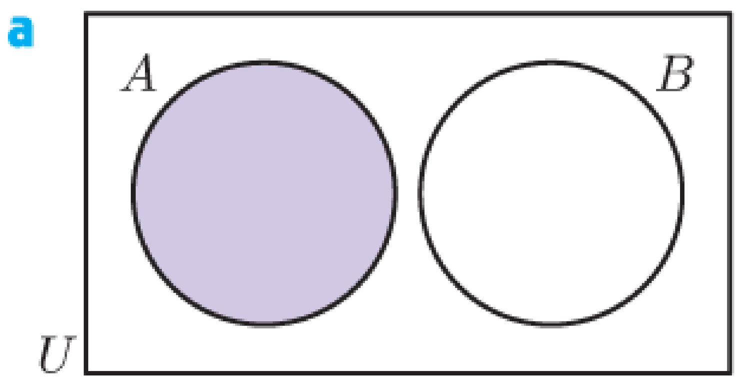
g



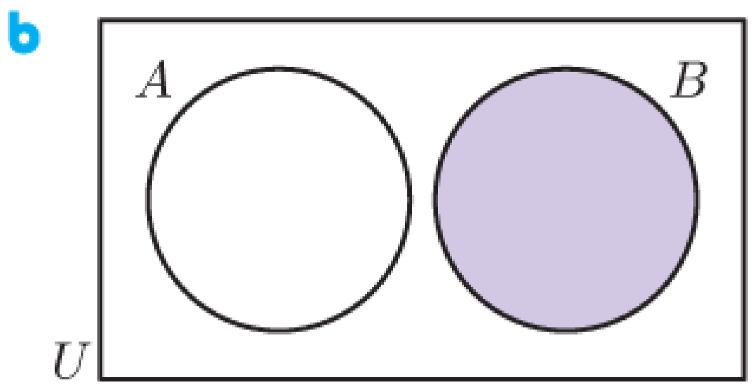
h



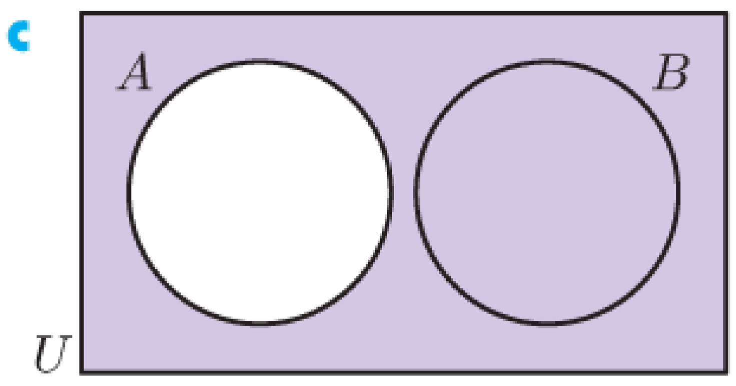
8 **a**



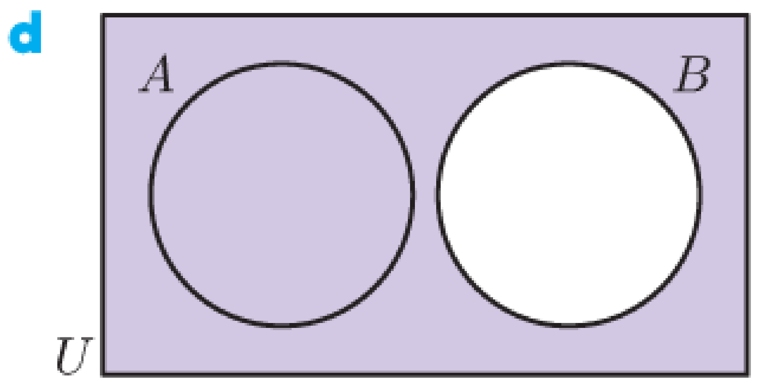
b



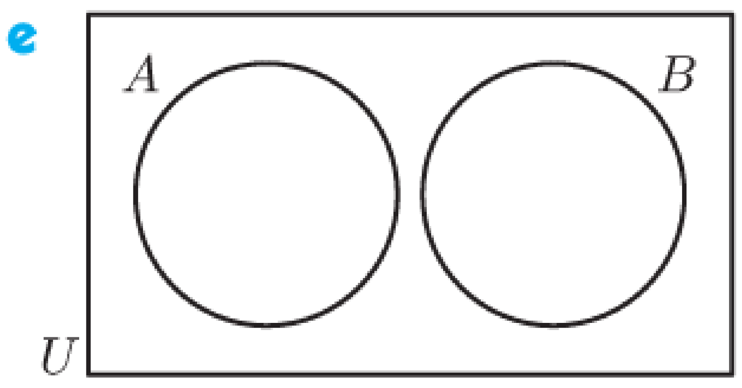
c



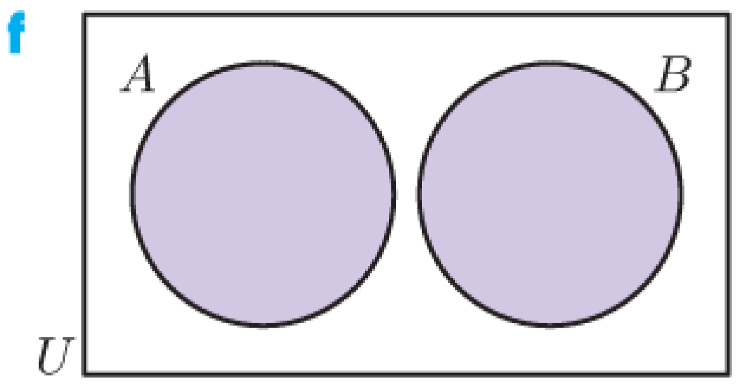
d



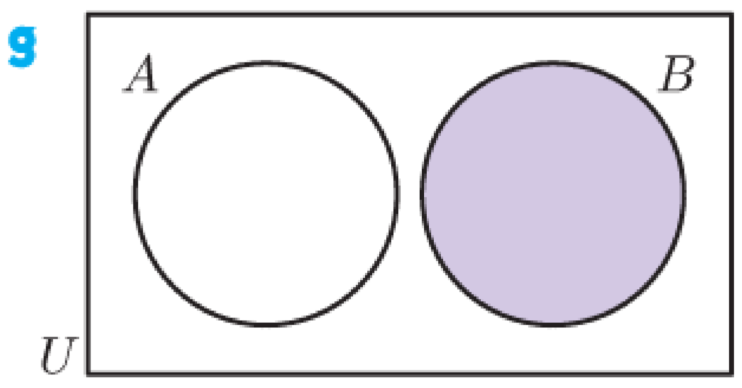
e



f



g



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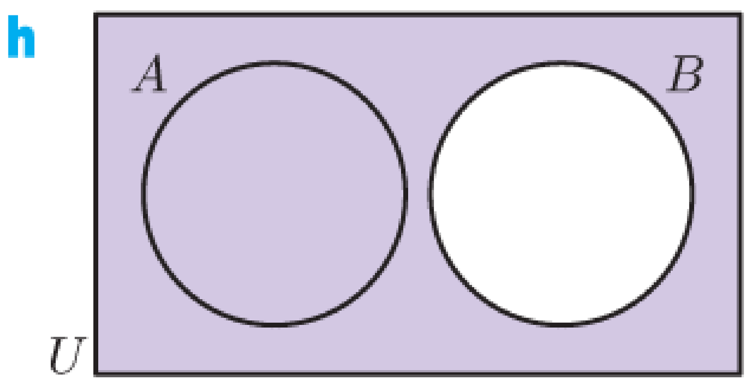
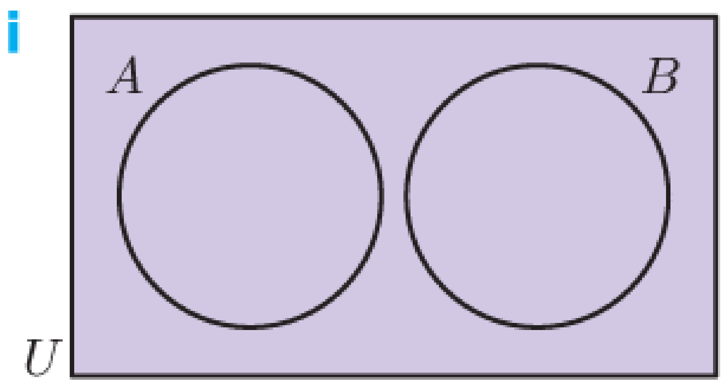
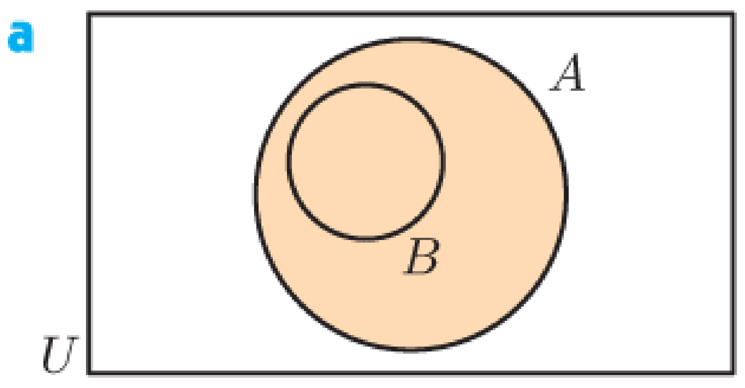


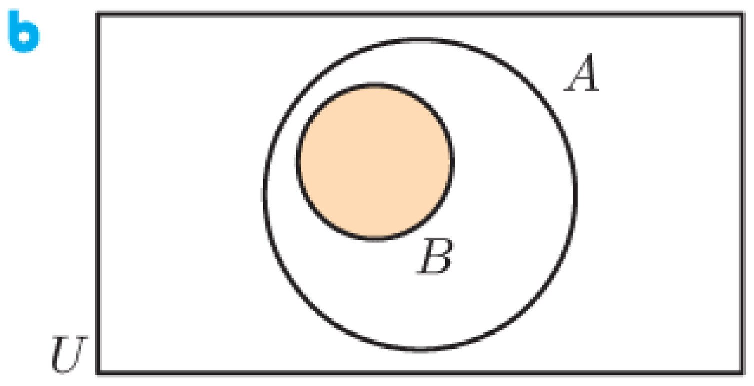
i



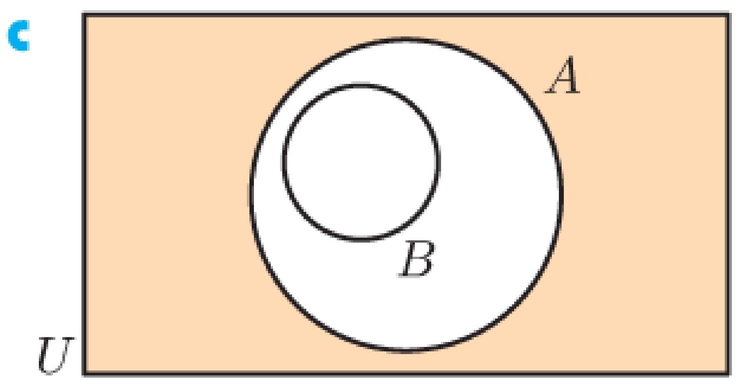
9 **a**



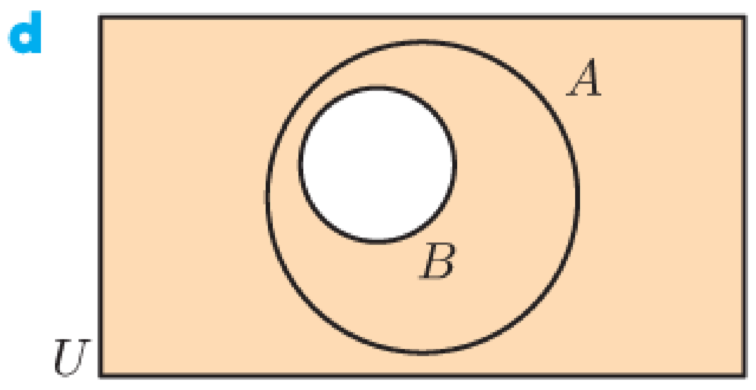
b



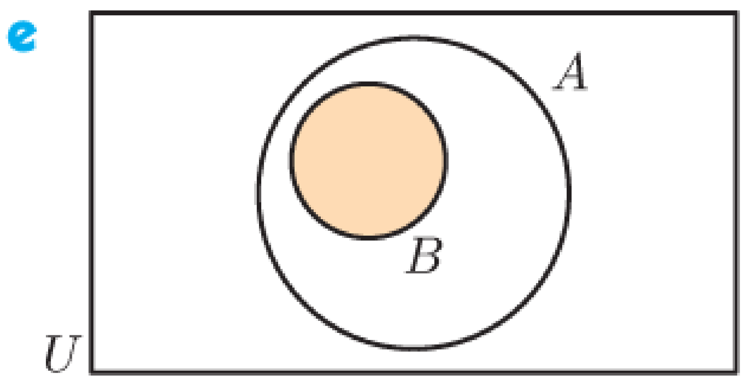
c



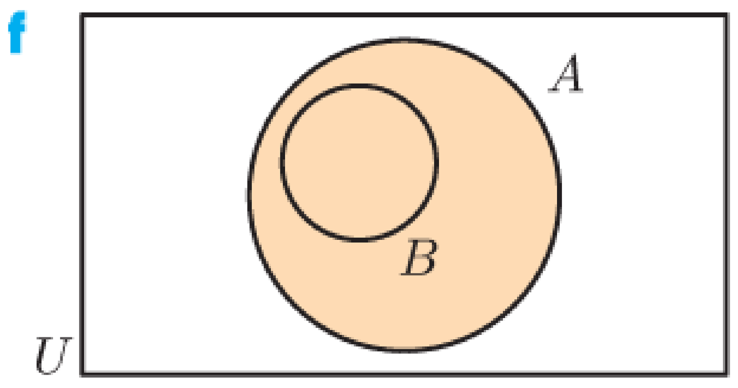
d



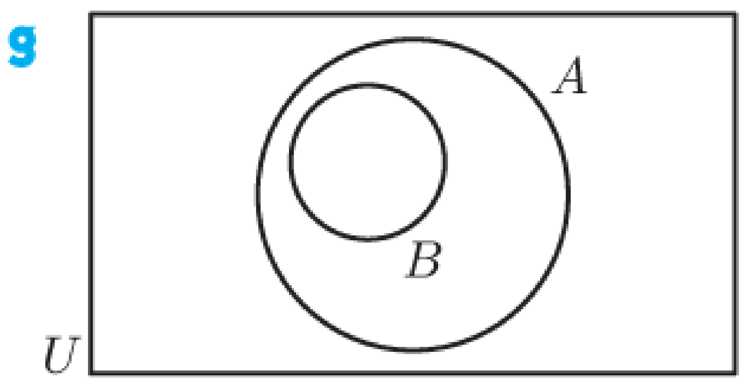
e



f



g



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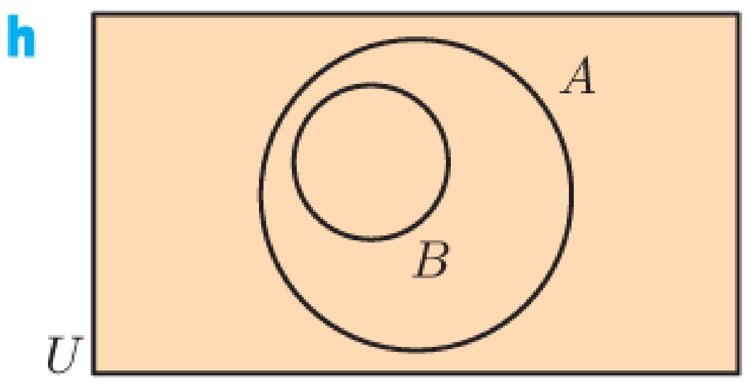
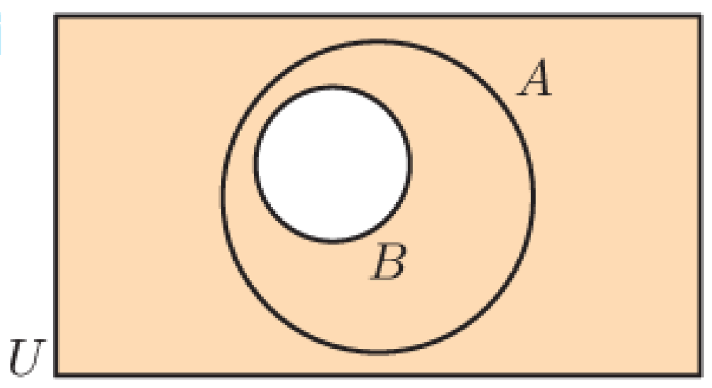
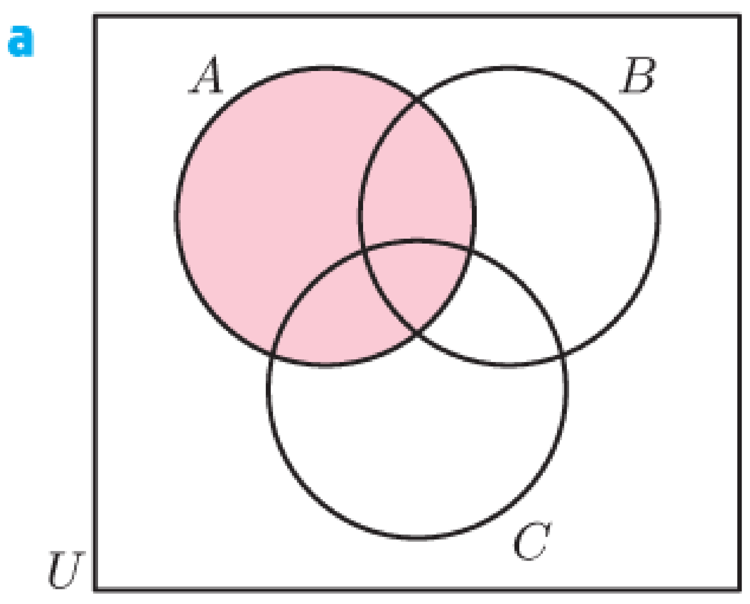


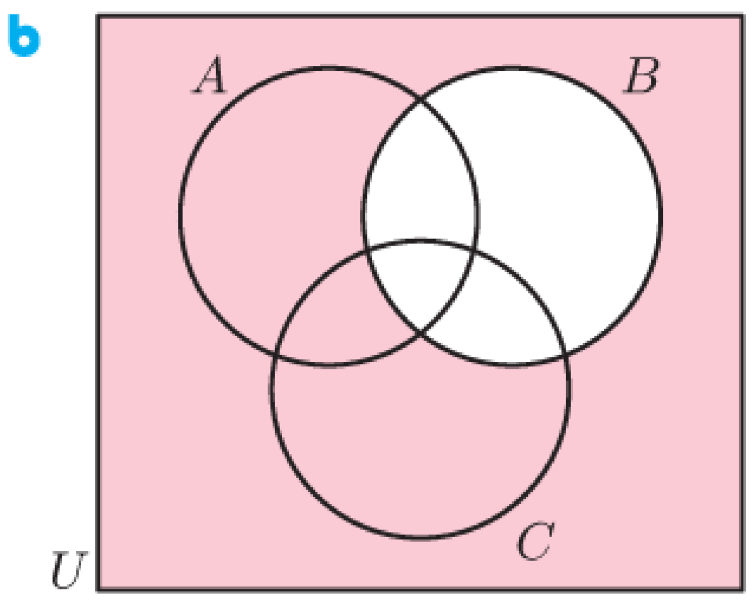
i



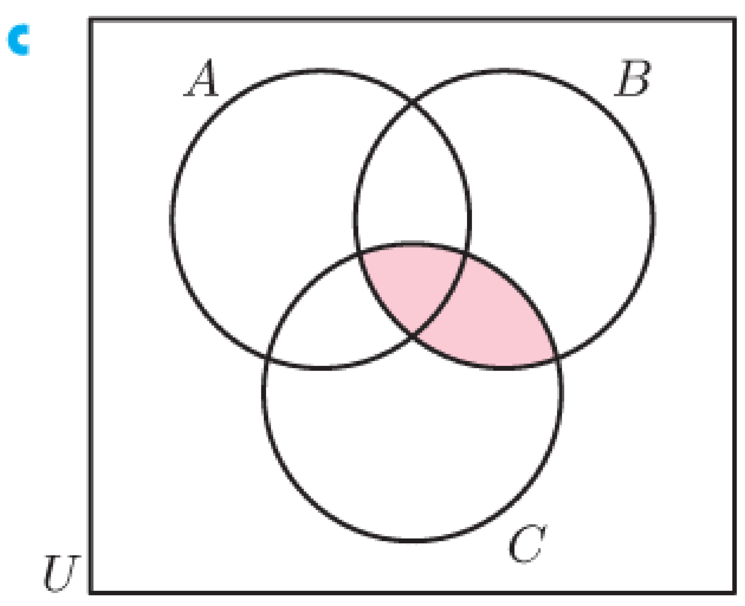
10 **a**



b



c



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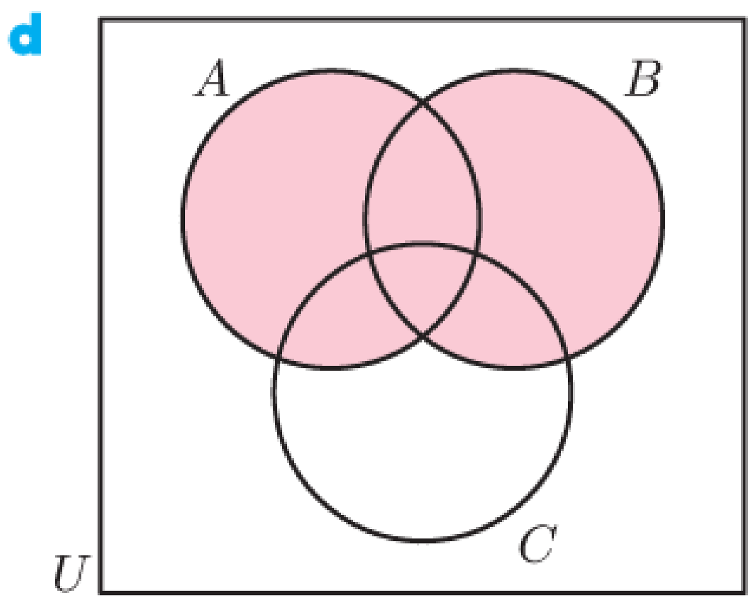
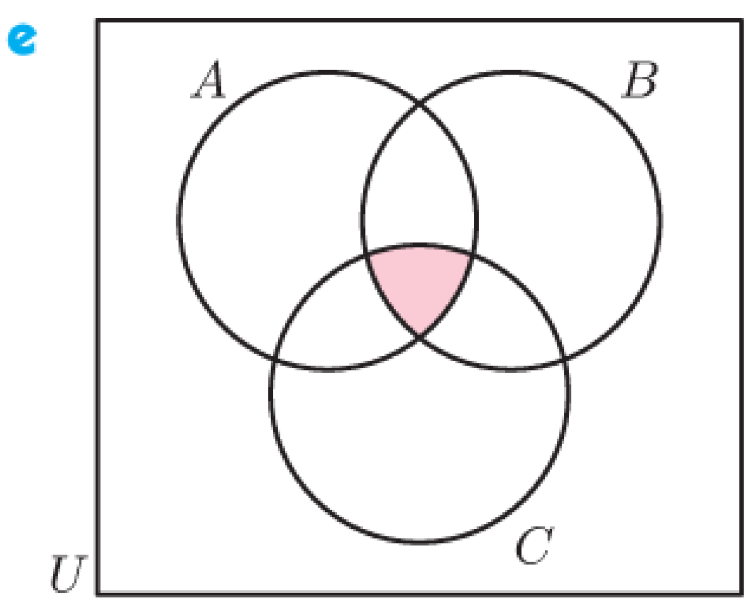
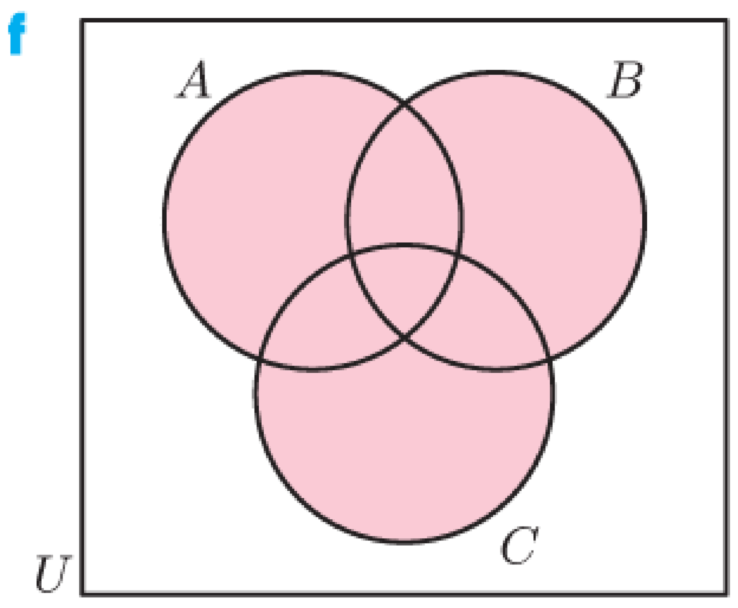


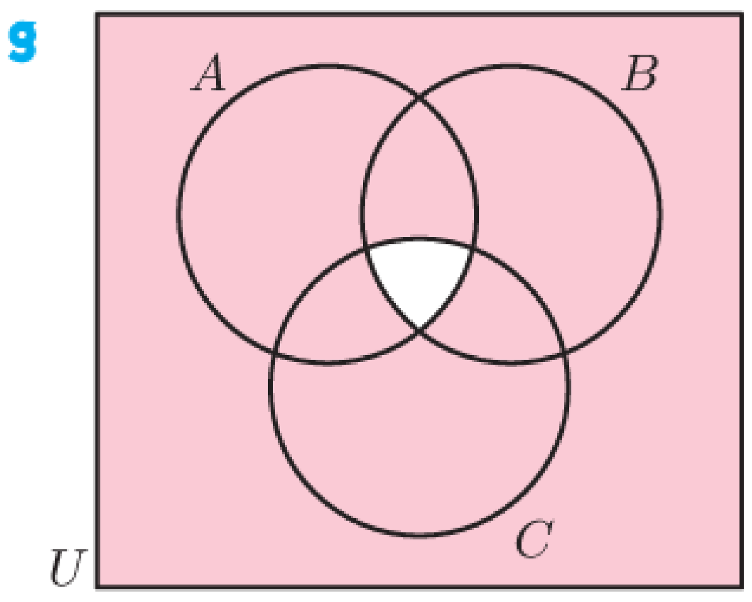
e



f



g



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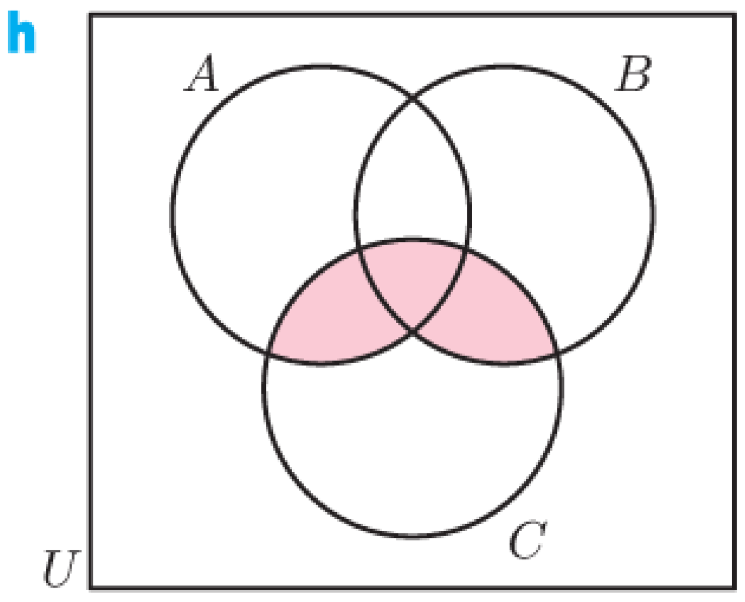
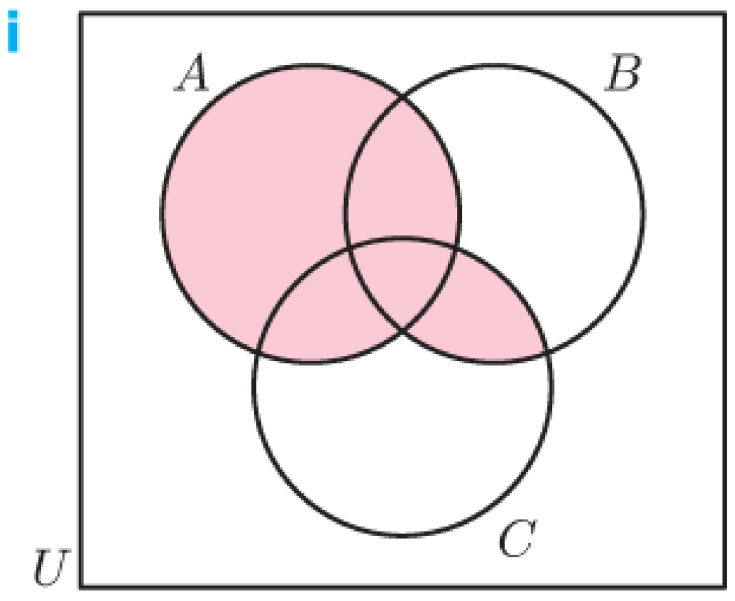
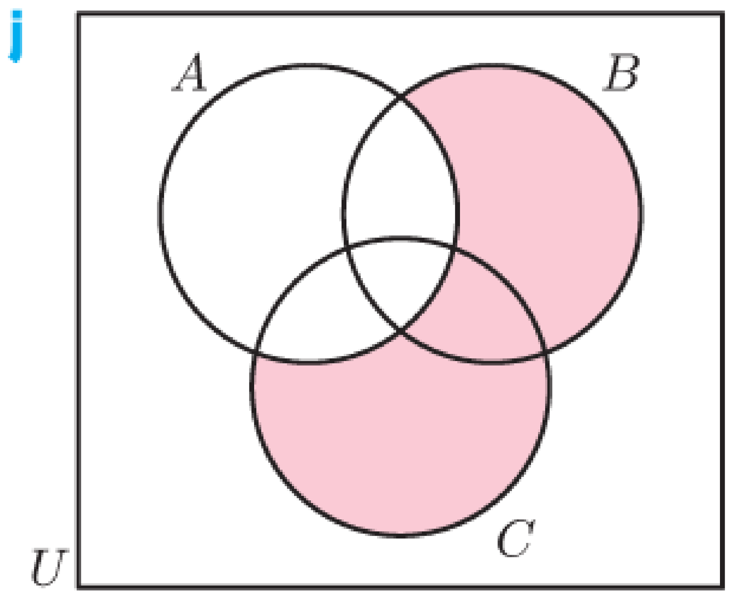


i



j



EXERCISE 2G

1 **a** 5 **b** 6 **c** 17 **d** 8 **e** 3 **f** 2

2 **a** **i** a **ii** $3a$ **iii** $2a+4$ **iv** $4a+4$
v $3a-5$ **vi** $5a-1$

b **i** $a=6$ **ii** $a=\frac{32}{5}=6.4$

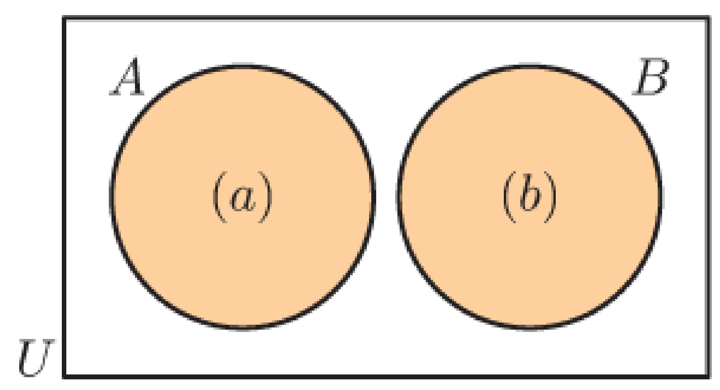
It is not possible to have a non-integer number of elements, as we have in **ii**.

$\therefore$ $n(U)$ can be equal to 29, but not equal to 31.

3 **a** $n(A)+n(B)-n(A\cap B)=(a+b)+(b+c)-b$
$=a+b+c$
$=n(A\cup B)$

b $n(A)-n(A\cap B)=(a+b)-b=a=n(A\cap B')$

4

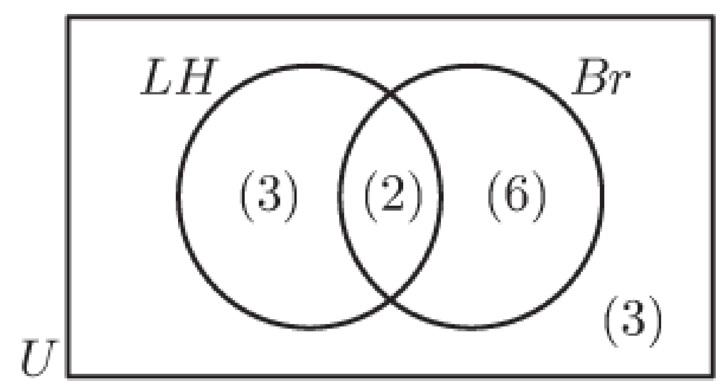


$n(A)+n(B)=a+b$
$=n(A\cup B)$

5 **a** 15 **b** 4 **6** **a** 18 **b** 6 **7** **a** 7 **b** 23

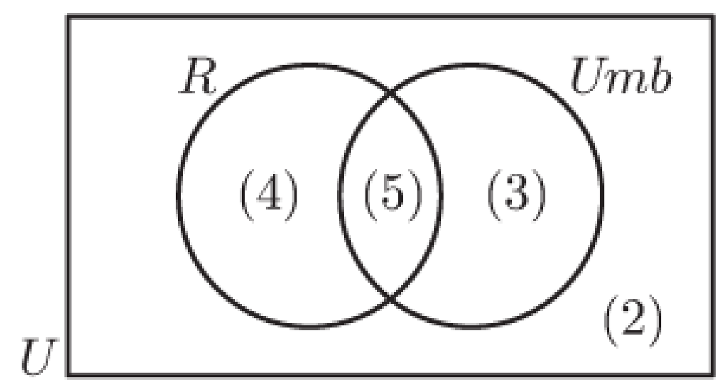
EXERCISE 2H

1 **a**



b **i** 9 cavies
ii 3 cavies
iii 3 cavies

2 **a**

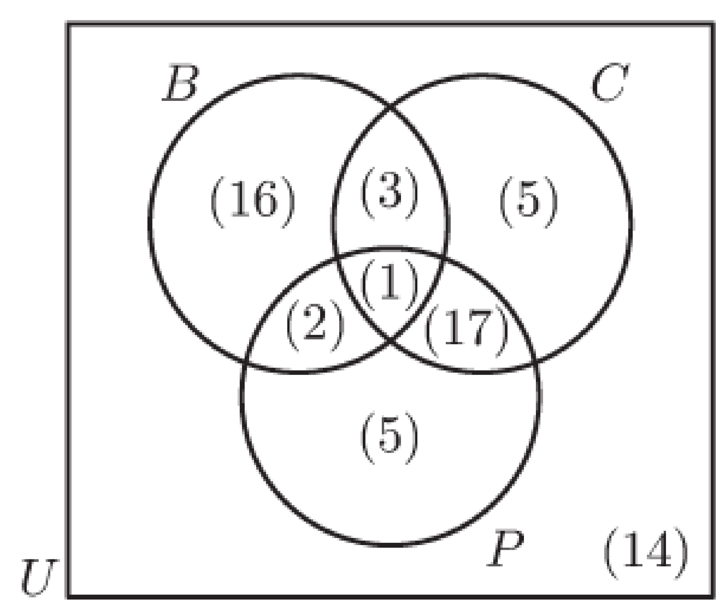


b **i** 4 days
ii 2 days

3 20 people **4** **a** 4 stalls **b** 27 stalls

5 **a** 10 movies **b** 4 movies

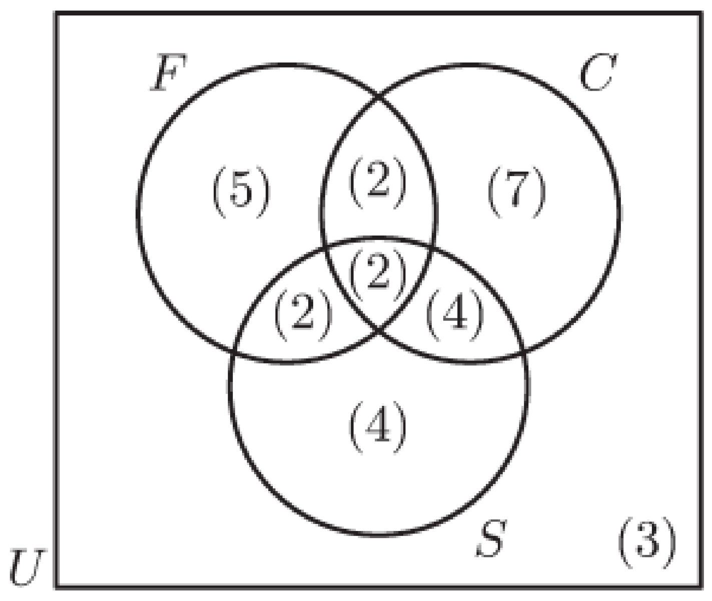
6 **a**



b **i** 16 students
ii 33 students
iii 14 students
iv 7 students

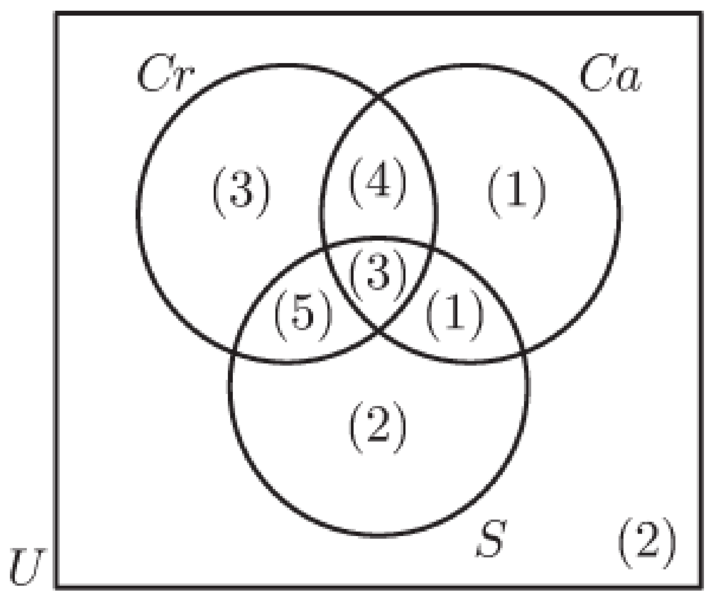
7 **a** 29 students **b** 6 students **c** 1 student **d** 11 students

8 **a**



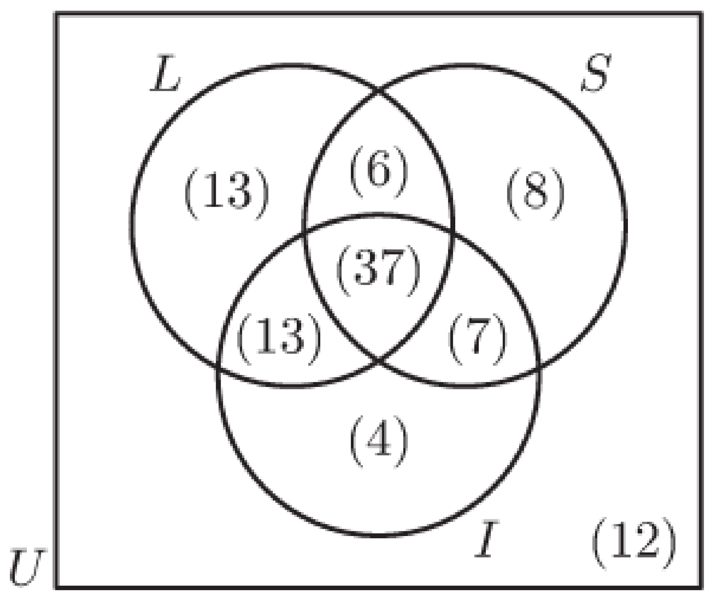
b **i** 2 students **ii** 4 students **iii** 4 students **iv** 16 students

9 **a**



b **i** 3 farms **ii** 4 farms **iii** 9 farms

10 **a**



b Yes, $n(U) - n(L \cup S \cup I) = 100 - 88 = 12$ nations.

c **i** 8 nations **ii** 30 nations **iii** 7 nations

REVIEW SET 2A

1 **a** $A = \{\text{V, E, N}\}$, $B = \{\text{D, I, A, G, R, M}\}$

b $n(A) = 3$, $n(B) = 6$

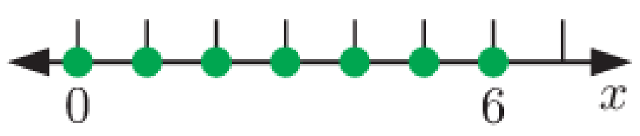
c $A \cap B = \varnothing$, "VENN" and "DIAGRAM" have no letters in common.

d **i** false **ii** true **iii** true

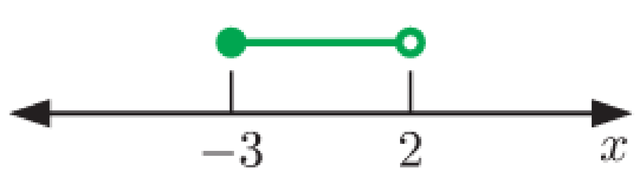
2 $A' = \{12, 18, 24, 30, 42, 48, 54, 60\}$

3 **a** true **b** false **c** true **d** false

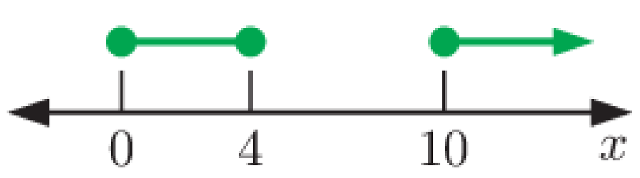
4 **a**



b



c



5 **a** **i** $P = \{1, 2, 3, 4, 6, 8, 12, 24\}$

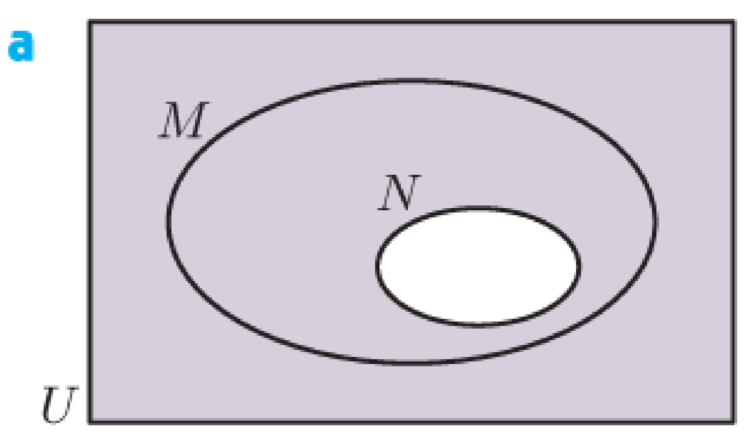
ii $Q = \{1, 2, 3, 5, 6, 10, 15, 30\}$

iii $P \cap Q = \{1, 2, 3, 6\}$

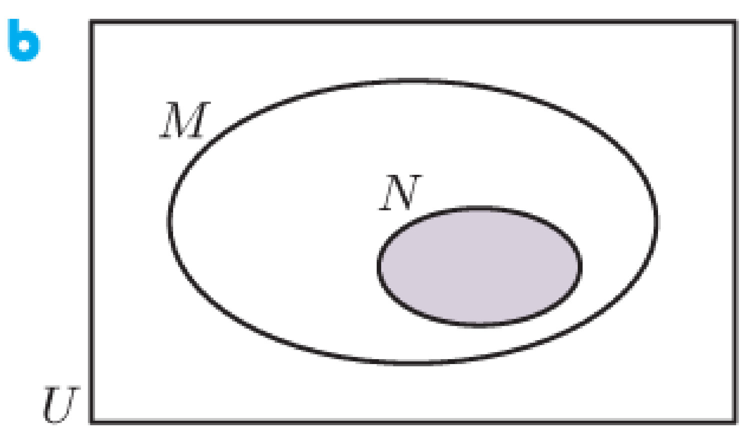
iv $P \cup Q = \{1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 24, 30\}$

b $n(P) + n(Q) - n(P \cap Q) = 8 + 8 - 4 = 12$
$= n(P \cup Q)$

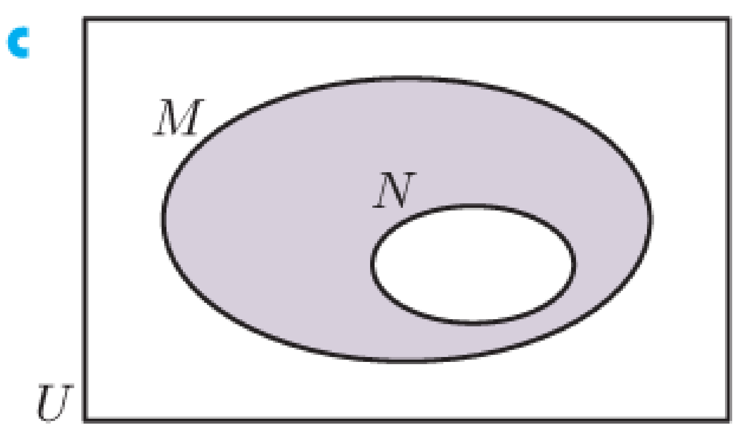
6 **a**



b



c

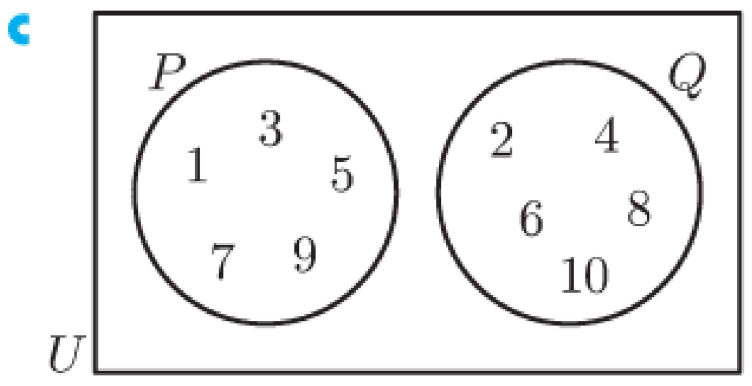


7 As $P \subseteq Q$, then all elements in P are also in Q.
An element which is not in Q must not be in P either.
$\therefore$ $P \cap Q' = \varnothing$, P and Q' are disjoint.

8 **a** $P = \{1, 3, 5, 7, 9\}$, $Q = \{2, 4, 6, 8, 10\}$

b P and Q are disjoint.

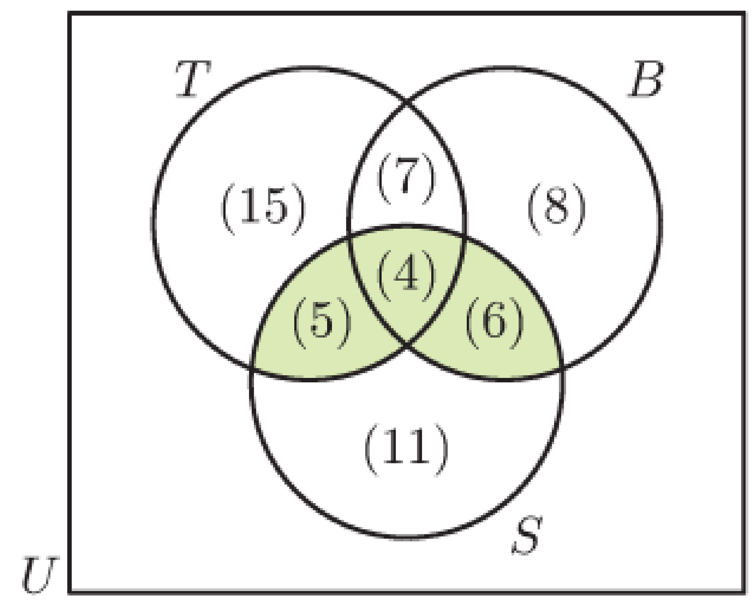
c



9 **a** 56 members

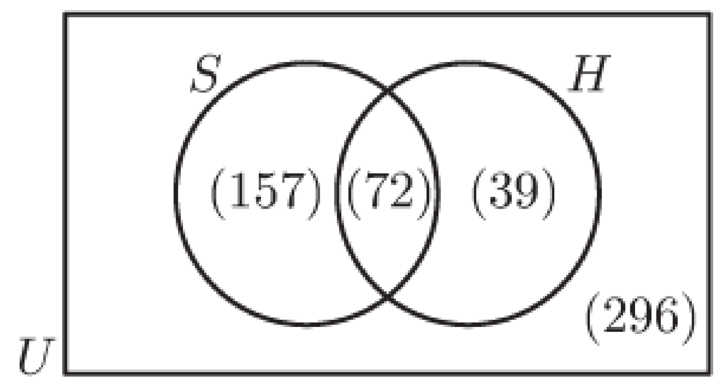
b **i** 8 members **ii** 25 members **iii** 5 members

c



d 11 members

10 **a**



b **i** 72 students **ii** 39 students **iii** 268 students

11 8 dishes

12 **a** 9 students **b** 7 students **c** 17 students

REVIEW SET 2B

1 $\varnothing$, $\{1\}$, $\{3\}$, $\{5\}$, $\{1, 3\}$, $\{1, 5\}$, $\{3, 5\}$, $\{1, 3, 5\}$

2 **a** $S \cap T = \varnothing$ **b** $s + t$

3 **a** $\{x \in \mathbb{R} \mid 5 < x < 12\}$, infinite

b $\{x \in \mathbb{Z} \mid -4 \leqslant x < 7\}$, finite

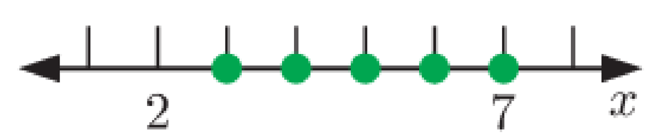
c $\{x \in \mathbb{N} \mid x > 45\}$, infinite

4 **a** $x \in\ ]2, 5]$ **b** $x \in [4, \infty[$

c $x \in\ ]-\infty, -3] \cup [1, \infty[$

5 **a** $S = \{3, 4, 5, 6, 7\}$

b



c 5

6 **a** $A \subseteq B$ **b** $A \subseteq B$ **c** $A \nsubseteq B$ **d** $A \subseteq B$

7 **a** $X' = \{\text{orange, yellow, green, blue}\}$

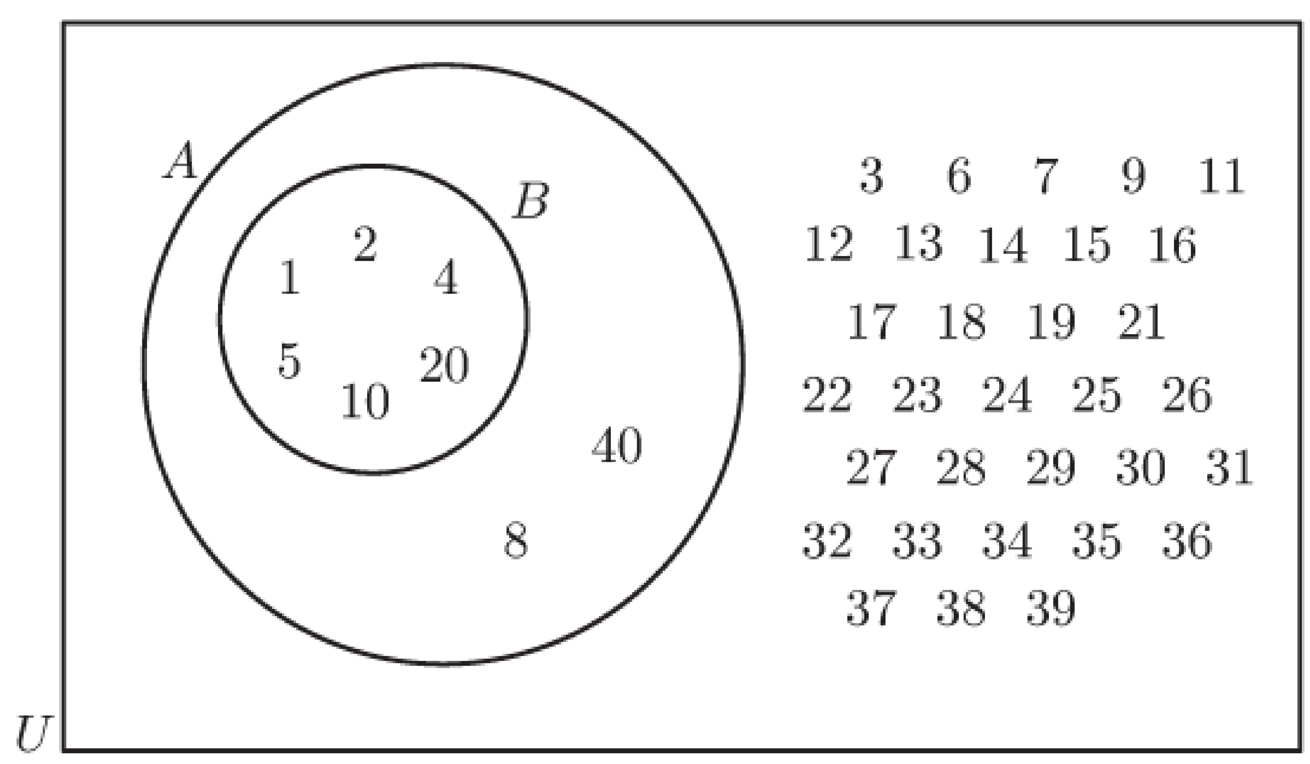
b $X' = \{-5, -3, -2, 0, 1, 2, 5\}$

c $X' = \{x \in \mathbb{Q} \mid x \geqslant -8\}$

8 **a** $A = \{1, 2, 4, 5, 8, 10, 20, 40\}$, $B = \{1, 2, 4, 5, 10, 20\}$

b $B \subset A$

c



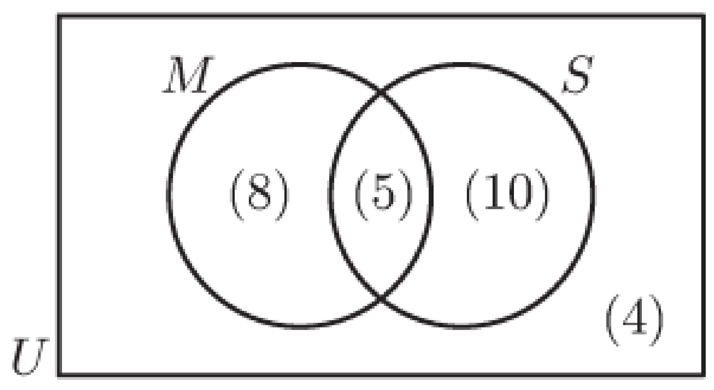
9 **a** $P = \{3, 4, 5, 6, 7, 8, 9\}$ **b** 7 **c** finite

d **i** 2 and 15 are in Q, but not in P

ii $R = \{3, 6, 9\}$, all these elements are in P, but $R \neq P$

e **i** $\{9\}$ **ii** $\{9\}$ **iii** $\{2, 3, 6, 9, 15\}$

10 **a**



b **i** 27 members **ii** 8 members **iii** 14 members

11 4 students

12 **a** 1 delegate **b** 7 delegates **c** 15 delegates

EXERCISE 3A

1 **a** 11 **b** $\sqrt{15}$ **c** $\sqrt{30}$ **d** 42
e 45 **f** $\sqrt{90}$ **g** $-\sqrt{540}$ **h** $\sqrt{288}$

2 **a** $\sqrt{6}$ **b** $\sqrt{6}$ **c** 2 **d** $\frac{1}{2}$
e $\sqrt{\frac{1}{3}}$ **f** $\sqrt{5}$ **g** $\frac{1}{4}$ **h** $\frac{1}{4}$

3 **a** $2\sqrt{3}$ **b** $2\sqrt{5}$ **c** $3\sqrt{3}$ **d** $3\sqrt{6}$
e $5\sqrt{2}$ **f** $4\sqrt{5}$ **g** $4\sqrt{6}$ **h** $6\sqrt{3}$

4 **a** $5\sqrt{2}$ **b** $-\sqrt{2}$ **c** $2\sqrt{5}$ **d** $8\sqrt{5}$
e $-2\sqrt{5}$ **f** $9\sqrt{3}$ **g** $-3\sqrt{6}$ **h** $3\sqrt{2}$

5 **a** $2\sqrt{3}$ **b** $8\sqrt{2}$ **c** $5\sqrt{6}$ **d** $10\sqrt{3}$
e $3\sqrt{3}$ **f** $-\sqrt{2}$

6 **a** $3\sqrt{2}-2$ **b** $12-14\sqrt{6}$ **c** $-8+10\sqrt{2}$
d $-12\sqrt{2}+36$ **e** $22+9\sqrt{2}$ **f** $22+14\sqrt{7}$
g $-7-\sqrt{3}$ **h** $34-30\sqrt{2}$ **i** $30\sqrt{5}-47$

7 **a** $11+6\sqrt{2}$ **b** $39-12\sqrt{3}$ **c** $46+6\sqrt{5}$
d $89-28\sqrt{10}$ **e** 2 **f** -23
g 7 **h** 218 **i** $10+6\sqrt{3}$

EXERCISE 3B

1 **a** $\frac{\sqrt{3}}{3}$ **b** $\frac{11\sqrt{3}}{3}$ **c** $\frac{\sqrt{6}}{9}$ **d** $6\sqrt{2}$
e $\frac{\sqrt{6}}{2}$ **f** $\frac{\sqrt{2}}{8}$ **g** $3\sqrt{5}$ **h** $-\frac{3\sqrt{5}}{5}$
i $\frac{\sqrt{5}}{15}$ **j** $3\sqrt{7}$ **k** $\frac{2\sqrt{11}}{11}$ **l** $\frac{\sqrt{3}}{9}$

2 **a** $\frac{3-\sqrt{2}}{7}$ **b** $\frac{6+2\sqrt{2}}{7}$ **c** $2\sqrt{6}+2$
d $\frac{\sqrt{21}-2\sqrt{3}}{3}$ **e** $-3-2\sqrt{2}$ **f** $2\sqrt{2}+4$
g $-7-3\sqrt{5}$ **h** $\frac{-38+11\sqrt{10}}{6}$ **i** $\frac{7+\sqrt{5}}{11}$
j $\frac{28+\sqrt{2}}{23}$ **k** $\frac{17+7\sqrt{7}}{3}$ **l** $\frac{\sqrt{11}-1}{5}$

3 **a** $3-2\sqrt{2}$ **b** $\frac{14}{17}-\frac{1}{34}\sqrt{2}$ **c** $3-2\sqrt{2}$
d $\frac{11}{49}+\frac{6}{49}\sqrt{2}$ **e** $3-2\sqrt{2}$ **f** $-\frac{7}{41}-\frac{2}{41}\sqrt{2}$
g $\frac{7}{529}+\frac{17}{529}\sqrt{2}$ **h** $\frac{45}{343}+\frac{29}{343}\sqrt{2}$

EXERCISE 3C

1 **a** $2^1=2,\ 2^2=4,\ 2^3=8,\ 2^4=16,\ 2^5=32,\ 2^6=64$
b $3^1=3,\ 3^2=9,\ 3^3=27,\ 3^4=81,\ 3^5=243,$
$3^6=729$
c $4^1=4,\ 4^2=16,\ 4^3=64,\ 4^4=256,\ 4^5=1024,$
$4^6=4096$

2 **a** -1 **b** 1 **c** 1 **d** -1 **e** 1 **f** -1
g -1 **h** -32 **i** -32 **j** -64 **k** 625 **l** -625

3 **a** 16 384 **b** 2401 **c** -3125 **d** -3125
e 262 144 **f** 262 144 **g** $-262\,144$
h 902.436 039 6 **i** $-902.436\,039\,6$ **j** $-902.436\,039\,6$

4 **a** $0.\overline{1}$ **b** $0.0\overline{27}$ **c** $0.\overline{012\,345\,679}$ **d** 1

Notice that $a^{-n}=\dfrac{1}{a^n}$ and $a^0=1$.

5 3 **6** 7

7 **a** $21+23+25+27+29=125=5^3$
b $43+45+47+49+51+53+55=343=7^3$
c $133+135+137+139+141+143+145+147+149$
$+151+153+155=1728=12^3$

EXERCISE 3D

1 **a** k^6 **b** c^{8+m} **c** r^{11} **d** 7^5
e m^6 **f** x^{3a-2} **g** 7^{6d} **h** m^{3t}
i 11^{2xy} **j** 7^{6-n} **k** x^{6s} **l** 3^{13}
m j^{12x} **n** z^{7-4t} **o** 13^{5cd} **p** w^{7p-1}
q k^{5t-3} **r** x^{3m-n}

2 **a** $4b^3$ **b** $6w^5$ **c** $4p^2$ **d** $30c^{11}$ **e** d^4 **f** $3ab^2$
g $4n^3$ **h** t^7 **i** $20s^2t^4$ **j** k^{11} **k** $\frac{3xy^3}{2}$ **l** b^9

3 **a** 2^2 **b** 2^{-2} **c** 2^3 **d** 2^{-3} **e** 2^5 **f** 2^{-5}
g 2^1 **h** 2^{-1} **i** 2^6 **j** 2^{-6} **k** 2^7 **l** 2^{-7}

4 **a** 3^2 **b** 3^{-2} **c** 3^3 **d** 3^{-3} **e** 3^1 **f** 3^{-1}
g 3^4 **h** 3^{-4} **i** 3^0 **j** 3^5 **k** 3^{-5}

5 **a** 2^{1+a} **b** 2^{2+b} **c** 2^{3+t} **d** 2^{2x+2} **e** 2^{n-1}
f 2^{c-2} **g** 2^{2m} **h** 2^{1+n} **i** 2^1 **j** 2^{3x-1}

6 **a** 3^{2+p} **b** 3^{3a} **c** 3^{1+2n} **d** 3^{3+d} **e** 3^{2+3t}
f 3^{y-1} **g** 3^{1-y} **h** 3^{2-3t} **i** 3^{3a-1} **j** 3^3

7 **a** 2^5 **b** 5^6 **c** 2^{4p} **d** 5^{a+2}
e 2^{5n} **f** 2^{3m-4n} **g** 5^{2p-4} **h** 3^{2t+4}
i 2^{10-5r} **j** 3^{3-y} **k** 2^{2k} **l** 5^{4-3a}

8 **a** 1 **b** $\frac{1}{3}$ **c** $\frac{1}{49}$ **d** $\frac{1}{x^3}$ **e** $\frac{6}{5}$ (or $1\frac{1}{5}$)
f 1 **g** $\frac{4}{7}$ **h** 6 **i** $\frac{9}{16}$ **j** $\frac{5}{2}$ (or $2\frac{1}{2}$)
k $\frac{27}{125}$ **l** $\frac{151}{5}$ (or $30\frac{1}{5}$)

9 **a** 3^{-2} **b** 2^{-4} **c** 5^{-3} **d** $3^1\times5^{-1}$
e $2^2\times3^{-3}$ **f** $2^{c-3}\times3^{-2}$
g $3^{2k}\times2^{-1}\times5^{-1}$ **h** $2^p\times3^{p-1}\times5^{-2}$

10 **a** $4a^2$ **b** $9n^2$ **c** $125m^3$ **d** m^3n^3
e $\frac{a^3}{8}$ **f** $\frac{9}{m^2}$ **g** $\frac{p^4}{q^4}$ **h** $\frac{t^2}{25}$

11 **a** $4a^2b^2$ **b** $4a^2$ **c** $36b^4$ **d** $-8a^3$
e $-27m^6n^6$ **f** $16a^4b^{16}$ **g** 1 $(a\neq0,\ b\neq0)$
h $\frac{m^4}{81n^4}$ **i** $\frac{x^3y^3}{8}$ **j** $\frac{-8a^6}{b^6}$ **k** $\frac{16a^6}{b^2}$ **l** $\frac{9p^4}{q^6}$

12 **a** x^5+x^3 **b** $x^4-2x^3+3x^2$ **c** x^5-x
d $x^5-x^4+2x^3-2x^2$ **e** $x^6-2x^4+x^2$
f x^3-2x^2+x **g** x^2+x-1
h x^4+2x+x^{-2} **i** x^4-x^{-2}

13 **a** $\frac{a}{b^2}$ **b** $\frac{1}{a^2b^2}$ **c** $\frac{4a^2}{b^2}$ **d** $\frac{1}{25m^4}$
e $\frac{9b^2}{a^4}$ **f** $\frac{1}{27x^3y^{12}}$ **g** $\frac{a^2}{bc^2}$ **h** $\frac{a^2c^2}{b}$
i a^3 **j** $\frac{b^3}{a^2}$ **k** $\frac{2}{ad^2}$ **l** $12am^3$

14 a a^{-n} b $5a^{-m}$ c b^n d 2^{3-n}
e 3^{n-2} f $3a^{m-4}$ g $a^n b^m$ h a^{-2n-2}

15 a x^{-2} b $2x^{-1}$ c $x + x^{-1}$ d $x^2 - 2x^{-3}$
e $x^{-1} + 3x^{-2}$ f $4x^{-1} - 5x^{-3}$
g $7x - 4x^{-1} + 5x^{-2}$ h $3x^{-1} - 2x^{-2} + 5x^{-4}$

16 a $1 + 3x^{-1}$ b $3x^{-1} - 2$ c $5x^{-2} - x^{-1}$
d $x^{-2} + 2x^{-3}$ e $x + 5x^{-1}$ f $x + 1 - 2x^{-1}$
g $2x - 3 + 4x^{-1}$ h $x - 3x^{-1} + 5x^{-2}$
i $5x^{-1} - 1 - x$ j $8x^{-1} + 5 - 2x^2$
k $16x^{-2} - 3x^{-1} + x$ l $5x^2 - 3 + x^{-1} + 6x^{-2}$

17 a $4x + 2x^2$ b $5x^2 - 4x^3$ c $6x^3 + 3x^4$
d $x^3 + 3x$ e $x^4 + x^3 - 4x^2$ f $x^6 - 3x^4 + 6x^3$
g $x^5 - 6x^3 + 10x^2$ h $x^5 + 4x^3 + x^2$

EXERCISE 3E

1 **C** and **D**

2 a 2.59×10^2 b 2.59×10^5 c 2.59×10^9
d 2.59×10^0 e 2.59×10^{-1} f 2.59×10^{-4}
g 4.07×10^1 h 4.07×10^3 i 4.07×10^{-2}
j 4.07×10^5 k 4.07×10^8 l 4.07×10^{-5}

3 a 4.745×10^7 kg b 3×10^{-3} m
c 2.599×10^6 hands d 4.7×10^{-7} m

4 a 4000 b 380 000 c 86 d 43 300 000
e 0.004 f 0.000 038 g 0.86 h 0.000 000 433

5 a 7 400 000 000 people b 0.0112 kg
c 0.000 000 5 m d 7 300 000 kg

6 a $4.5 \times 10^7 = 45\,000\,000$ b $3.8 \times 10^{-4} = 0.000\,38$
c $2.1 \times 10^5 = 210\,000$ d $4 \times 10^{-3} = 0.004$
e $6.1 \times 10^3 = 6100$ f $1.6 \times 10^{-6} = 0.000\,001\,6$
g $3.9 \times 10^4 = 39\,000$ h $6.7 \times 10^{-2} = 0.067$

7 a 4.964×10^{13} b 4×10^{-8} c 3.43×10^{-10}
d 1.6416×10^{10} e $4.121\,64 \times 10^{-3}$ f $\approx 5.27 \times 10^{-18}$
g $\approx 1.36 \times 10^2$ h $\approx 2.63 \times 10^{-6}$ i 1.728×10^9

8 7.5×10^7 peanuts **9** 2.61×10^{-6} m

10 a 1.15×10^{10} m
b We have assumed that we will always be on the side of the planet that is closest to the next planet, at the time when the planets are closest. It could take a very long time for these ideal conditions to occur.

11 a i $\approx 1.80 \times 10^{10}$ m ii $\approx 2.59 \times 10^{13}$ m
b $\approx 9.46 \times 10^{15}$ m c $\approx 3.99 \times 10^{16}$ m
d i $\approx 9.27 \times 10^{21}$ m ii $\approx 5.46 \times 10^{22}$
iii $\approx 9.46 \times 10^{12}$ hours (≈ 1.08 billion years)

12 a It allows us to write very small numbers without having to write and count lots of zeros.
b i ≈ 1839 times ii ≈ 1836 times iii ≈ 1.001 times
c 47 electrons, 60 neutrons d $\approx 2.18 \times 10^{21}$ electrons

REVIEW SET 3A

1 a $4\sqrt{5}$ b $-\sqrt{6}$ c $20\sqrt{3} - 15$
d $4 + 3\sqrt{2}$ e $86 - 60\sqrt{2}$ f 4

2 a -1 b 27 c $\frac{2}{3}$

3 a x^6 b 2^{-7} (or $\frac{1}{128}$) c $a^6 b^{18}$

4 a $\frac{1}{27}$ b $\dfrac{y}{x}$ c $\dfrac{b}{a}$ **5** a 3^3 b 3^{2t} c 2^{3-m}

6 a $\dfrac{5x}{y^2}$ b $\dfrac{1}{j^7}$ c $3g^3h^3$

7 a $\dfrac{t^3}{64s^3}$ b $1 \ (m \neq 0,\ n \neq 0)$ c $25p^6q^2$

8 a $x + 8x^{-1}$ b $4x^2 + x^3 + x^5$ c k^{-2x-6}

9 a a^6b^7 b $\dfrac{2}{3x}$ c $\dfrac{y^2}{5}$

10 a 460 000 000 000 b 1.9 c 0.0032

11 a 1.274×10^7 m b 1.2×10^{-4} m

12 313 sheets **13** 2.8×10^9 km

REVIEW SET 3B

1 a $-\sqrt{11}$ b $\sqrt{2}$ c $17 - 11\sqrt{3}$ d 28

2 a $\frac{2\sqrt{3}}{3}$ b $\frac{\sqrt{35}}{5}$ c $6 - 3\sqrt{3}$ d $\dfrac{4 - \sqrt{7}}{9}$

3 a m^4 b $1 \ (y \neq 0)$ c $\dfrac{w^2}{49z^2}$

4 a k^{x-2} b 11^{r-4} c 3^{2+b}

5 a ab^{-2} b jk^4l^{-a} c $x^{-3} - 2x^{-5}$

6 a 2^{-4} b 3^{k+4} c 5^{3a-b}

7 a $x - 5 + \dfrac{6}{x}$ b $x^6 + 2x^2 + \dfrac{1}{x^2}$ c $x^3 - 2x - \dfrac{3}{x}$

8 a $\dfrac{a^{18}}{64b^6}$ b $\dfrac{25}{d^8}$ c $2z^4$

9 a $\dfrac{1}{x^5}$ b $\dfrac{2}{a^2b^2}$ c $\dfrac{2a}{b^2}$

10 a 3^{3-2a} b 3^{6-8x}

11 a 143 000 km b 0.000 000 082 m

12 a $\approx 1.96 \times 10^{-5}$ s b ≈ 0.0110 s **13** 7500 sheets

EXERCISE 4A

1 a $x = \pm 4$ b $x = \pm\sqrt{7}$ c no real solutions
d $x = 0$ e $x = \pm\sqrt{5}$ f no real solutions

2 a $x = 3$ b $x = \pm 2$ c no real solutions
d $x = \sqrt[5]{-13}$ e $x = -2$ f $x = \sqrt[3]{7}$
g $x = \pm\sqrt[4]{6}$ h $x = \frac{2}{3}$ i $x = \pm\frac{1}{2}$
j $x = \sqrt[5]{\frac{1}{3}}$ k $x = \sqrt[3]{-6}$ l $x = \pm 3$

3 a $x = 7$ or -1 b no real solutions c $x = -4 \pm \sqrt{13}$
d $x = 7$ e $x = 4$ or -1 f $x = \dfrac{-1 \pm \sqrt{14}}{3}$
g $x = 0$ or $2\sqrt{2}$ h $x = \dfrac{\sqrt{3} \pm \sqrt{2}}{2}$ i $x = \dfrac{-1 \pm \sqrt{7}}{2}$

4 a $x = 1 + \sqrt[3]{17}$ b $x = -4$ c $x = 2 \pm \sqrt[4]{20}$
d no real solutions e $x = -4 + \sqrt[5]{-12}$ f $x = 0$ or $\frac{2}{3}$
g $x = \dfrac{3 \pm \sqrt[4]{15}}{2}$ h $x = -1$ i $x = 4 - \sqrt[3]{-22}$

5 a $x = 2$ b $x = 1 \pm \sqrt[4]{11}$ c $x = -2$

6 a $x = 6$ b $x = \pm 3$ c $x = -3$ d $x = \pm\frac{1}{7}$
e $x = \pm 2$ f $x = -\frac{1}{4}$ g no real solutions
h $x = \dfrac{5 + \sqrt[3]{5}}{2}$

EXERCISE 4B

1 a $x = 0$ b $a = 0$ or $b = 0$ c $x = 0$ or $y = 0$
d $a = 0$ or $b = 0$ or $c = 0$

2 **a** $x = 0$ or 5 **b** $x = 0$ or -3 **c** $x = -1$ or 3
d $x = 0$ or 7 **e** $x = -6$ or $\frac{3}{2}$ **f** $x = -\frac{1}{2}$ or $\frac{1}{2}$

3 **a** $x = 0$ or -5 **b** $x = 5$ **c** $x = \frac{1}{3}$ **d** $x = 5$ or $-\frac{2}{3}$
e $x = 0, -1,$ or 2 **f** $x = -2, -4,$ or $\frac{1}{2}$

4 **a** $a = 0,\ b \neq 0$ **b** $x = 0$ or $y = 0,\ z \neq 0$
c no solutions **d** $x = 0,\ y \neq 0$

EXERCISE 4C.1

1 **a** $x = 0$ or $-\frac{7}{4}$ **b** $x = 0$ or $-\frac{1}{3}$ **c** $x = 0$ or $\frac{7}{3}$
d $x = 0$ or $\frac{11}{2}$ **e** $x = 0$ or $\frac{8}{3}$ **f** $x = 0$ or $\frac{3}{2}$

2 **a** $x = 2$ or 3 **b** $x = 1$ **c** $x = -4$ or 2
d $x = -3$ or -4 **e** $x = -2$ or 4 **f** $x = 3$ or 7
g $x = 3$ **h** $x = -4$ or 3 **i** $x = -11$ or 3
j $x = -4$ or 1 **k** $x = -7$ or 5 **l** $x = -2$ or 5

3 **a** $x = \frac{2}{3}$ **b** $x = -\frac{1}{2}$ or 7 **c** $x = -\frac{2}{3}$ or 6
d $x = \frac{1}{3}$ or -2 **e** $x = \frac{3}{2}$ or 1 **f** $x = -\frac{2}{3}$ or -2
g $x = -\frac{2}{3}$ or 4 **h** $x = \frac{1}{2}$ or $-\frac{3}{2}$ **i** $x = -\frac{1}{4}$ or 3
j $x = -\frac{3}{4}$ or $\frac{5}{3}$ **k** $x = \frac{1}{7}$ or -1 **l** $x = -2$ or $\frac{28}{15}$

4 **a** $x = 2$ or 5 **b** $x = -3$ or 2 **c** $x = 0$ or $-\frac{3}{2}$
d $x = 1$ or 2 **e** $x = \frac{1}{2}$ or -1 **f** $x = 3$
g $x = 1$ or -2 **h** $x = 6$ or -4 **i** $x = 7$ or -5
j $x = 4$ or -2

EXERCISE 4C.2

1 **a** $x = 2 \pm \sqrt{3}$ **b** $x = -3 \pm \sqrt{7}$ **c** $x = 7 \pm \sqrt{3}$
d $x = 2 \pm \sqrt{7}$ **e** $x = -3 \pm \sqrt{2}$ **f** $x = 1 \pm \sqrt{7}$
g $x = -3 \pm \sqrt{11}$ **h** $x = 4 \pm \sqrt{6}$ **i** no real solutions

2 **a** $x = -1 \pm \frac{1}{\sqrt{2}}$ **b** $x = \frac{5}{2} \pm \frac{\sqrt{19}}{2}$ **c** $x = -2 \pm \sqrt{\frac{7}{3}}$
d $x = 1 \pm \sqrt{\frac{7}{3}}$ **e** $x = \frac{3}{2} \pm \sqrt{\frac{37}{20}}$ **f** $x = -\frac{1}{2} \pm \frac{\sqrt{6}}{2}$

3 **a** $x = \frac{2}{3} \pm \frac{\sqrt{10}}{3}$ **b** $x = -\frac{1}{10} \pm \frac{\sqrt{21}}{10}$ **c** $x = -\frac{5}{6} \pm \frac{\sqrt{13}}{6}$

4 $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EXERCISE 4C.3

1 **a** $x = 2 \pm \sqrt{7}$ **b** $x = -3 \pm \sqrt{2}$ **c** $x = 2 \pm \sqrt{3}$
d $x = -2 \pm \sqrt{5}$ **e** $x = 2 \pm \sqrt{2}$ **f** $x = \dfrac{1 \pm \sqrt{7}}{2}$
g $x = \dfrac{5 \pm \sqrt{37}}{6}$ **h** $x = 2 \pm \sqrt{10}$ **i** $x = \dfrac{7 \pm \sqrt{33}}{4}$

2 **a** $x = -2 \pm 2\sqrt{2}$ **b** $x = \dfrac{-5 \pm \sqrt{57}}{8}$ **c** $x = \dfrac{5 \pm \sqrt{13}}{2}$
d $x = \dfrac{-4 \pm \sqrt{7}}{9}$ **e** $x = \dfrac{-7 \pm \sqrt{97}}{4}$ **f** $x = \dfrac{1 \pm \sqrt{145}}{8}$
g $x = \dfrac{1 \pm \sqrt{7}}{2}$ **h** $x = \dfrac{1 \pm \sqrt{5}}{2}$ **i** $x = \dfrac{3 \pm \sqrt{17}}{4}$

EXERCISE 4C.4

1 **a** $\Delta = 13$ **b** 2 distinct irrational roots **c** $x = \frac{7}{2} \pm \frac{\sqrt{13}}{2}$

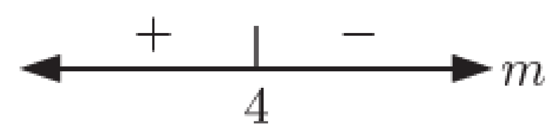
2 **a** $\Delta = 0$ **b** 1 root (repeated) **c** $x = \frac{1}{2}$

3 **a** $x^2 = -5$, $\therefore$ no real roots **b** $\Delta = -20$

4 **a** 2 distinct irrational roots **b** 2 distinct rational roots
c 2 distinct rational roots **d** 2 distinct irrational roots
e no real roots **f** a repeated root

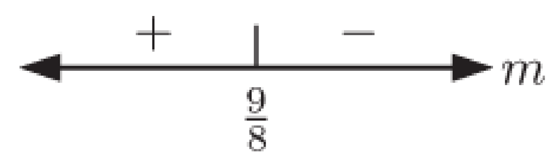
5 **a**, **c**, **d**, and **f**

6 **a** $\Delta = 16 - 4m$



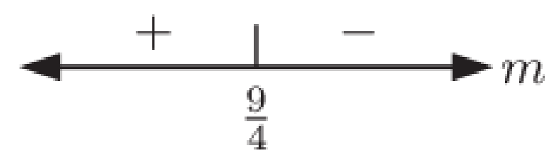
i $m = 4$ **ii** $m < 4$ **iii** $m > 4$

b $\Delta = 9 - 8m$



i $m = \frac{9}{8}$ **ii** $m < \frac{9}{8},\ m \neq 0$ **iii** $m > \frac{9}{8}$

c $\Delta = 9 - 4m$



i $m = \frac{9}{4}$ **ii** $m < \frac{9}{4},\ m \neq 0$ **iii** $m > \frac{9}{4}$

7 For $k = -8 + 4\sqrt{7}$, repeated root is $x = 1 - \frac{\sqrt{7}}{2}$.
For $k = -8 - 4\sqrt{7}$, repeated root is $x = 1 + \frac{\sqrt{7}}{2}$.

EXERCISE 4C.5

1 **a** **i** sum $= -4$, product $= -21$ **ii** roots are -7 and 3
b **i** sum $= 5$, product $= 5$ **ii** roots are $\frac{5}{2} \pm \frac{\sqrt{5}}{2}$
c **i** sum $= 3$, product $= \frac{5}{4}$ **ii** roots are $\frac{1}{2}$ and $\frac{5}{2}$
d **i** sum $= \frac{4}{3}$, product $= -\frac{2}{3}$ **ii** roots are $\frac{2}{3} \pm \frac{\sqrt{10}}{3}$

2 $k = -\frac{3}{5}$, roots are -1 and $\frac{1}{3}$

3 **a** $3\alpha = \dfrac{6}{a}$, $2\alpha^2 = \dfrac{a-2}{a}$
b $a = 4$, roots are $\frac{1}{2}$ and 1; or $a = -2$, roots are -1 and -2

4 $k = 4$, roots are $-\frac{1}{2}$ and $\frac{3}{2}$; or $k = 16$, roots are $-\frac{5}{4}$ and $\frac{3}{4}$

5 **a** $x^2 + 2x - 15 = 0$ **b** $x^2 - 4x + 1 = 0$

6 **a** $3x^2 - 2x - 4 = 0$ **b** $3x^2 + 4x - 16 = 0$

7 **a** $2x^2 - 13x + 17 = 0$
b $k(8x^2 - 10x - 1) = 0,\ k \in \mathbb{R},\ k \neq 0$

8 $k(x^2 + 7x - 44) = 0,\ k \in \mathbb{R},\ k \neq 0$

9 $k(4x^2 - 61x + 81),\ k \in \mathbb{R},\ k \neq 0$

10 $7x^2 - 48x + 64 = 0$

11 $k(8x^2 - 70x + 147) = 0,\ k \in \mathbb{R},\ k \neq 0$

EXERCISE 4D

1 **a** $x = -2$ or -7 **b** $x = 4$
c $x \approx 1.29$ or -1.54 **d** $x = 1.5$ or -2.5
e $x \approx 1.18$ or 2.82 **f** no real solutions
g $x \approx 0.360$ or 1.39 **h** $x \approx -5.99$ or 4.18

2 **a** $x = -3$ or -4 **b** $x \approx 1.85$ or -4.85
c $x \approx 0.847$ or -1.18 **d** no real solutions

3 **a** $x = -3, 0,$ or 3 **b** $x \approx -1.13$ **c** $x = 3, 2,$ or -4
d $x = 1$ **e** $x = 0.5, \approx 0.618,$ or ≈ -1.62
f $x \approx 4.36, 0.406,$ or -2.26

4 **a** no real solutions **b** $x \approx 1.34$ or -3.17
c $x = 1$ or -1 **d** no real solutions **e** $x \approx -2.27$ or 2.43
f $x \approx -3.36, -1.65, 0.192,$ or 2.82

5 **a** $x = 0, \approx 1.73,$ or -1.73 **b** $x \approx -0.811$

6 **a** $x^3 - 6x^2 + 2x - 6 = 0$ **b** $x \approx 5.83$

EXERCISE 4E

1 **a** $x = -2$ or 3 **b** $x = -2$ or 3

2 **a** $x \approx 3.21$ **b** $x \approx 0.387$ or -1.72 **c** $x \approx 2.46$
d $x \approx 5.17$ **e** $x \approx 1.52$ or 2.83 **f** $x \approx 3.56$ or -1.30

3 **a** $x \approx -1.59$ **b** $x \approx -0.861,\ 1.24,$ or 16
c $x \approx -2.62$ **d** $x \approx -0.572$ or 0.821
e $x \approx -2.24$ or 2.34 **f** $x \approx -0.577$ or 0.577

4 **a** **i** $x = 1$ or 5 **ii** $x = 3$ **iii** no real solutions
b **i** $k > -7$ **ii** $k = -7$ **iii** $k < -7$

REVIEW SET 4A

1 **a** $x = \pm\sqrt{19}$ **b** $x = -3$ or 7 **c** $x = 0$ or $2\sqrt{2}$

2 **a** no real solutions **b** $x = \frac{1}{3}$ **c** $x = 1 + \sqrt[5]{2}$

3 **a** $x = 0$ or -2 **b** $x = -3$ or $\frac{7}{2}$ **c** $x = -5, -1,$ or 6

4 **a** $x = 0$ or $\frac{5}{3}$ **b** $x = 5$ or -1 **c** $x = -3$
d $x = 1$ or 3 **e** $x = \frac{1}{3}$ or -2 **f** $x = -6$ or 9

5 **a** $x = -5$ or 4 **b** $x = 2$ or 3 **c** $x = -\frac{5}{2}$ or 7

6 **a** $x = \dfrac{7 \pm \sqrt{41}}{2}$ **b** no real solutions **c** $x = \dfrac{-1 \pm \sqrt{37}}{6}$

7 **a** $\Delta = 49$, 2 distinct rational roots **b** $x = -\frac{1}{2}$ or $\frac{2}{3}$

8 $\Delta = -7 \quad \therefore$ no real roots

9 **a** $m = \frac{9}{8}$ **b** $m < \frac{9}{8}$ **c** $m > \frac{9}{8}$

10 $k(2x^2 + 5x - 3) = 0,\ k \in \mathbb{R},\ k \neq 0$

11 $4x^2 + 3x - 2 = 0$ **12** $k = 3$, roots are $-\frac{1}{3}$ and 3

13 **a** $x = \frac{5}{2}, 1,$ or -2 **b** $x = 2, \frac{1}{3},$ or 0
c $x = 4, 3,$ or -5 **d** $x \approx 1.84$ or -6.92

14 **a** $x \approx 2.81$ **b** $x \approx 1.73$ **c** $x \approx -1.84$

REVIEW SET 4B

1 **a** $x = 0$ **b** $x = -\frac{5}{2}$ **c** $x = \sqrt{3} \pm 4$

2 **a** $x = \pm\frac{3}{2}$ **b** $x = \sqrt[5]{-18}$ **c** $x = \frac{1}{2}$ or $\frac{3}{2}$

3 **a** $p = 0,\ q \neq 0$ **b** $x = 0$ or $z = 0,\ y \neq 0$
c no solutions

4 **a** $x = 0$ or $\frac{5}{2}$ **b** $x = 0$ or 4 **c** $x = 1$ or 6
d $x = -2$ **e** $x = 6$ or -2 **f** $x = -\frac{5}{3}$ or 2

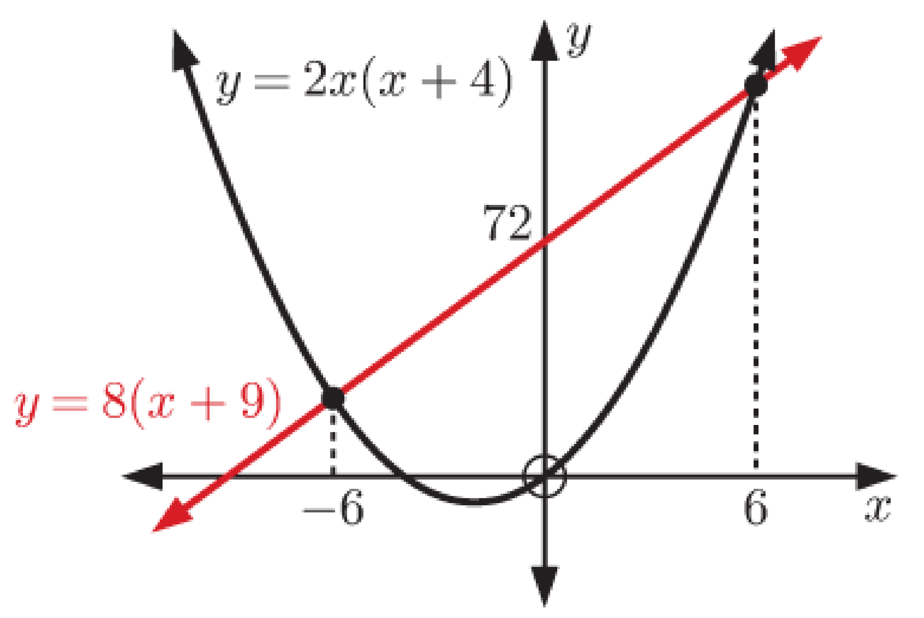
5 **a** $x^2 - 9 = 0,\quad x = \pm 3$
b $2x^2 + x - 3 = 0,\quad x = 1$ or $-\frac{3}{2}$
c $3x^2 - x - 2 = 0,\quad x = 1$ or $-\frac{2}{3}$

6 **a** $x = \dfrac{-5 \pm \sqrt{13}}{2}$ **b** $x = \dfrac{-11 \pm \sqrt{145}}{6}$

7 **a** $x = 3 \pm \sqrt{5}$ **b** $x = -2 \pm \frac{3}{\sqrt{2}}$ **c** $x = -\frac{3}{4}$ or 2

8 **a** $\Delta = 0$, 1 root (repeated)
b $\Delta = 41$, 2 distinct irrational roots
c $\Delta = -11$, no real roots

9 **a** **i** $x = \pm 6$
ii



$x = -6$ or 6

c **i** $k > 0$ **ii** $k = 0$ **iii** $k < 0$

11 $k = 8\sqrt{2}$, roots are $-\sqrt{2}$ and $-3\sqrt{2}$; or
$k = -8\sqrt{2}$, roots are $\sqrt{2}$ and $3\sqrt{2}$

12 $k(64x^2 - 135x - 27) = 0,\ k \in \mathbb{R},\ k \neq 0$

13 **a** $x = 5, 0,$ or -3 **b** $x \approx 4.93, 0.814,$ or -1.74
c $x \approx 2.39$ or 0.449

14 **a** $x \approx 2.81$ **b** $x \approx 1.85$ **c** $x \approx -2.15$ or 3.58

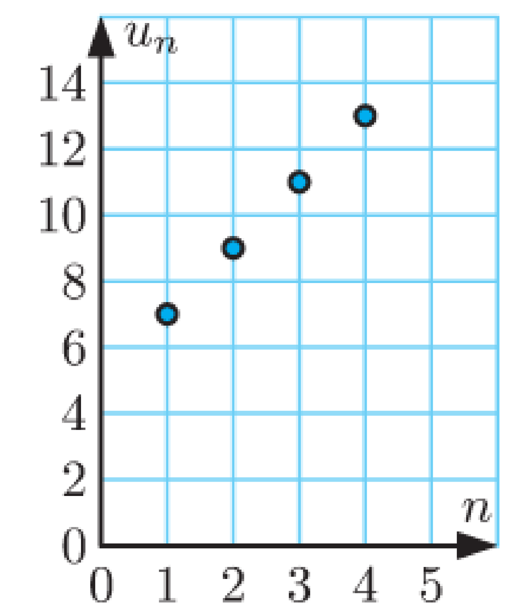
EXERCISE 5A

1 **a** 4, 13, 22, 31 **b** 45, 39, 33, 27 **c** 2, 6, 18, 54
d 96, 48, 24, 12

2 **a** $u_2 = 3$ **b** $u_5 = 11$ **c** $u_{10} = 29$

3 **a** We start with 4 and add 3 each time.
b $u_1 = 4,\ u_4 = 13$ **c** $u_8 = 25$

4 **a** $u_1 = 7,\ u_2 = 9,$
$u_3 = 11,\ u_4 = 13$
b



5 **a** $u_1 = 1$ **b** $u_5 = 13$ **c** $u_{27} = 79$

6 **a** **B** **b** $u_{20} = 390$

7 **a** The sequence starts at 8, and each term is 8 more than the previous term. The next two terms are 40 and 48.
b The sequence starts at 2, and each term is 3 more than the previous term. The next two terms are 14 and 17.
c The sequence starts at 36, and each term is 5 less than the previous term. The next two terms are 16 and 11.
d The sequence starts at 96, and each term is 7 less than the previous term. The next two terms are 68 and 61.
e The sequence starts at 1, and each term is 4 times the previous term. The next two terms are 256 and 1024.
f The sequence starts at 2, and each term is 3 times the previous term. The next two terms are 162 and 486.
g The sequence starts at 480, and each term is half the previous term. The next two terms are 30 and 15.
h The sequence starts at 243, and each term is one third of the previous term. The next two terms are 3 and 1.
i The sequence starts at 50 000, and each term is one fifth of the previous term. The next two terms are 80 and 16.

8 **a** Each term is the square of the term number; 25, 36, 49.
b Each term is the cube of the term number; 125, 216, 343.
c Each term is $n(n+1)$ where n is the term number; 30, 42, 56.

9 **a** 79, 75 **b** 1280, 5120 **c** 625, 1296
d 13, 17 **e** 16, 22 **f** 6, 12

10 **a** 2, 4, 6, 8, 10 **b** -1, 1, 3, 5, 7
c 13, 15, 17, 19, 21 **d** 1, 5, 9, 13, 17
e 2, 4, 8, 16, 32 **f** $3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}$
g -2, 4, -8, 16, -32 **h** 17, 11, 23, -1, 47

EXERCISE 5B.1

1 **a** **i** $u_1 = 19,\ d = 6$ **ii** $u_n = 6n + 13$ **iii** $u_{15} = 103$
b **i** $u_1 = 101,\ d = -4$ **ii** $u_n = 105 - 4n$
iii $u_{15} = 45$
c **i** $u_1 = 8,\ d = 1\frac{1}{2}$ **ii** $u_n = 1\frac{1}{2}n + 6\frac{1}{2}$ **iii** $u_{15} = 29$
d **i** $u_1 = 31,\ d = 5$ **ii** $u_n = 5n + 26$ **iii** $u_{15} = 101$
e **i** $u_1 = 5,\ d = -8$ **ii** $u_n = 13 - 8n$
iii $u_{15} = -107$

f **i** $u_1 = a,\ d = d$ **ii** $u_n = a + (n-1)d$
iii $u_{15} = a + 14d$

2 **a** $u_1 = 6,\ d = 11$ **b** $u_n = 11n - 5$
c $u_{50} = 545$ **d** yes, $u_{30} = 325$ **e** no

3 **a** $u_1 = 87,\ d = -4$ **b** $u_n = 91 - 4n$
c $u_{40} = -69$ **d** u_{97}

4 **b** $u_1 = 1,\ d = 3$ **c** $u_{57} = 169$ **d** $u_{150} = 448$

5 **b** $u_1 = 32,\ d = -\frac{7}{2}$ **c** $u_{75} = -227$ **d** $n \geqslant 68$

6 $u_{7692} = 100\,006$ **7** **b** $u_{200} = 1381$ **c** no

8 **a** $k = 17\frac{1}{2}$ **b** $k = 4$ **c** $k = 5$ **d** $k = \frac{3}{2}$
e $k = 7$ **f** $k = -4$ **g** $k = -2$ or 3 **h** $k = -1$ or 3

9 **a** $k = \frac{1}{2}$ or -2 **b** For $k = \frac{1}{2},\ d = -5$
For $k = -2,\ d = 15$

10 **a** $u_n = 6n - 1$ **b** $u_n = -\frac{3}{2}n + \frac{11}{2}$
c $u_n = -5n + 36$ **d** $u_n = -\frac{3}{2}n + \frac{1}{2}$

11 **b** $u_{30} = 815$ **12** $5, 6\frac{1}{4}, 7\frac{1}{2}, 8\frac{3}{4}, 10$

13 $-1, 3\frac{5}{7}, 8\frac{3}{7}, 13\frac{1}{7}, 17\frac{6}{7}, 22\frac{4}{7}, 27\frac{2}{7}, 32$

14 **a** $50, 48\frac{1}{2}, 47, 45\frac{1}{2}, 44$ **b** $u_{35} = -1$

15 **a** $k = 2$ **b** $u_n = \dfrac{3n-8}{2}$

16 **Hint:** Since $x > y > z > 0$, then $\dfrac{1}{x} < \dfrac{1}{y} < \dfrac{1}{z}$.

17 No, such a sequence would require $u_1 =$ some prime p, and $d \in \mathbb{Z}^+$. But then u_{p+1} must be a multiple of p.

18 **a** Month 1: 5 cars Month 4: 44 cars
Month 2: 18 cars Month 5: 57 cars
Month 3: 31 cars Month 6: 70 cars
b The total number of cars made increases by 13 each month. So, the common difference $d = 13$.
c 148 cars **d** 20 months

19 **a** Week 1: 2817 L Week 3: 2451 L
Week 2: 2634 L Week 4: 2268 L
b The amount in the tank decreases by the same amount (183 L) each week.
c in the 17th week

EXERCISE 5B.2

1 **a** 140.75 g **b** $u_n = 140.75n$

2 **a** 59.25 g **b** $u_n = 32 + 59.25n$ **c** $0 \leqslant n \leqslant 12$

3 **a** $u_n = 580 - 16n$ **b** $u_n = 9850 - \frac{7880}{29}n$

4 **a** 5.75 online friends per week **b** $u_n = 28.25 + 5.75n$
c No, the model is only intended to *estimate* the number of online friends. We can simply round to the nearest whole number.
d $143.25 \approx 143$ online friends

5 **a** $u_n = 1950 + 100n$
b **i** Catering is €100 per guest.
ii The venue hire is €1950 (with 0 guests).
c €10 450

EXERCISE 5C

1 **a** **i** $u_1 = 3,\ r = 2$ **ii** $u_n = 3 \times 2^{n-1}$ **iii** $u_9 = 768$
b **i** $u_1 = 2,\ r = 5$ **ii** $u_n = 2 \times 5^{n-1}$
iii $u_9 = 781\,250$
c **i** $u_1 = 512,\ r = \frac{1}{2}$ **ii** $u_n = 512 \times 2^{1-n}$
iii $u_9 = 2$
d **i** $u_1 = 1,\ r = 3$ **ii** $u_n = 3^{n-1}$ **iii** $u_9 = 6561$
e **i** $u_1 = 12,\ r = \frac{3}{2}$ **ii** $u_n = 12 \times (\frac{3}{2})^{n-1}$
iii $u_9 = \dfrac{3^9}{2^6} = \dfrac{19\,683}{64}$
f **i** $u_1 = \frac{1}{16},\ r = -2$ **ii** $u_n = \frac{1}{16}(-2)^{n-1}$
iii $u_9 = 16$

2 **a** $u_1 = 5,\ r = 2$ **b** $u_n = 5 \times 2^{n-1},\ u_{15} = 81\,920$

3 **a** $u_1 = 12,\ r = -\frac{1}{2}$
b $u_n = 12 \times (-\frac{1}{2})^{n-1},\ u_{13} = \frac{3}{1024}$

4 **a** $u_1 = 8,\ r = -\frac{3}{4}$ **b** $u_{10} \approx -0.601$

5 **a** $u_1 = 8,\ r = \frac{1}{\sqrt{2}}$ **b** **Hint:** $u_n = 2^3 \times (2^{-\frac{1}{2}})^{n-1}$

6 **a** $k = 6$ **b** $k = \frac{125}{2}$ **c** $k = \pm 14$ **d** $k = \pm 2$
e $k = \pm 36$ **f** $k = \pm 8$ **g** $k = 2$ **h** $k = -2$ or 4

7 **a** $k = -3$ or 4 **b** For $k = -3$, next term is $\frac{27}{2}$.
For $k = 4$, next term is 24.

8 **a** $u_n = 3 \times 2^{n-1}$ **b** $u_n = 32 \times (-\frac{1}{2})^{n-1}$
c $u_n = 3 \times (\pm\sqrt{2})^{n-1}$ **d** $u_n = 10 \times (\pm\sqrt{2})^{1-n}$

9 **a** $u_1 = 35\frac{5}{9},\ r = \frac{3}{2}$ **b** $u_{10} = 1366\frac{7}{8}$

10 **a** $u_9 = 13\,122$ **b** $u_{14} = 2916\sqrt{3} \approx 5050$
c $u_{18} = \frac{3}{32\,768} \approx 0.000\,091\,6$
d $u_{10} = -\frac{98\,415}{512} \approx -192$

11 2 primes
Consider $u_1 = 2,\ r = \frac{3}{2}$. The only other prime is $u_2 = 3$.

12 **a** $r = 2 \pm \sqrt{2}$
b For $r = 2 + \sqrt{2}$, geometric to arithmetic is $2 + 2\sqrt{2} : 1$.
For $r = 2 - \sqrt{2}$, geometric to arithmetic is $2 - 2\sqrt{2} : 1$.

EXERCISE 5D

1 **a** **i** ≈ 1553 ants **ii** ≈ 4823 ants **b** ≈ 12.2 weeks

2 **a** ≈ 278 **b** year 2057

3 **a** **i** ≈ 73 deer **ii** ≈ 167 deer **b** ≈ 30.5 years

4 **a** **i** ≈ 2860 **ii** $\approx 184\,000$ **b** ≈ 14.5 years

5 **a** ≈ 3.36 g **b** ≈ 10.2 more years

6 **a** €39 712.41 p.a. **b** €54 599.05 p.a.

EXERCISE 5E.1

1 £9367.58 **2** **a** €2233.58 **b** €233.58
3 \$716.38 **4** **a** \$20 977.42 **b** \$23 077.89
5 **a** €37 305.85 **b** €7305.85
6 \$11 222.90 **7** Bank A **8** £14 977
9 \$11 478 **10** \$22 054.85 **11** ¥3 000 340

EXERCISE 5E.2

1 **a** \$8487.20 **b** \$16 229.84 **c** \$27 672.16
2 **a** \$1218.99 **b** \$1485.95 **c** \$1811.36
3 \$16 236.48

EXERCISE 5E.3

1 **a** \$5567.55 **b** \$5246.43
2 **a** €23 651.79 **b** €20 691.02
3 **a** \$4782.47 **b** \$782.47 **c** \$3958.90
d The investment has not been effective. The real value of the investment after 6 years is less than what was originally invested.

4 **a** Real interest rate $= \frac{1.005}{1.001} \approx 1.003\,996 \approx 4\%$
b \$6602.66

5 $u_0\left(\dfrac{100+i}{100+r}\right)^{4y}$

EXERCISE 5E.4

1 €1280 **2** **a** €26 103.52 **b** €83 896.48
3 **a** ¥30 013 **b** ¥57 487 **4** 24.8%

EXERCISE 5E.5

1 74 614.60 pesos **2** \$6629.65
3 **a** \$9452.47 **b** \$12 482.59
4 **a** €6705.48 **b** €1705.48
5 **a** 2.82% p.a. **b** €4595.67 **6** \$1997.13
7 \$80 000 **8** \$108.69 **9** 2 years 9 months
10 13 years 3 months **11** 15 years **12** 14.5% p.a.
13 6.00% p.a. **14** 5.15% p.a. **15** 21.2% p.a.

EXERCISE 5F

1 **a** $S_3 = 18$ **b** $S_5 = 37$ **c** $S_{12} = 153$ **2** $u_5 = 7$

3 **a** **i** $S_n = \sum_{k=1}^{n}(8k-5)$ **ii** $S_5 = 95$
b **i** $S_n = \sum_{k=1}^{n}(47-5k)$ **ii** $S_5 = 160$
c **i** $S_n = \sum_{k=1}^{n} 12(\frac{1}{2})^{k-1}$ **ii** $S_5 = 23\frac{1}{4}$
d **i** $S_n = \sum_{k=1}^{n} 2(\frac{3}{2})^{k-1}$ **ii** $S_5 = 26\frac{3}{8}$
e **i** $S_n = \sum_{k=1}^{n} \frac{1}{2^{k-1}}$ **ii** $S_5 = 1\frac{15}{16}$
f **i** $S_n = \sum_{k=1}^{n} k^3$ **ii** $S_5 = 225$

4 **a** 24 **b** 27 **c** 10 **d** 25 **e** 168 **f** 310

5 $S_{20} = \sum_{k=1}^{20}(3k-1) = 610$

7 **a**
$$\begin{array}{l} 1 + \quad 2 \quad + \quad 3 \quad + \ldots + (n-1) + n \\ n + (n-1) + (n-2) + \ldots + \quad 2 \quad + 1 \end{array}$$
b $S_n = \dfrac{n(n+1)}{2}$ **c** $a = 16, \ b = 3$

8 $S_n = \sum_{k=1}^{n}(2k-1)$

9 **b**
$$\begin{aligned} (0+1)^3 &= 0^3 + 3(0)^2 + 3(0) + 1 \\ (1+1)^3 &= 1^3 + 3(1)^2 + 3(1) + 1 \\ (2+1)^3 &= 2^3 + 3(2)^2 + 3(2) + 1 \\ (3+1)^3 &= 3^3 + 3(3)^2 + 3(3) + 1 \\ &\ \vdots \\ (n+1)^3 &= n^3 + 3n^2 + 3n + 1 \end{aligned}$$

10 $\sum_{k=1}^{n}(k+1)(k+2) = \dfrac{n(n^2+6n+11)}{3}$,
$\sum_{k=1}^{10}(k+1)(k+2) = 570$

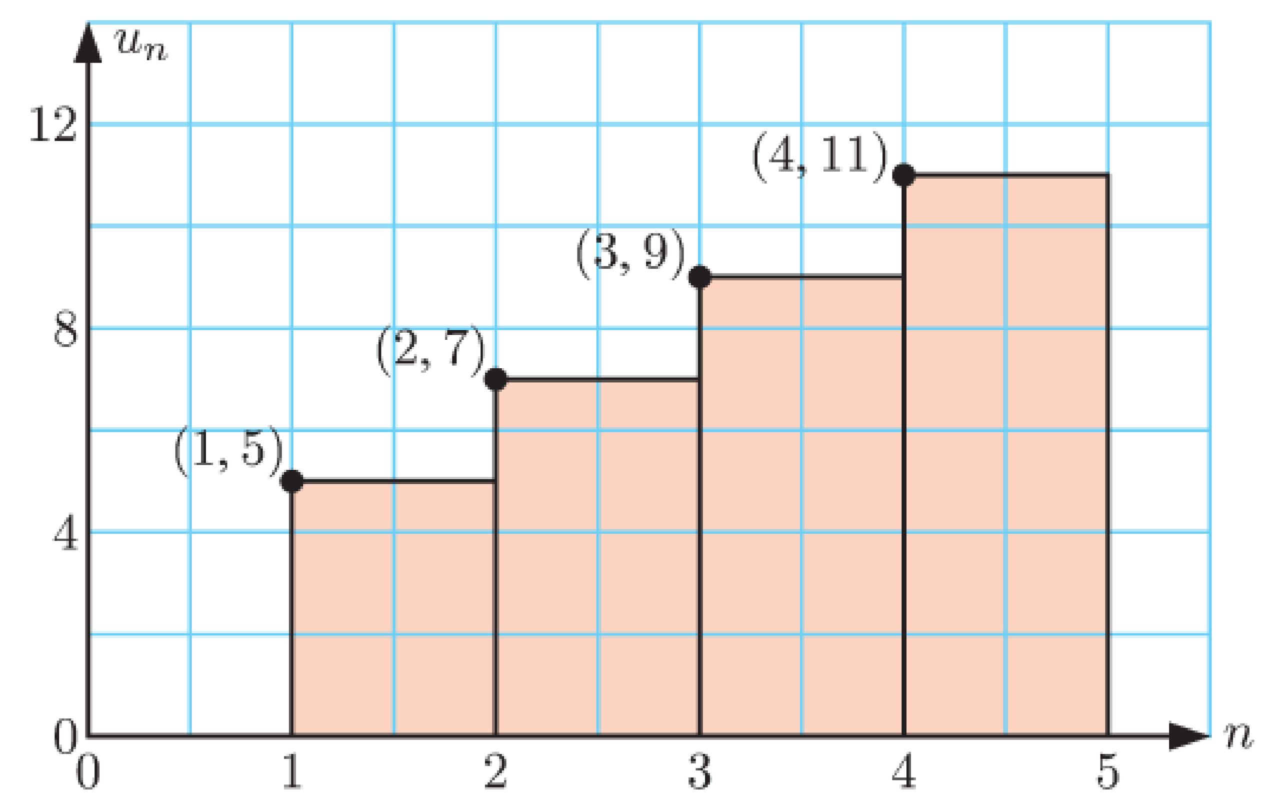
EXERCISE 5G

1 **a** 160 **b** 820 **c** $3087\frac{1}{2}$ **d** -1460
e -150 **f** -740
2 **a** 1749 **b** 184 **c** 2115 **d** $1410\frac{1}{2}$
3 **a** 160 **b** -630 **c** 135
4 $-115\frac{1}{2}$ **5** 18 layers
6 **a** **i** 38 laps **ii** 78 laps **b** 1470 laps
7 **a** \$450 **b** \$4125
8 **a** 65 seats **b** 1914 seats **c** 47 850 seats
9 **a** 14 025 **b** 71 071 **c** 3367 **d** 89 870
10 **a** $k = 5$ **b** $S_{25} = 2800$
11 $u_1 = 56, \ u_2 = 49$ **12** $S_{10} = 310$ **13** 8 terms
14 **a** $d = 3$ **b** $n = 11$ **15** 15 terms

16 **Hint:**
$$\begin{aligned} S_n &= \frac{n}{2}(2u_1 + (n-1)d) \\ &= \frac{n}{2}(2 \times 1 + (n-1) \times 1), \quad \text{and so on} \end{aligned}$$

17 **a** $u_n = 2n - 1$
b $S_n = \dfrac{n}{2}(u_1 + u_n) = \dfrac{n}{2}(1 + 2n - 1) = n^2$

18 10, 4, -2 or -2, 4, 10 **19** $u_8 = 32$
20 2, 5, 8, 11, 14 or 14, 11, 8, 5, 2
21 **a** $u_1 = 7, \ u_2 = 10$ **b** $u_{20} = 64$
22 $S_{80} = -80$ **23** $u_1 = -\frac{7}{2}, \ d = 2$

24 **a**


b S_n is the sum of the areas of the first n rectangles.
c **i** The left side of each rectangle increases in length by 2 units from the previous rectangle, $u_{n+1} = u_n + 2$.
ii The area of the $(n+1)$th rectangle is u_{n+1}.
S_{n+1} is the sum of the areas of the first n rectangles and the $(n+1)$th rectangle, $S_{n+1} = S_n + u_{n+1}$.

25 $15d$

EXERCISE 5H

1 **a** 6560 **b** 5115 **c** $\frac{3069}{128} \approx 24.0$
d $\approx 189\,134$ **e** $\frac{32\,769}{8192} \approx 4.00$ **f** ≈ 0.585

2 **a** $S_n = \dfrac{3+\sqrt{3}}{2}\left((\sqrt{3})^n - 1\right)$ **b** $S_n = 24\left(1 - (\frac{1}{2})^n\right)$
c $S_n = 1 - (0.1)^n$ **d** $S_n = \frac{40}{3}\left(1 - (-\frac{1}{2})^n\right)$

3 **a** 3069 **b** $\frac{4095}{1024} \approx 4.00$ **c** $-134\,217\,732$
4 **c** \$26 361.59
5 **a** The number of grains of wheat starts at 1, and each square has double the number of grains of the previous square.
c $u_n = 2^{n-1}$ **d** $(2^{64} - 1) \approx 1.84 \times 10^{19}$ grains of wheat
6 **a** \$5790 **b** $S_n = 100\,000((1.05)^n - 1)$ **c** $\approx$ \$40 710
7 £18 413.84

8 **a** $S_1 = \frac{1}{2}$, $S_2 = \frac{3}{4}$, $S_3 = \frac{7}{8}$, $S_4 = \frac{15}{16}$, $S_5 = \frac{31}{32}$

b $S_n = \frac{2^n - 1}{2^n}$ **c** $S_n = 1 - (\frac{1}{2})^n = \frac{2^n - 1}{2^n}$

d as $n \to \infty$, $S_n \to 1$

e As $n \to \infty$, the sum of the fractions approaches the area of a 1×1 unit square.

9 $u_4 = \frac{2}{3}$ or 54

10 Arithmetic: $u_{20} = 39$ or $7\frac{1}{3}$

Geometric: $u_{20} = 3^{19}$ or $\left(\frac{4}{3}\right)^{19}$

12 $n = 5$ **13** $n = 11$

14 **a** In 3 years she will earn \$183 000 under *Option B*, compared with \$126 100 under *Option A*.

b **i** $A_n = 40\,000 \times (1.05)^{n-1}$ **ii** $B_n = 59\,000 + 1000n$

c ≈ 13.1 years

e **i** graph 1 represents T_A, graph 2 represents T_B

ii P(22.3, 1 580 000) **iii** $0 \leqslant n \leqslant 22$

15 **a** $A_3 = \$8000 \times (1.03)^3 - (1.03)^2 R - 1.03R - R$

b $A_8 = \$8000 \times (1.03)^8 - (1.03)^7 R - (1.03)^6 R - (1.03)^5 R - (1.03)^4 R - (1.03)^3 R - (1.03)^2 R - (1.03)R - R$

$= 0$

c $R = \$1139.65$

EXERCISE 5I

1 **a** It is geometric with $u_1 = \frac{3}{10}$ and $r = \frac{1}{10}$, and we are adding all the terms. Therefore it is an infinite geometric series.

b Using **a**, $S = \dfrac{\frac{3}{10}}{1 - \frac{1}{10}} = \frac{3}{9} = \frac{1}{3}$ $\therefore$ $0.\overline{3} = \frac{1}{3}$

2 **a** $0.\overline{4} = \frac{4}{9}$ **b** $0.\overline{16} = \frac{16}{99}$ **c** $0.\overline{312} = \frac{104}{333}$

4 **a** 54 **b** 14.175

5 **a** 1 **b** $4\frac{2}{7}$ **6** $u_1 = 9$, $r = \frac{2}{3}$

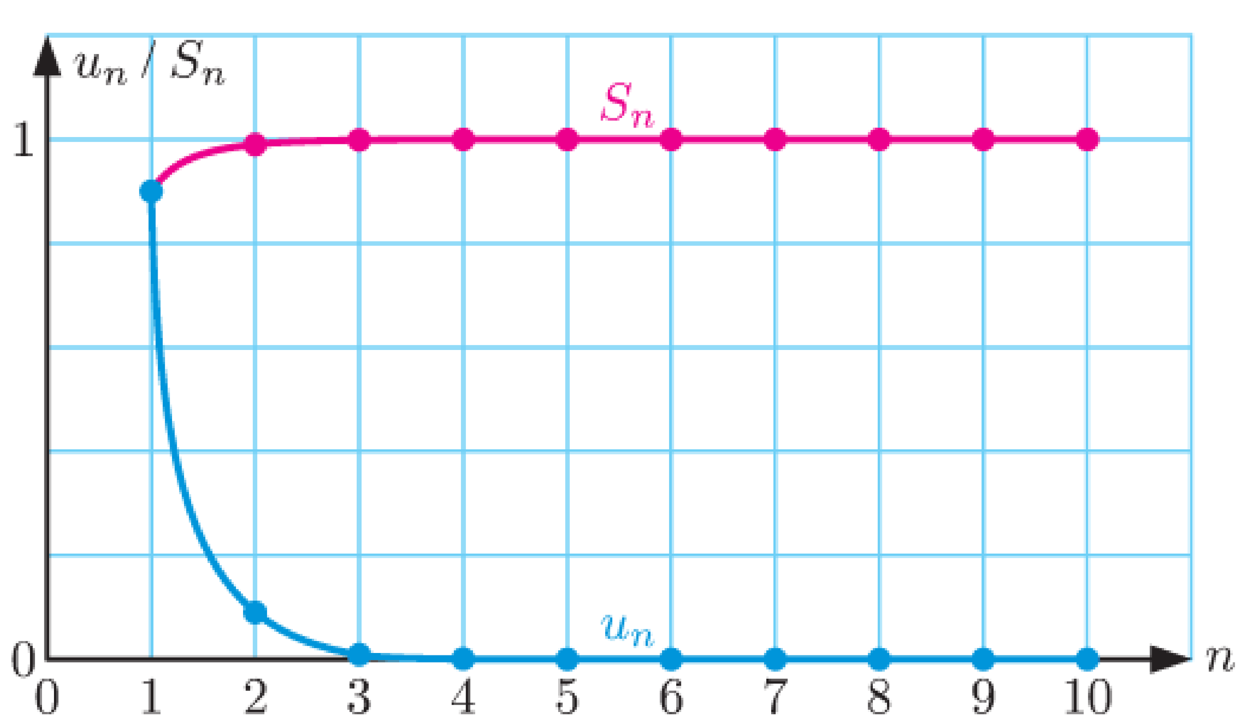
7 $u_1 = 8$, $r = \frac{1}{5}$ and $u_1 = 2$, $r = \frac{4}{5}$ **8** $S_5 = \frac{341}{16}$

9 **b** $S_n = 19 - 20(0.9)^n$ **c** 19 seconds **10** 70 cm

11 **a** $0.\overline{9} = \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + ...$ which is geometric with $u_1 = \frac{9}{10}$ and $r = \frac{1}{10}$

$\therefore$ $0.\overline{9} = S = \dfrac{\frac{9}{10}}{1 - \frac{1}{10}} = 1$

c



12 $x = \frac{1}{2}$

13 $u_1 + u_2 + u_3 + u_4 + ...$ has common ratio r, $0 < r < 1$

a $u_1 - u_2 + u_3 - u_4 + ...$ has common ratio $-r$

$\therefore$ $-1 < -r < 1$ and so the series is convergent.

$\sqrt{u_1} + \sqrt{u_2} + \sqrt{u_3} + \sqrt{u_4} + ...$ has common ratio $\sqrt{r}$

$\therefore$ $-1 < \sqrt{r} < 1$ and so the series is convergent.

b $\frac{81}{8}$

REVIEW SET 5A

1 **a** $u_2 = 9$ **b** $u_6 = 19$ **c** $S_4 = 37$

2 $k = -\frac{11}{2}$ **3** **b** $u_1 = 6$, $r = \frac{1}{2}$ **c** $u_{16} \approx 0.000\,183$

4 $u_n = \frac{1}{6} \times 2^{n-1}$ or $-\frac{1}{6} \times (-2)^{n-1}$

5 23, 21, 19, 17, 15, 13, 11, 9

6 **a** ≈ 45.7 mL **b** $u_n \approx 45.7n$ **c** ≈ 594 mL

7 **a** $10\frac{4}{5}$ **b** $16 + 8\sqrt{2}$ **8** **a** 1272 **b** $302\frac{1}{2}$

9 **a** 2011: 630 000 sheets, 2012: 567 000 sheets

b $\approx 5\,630\,000$ sheets

10 **a** $1 + 4 + 9 + 16 + 25 + 36 + 49 = 140$

b $\frac{4}{3} + \frac{5}{4} + \frac{6}{5} + \frac{7}{6} = \frac{99}{20}$

11 **a** $u_n = 3n + 1$ **12** **a** £15 425.20 **b** £15 453.77

13 \$4800 **14** **a** €6622.87 **b** €13 313.28

15 **a** \$24 076.91 **b** \$22 822.20

16 $u_n = 33 - 5n$, $S_n = \frac{n}{2}(61 - 5n)$

17 **a** 17 terms **b** $\frac{131\,071}{512} \approx 255.998$

18 $u_1 = 54$, $r = \frac{2}{3}$ or $u_1 = 150$, $r = -\frac{2}{5}$

$|r| < 1$ in both cases, so the series will converge.

For $u_1 = 54$, $r = \frac{2}{3}$, $S = 162$.

For $u_1 = 150$, $r = -\frac{2}{5}$, $S = 107\frac{1}{7}$.

19 **a** The number of cigarettes Tim smokes decreases by 5 each week, with 115 in the first week. The common difference $d = -5$.

b 24 weeks **c** 1380 cigarettes

20 **a** $\frac{5}{2}\sqrt{2}$

b **i** $S_{10} = 310 + 155\sqrt{2}$ **ii** $S = 320 + 160\sqrt{2}$

21 $u_1 = 3$ **22** $r = 4$

23 $x = 3$, $y = -1$, $z = \frac{1}{3}$ or $x = \frac{1}{3}$, $y = -1$, $z = 3$

24 **a** \$192 000

b **i** \$1000, \$1600, \$2200 **ii** \$189 600

c **i** \$500, \$600, \$720 **ii** \$196 242.12

d Option 3 **e** \$636.97

25 $x = \frac{3}{2}$ ($x = -\frac{6}{7}$ gives a divergent series)

REVIEW SET 5B

1 **a** $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$, $\frac{1}{243}$ **b** 17, 22, 27, 32, 37

c $\frac{4}{3}$, 1, $\frac{4}{5}$, $\frac{2}{3}$, $\frac{4}{7}$

2 **b** $u_1 = 63$, $d = -5$ **c** $u_{37} = -117$ **d** $u_{54} = -202$

3 **a** $u_n = 73 - 6n$ **b** $u_{34} = -131$

4 **a** $S_{12} = 432$ **b** $S_{12} = \frac{12\,285}{256} \approx 48.0$

5 **a** $u_n = \frac{25}{6}n - \frac{265}{6}$

b Stacy makes $\approx$ £4.17 per customer. The setup fee for the stand $\approx$ £44.17.

c $\approx$ £105.83

6 $u_{12} = 10\,240$ **7** 27 metres

8 **a** \$8337.11 **b** \$8369.33 **c** \$8376.76

9 **a** $k = \pm\frac{2\sqrt{3}}{3}$ **b** For $k = \frac{2\sqrt{3}}{3}$, $r = \frac{\sqrt{3}}{6}$

For $k = -\frac{2\sqrt{3}}{3}$, $r = -\frac{\sqrt{3}}{6}$

10 **a** 35.5 km **b** 1183 km

11 **a** $\frac{1331}{2100} \approx 0.634$ **b** $\frac{98}{15} \approx 6.53$

12 $u_{11} = \frac{8}{19\,683} \approx 0.000\,406$ **13** 3.80% p.a.

14 182 months (15 years 2 months)

15 **a** \$10 069.82 **b** \$7887.74 **16** \$2174.63

17 **a** 70 **b** ≈ 241 **c** $\frac{64}{1875} \approx 0.0341$

18 **a** $u_n = \frac{3}{4} \times 2^{n-1}$ **b** $S_{15} = 24\,575\frac{1}{4}$

19 **a** ≈ 3470 iguanas **b** year 2029

20 **a** $0 < x < 1$ (we require $|2x - 1| < 1$) **b** $35\frac{5}{7}$

21 **a** The sequence is $2^{u_1}, 2^{u_1+d}, 2^{u_1+2d}, \ldots$
or $2^{u_1}, 2^d 2^{u_1}, (2^d)^2 2^{u_1}, \ldots$
which is geometric.

b $\frac{32}{7}$

22 **a** \$82 539.08

b

n (years)	0	1	2	3	4
V_n (\$)	100 000	106 000	112 360	119 101.60	126 247.70

c $V_n = 100\,000 \times (1.06)^n$ dollars

d $S_n = 6000n$ dollars

e

n (years)	0	1	2	3	4
T_n (\$)	100 000	112 000	124 360	137 101.60	150 247.70

f 19 years

23 $47\frac{6}{7}$ or $31\frac{1}{7}$ **24** $S_n = \dfrac{2 - 2^{\frac{1}{n+1}}}{2^{\frac{1}{n+1}} - 1}$

25 **a** $r = 4$

b **Hint:** If u_1 is the first term of the arithmetic sequence, show that $(u_1 + 7d) \times 4 = u_1 + 23d$.

EXERCISE 6A

1 **a** ≈ 57.2 mm **b** ≈ 33.5 cm **c** ≈ 40.5 m
d ≈ 138 cm

2 ≈ 41.4 cm **3** ≈ 68.5 mm

4 **a** ≈ 133 cm^2 **b** ≈ 9.62 m^2 **c** ≈ 58.5 cm^2
d ≈ 192 cm^2

5 ≈ 5.26 cm **6** ≈ 21.5 cm

7 **a** ≈ 191 m **b** ≈ 6.04 m s^{-1}

8 **a** $8\sqrt{2} \approx 11.3$ mm **b** $8\pi(1 + \sqrt{2}) \approx 60.7$ mm
c 128 mm^2

9 **c** $r = 0.98$ m, $\theta \approx 58.5$ **d** ≈ 1.29 m

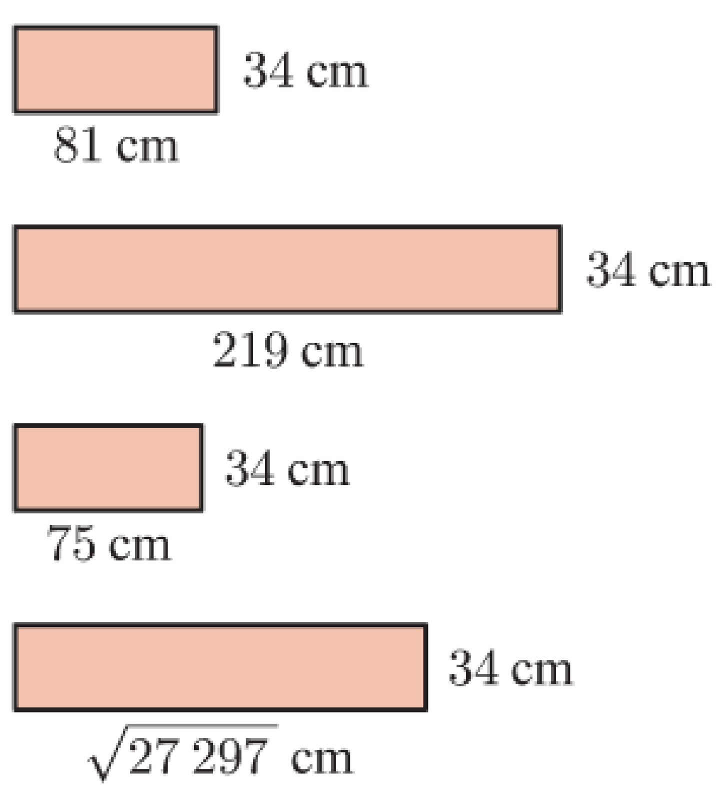
EXERCISE 6B.1

1 **a** 5.7802 m^2 **b** ≈ 112 cm^2 **c** ≈ 14.9 cm^2

2 **a** 1440 cm^2 **b** ≈ 51.6 cm^2 **c** ≈ 181 m^2

3 **a** 23 814 cm^2

b



area = 2754 cm^2

area = 7446 cm^2

area = 2550 cm^2

area $\approx$ 5617 cm^2

c $\approx$ €540

4 **a** 26 940 cm^2 **b** ≈ 407 m^2 **5** ≈ 2310 cm^2

6 **a** $(10x^2 + 12x)$ cm^2 **b** $(1 + \sqrt{3})x^2$ cm^2

EXERCISE 6B.2

1 **a** ≈ 1005.3 cm^2 **b** ≈ 63.6 km^2 **c** ≈ 188.5 cm^2
d ≈ 549.8 m^2 **e** ≈ 1068.1 cm^2 **f** ≈ 84.8 cm^2

2 **a** ≈ 2210 cm^2 **b** ≈ 66.5 m^2 **c** $\approx 14\,800$ mm^2
d ≈ 12.1 cm^2

3 **a** $s \approx 5.39$ **b** ≈ 46.4 m^2 **c** $\approx$ \$835.24

4 **a** ≈ 50.3 m^2 **b** $\approx$ \$1166.16 **c** ≈ 150.8 m^2
d $\approx$ \$2789.73 **e** $\approx$ \$3960

5 ≈ 266 cm^2 **6** **a** $SA = 4\pi r^2$ **b** ≈ 5.40 m

7 **a** $SA = 3\pi r^2$ **b** **i** ≈ 4.50 cm **ii** ≈ 4.24 cm

8 **a** $SA = 6\pi x^2$ cm^2 **b** $SA = 3\pi r^2$ cm^2
c $SA = \pi x^2(1 + \sqrt{5})$ cm^2

9 **a** 4 cm **b** ≈ 25.1 cm **c** ≈ 84.1 mm

10 **a** ≈ 34.7 m^2 **b** ≈ 285.4 m^2 **c** ≈ 62.8 cm^2

11 $\approx 24\,600$ km **12** **a** $\dfrac{\theta\pi s}{180}$ **b** $\theta = \dfrac{360r}{s}$

EXERCISE 6C.1

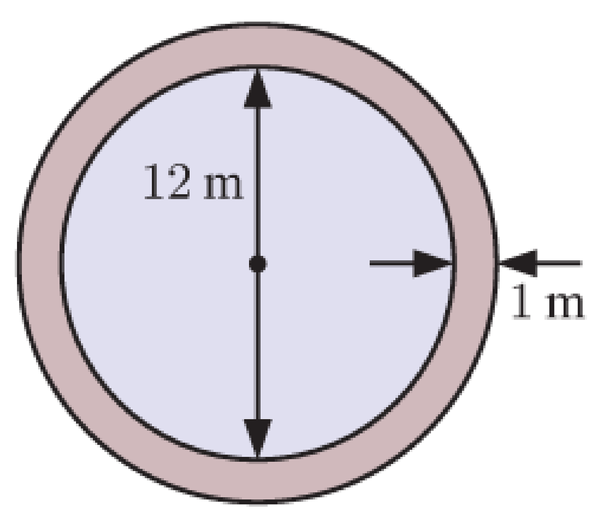
1 **a** 25.116 cm^3 **b** 373 cm^3 **c** 765.486 cm^3
d ≈ 2940 cm^3 **e** ≈ 3.13 m^3 **f** 1440 cm^3

2 **a** 648 000 000 mm^3 **b** ≈ 11.6 m^3 **c** 156 cm^3

3 **a** 0.5 m **b** 0.45 m **c** ≈ 0.373 m^3

4 **a** 7.176 m^3 **b** \$972

5 **a**



b ≈ 40.8 m^2
c ≈ 4.08 m^3

6 ≈ 81.1 tonnes

7 **a** 2 trailer loads **b** \$174.60
c **i** 2 loads **ii** \$95.90 **d** \$270.50

8 **a** 100 cm **b** 1 500 000 cm^3 (or 1.5 m^3) **c** 95 000 cm^2

9 **a** $\frac{8}{3} \approx 2.67$ cm **b** ≈ 3.24 cm **c** ≈ 1.74 cm

10 ≈ 12.7 cm

EXERCISE 6C.2

1 **a** ≈ 463 cm^3 **b** ≈ 4.60 cm^3 **c** ≈ 26.5 cm^3
d ≈ 1870 m^3 **e** ≈ 155 m^3 **f** ≈ 226 cm^3

2 **a** $\approx 29\,000$ m^3 **b** 480 m^3 **c** ≈ 497 cm^3

3 **a** ≈ 11.9 m^3 **b** 5.8 m **c** ≈ 1.36 m^3 more
e The hemispherical design, as it holds more concrete and is shorter.

4 **a** ≈ 4.46 cm **b** ≈ 2.60 m **c** ≈ 5.60 cm

6 **a** **i** ≈ 67.0 cm^3 **ii** ≈ 113 m^3
b $V = \frac{2}{3}\pi r^3$ This is half the volume of a sphere because when $h = r$, the cap is a hemisphere.

EXERCISE 6D

1 **a** 12.852 kL **b** ≈ 61.2 kL **c** ≈ 68.0 kL

2 **a** $\approx 12\,200$ cm^3 **b** ≈ 12.2 L **3** 594 425 kL

4 **a** ≈ 954 mL **b** 4.92 kL **c** 5155 tins **d** \$18 042.50

5 ≈ 0.553 m (or ≈ 55.3 cm)

6 **a** 1.32 m^3 **b** 1.32 kL **c** ≈ 10.5 cm **7** ≈ 7.8 cm

8 **a** ≈ 252 mL **b** **i** ≈ 189 mL **ii** 3.25 cm

9 35 truck loads

10 **a** $\approx 110\,000$ mm^3

b The external surface area and internal surface area of a container may be different.

c **i** $1\,870\,000$ mm^3 **ii** 1.87 L **iii** $\approx 502\,000$ mm^3

REVIEW SET 6A

1 **a** ≈ 18.3 cm **b** ≈ 38.3 cm **c** ≈ 91.6 cm^2

2 ≈ 10.4 cm

3 **a** ≈ 377.0 cm^2 **b** ≈ 339.8 cm^2 **c** ≈ 201.1 cm^2

4 **a** 71 m^2 **b** \$239.25

5 **a** ≈ 4.99 m^3 **b** 853 cm^3 **c** ≈ 0.452 m^3

6 ≈ 3.22 m^3 **7** $\approx 82\,400$ cm^3 **8** ≈ 1470 m^3

9 **a** 734.44 mL **b** ≈ 198 L **10** ≈ 68.4 mm

11 **a** height $= 3.3$ m $- 1.8$ m $- 0.8$ m $= 0.7$ m $= 70$ cm

b ≈ 1.06 m **c** ≈ 15.7 m^2

d **Hint:** Volume of silo
$=$ volume of hemisphere $+$ volume of cylinder
$+$ volume of cone

e ≈ 5.2 kL

REVIEW SET 6B

1 **a** $\theta° \approx 76.6°$ **b** ≈ 14.3 cm^2

2 **a** ≈ 29.1 cm **b** ≈ 25.1 cm^2

3 **a** ≈ 84.7 cm^2 **b** ≈ 7110 mm^2 **c** ≈ 8.99 m^2

4 ≈ 23.5 m^2 **5** ≈ 434 cm^2

6 **a** ≈ 164 cm^3 **b** 120 m^3 **c** $\approx 10\,300$ mm^3

7 **a** 0.52 m^3 **b** 5.08 m^2 **8** ≈ 5680 L **9** ≈ 1.03 m

10 **a** $\approx 6.08 \times 10^{18}$ m^2 **b** $\approx 1.41 \times 10^{27}$ m^3

11 **a** ≈ 56.5 cm^3 **b** 3 cm **c** ≈ 96.5 cm^2

EXERCISE 7A

1 **a** **i** $\frac{4}{5}$ **ii** $\frac{3}{5}$ **iii** $\frac{4}{3}$

b **i** $\frac{5}{8}$ **ii** $\frac{\sqrt{39}}{8}$ **iii** $\frac{5}{\sqrt{39}}$

c **i** $\frac{7}{\sqrt{65}}$ **ii** $\frac{4}{\sqrt{65}}$ **iii** $\frac{7}{4}$

d **i** $\frac{5}{\sqrt{61}}$ **ii** $\frac{6}{\sqrt{61}}$ **iii** $\frac{5}{6}$

2 **a** XY ≈ 4.9 cm, XZ ≈ 3.3 cm, YZ ≈ 5.9 cm

b **i** ≈ 0.83 **ii** ≈ 0.56 **iii** ≈ 1.48

3 **a** **Hint:** Base angles of an isosceles triangle are equal, and sum of all angles in a triangle is $180°$.

b AB $= \sqrt{2} \approx 1.41$ m

c **i** $\frac{1}{\sqrt{2}} \approx 0.707$ **ii** $\frac{1}{\sqrt{2}} \approx 0.707$ **iii** 1

4 The OPP and ADJ sides will always be smaller than the HYP. So, the sine and cosine ratios will always be less than or equal to 1.

5 **a** **i** $\dfrac{a}{c}$ **ii** $\dfrac{b}{c}$ **iii** $\dfrac{a}{b}$ **iv** $\dfrac{b}{c}$ **v** $\dfrac{a}{c}$ **vi** $\dfrac{b}{a}$

b $A = 90° - B$

c **i** $\sin\theta = \cos(90° - \theta)$ **ii** $\cos\theta = \sin(90° - \theta)$

iii $\tan\theta = \dfrac{1}{\tan(90° - \theta)}$

6 **a** ≈ 7.50 m **b** ≈ 7.82 cm **c** ≈ 4.82 cm

d ≈ 5.17 m **e** ≈ 6.38 m **f** ≈ 4.82 cm

7 **a** $x \approx 3.98$ **b** **i** $y \approx 4.98$ **ii** $y \approx 4.98$

8 **a** $x \approx 2.87$, $y \approx 4.10$ **b** $x \approx 16.40$, $y \approx 18.25$

c $x \approx 10.77$, $y \approx 14.50$

9 **a** perimeter ≈ 23.2 cm, area ≈ 22.9 cm^2

b perimeter ≈ 17.0 cm, area ≈ 10.9 cm^2

10 ≈ 21.7 cm

EXERCISE 7B

1 **a** $\theta \approx 53.1°$ **b** $\theta \approx 45.6°$ **c** $\theta \approx 13.7°$

d $\theta \approx 52.4°$ **e** $\theta \approx 76.1°$ **f** $\theta \approx 36.0°$

2 **a** $\theta \approx 56.3°$ **b** **i** $\phi \approx 33.7°$ **ii** $\phi \approx 33.7°$

3 **a** $\theta \approx 39.7°$, $\phi \approx 50.3°$ **b** $\alpha \approx 38.9°$, $\beta \approx 51.1°$

c $\theta \approx 61.5°$, $\phi \approx 28.5°$

4 **a** The triangle cannot be drawn with the given dimensions.

b The triangle cannot be drawn with the given dimensions.

c The result is not a triangle, but a straight line of length 9.3 m.

5 **a** $x \approx 2.65$, $\theta \approx 37.1°$

b $x \approx 6.16$, $\theta \approx 50.3°$, $y \approx 13.0$

6 $\approx 135°$ **7** $\alpha \approx 6.92$

EXERCISE 7C

1 **a** $x \approx 4.13$ **b** $\alpha \approx 75.5°$ **c** $\beta \approx 41.0°$

d $x \approx 6.29$ **e** $\theta \approx 51.9°$ **f** $x \approx 12.6$

2 $\approx 22.4°$ **3** ≈ 11.8 cm

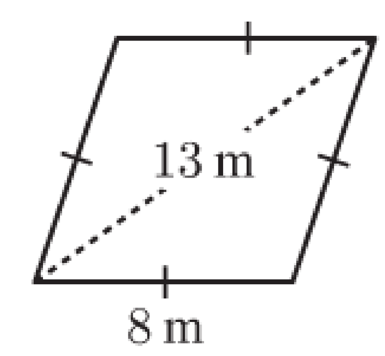
4 **a** ≈ 27.2 cm^2 **b** ≈ 153 m^2 **5** $\approx 119°$

6 ≈ 36.5 cm **7** **a** $x \approx 45.4$ **b** $x \approx 2.24$

8 **a** $x \approx 3.44$ **b** $\alpha \approx 51.5°$

9 **a** ≈ 12.3 cm^2 **b** ≈ 14.3 cm^2

10 **a**



b ≈ 9.33 m

c $\approx 71.3°$

11 **a** ≈ 2.59 cm **b** ≈ 8.46 cm

12 **a** $\theta \approx 36.9°$ **b** $r \approx 11.3$ **c** $\alpha \approx 61.9°$

13 ≈ 7.99 cm **14** $\approx 89.2°$ **15** $\approx 47.2°$ **16** ≈ 6.78 cm^2

EXERCISE 7D

1 ≈ 18.3 m **2** **a** ≈ 46.4 m **b** ≈ 259 m

3 $\approx 1.58°$ **4** **a** $\approx 26.4°$ **b** $\approx 26.4°$

5 ≈ 142 m **6** $\theta \approx 12.6°$ **7** ≈ 9.56 m

8 ≈ 46.7 m **9** $\beta \approx 129°$ **10** ≈ 10.9 m

11 ≈ 104 m **12** ≈ 962 m **13** ≈ 3.17 km

14 ≈ 43.8 m **15** **a** ≈ 18.4 cm **b** $\approx 35.3°$

16 **a** ≈ 10.8 cm **b** $\approx 36.5°$ **c** ≈ 9.49 cm **d** $\approx 40.1°$

17 **a** ≈ 82.4 cm **b** ≈ 77.7 L

18 **a** **i** 2 m **ii** ≈ 2.01 m **b** $\approx 6.84°$

19 **a** ≈ 10.2 m **b** no **20** **a** ≈ 73.4 m **b** $\approx 16.2°$

21 $\approx 67.0°$

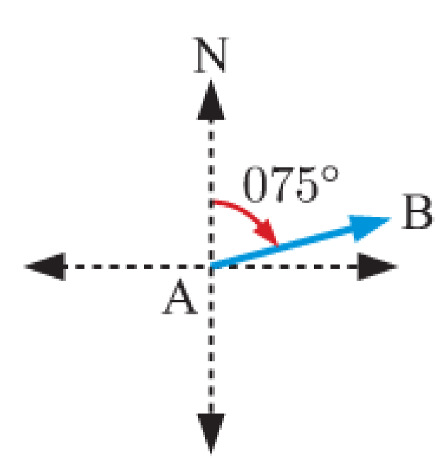
22 **a** ≈ 1.49 m^3 **b** ≈ 0.331 m^3 **c** ≈ 88.9 cm^3

23 **a** **Hint:** Consider

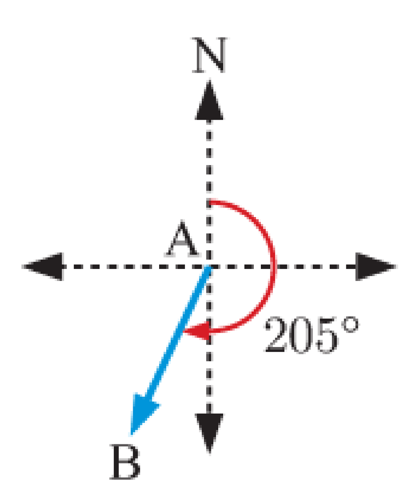
b ≈ 0.285 arc seconds

EXERCISE 7E

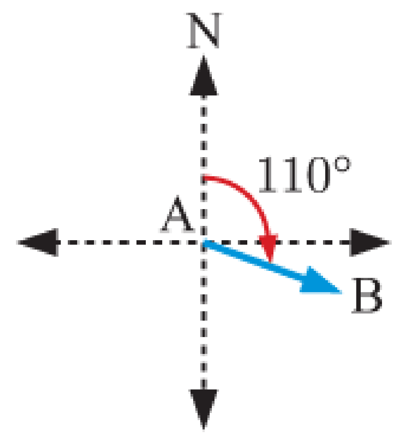
1 **a**



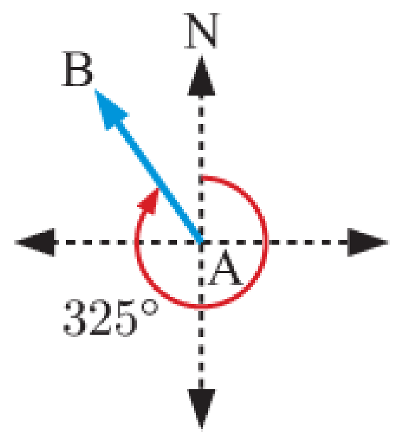
b



c



d

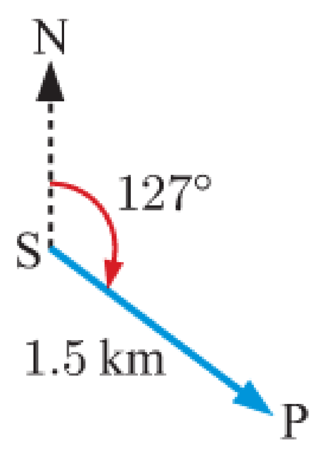


2 **a** 126° **b** 245° **c** 152° **d** 308°

3 **a** 072° **b** 252° **c** 162° **d** 342°
e 113° **f** 293°

4 ≈ 125° **5** **a** ≈ 224 m **b** ≈ 333° **c** ≈ 153°

6 **a**



b ≈ 1.20 km **c** ≈ 0.903 km

7 ≈ 2.41 km **8** ≈ 12.6 km

9 **a** ≈ 854 m **b** ≈ 203°

10 ≈ 73.3 km on the bearing ≈ 191°

11 ≈ 17.8 km on the bearing ≈ 162°

12 **a** ≈ 046.6° **b** ≈ 4.22 km

EXERCISE 7F

1 **a** **i** [EH] **ii** [EF] **iii** [EG] **iv** [FH]
b **i** [MR] **ii** [MN]

2 **a** **i** $\widehat{AFE}$ **ii** $\widehat{BMF}$ **iii** $\widehat{ADE}$ **iv** $\widehat{BNF}$
b **i** $\widehat{BAM}$ **ii** $\widehat{BNM}$ **iii** $\widehat{EAN}$

3 **a** **i** ≈ 36.9° **ii** ≈ 25.1° **iii** ≈ 56.3° **iv** ≈ 29.1°
b **i** ≈ 33.7° **ii** ≈ 33.7° **iii** ≈ 25.2° **iv** ≈ 30.8°
c **i** ≈ 59.0° **ii** ≈ 22.0° **iii** ≈ 22.6°
d **i** ≈ 64.9° **ii** ≈ 71.7°

4 ≈ 31.7°

REVIEW SET 7A

1 **a** 10 cm **b** $\frac{6}{10} = \frac{3}{5}$ **c** $\frac{8}{10} = \frac{4}{5}$ **d** $\frac{6}{8} = \frac{3}{4}$

2 **a** $x \approx 3.51$ **b** $x \approx 51.1$ **c** $x \approx 5.62$

3 ≈ 43.4 cm² **4** $\theta = 33°$, $x \approx 3.90$, $y \approx 7.15$

5 $\theta \approx 8.19°$ **6** ≈ 124°

7 **a** $x \approx 2.8$ **b** $x \approx 4.2$ **c** $x \approx 5.2$

8 ≈ 13.5 m **9** **a** 118° **b** 231° **c** 329°

10 13 km on the bearing ≈ 203° from the helipad.

11 ≈ 8.74° **12** ≈ 0.607 L **13** **a** ≈ 53.1° **b** ≈ 62.1°

REVIEW SET 7B

1 **a** AB ≈ 4.5 cm, AC ≈ 2.2 cm, BC ≈ 5.0 cm
b **i** ≈ 0.44 **ii** ≈ 0.90 **iii** ≈ 0.49

2 **a** $\theta \approx 34.8°$ **b** $\theta \approx 39.7°$ **c** $\theta \approx 36.0°$

3 AB ≈ 120 mm, AC ≈ 111 mm

4 $x \approx 25.7$, $\theta \approx 53.6°$, $\alpha \approx 36.4°$

5 **a** ≈ 200 cm **b** ≈ 1500 cm² **6** ≈ 2.54 cm

7 ≈ 204 m **8** **a** 90° **b** ≈ 33.9°

9 ≈ 3.91 km on the bearing ≈ 253° from his starting point.

10 ≈ 5.46 km **11** ≈ 485 m³

12 **a** ≈ 14.4° **b** ≈ 18.9° **c** ≈ 21.8°

13 **a** **i** ≈ 27.6 cm **ii** ≈ 23.3 cm **b** ≈ 6010 cm³

EXERCISE 8A

1 **a** $\frac{\pi}{2}$ **b** $\frac{\pi}{3}$ **c** $\frac{\pi}{6}$ **d** $\frac{\pi}{10}$ **e** $\frac{\pi}{20}$
f $\frac{3\pi}{4}$ **g** $\frac{5\pi}{4}$ **h** $\frac{3\pi}{2}$ **i** 2π **j** 4π
k $\frac{7\pi}{4}$ **l** 3π **m** $\frac{\pi}{5}$ **n** $\frac{4\pi}{9}$ **o** $\frac{23\pi}{18}$

2 **a** $\approx 0.641^c$ **b** $\approx 2.39^c$ **c** $\approx 5.55^c$ **d** $\approx 3.83^c$
e $\approx 6.92^c$

3 **a** 36° **b** 108° **c** 135° **d** 10° **e** 20°
f 140° **g** 18° **h** 27° **i** 210° **j** 22.5°

4 **a** ≈ 114.59° **b** ≈ 87.66° **c** ≈ 49.68°
d ≈ 182.14° **e** ≈ 301.78°

5 **a**

Degrees	0	45	90	135	180	225	270	315	360
Radians	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π

b

Deg.	0	30	60	90	120	150	180	210	240	270	300	330	360
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π

EXERCISE 8B

1 **a** 7 cm **b** 12 cm **c** ≈ 13.0 m

2 **a** 6 cm² **b** 48 cm² **c** ≈ 8.21 cm²

3 **a** arc length ≈ 49.5 cm, area ≈ 223 cm²
b arc length ≈ 23.0 cm, area ≈ 56.8 cm²

4 **a** $\approx 0.686^c$ **b** 0.6^c

5 **a** $\theta = 0.75^c$, area = 24 cm²
b $\theta = 1.68^c$, area = 21 cm²
c $\theta \approx 2.32^c$, area = 126.8 cm²

6 **a** ≈ 3.15 m **b** ≈ 9.32 m²

7 **a** ≈ 5.91 cm **b** ≈ 18.9 cm

8 **a** $\alpha \approx 0.3218^c$ **b** $\theta \approx 2.498^c$ **c** ≈ 387 m²

9 **a** ≈ 11.7 cm **b** $r \approx 11.7$ **c** ≈ 37.7 cm **d** $\theta \approx 3.23^c$

10 ≈ 25.9 cm **11** **b** ≈ 2 h 24 min **12** ≈ 227 m²

13 **a** $\alpha \approx 5.739$ **b** $\theta \approx 168.5$ **c** $\phi \approx 191.5$
d ≈ 71.62 cm

14 **a** 4 cm **b** **i** ≈ 2.16 cm² **ii** ≈ 29.3 cm²

15 **a** **Hint:** Let the largest circle have radius r_1, and use a right angled triangle to show that $\sin\frac{\pi}{6} = \frac{r_1}{10 - r_1}$.
b $\frac{25\pi}{2}$ units² **c** $\frac{3}{4}$

EXERCISE 8C

1

θ (degrees)	0°	90°	180°	270°	360°	450°
θ (radians)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$
sine	0	1	0	−1	0	1
cosine	1	0	−1	0	1	0
tangent	0	undef.	0	undef.	0	undef.

2 **a** **i** A(cos 26°, sin 26°), B(cos 146°, sin 146°), C(cos 199°, sin 199°)
ii A(0.899, 0.438), B(−0.829, 0.559), C(−0.946, −0.326)
b **i** A(cos 123°, sin 123°), B(cos 251°, sin 251°), C(cos(−35°), sin(−35°))
ii A(−0.545, 0.839), B(−0.326, −0.946), C(0.819, −0.574)

3 **a** **i** $\frac{1}{\sqrt{2}} \approx 0.707$ **ii** $\frac{\sqrt{3}}{2} \approx 0.866$

b

θ (degrees)	$30°$	$45°$	$60°$	$135°$	$150°$	$240°$	$315°$
θ (radians)	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{4\pi}{3}$	$\frac{7\pi}{4}$
sine	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$
cosine	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
tangent	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	$\sqrt{3}$	-1

4

Quadrant	*Degree measure*	*Radian measure*	$\cos\theta$	$\sin\theta$	$\tan\theta$
1	$0° < \theta < 90°$	$0 < \theta < \frac{\pi}{2}$	$+$ve	$+$ve	$+$ve
2	$90° < \theta < 180°$	$\frac{\pi}{2} < \theta < \pi$	$-$ve	$+$ve	$-$ve
3	$180° < \theta < 270°$	$\pi < \theta < \frac{3\pi}{2}$	$-$ve	$-$ve	$+$ve
4	$270° < \theta < 360°$	$\frac{3\pi}{2} < \theta < 2\pi$	$+$ve	$-$ve	$-$ve

5 **a** 1 and 4 **b** 2 and 3 **c** 3 **d** 2

6 **a** $\cos 400° = \cos(360 + 40)° = \cos 40°$

b $\sin\frac{5\pi}{7} = \sin\left(\frac{5\pi}{7} + 2\pi\right) = \sin\frac{19\pi}{7}$

c $\tan\frac{13\pi}{8} = \tan\left(\frac{13\pi}{8} - 3\pi\right) = \tan\left(-\frac{11\pi}{8}\right)$

7 **B** and **D** **8** **B** and **E**

9 **a** **i** ≈ 0.985 **ii** ≈ 0.985 **iii** ≈ 0.866 **iv** ≈ 0.866
v 0.5 **vi** 0.5 **vii** ≈ 0.707 **viii** ≈ 0.707

b $\sin(180° - \theta) = \sin\theta$ **c** $\sin(\pi - \theta) = \sin\theta$

d The points have the same y-coordinate.

e **i** $135°$ **ii** $129°$ **iii** $\frac{2\pi}{3}$ **iv** $\frac{5\pi}{6}$

10 **a** **i** ≈ 0.342 **ii** ≈ -0.342 **iii** 0.5
iv -0.5 **v** ≈ 0.906 **vi** ≈ -0.906
vii ≈ 0.174 **viii** ≈ -0.174

b $\cos(180° - \theta) = -\cos\theta$ **c** $\cos(\pi - \theta) = -\cos\theta$

d The x-coordinates of the points have the same magnitude but are opposite in sign.

e **i** $140°$ **ii** $161°$ **iii** $\frac{4\pi}{5}$ **iv** $\frac{3\pi}{5}$

11 $\tan(\pi - \theta) = -\tan\theta$

12 **a** ≈ 0.6820 **b** ≈ 0.8572 **c** ≈ -0.7986
d ≈ 0.9135 **e** ≈ 0.9063 **f** ≈ -0.6691

13 **a**

θ^c	$\sin\theta$	$\sin(-\theta)$	$\cos\theta$	$\cos(-\theta)$
0.75	≈ 0.682	≈ -0.682	≈ 0.732	≈ 0.732
1.772	≈ 0.980	≈ -0.980	≈ -0.200	≈ -0.200
3.414	≈ -0.269	≈ 0.269	≈ -0.963	≈ -0.963
6.25	≈ -0.0332	≈ 0.0332	≈ 0.999	≈ 0.999
-1.17	≈ -0.921	≈ 0.921	≈ 0.390	≈ 0.390

b $\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = \cos\theta$

c Q has coordinates $(\cos(-\theta), \sin(-\theta))$ or $(\cos\theta, -\sin\theta)$ (since it is the reflection of P in the x-axis)
$\therefore$ $\cos(-\theta) = \cos\theta$ and $\sin(-\theta) = -\sin\theta$

d $\cos(2\pi - \theta) = \cos(-\theta) = \cos\theta$
$\sin(2\pi - \theta) = \sin(-\theta) = -\sin\theta$

e $\tan(2\pi - \theta) = -\tan\theta$

14 **a** The angle between [OP] and the positive x-axis is $\left(\frac{\pi}{2} - \theta\right)$.
$\therefore$ P is $\left(\cos\left(\frac{\pi}{2} - \theta\right), \sin\left(\frac{\pi}{2} - \theta\right)\right)$

b **i** In $\triangle$OXP, $\sin\theta = \frac{XP}{OP} = \frac{XP}{1}$
$\therefore$ $XP = \sin\theta$

ii In $\triangle$OXP, $\cos\theta = \frac{OX}{OP} = \frac{OX}{1}$
$\therefore$ $OX = \cos\theta$

c **i** $\cos\left(\frac{\pi}{2} - \theta\right) = XP = \sin\theta$

ii $\sin\left(\frac{\pi}{2} - \theta\right) = OX = \cos\theta$

d **i** $\cos\frac{\pi}{5} = \sin\left(\frac{\pi}{2} - \frac{\pi}{5}\right) = \sin\frac{3\pi}{10} \approx 0.809$

ii $\sin\frac{\pi}{8} = \cos\left(\frac{\pi}{2} - \frac{\pi}{8}\right) = \cos\frac{3\pi}{8} \approx 0.383$

e $\tan\left(\frac{\pi}{2} - \theta\right) = \frac{1}{\tan\theta}$

EXERCISE 8D

1

	a	**b**	**c**	**d**	**e**
$\sin\theta$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$
$\cos\theta$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	-1	$-\frac{1}{\sqrt{2}}$
$\tan\theta$	1	-1	-1	0	1

2

	a	**b**	**c**	**d**	**e**
$\sin\beta$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
$\cos\beta$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\tan\beta$	$\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	$\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$

3 **a** $\cos\frac{2\pi}{3} = -\frac{1}{2}$, $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$, $\tan\frac{2\pi}{3} = -\sqrt{3}$

b $\cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$, $\sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$, $\tan\left(-\frac{\pi}{4}\right) = -1$

4 **a** $\cos\frac{\pi}{2} = 0$, $\sin\frac{\pi}{2} = 1$ **b** $\tan\frac{\pi}{2}$ is undefined

5 **a** $\frac{3}{4}$ **b** $\frac{1}{4}$ **c** $\frac{1}{4}$ **d** $-\frac{1}{4}$ **e** 1 **f** $\sqrt{2}$
g $\frac{1}{2}$ **h** $\frac{1}{2}$ **i** 2 **j** -1 **k** $-\sqrt{3}$ **l** $-\sqrt{3}$

6 **a** $\frac{\pi}{6}, \frac{5\pi}{6}$ **b** $\frac{\pi}{3}, \frac{2\pi}{3}$ **c** $\frac{\pi}{4}, \frac{7\pi}{4}$ **d** $\frac{2\pi}{3}, \frac{4\pi}{3}$
e $\frac{3\pi}{4}, \frac{5\pi}{4}$ **f** $\frac{4\pi}{3}, \frac{5\pi}{3}$

7 **a** $\frac{\pi}{4}, \frac{5\pi}{4}$ **b** $\frac{3\pi}{4}, \frac{7\pi}{4}$ **c** $\frac{\pi}{3}, \frac{4\pi}{3}$ **d** $0, \pi, 2\pi$
e $\frac{\pi}{6}, \frac{7\pi}{6}$ **f** $\frac{2\pi}{3}, \frac{5\pi}{3}$

8 **a** $\frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}$ **b** $\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$ **c** $\frac{3\pi}{2}, \frac{7\pi}{2}$

9 **a** $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ **b** $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$ **c** $\theta = \pi$
d $\theta = \frac{\pi}{2}$ **e** $\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$ **f** $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$
g $\theta = 0, \pi, 2\pi$ **h** $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
i $\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$ **j** $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

10 **a** $\theta = k\pi$, $k \in \mathbb{Z}$ **b** $\theta = \frac{\pi}{2} + k\pi$, $k \in \mathbb{Z}$

EXERCISE 8E

1 **a** $\cos\theta = \pm\frac{\sqrt{3}}{2}$ **b** $\cos\theta = \pm\frac{2\sqrt{2}}{3}$ **c** $\cos\theta = \pm 1$
d $\cos\theta = 0$

2 **a** $\sin\theta = \pm\frac{3}{5}$ **b** $\sin\theta = \pm\frac{\sqrt{7}}{4}$ **c** $\sin\theta = 0$
d $\sin\theta = \pm 1$

3 **a** $\sin\theta = \frac{\sqrt{5}}{3}$ **b** $\cos\theta = -\frac{\sqrt{21}}{5}$ **c** $\cos\theta = \frac{4}{5}$
d $\sin\theta = -\frac{12}{13}$

4 **a** $\tan\theta = -\frac{1}{2\sqrt{2}}$ **b** $\tan\theta = -2\sqrt{6}$ **c** $\tan\theta = \frac{1}{\sqrt{2}}$
d $\tan\theta = -\frac{\sqrt{7}}{3}$

5 **a** $\sin\theta = \frac{2}{\sqrt{13}}$, $\cos\theta = \frac{3}{\sqrt{13}}$ **b** $\sin\theta = \frac{4}{5}$, $\cos\theta = -\frac{3}{5}$
c $\sin\theta = -\sqrt{\frac{5}{14}}$, $\cos\theta = -\frac{3}{\sqrt{14}}$
d $\sin\theta = -\frac{12}{13}$, $\cos\theta = \frac{5}{13}$

6 $\sin\theta = \dfrac{-k}{\sqrt{k^2+1}}$, $\cos\theta = \dfrac{-1}{\sqrt{k^2+1}}$

EXERCISE 8F

1 **a** $\theta \approx 76.0°$ or $256°$ **b** $\theta \approx 33.9°$ or $326.1°$
c $\theta \approx 36.9°$ or $143.1°$ **d** $\theta = 90°$ or $270°$
e $\theta \approx 81.5°$ or $261.5°$ **f** $\theta \approx 83.2°$ or $276.8°$

2 **a** $\theta \approx 0.322$ or 3.46 **b** $\theta \approx 1.13$ or 5.16
c $\theta \approx 0.656$ or 2.49 **d** $\theta \approx 1.32$ or 4.97
e $\theta \approx 0.114$ or 3.26 **f** $\theta \approx 0.167$ or 2.97

3 **a** $\theta \approx 1.82$ or 4.46 **b** $\theta = 0, \pi$, or 2π
c $\theta \approx 1.88$ or 5.02 **d** $\theta \approx 3.58$ or 5.85
e $\theta \approx 0.876$ or 4.02 **f** $\theta \approx 0.674$ or 5.61
g $\theta \approx 0.0910$ or 3.05 **h** $\theta \approx 2.19$ or 4.10

4 **a** $\theta \approx -95.7°$ or $95.7°$ **b** $\theta \approx 53.1°$ or $126.9°$
c $\theta \approx -56.3°$ or $123.7°$ **d** $\theta \approx -36.9°$ or $36.9°$
e $\theta \approx -39.8°$ or $140.2°$ **f** $\theta \approx -140.5°$ or $-39.5°$

5 **a** $\theta \approx 1.27$ or 5.02
b For $\theta \approx 1.27$: $\sin\theta = \frac{\sqrt{91}}{10}$, $\tan\theta = \frac{\sqrt{91}}{3}$
For $\theta \approx 5.02$: $\sin\theta = -\frac{\sqrt{91}}{10}$, $\tan\theta = -\frac{\sqrt{91}}{3}$

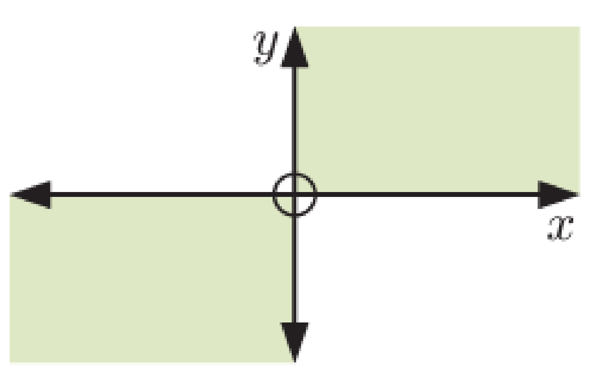
EXERCISE 8G

1 **a** $y = \sqrt{3}x$ **b** $y = x$ **c** $y = -\frac{1}{\sqrt{3}}x$
2 **a** $y = \sqrt{3}x + 2$ **b** $y = -\sqrt{3}x$ **c** $y = \frac{1}{\sqrt{3}}x - 2$
3 **a** $\theta \approx 1.25$ **b** $\theta \approx -0.983$ **c** $\theta \approx -0.381$
4 **a** $\theta \approx 23.2°$ **b** $\theta \approx 117°$ **c** $\theta \approx -11.3°$

REVIEW SET 8A

1 **a** $\frac{2\pi}{3}$ **b** $\frac{5\pi}{4}$ **c** $\frac{5\pi}{6}$ **d** 3π

2



3 **a** $(0.766, -0.643)$ **b** $(-0.956, 0.292)$
c $(0.778, 0.629)$ **d** $(0.866, -0.5)$

4 12 cm **5** **a** $\frac{\pi}{3}$ **b** $15°$ **c** $84°$

6 **a** ≈ 0.358 **b** ≈ -0.035 **c** ≈ 0.259 **d** ≈ 1.072

7 **a** $\cos 360° = 1$, $\sin 360° = 0$
b $\cos(-\pi) = -1$, $\sin(-\pi) = 0$

8 **a** $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$, $\cos\frac{2\pi}{3} = -\frac{1}{2}$, $\tan\frac{2\pi}{3} = -\sqrt{3}$
b $\sin\frac{8\pi}{3} = \frac{\sqrt{3}}{2}$, $\cos\frac{8\pi}{3} = -\frac{1}{2}$, $\tan\frac{8\pi}{3} = -\sqrt{3}$

9 **a** **i** $60°$ **ii** $\frac{\pi}{3}$ **b** $\frac{\pi}{3}$ cm **c** $\frac{\pi}{6}$ cm^2

10 $\tan x = \frac{1}{\sqrt{15}}$ **11** $\sin\theta = \pm\frac{\sqrt{7}}{4}$

12 **a** $\frac{\sqrt{3}}{2}$ **b** 0 **c** $\frac{1}{2}$ **13** **a** $\frac{2}{\sqrt{13}}$ **b** $-\frac{3}{\sqrt{13}}$

14 $\tan\theta = \frac{\sqrt{6}}{\sqrt{11}}$

16 **a** $\theta \approx 0.841$ or 5.44 **b** $\theta \approx 3.39$ or 6.03
c $\theta \approx 1.25$ or 4.39

17 **a** $y = \frac{1}{\sqrt{3}}x$ **b** $y = \sqrt{3}x + 3$

REVIEW SET 8B

1 **a** $72°$ **b** $\approx 83.65°$ **c** $\approx 24.92°$ **d** $\approx -302.01°$

2 ≈ 111 cm^2 **3** $\approx 103°$

4 radius ≈ 8.79 cm, area ≈ 81.0 cm^2 **5** 4.5 cm or 6 cm

6 **a** $\cos\frac{3\pi}{2} = 0$, $\sin\frac{3\pi}{2} = -1$
b $\cos\left(-\frac{\pi}{2}\right) = 0$, $\sin\left(-\frac{\pi}{2}\right) = -1$

7 **a** $\sin(\pi - p) = m$ **b** $\sin(p + 2\pi) = m$
c $\cos p = \sqrt{1 - m^2}$ **d** $\tan p = \dfrac{m}{\sqrt{1-m^2}}$

8 **a** $150°, 210°$ **b** $45°, 135°$ **c** $120°, 300°$

9 **a** $\theta = \pi$ **b** $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

10 **a** $133°$ **b** $\frac{14\pi}{15}$ **c** $174°$

11 perimeter ≈ 34.1 cm, area ≈ 66.5 cm^2

13 **a** $\frac{\sqrt{7}}{4}$ **b** $-\frac{\sqrt{7}}{3}$ **c** $\frac{3}{4}$

14 **a** $2\frac{1}{2}$ **b** $1\frac{1}{2}$ **c** $-\frac{1}{2}$ **d** 3

17 **a** $\theta \approx 0.322$ **b** $\theta \approx 1.95$

EXERCISE 9A

1 **a** ≈ 28.9 cm^2 **b** ≈ 384 km^2 **c** 20 m^2
2 **a** ≈ 18.7 cm^2 **b** ≈ 28.3 cm^2 **c** ≈ 52.0 m^2
3 $x \approx 19.0$ **4** **a** ≈ 166 cm^2 **b** ≈ 1410 cm^2
5 ≈ 18.9 cm^2 **6** ≈ 137 cm^2
7 **a** ≈ 71.616 m^2 **b** ≈ 8.43 m
8 ≈ 374 cm^2 **9** ≈ 7.49 cm **10** ≈ 11.9 m
11 **a** $\approx 48.6°$ or $\approx 131.4°$ **b** $\approx 42.1°$ or $\approx 137.9°$
12 $\frac{1}{4}$ is not covered
13 **a** ≈ 36.2 cm^2 **b** ≈ 62.8 cm^2 **c** ≈ 40.4 mm^2
d ≈ 19.3 cm^2
14 ≈ 4.69 cm^2

EXERCISE 9B

1 **a** ≈ 28.8 cm **b** ≈ 3.38 km **c** ≈ 14.2 m
2 **a** $\theta \approx 82.8°$ **b** $\theta \approx 54.8°$ **c** $\theta \approx 98.2°$
3 $B\widehat{A}C \approx 52.0°$, $A\widehat{B}C \approx 59.3°$, $A\widehat{C}B \approx 68.7°$
4 **a** $\approx 112°$ **b** ≈ 16.2 cm^2
5 **a** $\approx 40.3°$ **b** $\approx 107°$
6 **a** $\cos\theta = 0.65$ **b** $x \approx 3.81$
7 **a** $\theta \approx 75.2°$ **b** ≈ 6.30 m
8 **a** DB ≈ 4.09 m, BC ≈ 9.86 m
b $A\widehat{B}E \approx 68.2°$, $D\widehat{B}C \approx 57.5°$ **c** ≈ 17.0 m^2
9 **b** $x = 3 + \sqrt{22}$
10 **a** $x \approx 10.8$ **b** $x \approx 2.77$ **c** $x \approx 2.89$
11 $x \approx 1.41$ or 7.78 **12** BD ≈ 12.4 cm
13 $\theta \approx 71.6°$ **14** ≈ 6.40 cm
15 **a** $x = 2$ **b** $4\sqrt{6}$ cm^2 **16** $\approx 63°, 117°, 36°, 144°$

EXERCISE 9C.1

1 **a** $x \approx 28.4$ **b** $x \approx 13.4$ **c** $x \approx 3.79$
d $x \approx 10.3$ **e** $x \approx 4.49$ **f** $x \approx 7.07$

2 **a** $a \approx 21.3$ cm **b** $b \approx 76.9$ cm **c** $c \approx 5.09$ cm

3 **a** $\widehat{BAC} = 74°$, $AB \approx 7.99$ cm, $BC \approx 9.05$ cm

b $\widehat{XZY} = 108°$, $XZ \approx 13.5$ cm, $XY \approx 26.5$ cm

4 $x \approx 17.7$, $y \approx 33.1$ **5** $x = 11 + \frac{11}{2}\sqrt{2}$

EXERCISE 9C.2

1 **a** $x \approx 9.85$ **b** $x \approx 41.3$ **c** $x \approx 52.8$

2 $C \approx 62.1°$ or $C \approx 117.9°$

3 **a** $\widehat{BAC} \approx 30.9°$ **b** $\widehat{ABC} \approx 28.7°$

c $\widehat{ACB} \approx 30.1°$ **d** $\widehat{BAC} \approx 46.6°$

e $\widehat{ABC} \approx 55.5°$ or $124.5°$ **f** $\widehat{ACB} \approx 25.4°$ or $154.6°$

4 **a** We find that $\sin x \approx 1.04$ which has no solutions.

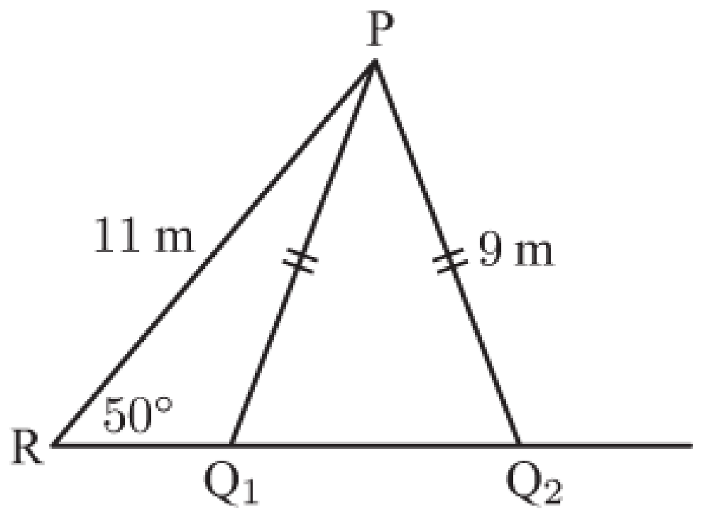
b The triangle cannot be drawn with the given dimensions.

5 **a** **i** $\widehat{ACB} \approx 22.9°$ **ii** $\widehat{BAC} \approx 127.1°$ **b** ≈ 25.1 cm^2

6 No, the angle opposite the 9.8 cm side has a sine of 1.05, which is impossible.

7 **a** $\approx 69.4°$ or $\approx 110.6°$

b



c For $\widehat{PQR} \approx 69.4°$:

i $\approx 60.6°$

ii ≈ 43.1 m^2

iii ≈ 30.2 m

For $\widehat{PQR} \approx 110.6°$:

i $\approx 19.4°$

ii ≈ 16.5 m^2

iii ≈ 23.9 m

EXERCISE 9D

1 ≈ 17.7 m **2** ≈ 207 m **3** ≈ 10.1 km

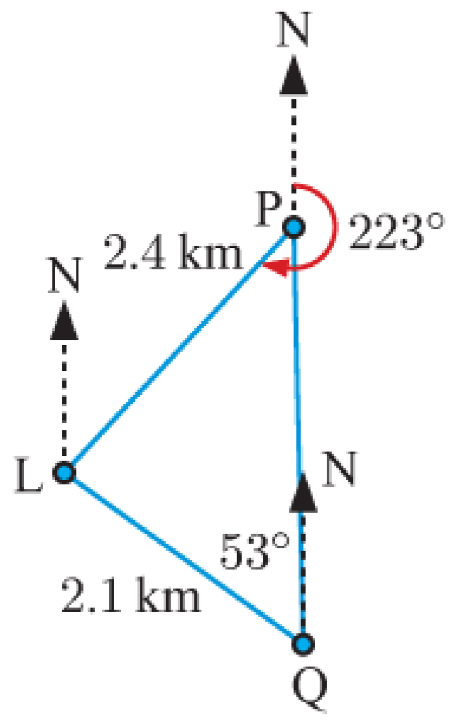
4 $\approx 23.9°$ **5** ≈ 37.6 km

6 **a** ≈ 5.63 km **b** on the bearing $\approx 115°$

c **i** Esko **ii** ≈ 7.37 min (≈ 7 min 22 s) **d** $\approx 295°$

7 $\approx 9.38°$ **8** ≈ 69.1 m **9** **a** ≈ 38.0 m **b** ≈ 94.0 m

10 **a**



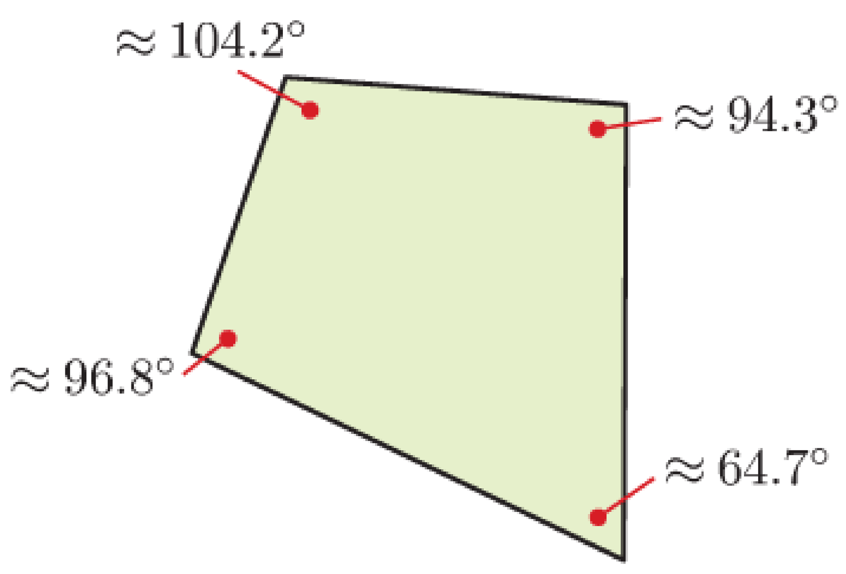
b ≈ 2.98 km

c $\approx 179°$

11 **a** $\approx 55.1°$ **b** $\approx 50.3°$ **12** $\approx 65.6°$ **13** ≈ 9.12 km

14 **a** ≈ 74.9 km^2 **b** ≈ 7490 ha **15** ≈ 85.0 mm

16



Area $\approx 13\,100$ m^2

17 ≈ 7400 m^2 **18** ≈ 2.52 km^2 **19** $\approx 32.2°$ and $\approx 87.8°$

20 ≈ 29.2 m **21** **a** ≈ 3.97 km **b** ≈ 1.13 km

22 **b** ≈ 1.467 cm

REVIEW SET 9A

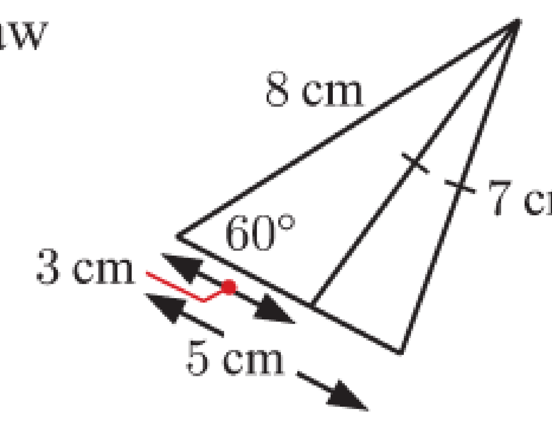
1 **a** ≈ 26.8 cm^2 **b** 14 km^2 **c** ≈ 33.0 m^2

2 ≈ 22.7 cm^2 **3** **a** ≈ 10.5 cm **b** ≈ 11.6 m

4 **a** $x \approx 9.24$ **b** $\theta \approx 59.2°$ **c** $x \approx 6.28$

5 ≈ 113 cm^2 **6** ≈ 51.6 cm^2

7 **a** $x = 3$ or 5 **b** Kady can draw 2 triangles:



8 **b** $x \approx 19.3$ **c** ≈ 43.3 cm

9 ≈ 7.21 cm, ≈ 11.2 cm, ≈ 12.5 cm

10 $\approx 74.4°$ **11** $x \approx 18.5$, $y \approx 13.8$ **12** 42 km

13 **a** $\approx 69.5°$ or $\approx 110.5°$

b For $\widehat{ABC} \approx 69.5°$, area ≈ 16.3 cm^2.

For $\widehat{ABC} \approx 110.5°$, area ≈ 8.09 cm^2.

14 ≈ 577 m

REVIEW SET 9B

1 **a** $x \approx 34.1$ **b** $x \approx 18.9$ **2** $\approx 47.5°$ or $132.5°$

3 **a** $\theta \approx 29.9°$ **b** $\theta \approx 103°$

4 **a** $AC \approx 12.6$ cm, $\widehat{BAC} \approx 48.6°$, $\widehat{ACB} \approx 57.4°$

b $\widehat{PRQ} = 51°$, $PQ \approx 7.83$ cm, $QR \approx 7.25$ cm

c $\widehat{YXZ} \approx 78.3°$, $\widehat{XYZ} \approx 55.5°$, $\widehat{XZY} \approx 46.2°$

5 **a** $x \approx 6.93$ **b** $x \approx 11.4$ **c** $x \approx 7.16$ **d** $x \approx 34.7$

6 ≈ 17.7 m **7** ≈ 7.32 m

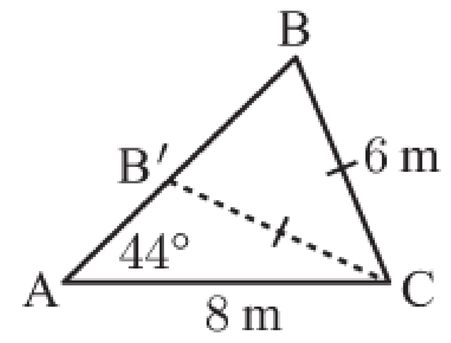
8 perimeter ≈ 578 m, area $\approx 15\,000$ m^2

9 ≈ 560 m on the bearing $\approx 079.7°$

10 $\widehat{BAD} \approx 90.5°$, $\widehat{BCD} \approx 94.3°$, $\widehat{ADC} \approx 70.2°$

11 $Q \approx 39.7°$ **12** **a** $\approx 10\,600$ m^2 **b** ≈ 1.06 ha

13 **a** The information given could give two triangles:



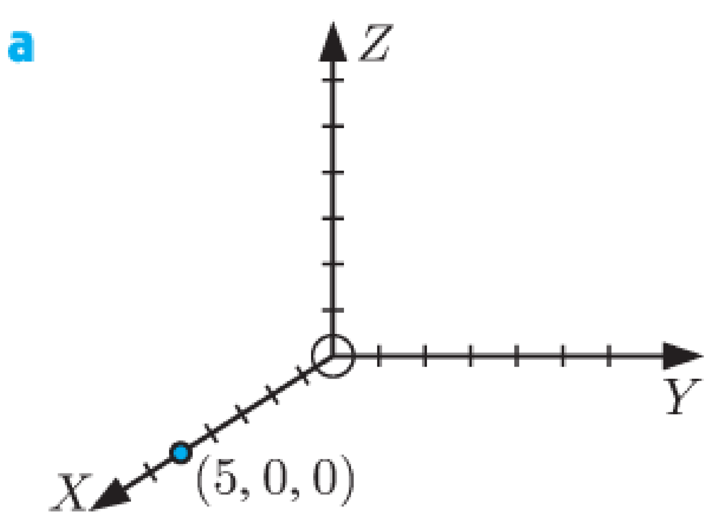
b ≈ 2.23 m^3

14 **a** **i** ≈ 20.8 mm **ii** ≈ 374 mm^2 **b** ≈ 2270 mm^3

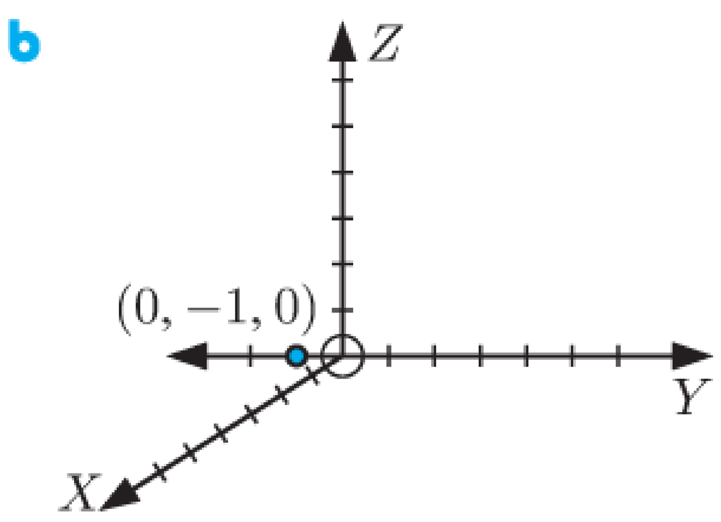
15 **a** **Hint:** Let $\widehat{BAC} = \theta$, so $A = \frac{1}{2}bc\sin\theta$.

EXERCISE 10A

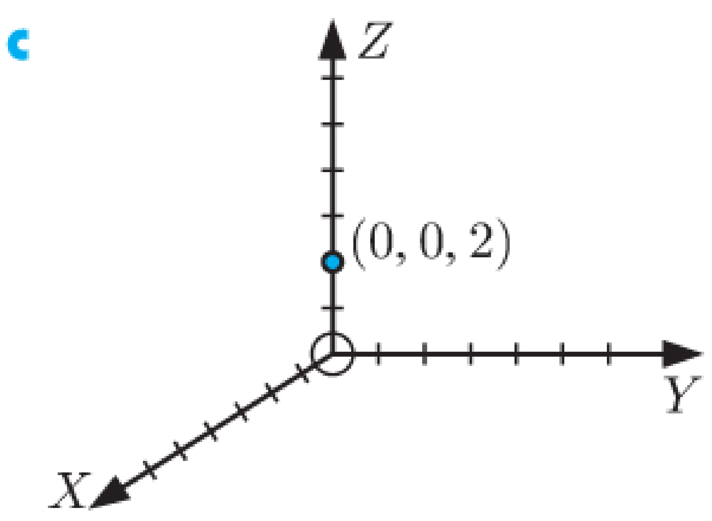
1 **a**



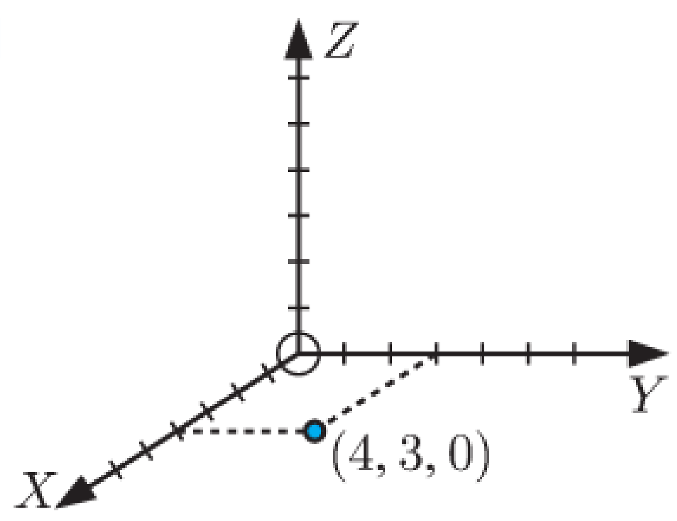
b



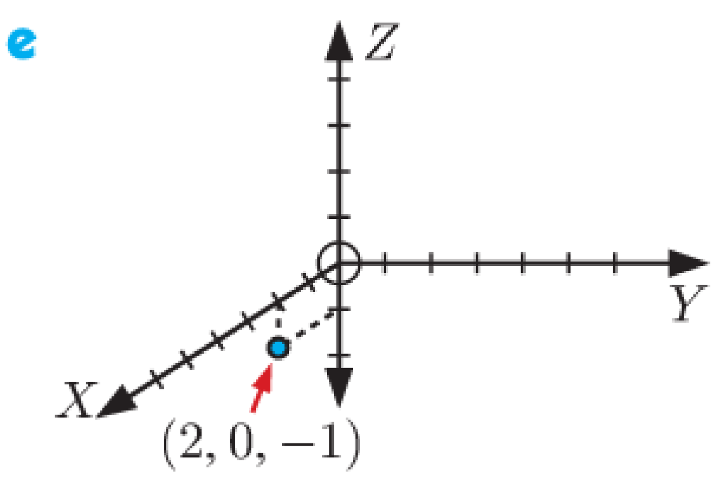
c



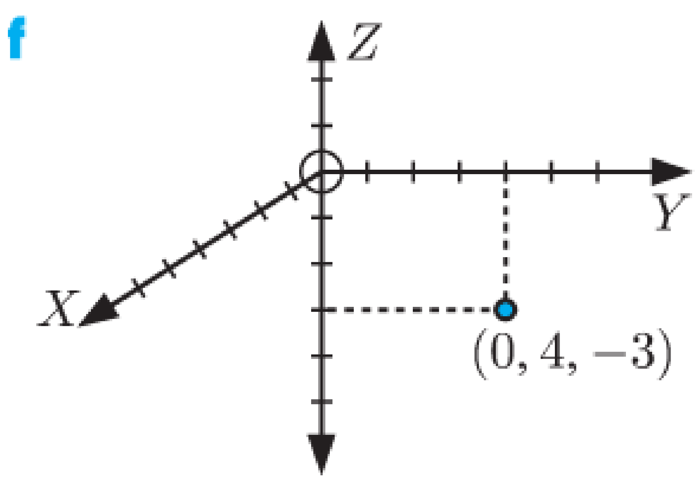
d



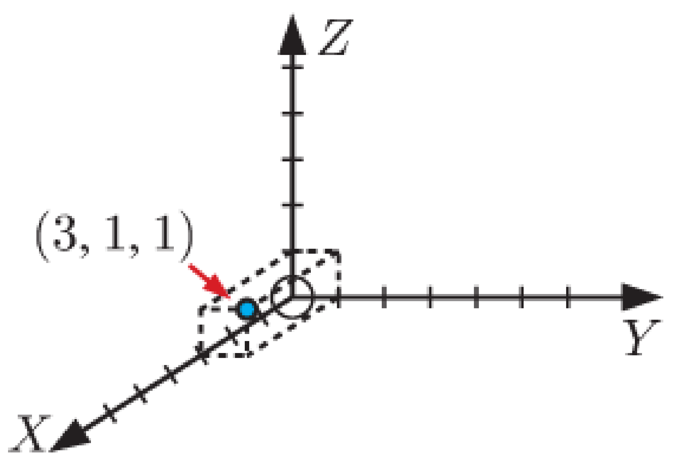
e



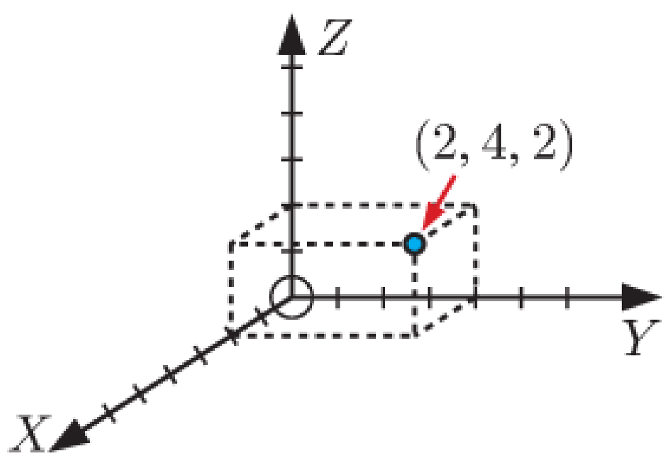
f



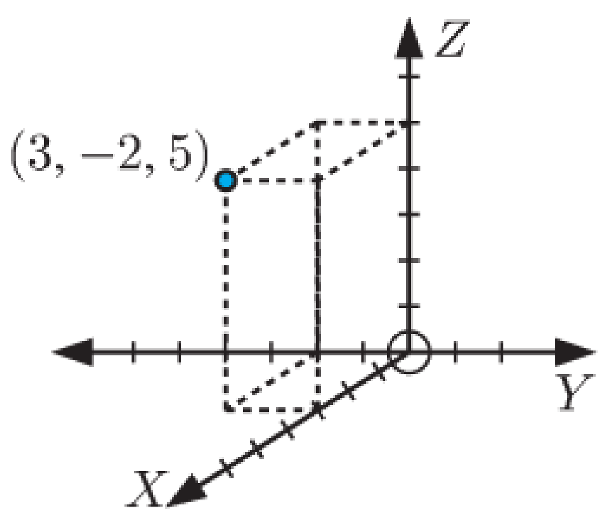
g



h



i



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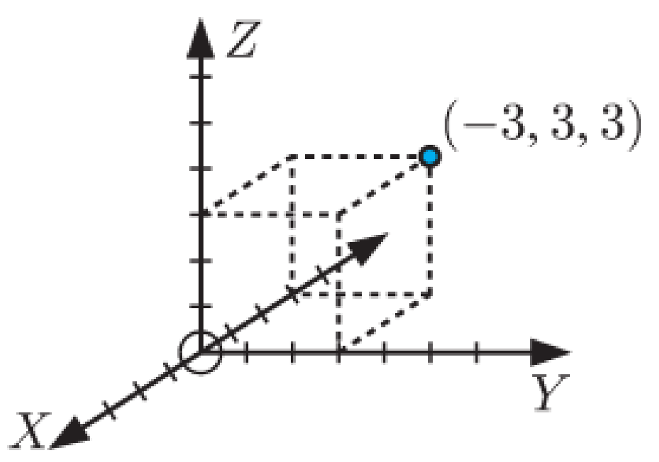
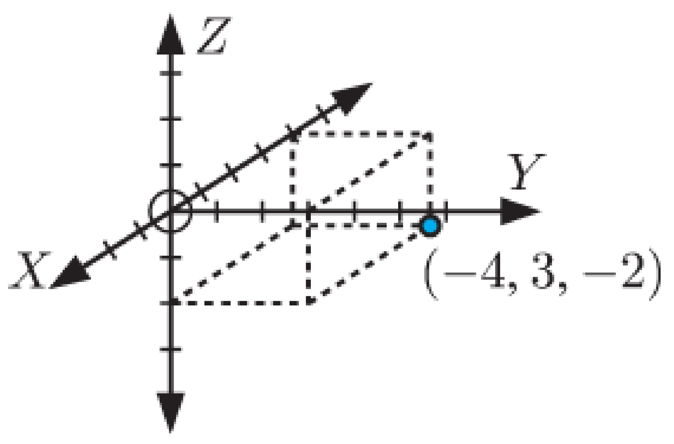
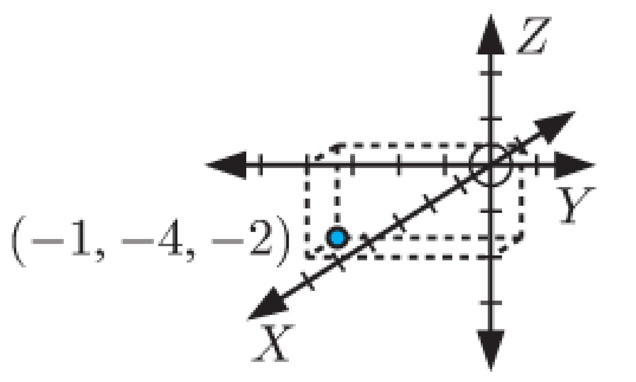


k



l



2 **a** **i** $2\sqrt{14}$ units **ii** $(3, -2, 1)$
b **i** $2\sqrt{5}$ units **ii** $(2, 1, -1)$
c **i** $2\sqrt{6}$ units **ii** $(3, -2, 1)$
d **i** $\sqrt{69}$ units **ii** $(-4, \frac{7}{2}, 4)$
e **i** $5\sqrt{2}$ units **ii** $(\frac{3}{2}, 3, \frac{1}{2})$
f **i** $\sqrt{83}$ units **ii** $(-\frac{3}{2}, \frac{9}{2}, -\frac{1}{2})$

3 **a** isosceles with AB = AC = $\sqrt{101}$ units **b** scalene

4 AB = $\sqrt{342}$ units, AC = $\sqrt{72}$ units, BC = $\sqrt{414}$ units
$AB^2 + AC^2 = (\sqrt{342})^2 + (\sqrt{72})^2 = 414 = BC^2$
$\therefore$ triangle ABC is right angled.

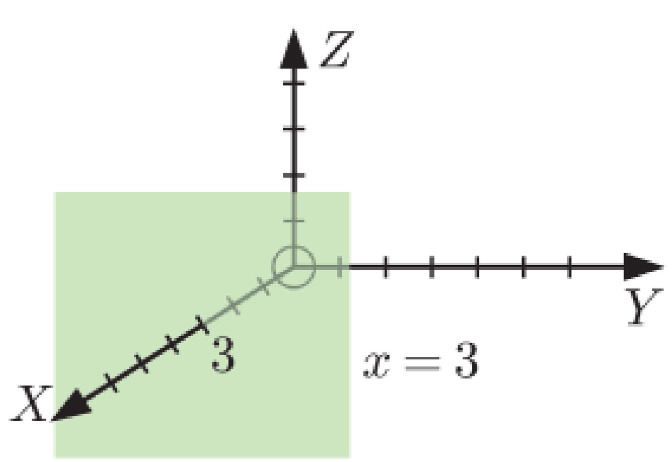
5 **a** $M(\frac{7}{2}, -2, 3)$, $N(\frac{1}{2}, -5, -1)$
b PR = $2\sqrt{34}$ units, MN = $\sqrt{34}$ units = $\frac{1}{2}$ PR

6 B(−7, 11, 3) **7** P(−2, 5, 0), Q(4, −1, 3), R(1, 7, 6)

8 $k = 2 \pm \sqrt{23}$

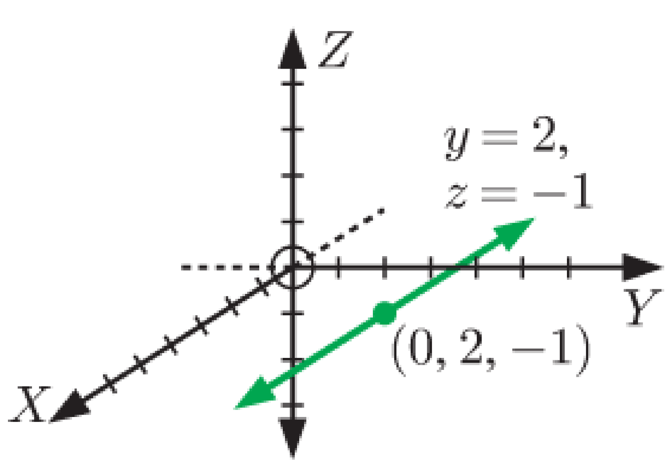
9 **a** $x^2 + y^2 + z^2 = 9$, P lies on a sphere with centre (0, 0, 0) and radius 3 units.
b $(x-2)^2 + (y-5)^2 + (z-4)^2 = 1$, P lies on a sphere with centre (2, 5, 4) and radius 1 unit.

10 **a**



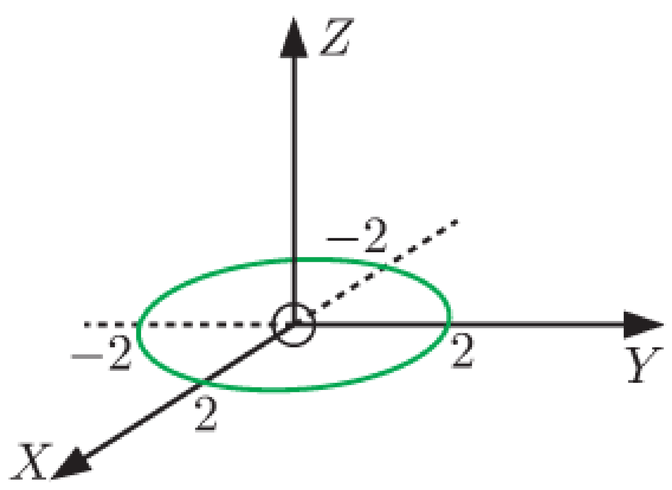
A plane parallel to the YOZ plane, passing through (3, 0, 0).

b



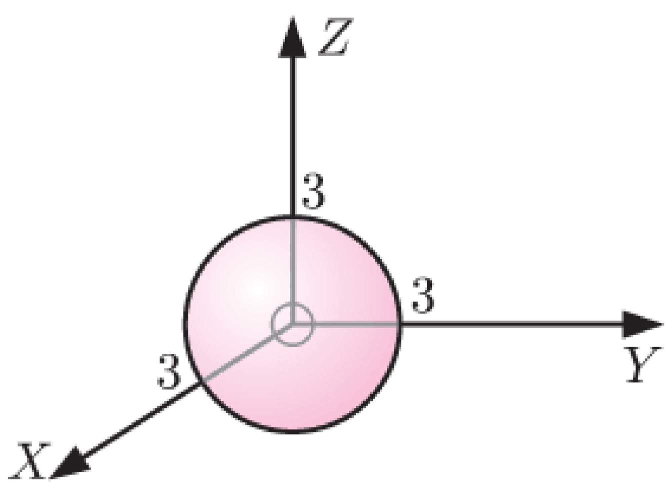
A line parallel to the X-axis, passing through (0, 2, −1).

c



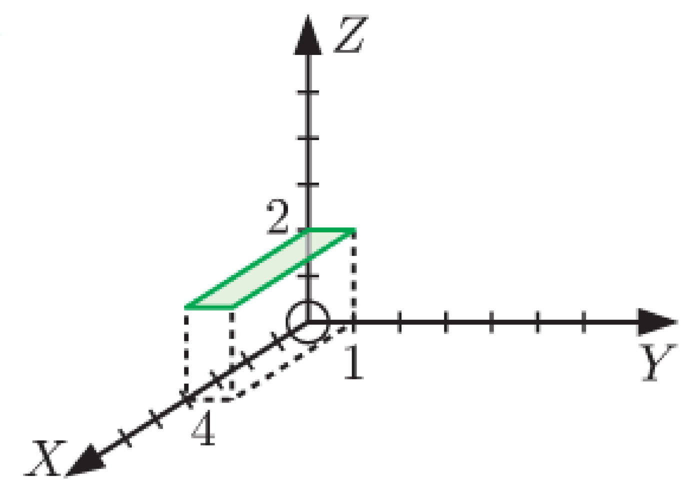
A circle in the XOY plane, centre (0, 0, 0), radius 2 units.

d



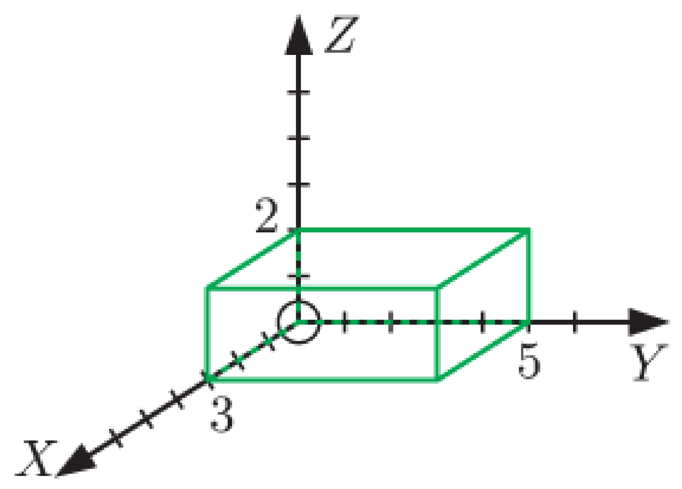
A sphere, centre (0, 0, 0), radius 3 units.

e



A 4 by 1 rectangular plane 2 units above the XOY plane (as shown).

f



All points on and within a $3 \times 5 \times 2$ rectangular prism (as shown).

EXERCISE 10B

1 **a** 27 units^3 **b** 60 units^3 **c** 40 units^3

2 **a** D(−7, 0, 3), E(−7, 4, 0) **b** 42 units^3
c 5 units **d** 96 units^2

3 **a** The centre of the base is (3, 3, 0) which is directly below the apex.
b 108 units^3
c **i** M(6, 3, 0) **ii** $3\sqrt{10}$ units **iii** $36(1+\sqrt{10})$ units^2

4 volume = 720 units^3, surface area = 564 units^2

5 **a** $\sqrt{41}$ units **b** 82π units^3 **c** $\sqrt{77}$ units
d ≈ 305 units^2

6 **a** $\sqrt{238}$ units **b** $\approx 15\,400$ units^3

7 **a** (−3, 4, −3) **b** $\sqrt{38}$ units
c volume ≈ 981 units^3, surface area ≈ 478 units^2

8 **a** 4 units **b** $k = -3 \pm \sqrt{6}$ **c** ≈ 88.0 units^2

9 **a** D(−4, −2, 2) **b** 16.8 units **c** ≈ 190 units^3

10 **a** **i** A(10, 40, 0), B(50, 160, 0), C(110, 140, 0), D(70, 20, 0)
ii (60, 90, 15)
b 40 000 m^3 **c** ≈ 8540 m^2

11 2 461 200 m^3

EXERCISE 10C

1 **a** $\approx 50.2°$ **b** $\approx 48.1°$

2 **a** M(3, 3, 0) **b** $\approx 25.4°$ **c** $\approx 50.2°$

3 **a** M(4, 6, 0) **b** $\approx 66.5°$ **c** **i** $\approx 35.0°$ **ii** $\approx 44.1°$

4 **a** M(2, 4, 0) **b** **i** $\approx 68.2°$ **ii** $\approx 60.5°$

5 **a** M(−4, 3, 5) **b** **i** $\approx 30.3°$ **ii** 45° **c** $\approx 34.4°$

6 **a** **i** $3\sqrt{3}$ units **ii** 3 units **iii** $\sqrt{38}$ units **b** $\approx 29.1°$

7 **a** $\approx 67.3°$ **b** $\approx 22.3°$

8 **a** $\approx 53.6°$ **b** ≈ 10.8 units^2 **9** $k = -1$

10 **a** $M(\frac{5}{2}, 3, 5)$ **b** **i** $\approx 66.8°$ **ii** $\approx 128°$ **iii** $\approx 76.0°$

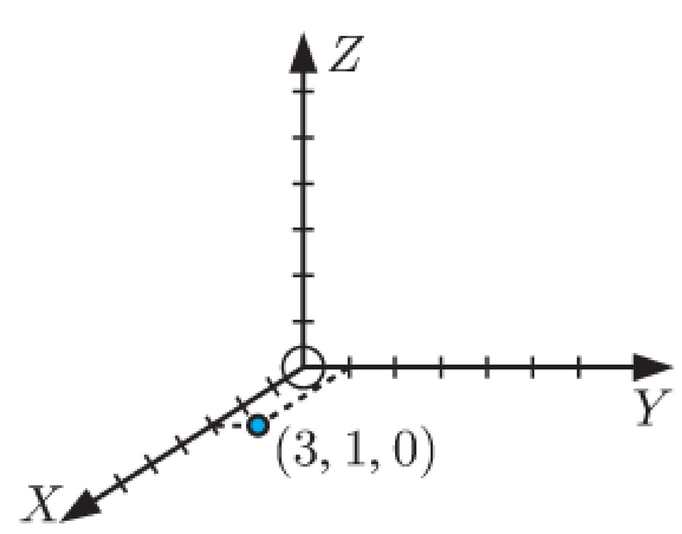
11 **a** The bird is at (30, 20, 10).
b **i** $10\sqrt{6}$ m ≈ 24.5 m **ii** $\approx 24.1°$

12 **a** $(-3, 4, \frac{1}{2})$ **b** $\frac{\sqrt{101}}{2} \approx 5.02$ km **c** 135° **d** $\approx 5.05°$

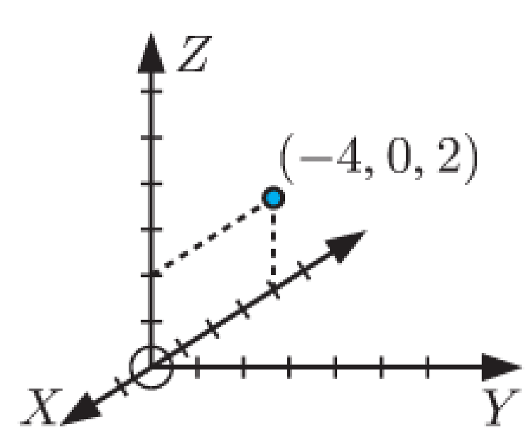
13 **a** $\frac{\sqrt{201}}{2} \approx 7.09$ km **b** Jack, Gabriel, Malina
c **i** $\approx 1.71°$ **ii** $\approx 7.64°$

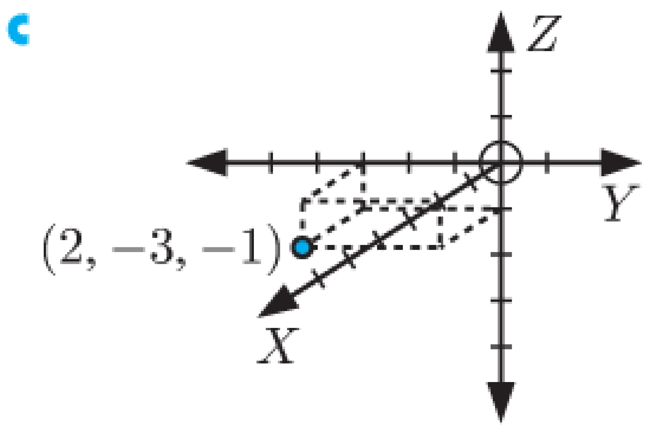
REVIEW SET 10A

1 **a**



b



c 

2 **a** **i** 6 units **ii** $(-1, -4, 1)$

b **i** $3\sqrt{10}$ units **ii** $(-\frac{5}{2}, -3, \frac{7}{2})$

3 isosceles with AB = AC = $\sqrt{41}$ units

4 **a** 128 units^3 **b** M(8, 4, 0) **c** $2\sqrt{13}$ units

d $32(2+\sqrt{13})$ units^2 ≈ 179 units^2 **e** $\approx 29.0°$

5 **a** $\sqrt{29}$ units

b volume ≈ 327 units^3, surface area ≈ 273 units^2

6 **a** M(5, 4, 3) **b** $\approx 64.9°$ **c** **i** $\approx 43.1°$ **ii** $\approx 25.1°$

7 $\approx 61.4°$ **8** **a** $k = 2$ **b** ≈ 68.9 units^2

9 **a** P(2, 7, −2.5), Q(8, 3, −2.9) **b** ≈ 7.22 m

c $\approx 3.17°$

REVIEW SET 10B

1 **a** **i** $\sqrt{41}$ units **ii** $(-2, 3, \frac{9}{2})$

b **i** $\sqrt{83}$ units **ii** $(-\frac{9}{2}, \frac{5}{2}, \frac{5}{2})$

2 **a** PQ = $\sqrt{14}$ units, PR = $\sqrt{45}$ units, QR = $\sqrt{59}$ units

$PQ^2 + PR^2 = (\sqrt{14})^2 + (\sqrt{45})^2 = 59 = QR^2$

∴ triangle PQR is right angled.

b $\approx 60.8°$

3 $k = 1 \pm \sqrt{30}$

4 **a** 96 units^3 **b** $2\sqrt{13}$ units

c $(104 + 16\sqrt{13}) \approx 162$ units^2

5 **a** $(-1, 0, -1)$ **b** $3\sqrt{5}$ units

c volume ≈ 1260 units^3, surface area ≈ 565 units^2

6 **a** $\approx 21.4°$ **b** ≈ 3.53 units^2

7 **a** M(6, 9, 5) **b** $\approx 71.6°$ **c** **i** $\approx 54.2°$ **ii** $\approx 36.7°$

8 **a** H$(2, -4, \frac{1}{5})$ **b** ≈ 4.48 km

c **i** M$(-4, 1, \frac{1}{2})$ **ii** ≈ 7.82 km **iii** $\approx 2.20°$

9 **a** R(0, 0, 3) **b** $\widehat{PRO} \approx 46.5°$, $\widehat{QRO} \approx 36.7°$

c $\widehat{PRQ} \approx 60.6°$

EXERCISE 11A

1 **a** ≈ 0.78 **b** ≈ 0.22

2 **a** ≈ 0.487 **b** ≈ 0.051 **c** ≈ 0.731

3 **a** 43 days **b** **i** ≈ 0.0465 **ii** ≈ 0.186 **iii** ≈ 0.465

4 **a** ≈ 0.0895 **b** ≈ 0.126

5 **a** ≈ 0.265 **b** ≈ 0.861 **c** ≈ 0.222

6 **a** ≈ 0.146 **b** ≈ 0.435 **c** ≈ 0.565

7 **a** **i** ≈ 0.171 **ii** ≈ 0.613 **b** ≈ 0.366 **c** ≈ 0.545

EXERCISE 11B

1 **a** 7510 **b** **i** ≈ 0.325 **ii** ≈ 0.653 **iii** ≈ 0.243

2 **a**

	Junior	*Middle*	*Senior*	*Total*
Sport	131	164	141	436
No sport	28	81	176	285
Total	159	245	317	721

b **i** $\frac{436}{721} \approx 0.605$ **ii** $\frac{131}{721} \approx 0.182$ **iii** $\frac{257}{721} \approx 0.356$

3 **a** **i** $\frac{743}{1235} \approx 0.602$ **ii** $\frac{148}{1235} \approx 0.120$ **iii** $\frac{1085}{1235} \approx 0.879$

iv $\frac{795}{1235} \approx 0.644$

b $\frac{52}{492} \approx 0.106$ **c** $\frac{518}{862} \approx 0.601$

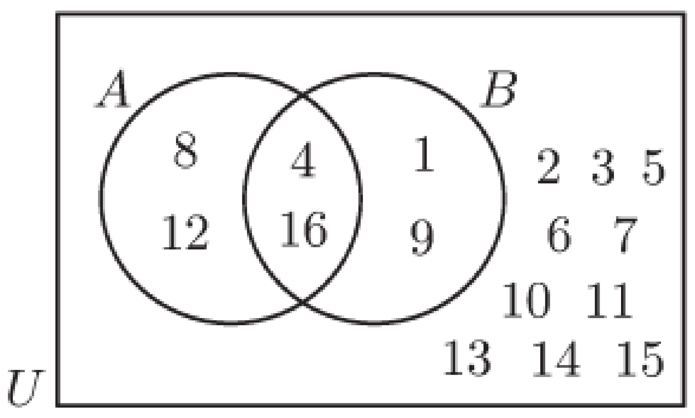
EXERCISE 11C

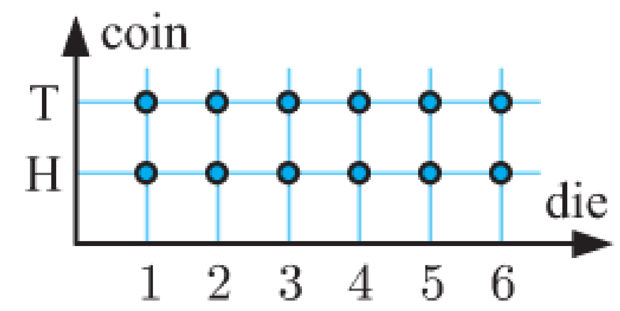
1 **a** {A, B, C, D} **b** {1, 2, 3, 4, 5, 6, 7, 8}

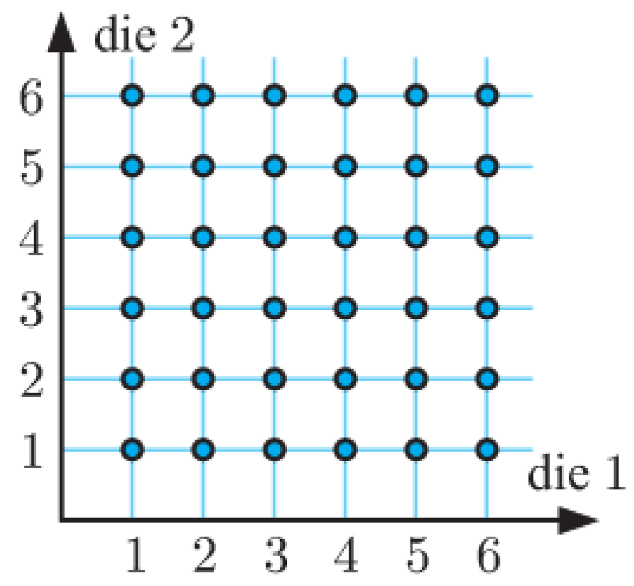
c {MM, MF, FM, FF}

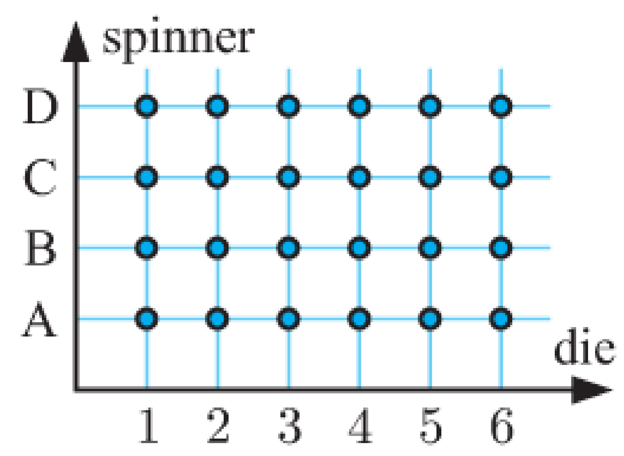
2 **a** $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$

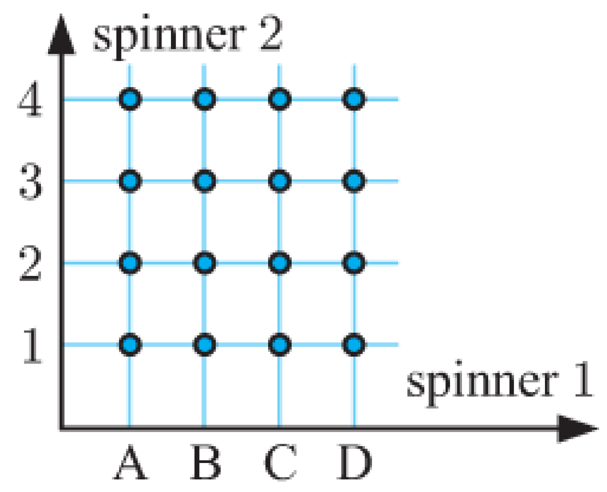
b **i** $A = \{4, 8, 12, 16\}$ **ii** $B = \{1, 4, 9, 16\}$

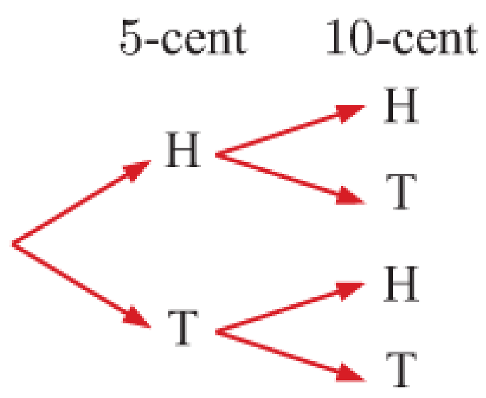
c 

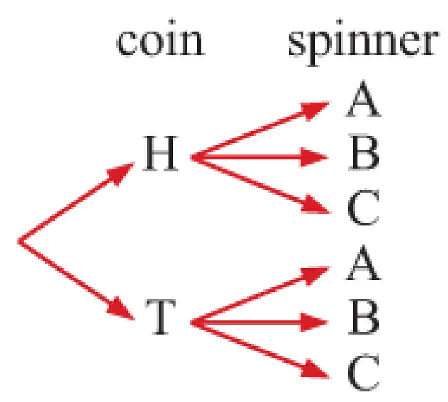
3 **a** 

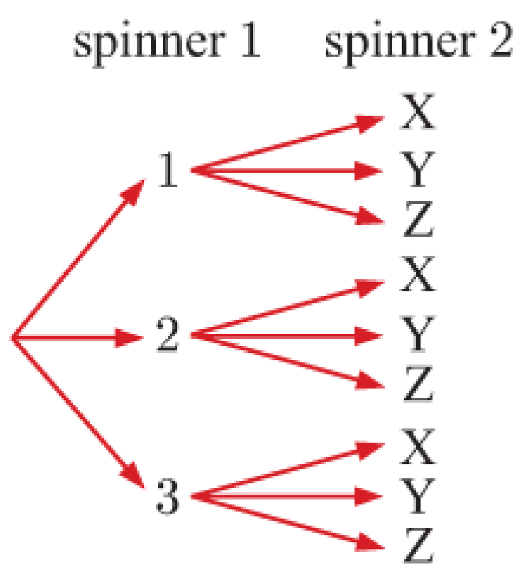
b 

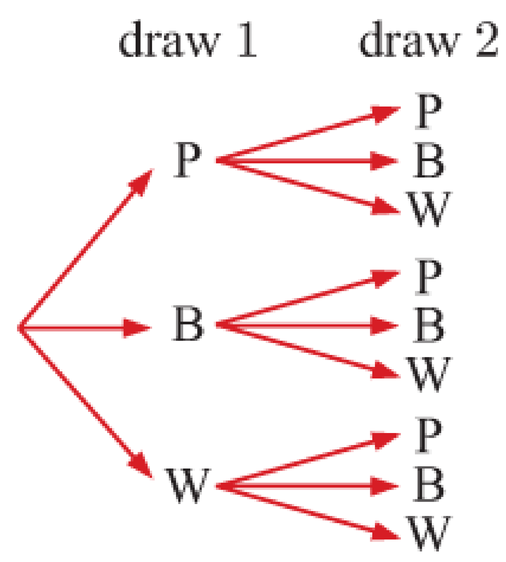
c 

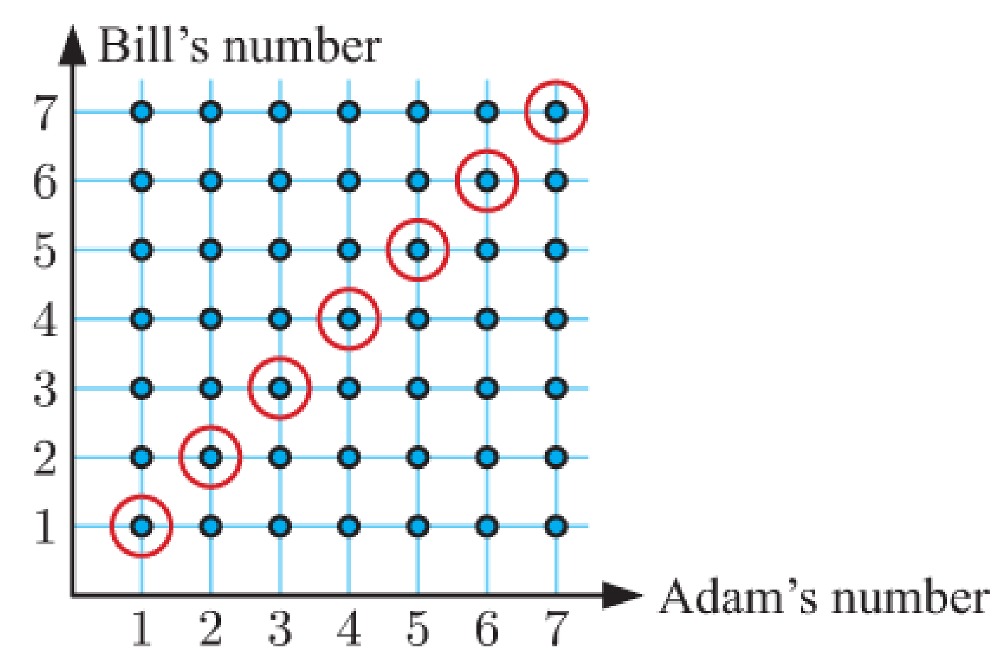
d 

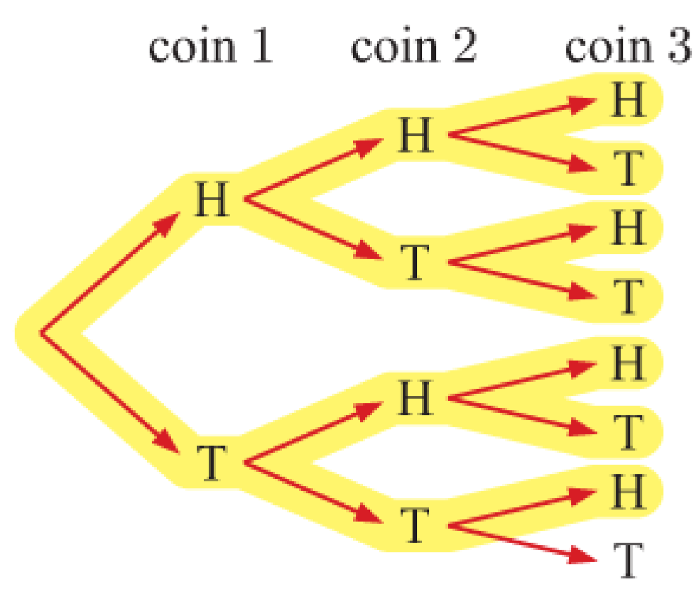
4 **a** 

b 

c 

d 

5 

6 

EXERCISE 11D

1 **a** $\frac{1}{5}$ **b** $\frac{1}{3}$ **c** $\frac{7}{15}$ **d** $\frac{4}{5}$ **e** $\frac{1}{5}$ **f** $\frac{8}{15}$

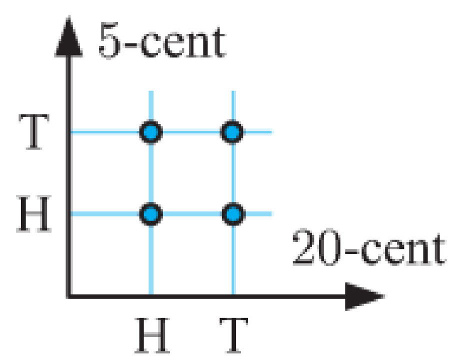
2 **a** $\frac{1}{2}$ **b** $\frac{2}{25}$ **c** $\frac{39}{200}$

3 **a** $\frac{1}{4}$ **b** $\frac{1}{9}$ **c** $\frac{4}{9}$ **d** $\frac{1}{18}$ **e** $\frac{1}{6}$ **f** $\frac{13}{36}$
g $\frac{1}{12}$ **h** $\frac{1}{3}$

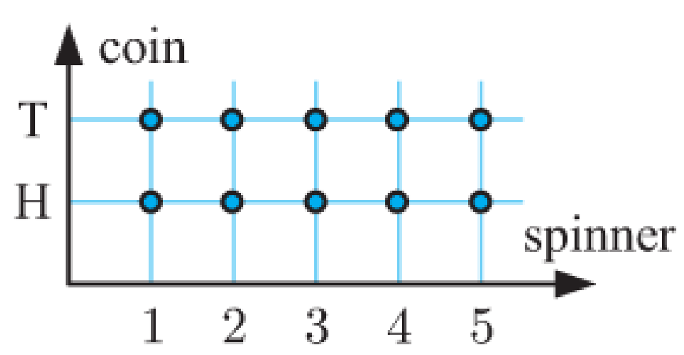
4 **a** $\frac{1}{7}$ **b** $\frac{2}{7}$ **c** $\frac{124}{1461}$ **d** $\frac{237}{1461}$
e $\frac{729}{1461}$ {remember leap years}

5 **a** {GGG, GGB, GBG, BGG, GBB, BGB, BBG, BBB}
b **i** $\frac{1}{8}$ **ii** $\frac{1}{8}$ **iii** $\frac{1}{8}$ **iv** $\frac{3}{8}$ **v** $\frac{1}{2}$ **vi** $\frac{7}{8}$

6 **a** {ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, BACD, BADC, BCAD, BCDA, BDAC, BDCA, CABD, CADB, CBAD, CBDA, CDAB, CDBA, DABC, DACB, DBAC, DBCA, DCAB, DCBA}
b **i** $\frac{1}{2}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{2}$ **iv** $\frac{1}{2}$

7 **a**


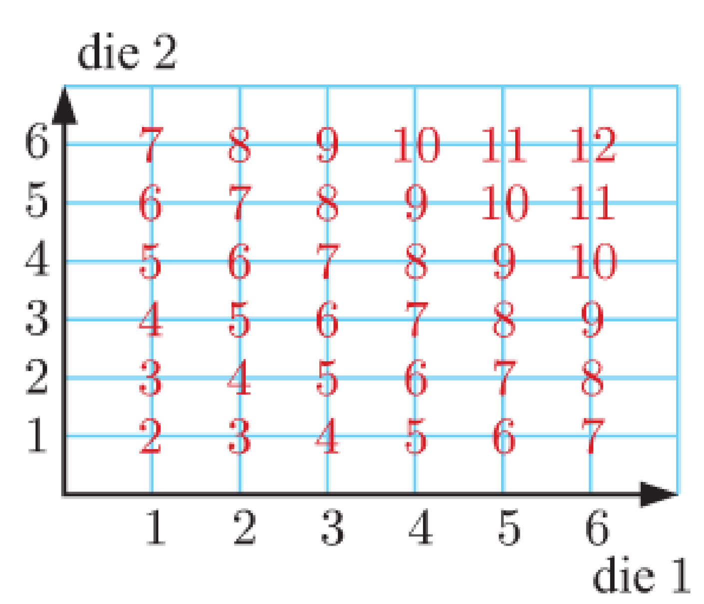
b **i** $\frac{1}{4}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{2}$ **iv** $\frac{3}{4}$

8 **a**


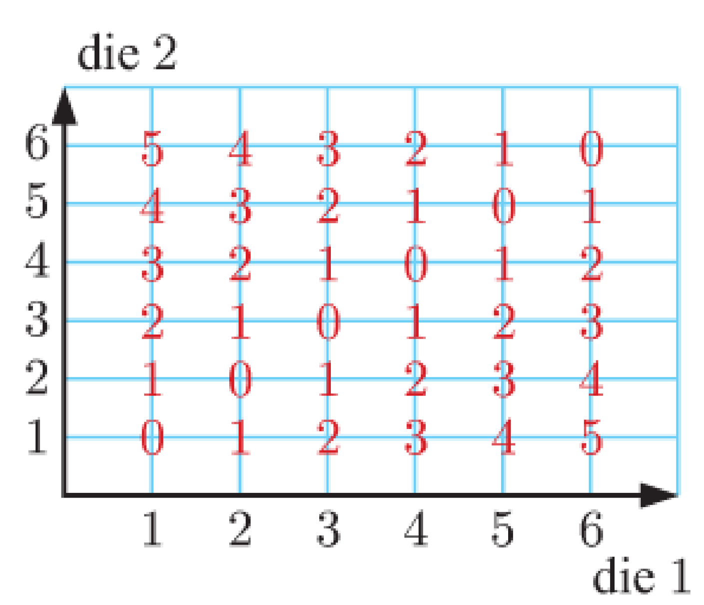
b **i** $\frac{1}{10}$ **ii** $\frac{3}{10}$ **iii** $\frac{2}{5}$ **iv** $\frac{3}{5}$

9 **a** $\frac{1}{36}$ **b** $\frac{1}{18}$ **c** $\frac{5}{9}$ **d** $\frac{11}{36}$ **e** $\frac{5}{18}$ **f** $\frac{25}{36}$

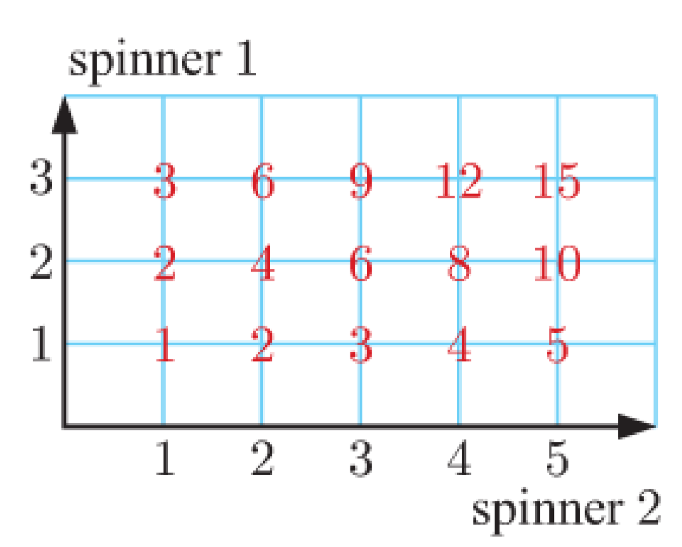
10 **a** Both grids show the sample space correctly, although **B** is more useful for calculating probabilities.
b $\frac{1}{6}$

11 **a**


b **i** $\frac{2}{36} = \frac{1}{18}$ **ii** $\frac{5}{36}$ **iii** $\frac{9}{36} = \frac{1}{4}$ **iv** $\frac{10}{36} = \frac{5}{18}$ **v** $\frac{10}{36} = \frac{5}{18}$ **vi** $\frac{26}{36} = \frac{13}{18}$

12 **a**


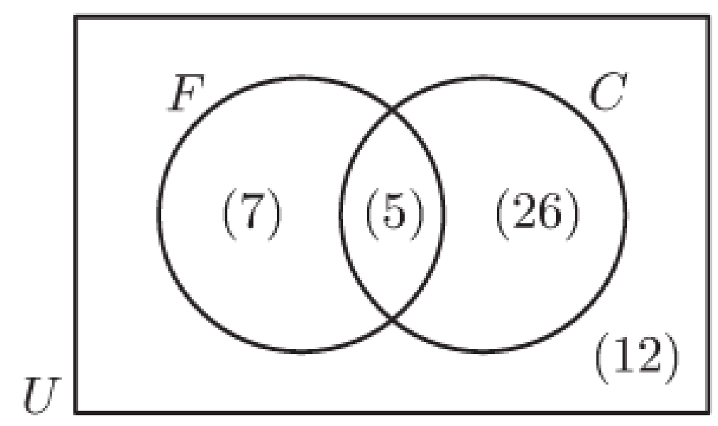
b **i** $\frac{6}{36} = \frac{1}{6}$ **ii** $\frac{8}{36} = \frac{2}{9}$ **iii** $\frac{18}{36} = \frac{1}{2}$ **iv** $\frac{6}{36} = \frac{1}{6}$ **v** $\frac{24}{36} = \frac{2}{3}$

13 **a**


b **i** $\frac{2}{15}$ **ii** $\frac{7}{15}$ **iii** $\frac{6}{15} = \frac{2}{5}$

14 **a** $\frac{3}{17}$ **b** $\frac{14}{17}$ **15** **a** $\frac{9}{65}$ **b** $\frac{4}{65}$ **c** $\frac{4}{5}$

16 **a** $\frac{17}{29}$ **b** $\frac{26}{29}$ **c** $\frac{5}{29}$ **17** **a** $\frac{37}{50}$ **b** $\frac{2}{5}$ **c** $\frac{17}{50}$

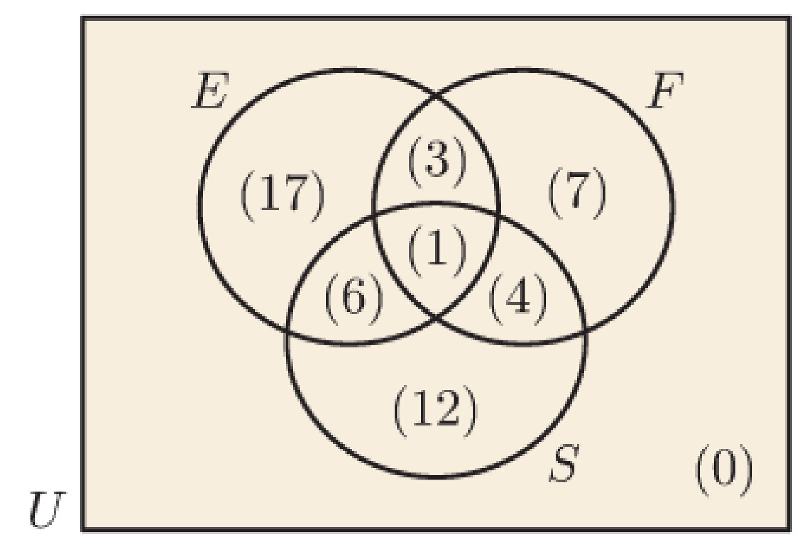
18 **a**


b **i** $\frac{19}{25}$ **ii** $\frac{13}{25}$ **iii** $\frac{6}{25}$

19 **a** $\frac{19}{40}$ **b** $\frac{1}{2}$ **c** $\frac{4}{5}$ **d** $\frac{5}{8}$ **e** $\frac{13}{40}$

20 **a** $\frac{7}{15}$ **b** $\frac{1}{15}$ **c** $\frac{2}{15}$

21 **a** $k = 5$
b **i** $\frac{7}{30}$ **ii** $\frac{11}{60}$ **iii** $\frac{7}{60}$ **iv** $\frac{53}{60}$ **v** $\frac{7}{60}$
vi $\frac{2}{15}$ **vii** $\frac{41}{60}$ **viii** $\frac{31}{60}$

22 **a**


b **i** $\frac{27}{50}$ **ii** $\frac{3}{10}$ **iii** $\frac{8}{25}$ **iv** $\frac{1}{5}$ **v** $\frac{2}{25}$

23 **a** $a = 3, \ b = 3$
b **i** $\frac{3}{10}$ **ii** $\frac{1}{10}$ **iii** $\frac{7}{40}$ **iv** $\frac{3}{8}$ **v** $\frac{5}{8}$

EXERCISE 11E

1 27 saves **2** $\approx$ 16 times **3** **a** $\frac{1}{4}$ **b** 50 occasions

4 15 days **5** 30 occasions

6 **a** **i** ≈ 0.55 **ii** ≈ 0.29 **iii** ≈ 0.16
b **i** $\approx$ 4125 people **ii** $\approx$ 2175 people
iii $\approx$ 1200 people

EXERCISE 11F

1 $P(A \cup B) = 0.55$ **2** $P(B) = 0.6$ **3** $P(X \cap Y) = 0.2$

4 **a** $P(A \cap B) = 0$ **b** A and B are mutually exclusive.

5 $P(A) = 0.35$

6 **a** yes
b **i** $P(A) = \frac{4}{15}$ **ii** $P(B) = \frac{7}{15}$ **iii** $P(A \cup B) = \frac{11}{15}$

7 **a** $\frac{11}{25}$ **b** $\frac{12}{25}$ **c** $\frac{8}{25}$ **d** $\frac{7}{25}$ **e** $\frac{4}{25}$ **f** $\frac{23}{25}$
g not possible **h** $\frac{11}{25}$ **i** not possible **j** $\frac{12}{25}$

8 $P(A \cup B) = 1$
Hint: Show $P(A' \cup B') = 2 - P(A \cup B)$

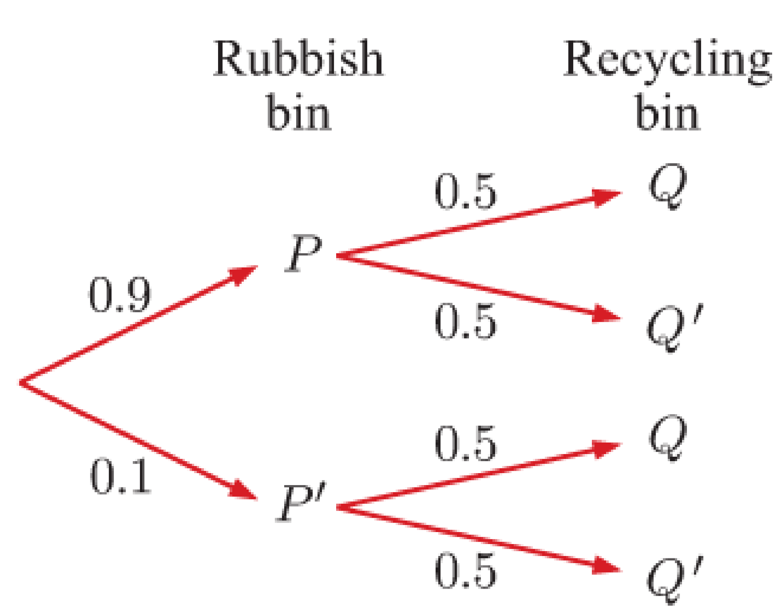
EXERCISE 11G

1 **a** $\frac{1}{24}$ **b** $\frac{1}{6}$ **2** **a** $\frac{1}{8}$ **b** $\frac{1}{8}$

3 **a** 0.0096 **b** 0.8096

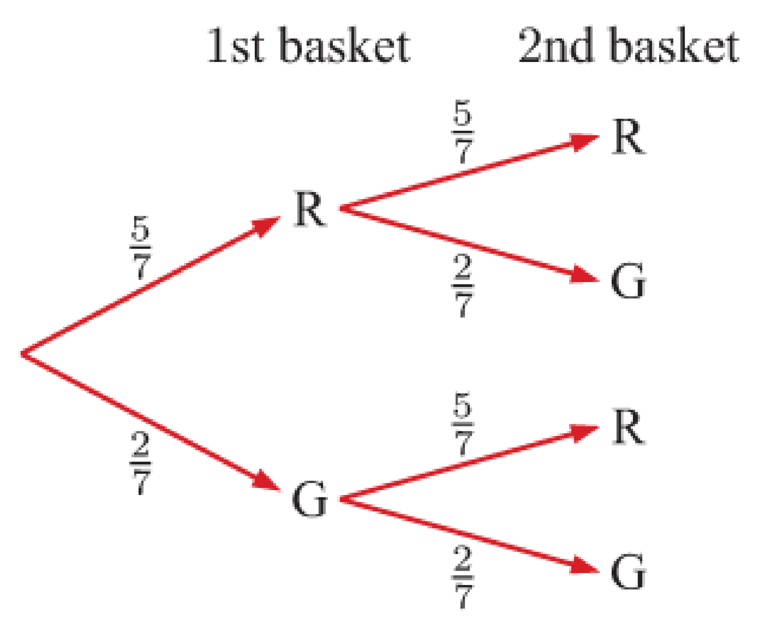
4 **a** 0.56 **b** 0.06 **c** 0.14 **d** 0.24

5 **a** $\frac{8}{125}$ **b** $\frac{12}{125}$ **c** $\frac{27}{125}$

6 **a**


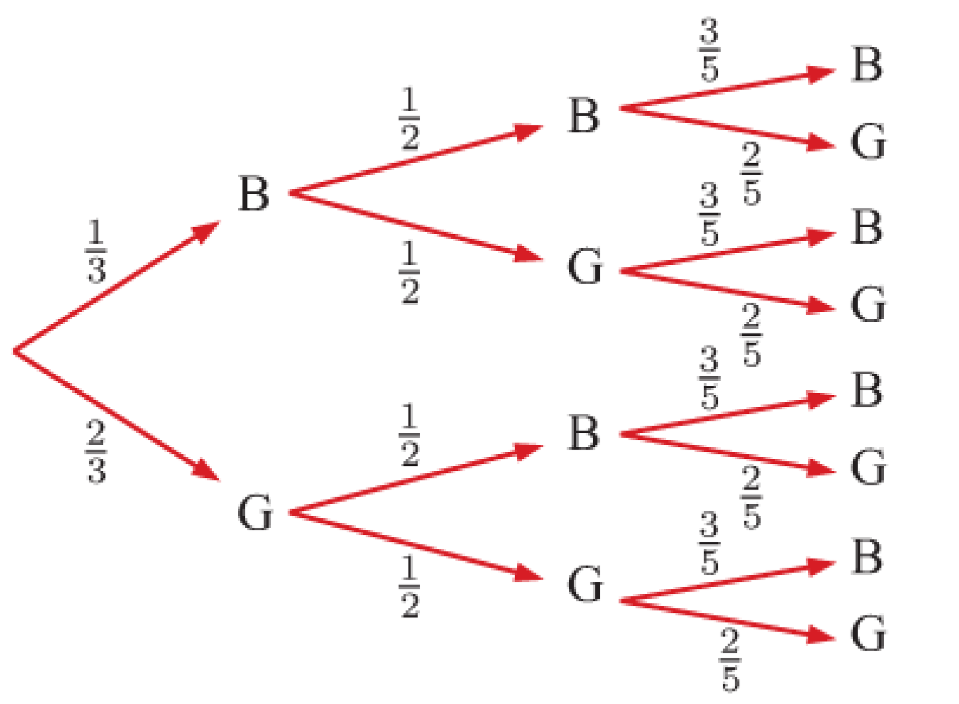
b **i** 0.45 **ii** 0.05

7 **a**



b **i** $\frac{25}{49}$ **ii** $\frac{20}{49}$

8 **a**



b **i** $\frac{1}{10}$ **ii** $\frac{1}{5}$ **iii** $\frac{13}{15}$

9 **a** $2p^2 - p^4$ **b** $p \approx 0.541$

10 Penny - Quentin - Penny
To win 2 matches in a row, Kane must win the middle match, so he should play against the weaker player in this match.

EXERCISE 11H

1 **a** $\frac{7}{15}$ **b** $\frac{7}{30}$ **2** **a** $\frac{2}{15}$ **b** $\frac{4}{15}$ **3** **a** $\frac{14}{55}$ **b** $\frac{1}{55}$

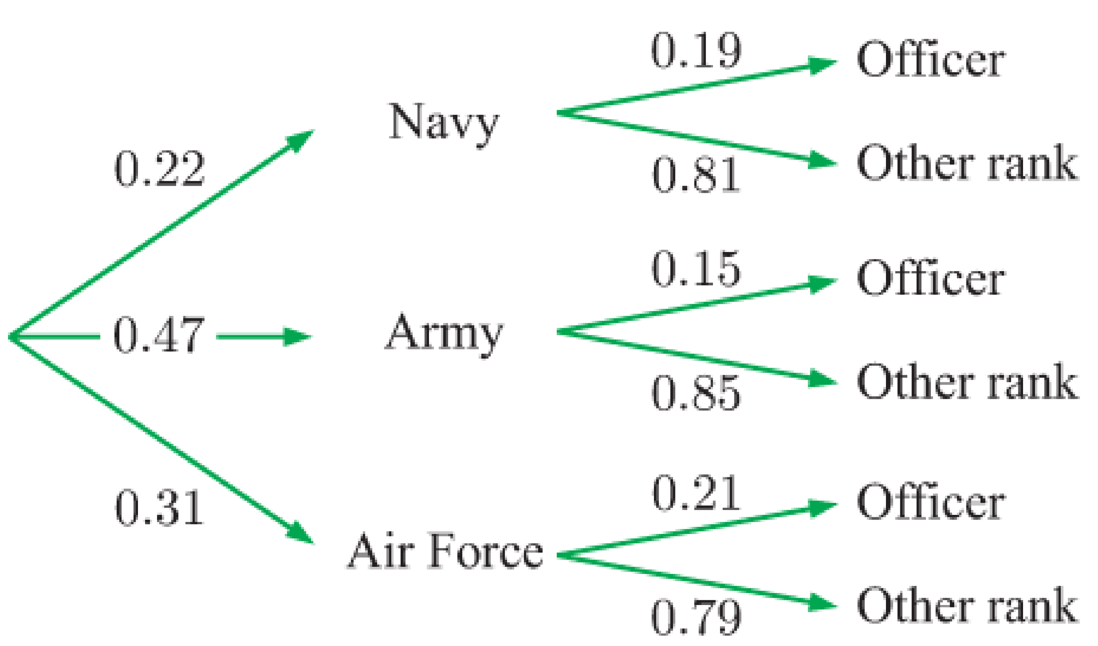
4 **a** $\frac{3}{100}$ **b** $\frac{3}{100} \times \frac{2}{99} \approx 0.000\,606$

c $\frac{3}{100} \times \frac{2}{99} \times \frac{1}{98} \approx 0.000\,006\,18$

d $\frac{97}{100} \times \frac{96}{99} \times \frac{95}{98} \approx 0.912$

5 **a** $\frac{4}{7}$ **b** $\frac{2}{7}$ **6** **a** $\frac{10}{21}$ **b** $\frac{1}{21}$

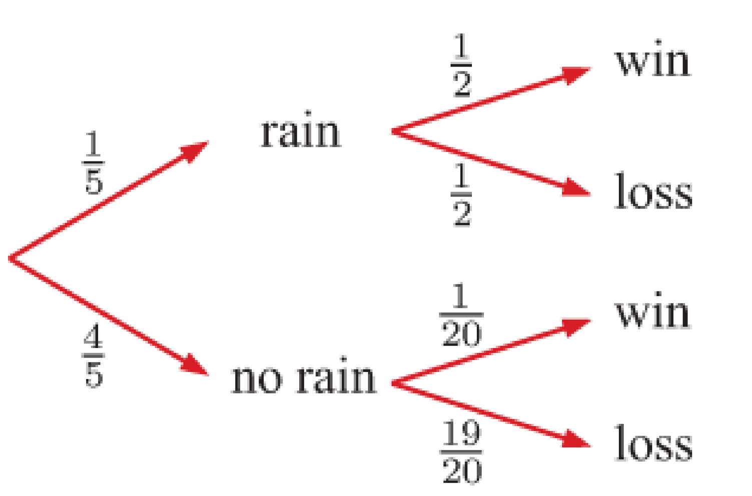
7 **a**



b **i** 0.1774 **ii** 0.9582 **iii** 0.8644

8 **a** 0.28 **b** 0.24

9 **a**



b $\frac{7}{50}$

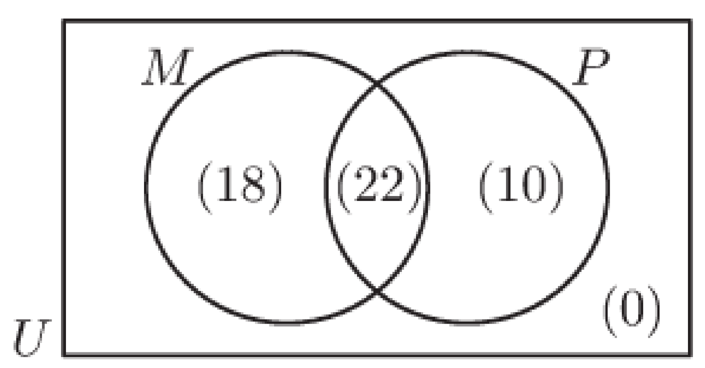
10 0.032 **11** $\frac{9}{38}$ **12** **a** $\frac{11}{30}$ **b** $\frac{19}{30}$

13 $\frac{187}{460} \approx 0.407$ **14** **a** $\frac{325}{833} \approx 0.390$ **b** $\frac{787}{833} \approx 0.945$

EXERCISE 11I

1 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** 0 **2** $\frac{1}{2}$

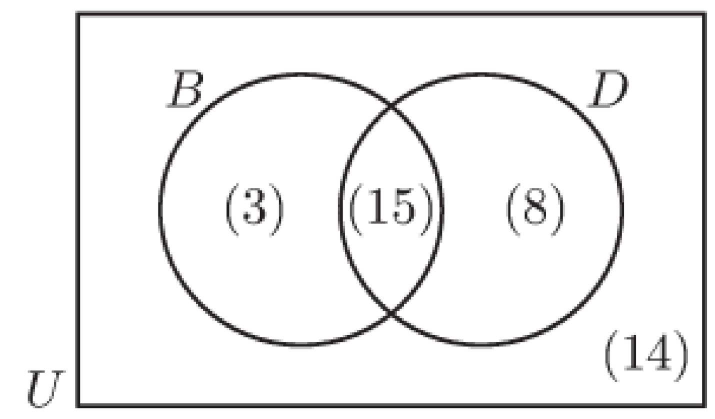
3 **a**



22 study both

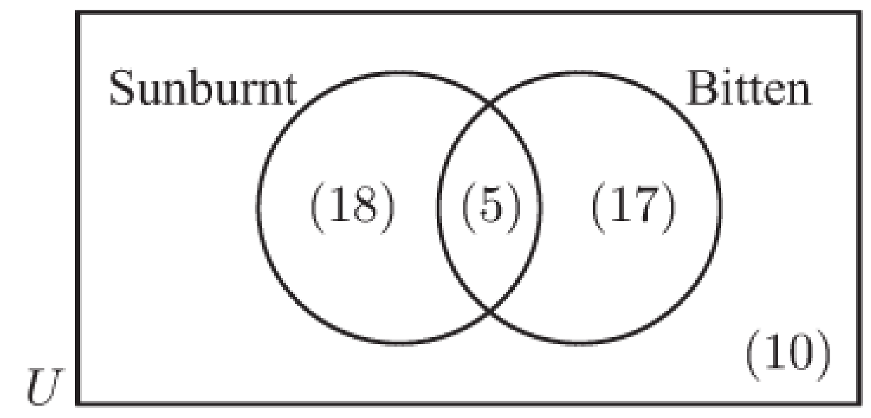
b **i** $\frac{9}{25}$ **ii** $\frac{11}{20}$

4 **a**



b **i** $\frac{3}{8}$ **ii** $\frac{15}{23}$

5 **a**



b **i** $\frac{14}{25}$ **ii** $\frac{4}{5}$ **iii** $\frac{5}{23}$ **iv** $\frac{9}{14}$

6 $\frac{7}{8}$

7 **a** $\frac{13}{20}$ **b** $\frac{7}{20}$ **c** $\frac{11}{50}$ **d** $\frac{7}{25}$ **e** $\frac{4}{7}$ **f** $\frac{1}{4}$

8 **a** $\frac{3}{5}$ **b** $\frac{2}{3}$ **9** **a** $\frac{23}{50}$ **b** $\frac{14}{23}$

10 **a** $\frac{10}{17}$ **b** $\frac{70}{163}$ **11** $\frac{7}{15}$

EXERCISE 11J

1 $P(R \cap S) = 0.4 + 0.5 - 0.7 = 0.2$ and $P(R) \times P(S) = 0.2$
$\therefore$ R and S are independent events.

2 **a** **i** $\frac{7}{30}$ **ii** $\frac{7}{12}$ **iii** $\frac{7}{10}$

b No, as $P(A \mid B) \neq P(A)$.

3 **a** 0.35 **b** 0.85 **c** 0.15 **d** 0.15 **e** 0.5

4 **Hint:** Show $P(A' \cap B') = P(A')\,P(B')$ using a Venn diagram and $P(A \cap B)$.

5 $P(B) = 0$ **6** 0.9

7 **a** $P(D) = \frac{89}{400}$ **b** No, as $P(D \mid C) \neq P(D)$.

8 $P(A \cup B) = 1$ or $P(A \cap B) = 0$

EXERCISE 11K

1 **a** 0.0435 **b** ≈ 0.598 **2** **a** ≈ 0.773 **b** ≈ 0.556

3 $\frac{10}{13}$ **4** ≈ 0.424 **5** 0.0137 **6** $\frac{15}{83}$ **7** $\frac{99}{148}$

8 **a** $\frac{9}{19}$ **b** $\frac{10}{19}$ **10** **a** 0.95 **b** ≈ 0.306 **c** 0.6

11 **a** 0.104 **b** ≈ 0.267 **c** ≈ 0.0168

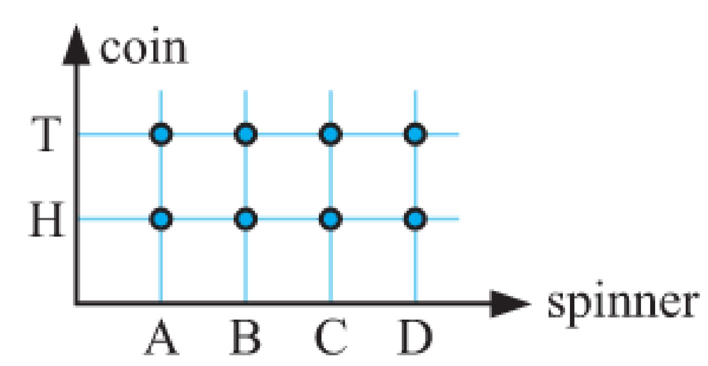
12 **a** $P(L \mid T) = \frac{46}{205}$ **b** $P(T \mid L) = \frac{46}{57}$

c Bayes' theorem tells us that $P(L \mid T) = P(T \mid L)\,\frac{P(L)}{P(T)}$.
Our answers to **a** and **b** differ since $P(L) \neq P(T)$.

REVIEW SET 11A

1 **a** ≈ 0.13 **b** ≈ 0.53

2 **a**



b **i** $\frac{3}{8}$ **ii** $\frac{1}{8}$ **iii** $\frac{5}{8}$

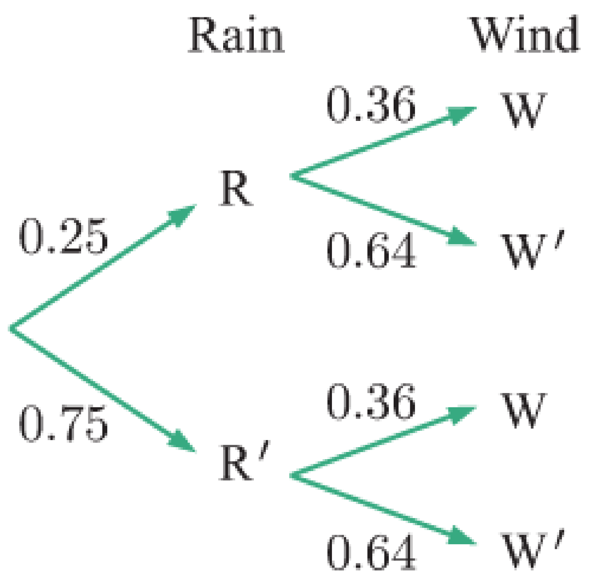
3 **a** Two events are independent if the occurrence of either event does not affect the probability that the other occurs. For A and B independent, $P(A \cap B) = P(A) \times P(B)$.

b Two events A and B are mutually exclusive if they have no common outcomes. $P(A \cup B) = P(A) + P(B)$

4 0.496

5 **a** $P(A \cup B) = x + 0.57$ **b** $x = 0.16$

6 **a**

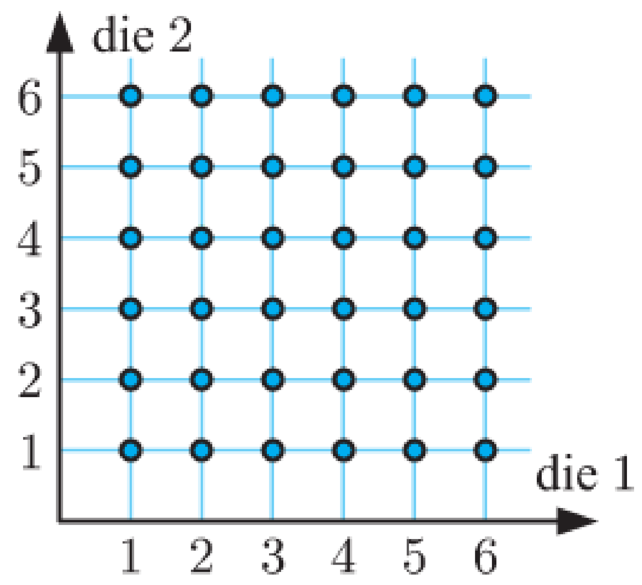


b **i** 0.09 **ii** 0.52

c It is assumed that the events are independent.

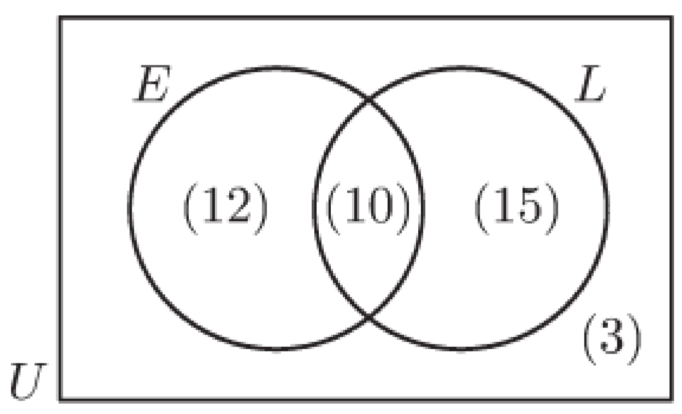
7 **a** 0 **b** 0.45 **c** 0.8

8 **a**



b **i** $\frac{2}{9}$ **ii** $\frac{5}{12}$

9 **a**



b **i** $\frac{1}{4}$ **ii** $\frac{37}{40}$ **iii** $\frac{2}{5}$

10 4350 seeds **11** **a** $\frac{25}{144}$ **b** $\frac{25}{72}$ **c** $\frac{7}{16}$ **d** $\frac{4}{9}$

12 ≈ 0.127

13 **a**

	Female	*Male*	*Total*
Smoker	20	40	60
Non-smoker	70	70	140
Total	90	110	200

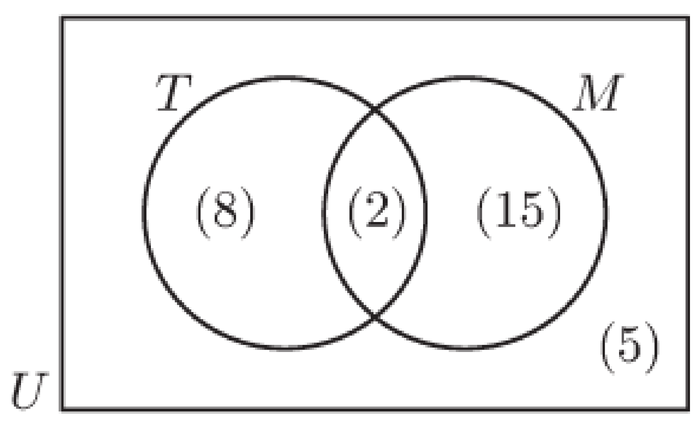
b **i** $\frac{7}{20}$ **ii** $\frac{1}{2}$

c **i** ≈ 0.121 **ii** ≈ 0.422

14 $\frac{69}{95}$ **15** **a** $\frac{1}{5}$ **b** $P(B \mid A) \neq P(B)$ **c** $\frac{2}{3}$ **16** $\frac{5}{324}$

REVIEW SET 11B

1 **a**



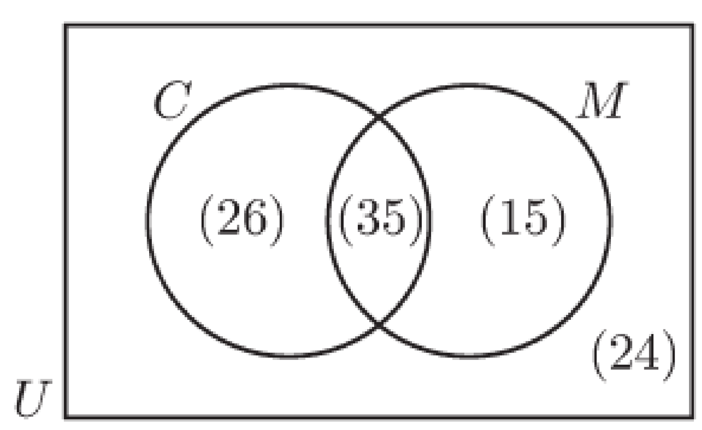
b **i** $\frac{1}{15}$ **ii** $\frac{2}{17}$

2 0.9975

3 **a** $P(A \cap B) = 0.28$ which is not equal to 0.
$\therefore$ A and B are not mutually exclusive.

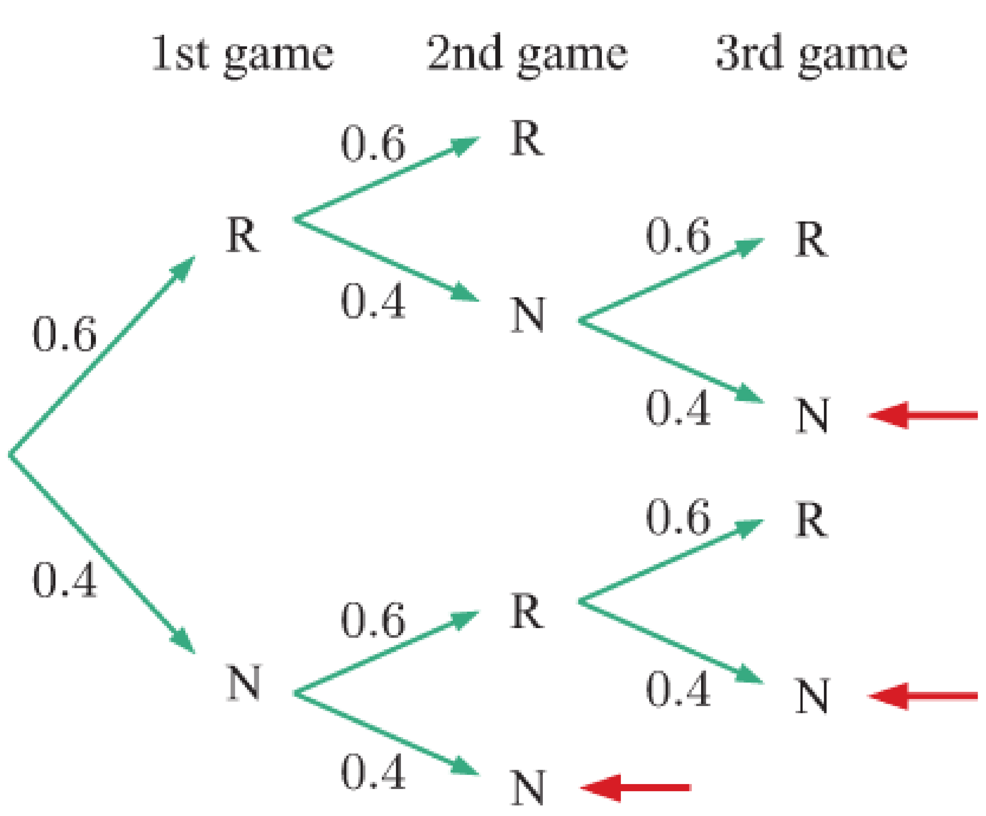
b $P(A \cup B) = 0.82$

4 $\frac{5}{9}$ **5** **a**



b $\frac{35}{61}$

6 **a**



b P(N wins) $= 0.352$

7 **a** 0.93 **b** 0.8 **c** 0.2 **d** 0.65

8 **a** $\frac{4}{500} \times \frac{3}{499} \times \frac{2}{498} \approx 0.000\,000\,193$

b $1 - \frac{496}{500} \times \frac{495}{499} \times \frac{494}{498} \approx 0.0239$

9 **a** 0.2588 **b** ≈ 0.703

10 **a** $P(B) = \frac{1}{3}$ **b** **i** $\frac{16}{21}$ **ii** $\frac{13}{21}$

11 $\frac{4}{9}$ **12** **a** ≈ 0.660 **b** $\frac{100}{7} \approx 14.3$ or 14 pieces

13 **a** $\frac{31}{70}$ **b** $\frac{21}{31}$ **14** $\frac{1}{2}$

15 **a** **i** 100 balloons **ii** 33 balloons

b **i** $\frac{19}{25}$ **ii** $\frac{37}{50}$

c **i** $\frac{17}{66}$ **ii** $\frac{2701}{4950}$ **iii** $\frac{25}{66}$ **iv** $\frac{29}{66}$

d **i** $\frac{1}{980}$ **ii** $\frac{17}{308}$

16 **b** ≈ 0.988 **c** **i** ≈ 0.547 **ii** ≈ 0.266 **d** females

e A 20 year old is expected to live much longer than 30 more years, so it is unlikely the insurance company will have to pay out the policy. A 50 year old however is expected to live for only another 26.45 years (males) or 31.59 years (females), so the insurance company may have to pay out the policy.

g For "third world" countries with poverty, lack of sanitation, and so on, the tables would show a significantly lower life expectancy.

EXERCISE 12A

1 This sample is too small to draw reliable conclusions from.

2
- The sample size is very small and may not be representative of the whole population.
- The sample was taken in a Toronto shopping mall. People living outside of the city are probably not represented.

3 **a** The sample is likely to under-represent full-time weekday working voters.

b The members of the golf club may not be representative of the whole electorate.

c Only people who catch the train in the morning such as full-time workers or students will be sampled.

d The voters in the street may not be representative of those in the whole electorate.

4 **a** The sample size is too small.

b With only 10 sheep being weighed, any errors in the measuring of weights will have more impact on the results.

5 **a** The whole population is being considered, not just a sample. There will be no sampling error as this is a census.

b measurement error

6 **a** Many of the workers may not return or even complete the survey.

b There may be more responses to the survey as many workers would feel that it is easier to complete a survey online rather than on paper and mailing it back. Responses would also be received more quickly however some workers may not have internet access and will therefore be unable to complete the survey.

7 **a** Yes; members with strong negative opinions regarding the management structure of the organisation are more likely to respond.

b No; the feedback from the survey is still valid. Although it might be biased, the feedback might bring certain issues to attention.

EXERCISE 12B

1 **Note:** Sample answers only - many answers are possible.

a 12, 6, 23, 10, 21, 25

b 11, 2, 10, 17, 24, 14, 25, 1, 21, 7
c 14, 24, 44, 34, 27, 1 **d** 166, 156, 129, 200, 452

2 **a** 17, 67, 117, 167, 217 **b** 1600 blocks of chocolate

3 **a** Select 5 random numbers between 1 and 365 inclusive. For example, 65, 276, 203, 165, and 20 represent 6th March, 3rd October, 22nd July, 14th June, and 20th January.
b Select a random number between 1 and 52 inclusive. Take the week starting on the Monday that lies in that week.
c Select a random number between 1 and 12 inclusive.
d Select 3 random numbers between 1 and 12 inclusive.
e Select a random number between 1 and 10 inclusive for the starting month.
f Select 4 random numbers between 1 and 52 inclusive. Choose the Wednesday that lies in that week.

4 **a** convenience sampling
b The people arriving first will spend more time at the show, and so are more likely to spend more than €20. Also, the sample size is relatively small.
c For example, systematic sample of every 10th person through the gate.

5 **a** systematic sampling **b** 14 days
c Only visitors who use the library on Mondays will be counted. Mondays may not be representative of all of the days.

6 **a** 160 members
b 20 tennis members, 15 lawn bowls members, 5 croquet members

7 1 departmental manager, 3 supervisors, 9 senior sales staff, 13 junior sales staff, 4 shelf packers

8 **a** It is easier for Mona to survey her own home room class, so this is a convenience sample.
b Mona's sample will not be representative of all of the classes in the school. Mona's survey may be influenced by her friends in her class.
c For example, a stratified sample of students from every class.

9 **a** Not all students selected for the sample will be comfortable discussing the topic.
b quota sample

10 **a** All students in Years 11 and 12 were asked, not just a sample.
b 0.48
c **i** Sample too small to be representative.
ii Sample too small to be representative.
iii Valid but unnecessarily large sample size.
iv Useful and valid technique.
v Useful and valid technique.
vi Useful and valid technique.
d **v** is simple random sampling, while **iii** and **iv** are systematic sampling, and **vi** is stratified or quota sampling.

EXERCISE 12C

Note: Sample answers only - many answers are possible.

1 **a** It does not allow for colours which are different from those given.
b What colour is your shirt?
c For example, one person might interpret a colour as blue whereas another person may interpret it as purple. A shirt may also be more than one colour which could lead to difficulties in interpreting the colour.

2 **a** The question could be interpreted as:
- "Do you have any medically diagnosed allergies?"
- "Do you have any life threatening allergies?"
- "Do you have any food allergies?"
- "Do you think you have any allergies?"

The question also does not specify if it is a structured yes/no type of question or if the respondent should list specific allergies.
b "Do you have any food or other type of allergies (medically diagnosed or otherwise), and if so, what are they?"

3 **a** The question could be interpreted as:
- "Do you have any animals in your household?"
- "Do you have any animals in your care at home or elsewhere?"

The question does not specify if it is a structured yes/no type of question or if it includes livestock or only domestic animals.
b "Do you have any domesticated animals in your household (not including livestock), and if so, what are they?"

4 **a** The journalist's question is misleading as it only mentions the proposed cuts to education, not the proposal to move those funds to health. This may produce a measurement error as the respondents are unlikely to give their views about the whole proposal.
b For example, "What are your views about the Government's proposal to move funding from education to health?".

5 **a** Many respondents would not be comfortable giving their address to someone they do not know.
b The question could be more specific, asking for the general area or suburb only. For example:
- "Which suburb do you live in?"
- "Which state do you live in?"

Giving a reason why this information is needed will also improve the response rate.

6 **a** **i** The question contains a double negative which could confuse respondents. The word "infectious" suggests that *not* immunising is undesirable behaviour, thus the question is biased.
ii "Have you been immunised against meningococcal disease?"
b **i** The question asks for two things:
- whether climate change is a major issue
- the respondent's political opinion on climate change.

It is not clear whether the respondent's response will reflect their general or political opinion on the issue.
The phrase "thrown around by politicians" is also rather emotive, thus the question is biased.
ii "Do you believe that climate change is an important issue?"
c **i** The question uses a positive fact about fair trade cocoa to try to persuade the respondent into answering "yes". So the question is biased.
It is also very long and takes a long time to get to the point.
ii "Do you believe that fair trade certified chocolate should be more expensive than uncertified chocolate?"

EXERCISE 12D

1 **a** discrete; 0, 1, 2, 3, ...
b categorical; red, yellow, orange, green
c continuous; 0 - 15 minutes
d continuous; 0 - 25 m
e categorical; Ford, BMW, Renault **f** discrete; 1, 2, 3, ...
g categorical; Australia, Hawaii, Dubai
h discrete; 0.0 - 10.0 **i** continuous; 0 - 4 L
j continuous; 0 - 80 hours **k** continuous; $-20°C$ - $35°C$

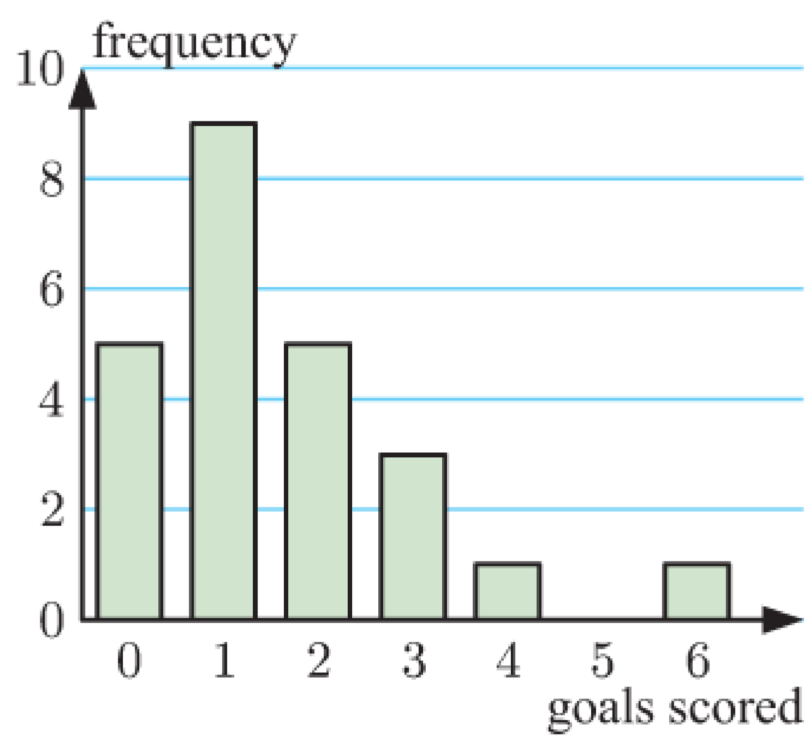
l categorical; cereal, toast, fruit, rice, eggs
m discrete; 0, 1, 2, ...

2 *Name*: categorical, *Age*: continuous, *Height*: continuous, *Country*: categorical, *Wins*: discrete, *Speed*: continuous, *Ranking*: discrete, *Prize money*: discrete

EXERCISE 12E

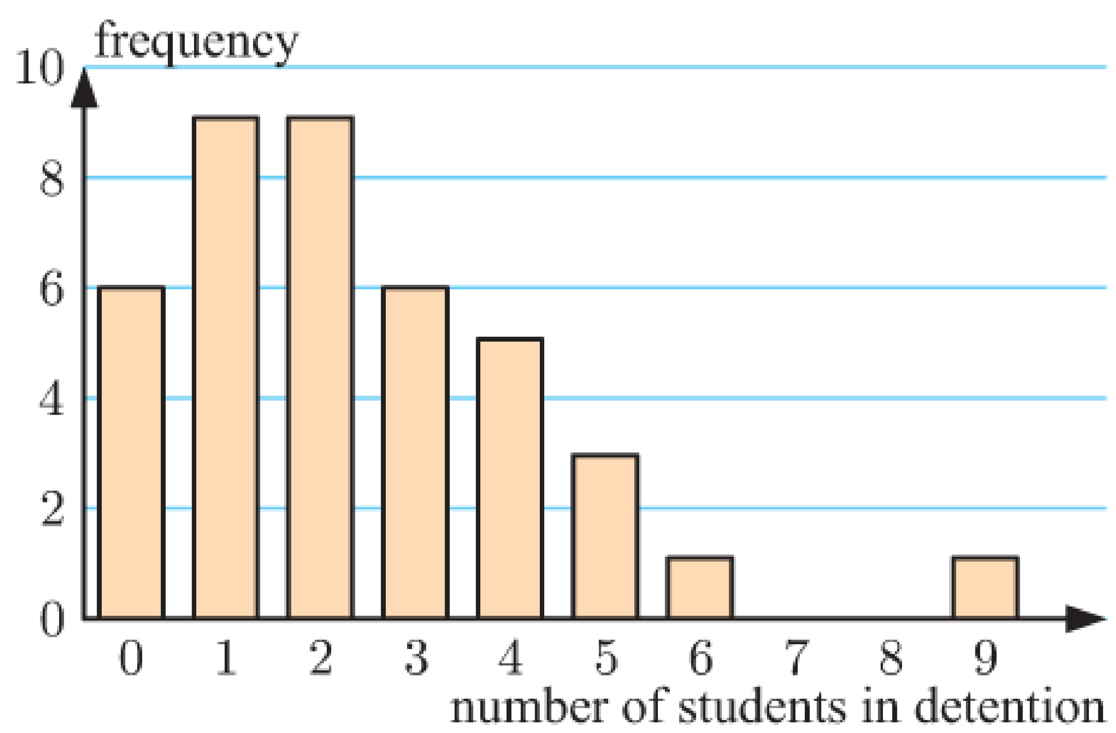
1 **a** the number of goals scored in a game
b variable is counted, not measured
c

Goals scored	*Tally*	*Frequency*	*Rel. Frequency*
0	~~||||~~	5	≈ 0.208
1	~~||||~~ ||||	9	0.375
2	~~||||~~	5	≈ 0.208
3	|||	3	0.125
4	|	1	≈ 0.042
5		0	0
6	|	1	≈ 0.042
	Total	24	

d


e 1 goal

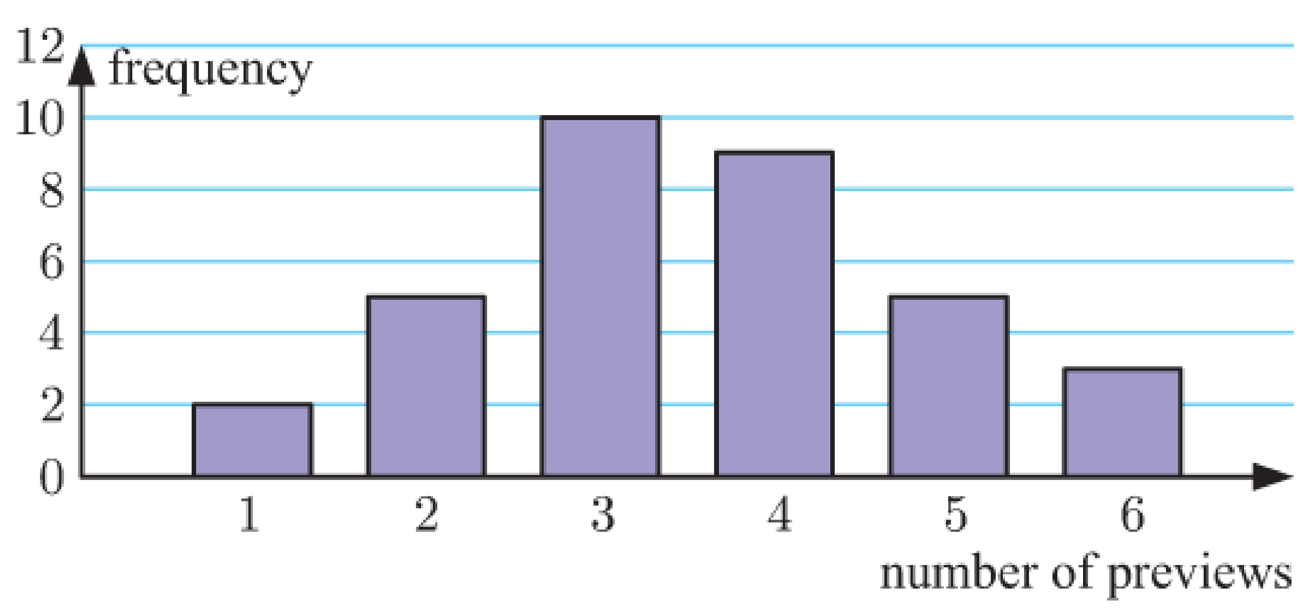
f positively skewed, one outlier (6 goals) **g** ≈ 20.8%

2 **a**


b 1 and 2 **c** positively skewed, one outlier (9 students)
d $12\frac{1}{2}\%$

3 **a**

Number of previews	*Tally*	*Frequency*
1	||	2
2	~~||||~~	5
3	~~||||~~ ~~||||~~	10
4	~~||||~~ ||||	9
5	~~||||~~	5
6	|||	3
	Total	34

b


c 3 previews **d** symmetrical, no outliers **e** ≈ 79.4%

4 **a** 45 people **b** 1 time **c** 8 people **d** 20%
e positively skewed, no outliers

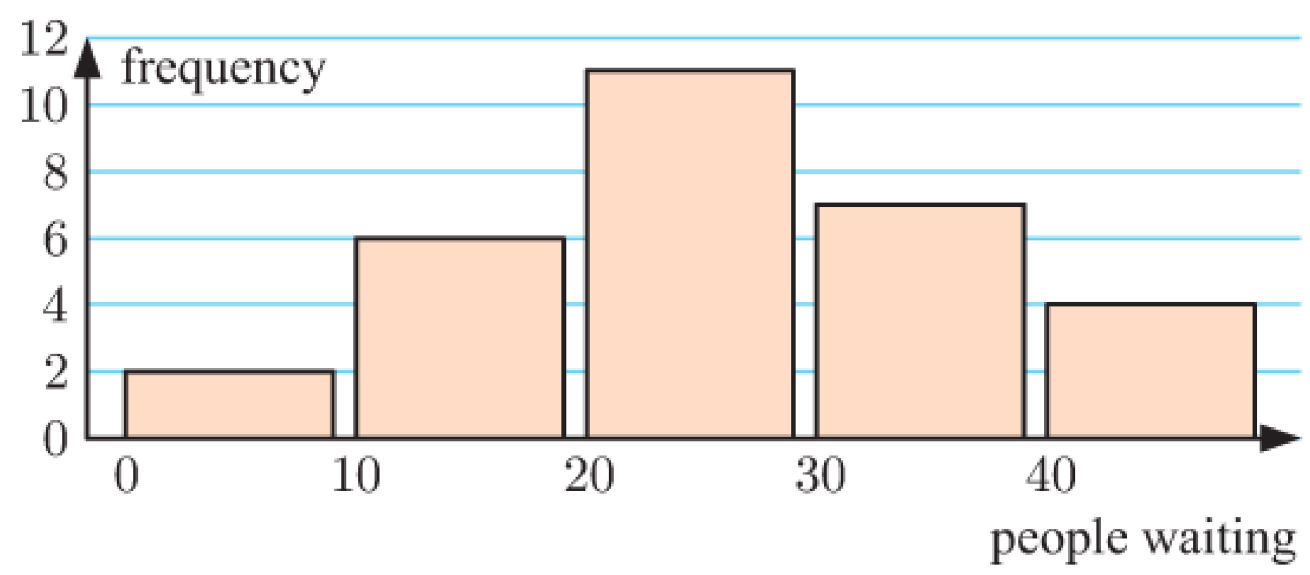
EXERCISE 12F

1 **a** 37 businesses **b** 40 - 49 employees
c negatively skewed **d** ≈ 37.8%
e No, only that it was in the interval 50 - 59 employees.

2 **a**

People waiting	*Tally*	*Frequency*	*Rel. Freq.*
0 - 9	||	2	≈ 0.067
10 - 19	~~||||~~ |	6	0.200
20 - 29	~~||||~~ ~~||||~~ |	11	≈ 0.367
30 - 39	~~||||~~ ||	7	≈ 0.233
40 - 49	||||	4	≈ 0.133
	Total	30	

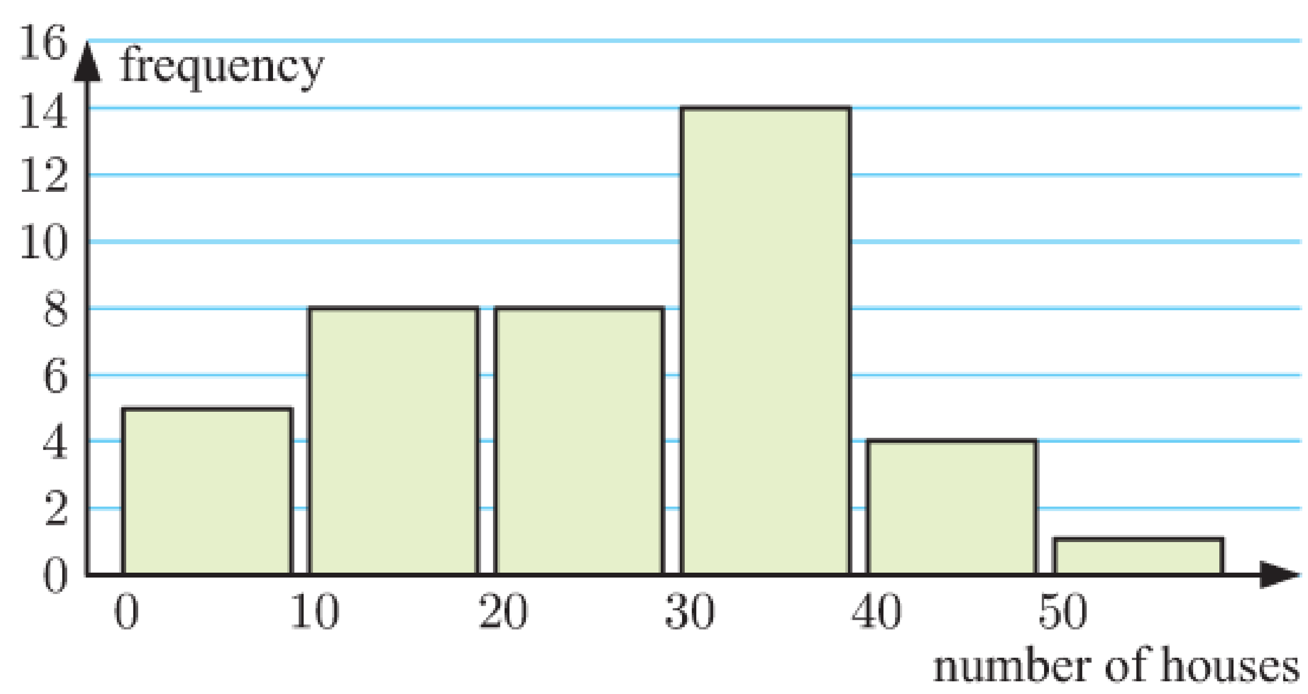
b 2 days **c** ≈ 36.7%

d


e 20 - 29 people

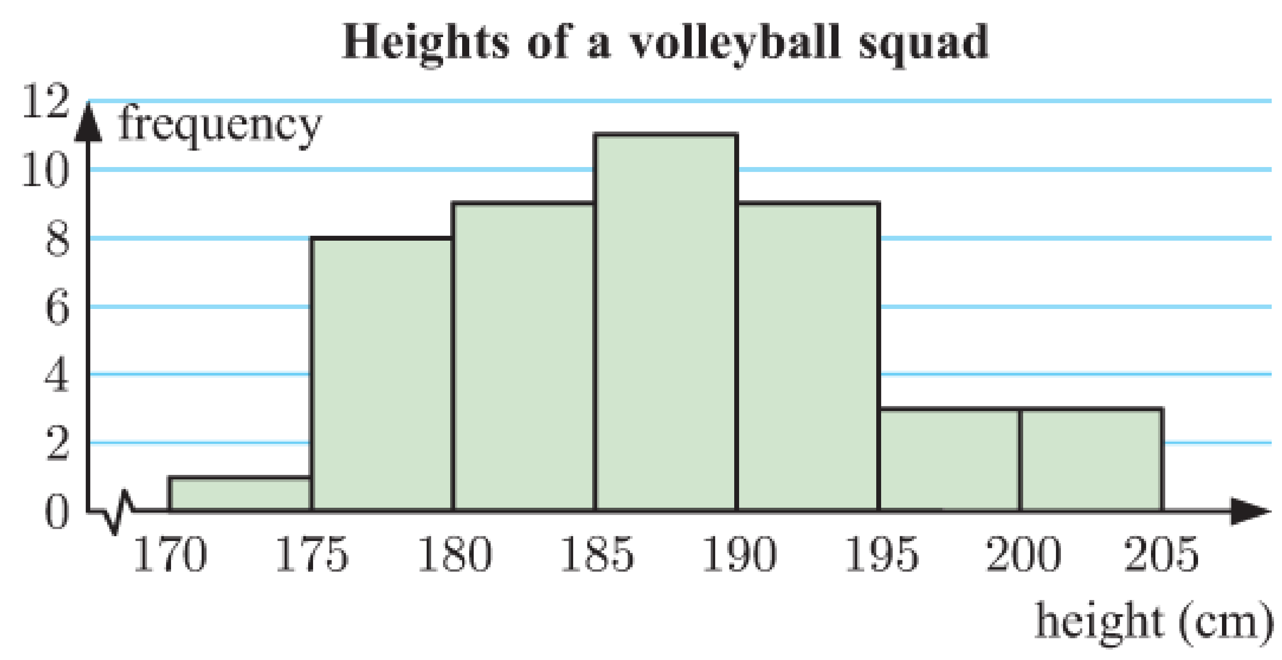
3 **a**

Number of houses	*Tally*	*Frequency*
0 - 9	~~||||~~	5
10 - 19	~~||||~~ |||	8
20 - 29	~~||||~~ |||	8
30 - 39	~~||||~~ ~~||||~~ ||||	14
40 - 49	||||	4
50 - 59	|	1
	Total	40

b


c 30 - 39 houses **d** 67.5%

EXERCISE 12G

1 **a** Height is measured on a continuous scale.
b


c $185 \leqslant H < 190$ cm. This is the class of values that appears most often.

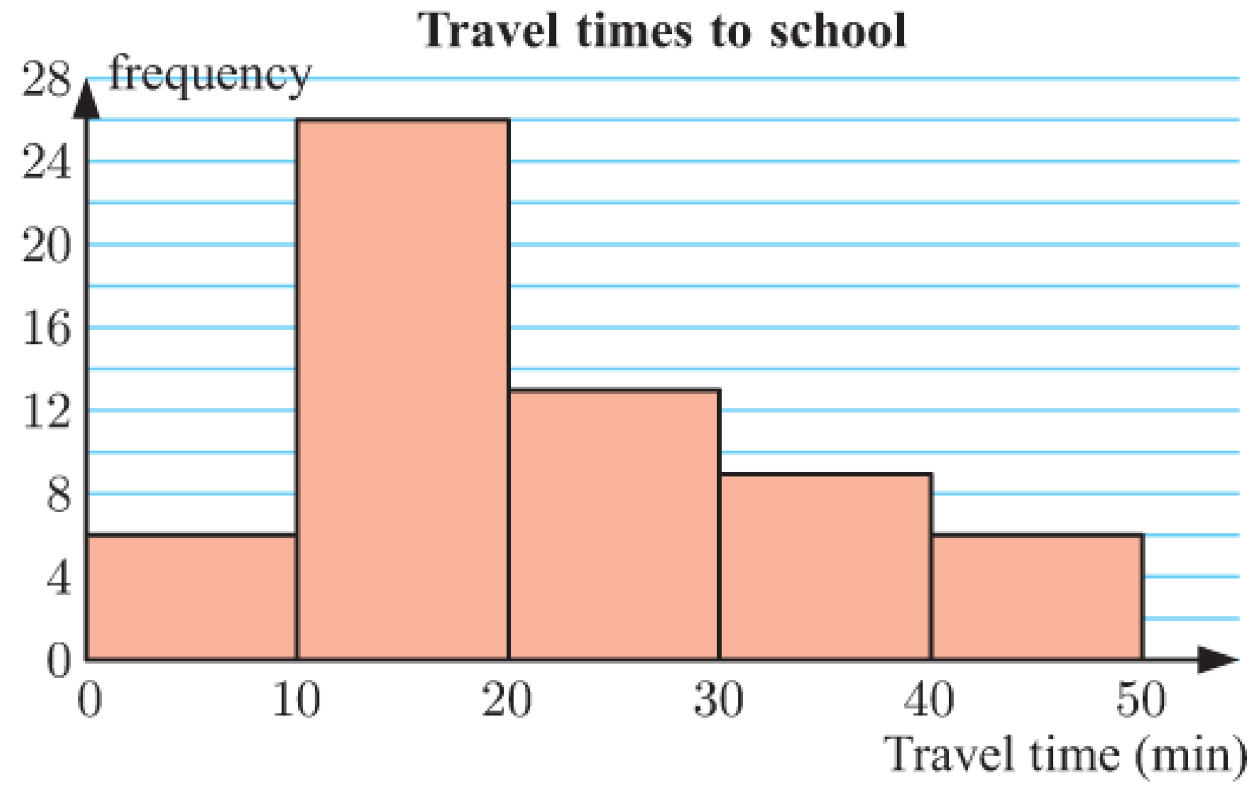
d slightly positively skewed

2 **a** continuous

b

Travel time (min)	*Tally*	*Frequency*
$0 \leqslant t < 10$	𝍸 |	6
$10 \leqslant t < 20$	𝍸 𝍸 𝍸 𝍸 𝍸 |	26
$20 \leqslant t < 30$	𝍸 𝍸 |||	13
$30 \leqslant t < 40$	𝍸 ||||	9
$40 \leqslant t < 50$	𝍸 |	6
	Total	60

c

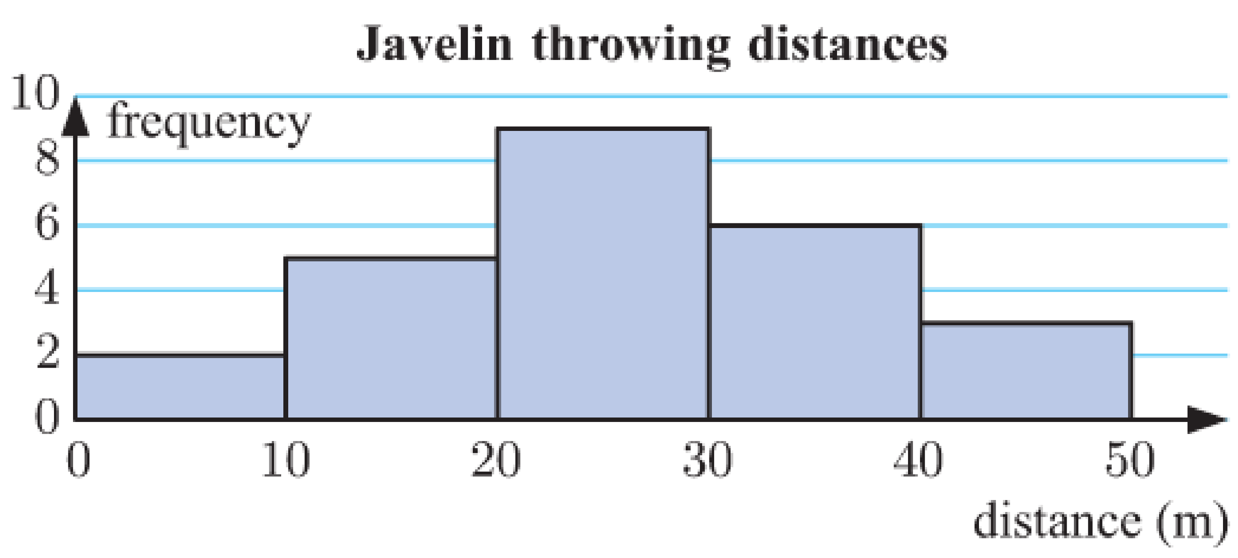


d positively skewed **e** $10 \leqslant t < 20$ minutes

3 **a, b**

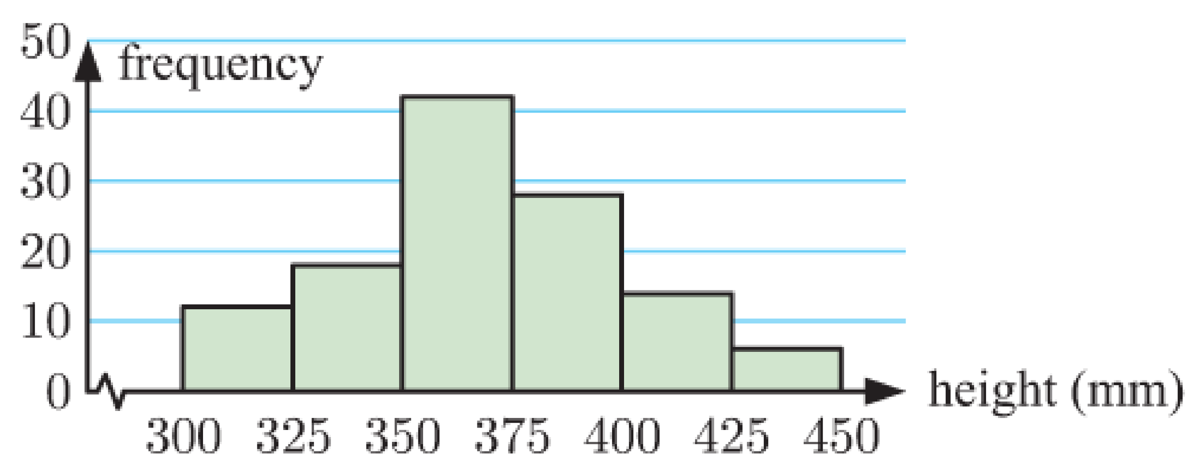
Distance (m)	*Tally*	*Frequency*
$0 \leqslant d < 10$	||	2
$10 \leqslant d < 20$	𝍸	5
$20 \leqslant d < 30$	𝍸 ||||	9
$30 \leqslant d < 40$	𝍸 |	6
$40 \leqslant d < 50$	|||	3
	Total	25

c



d $20 \leqslant d < 30$ m **e** 36%

4 **a** **Heights of 6-month old seedlings at a nursery**



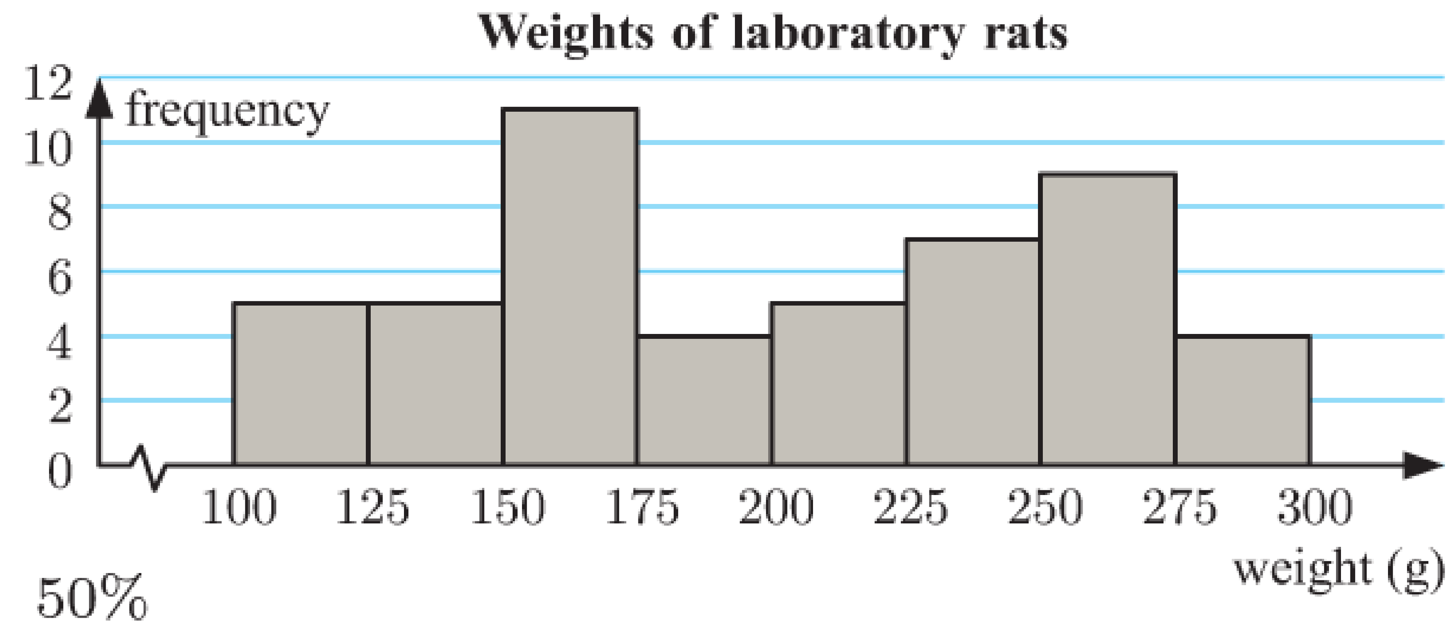
b 20 seedlings **c** $\approx 58.3\%$

d **i** ≈ 1218 seedlings **ii** ≈ 512 seedlings

5 **a, b**

Weight (g)	*Tally*	*Frequency*
$100 \leqslant w < 125$	𝍸	5
$125 \leqslant w < 150$	𝍸	5
$150 \leqslant w < 175$	𝍸 𝍸 |	11
$175 \leqslant w < 200$	||||	4
$200 \leqslant w < 225$	𝍸	5
$225 \leqslant w < 250$	𝍸 ||	7
$250 \leqslant w < 275$	𝍸 ||||	9
$275 \leqslant w < 300$	||||	4
	Total	50

c



d 50%

REVIEW SET 12A

1 **a** Students studying Italian may have an Italian background so surveying these students may produce a biased result.

b For example, Andrew could survey a randomly selected group of students as they entered the school grounds one morning.

2 **a** It would be too time consuming and expensive.

b

Age range	< 18	18 - 39	40 - 54	55 - 70	> 70
Sample size	50	82	123	69	26

3 **a** discrete **b** continuous **c** categorical
d categorical **e** categorical **f** continuous
g continuous **h** discrete **i** discrete

4 **a** convenience sampling

b Yes, the sample will be biased as people are more likely to be drinking on a Saturday night. It is sensible for this sample to be biased since drink-driving is illegal.

5 **a** The question could be interpreted as:

- "Do you consider yourself to be healthy?"
- "Are you not currently suffering from any health conditions?"
- "Do you eat a balanced diet and exercise regularly?"
- "Do you take any medication for any health conditions?"

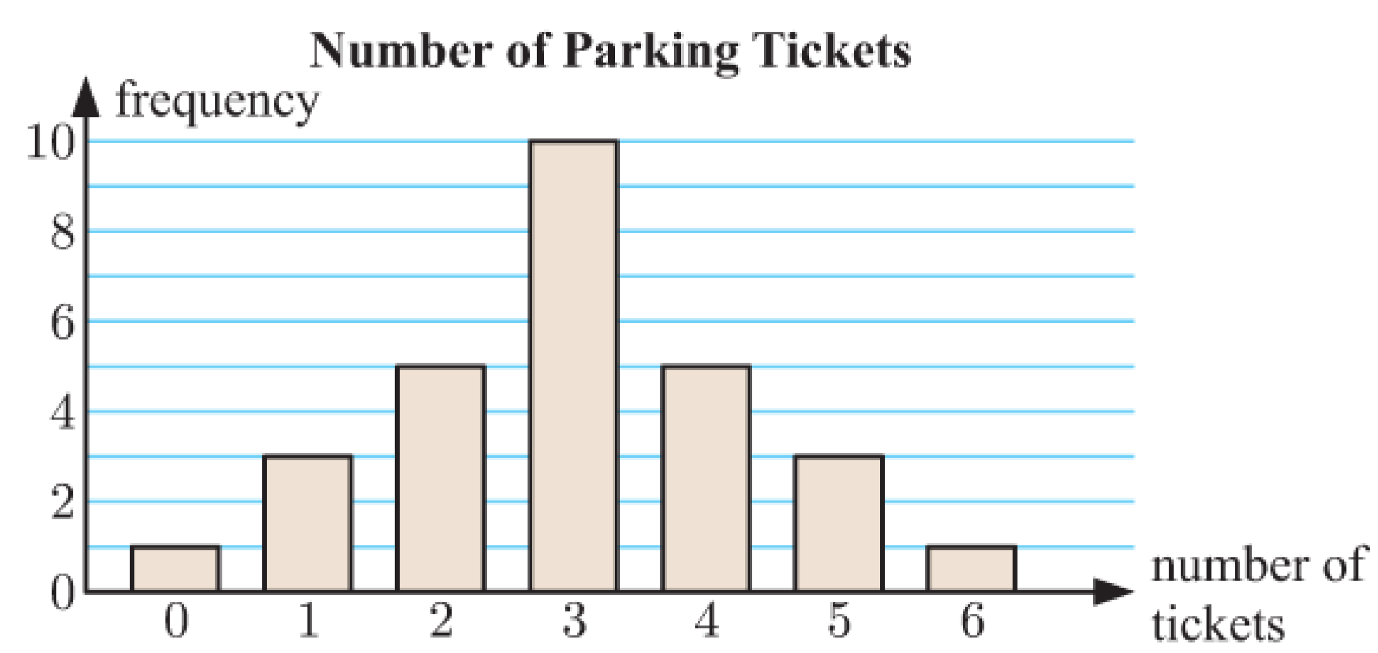
b "Do you eat a balanced diet and exercise regularly?"

6 **a** discrete **b** 1 round **c** positively skewed

7 **a**

Number of tickets	*Tally*	*Frequency*
0	|	1
1	|||	3
2	𝍸	5
3	𝍸 𝍸	10
4	𝍸	5
5	|||	3
6	|	1

b



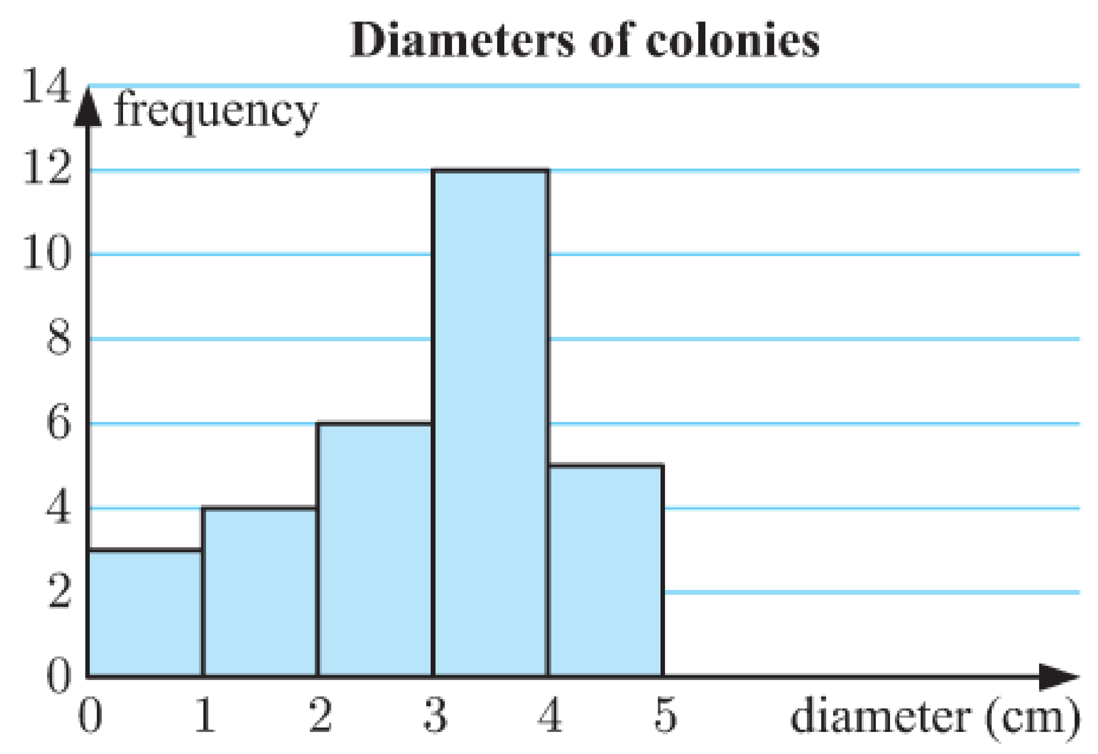
c The data is symmetric with no outliers.

8 **a** continuous

b

Diameter (*d* cm)	*Tally*	*Frequency*
$0 \leqslant d < 1$	|||	3
$1 \leqslant d < 2$	||||	4
$2 \leqslant d < 3$	𝍸 |	6
$3 \leqslant d < 4$	𝍸 𝍸 ||	12
$4 \leqslant d < 5$	𝍸	5
	Total	30

c



d $3 \leqslant d < 4$ cm **e** slightly negatively skewed

REVIEW SET 12B

1 **a** discrete **b** continuous **c** discrete

2 **a** systematic sampling

b A house will be visited if the last digit in its number is equal to the random number chosen by the promoter, with the random number 10 corresponding to the digit 0. Each house therefore has a 1 in 10 chance of being visited.

c Once the first house number has been chosen, the remaining houses chosen must all have the same second digit in their house number, that is, they are not randomly chosen. For example, it is impossible for two consecutively numbered houses to be selected for the sample.

3 **a** Petra's teacher colleagues are quite likely to ignore the emailed questionnaire as emails are easy to ignore.

b It is likely that the teachers who have responded will have strong opinions either for or against the general student behaviour. These responses may therefore not be representative of all teachers' views.

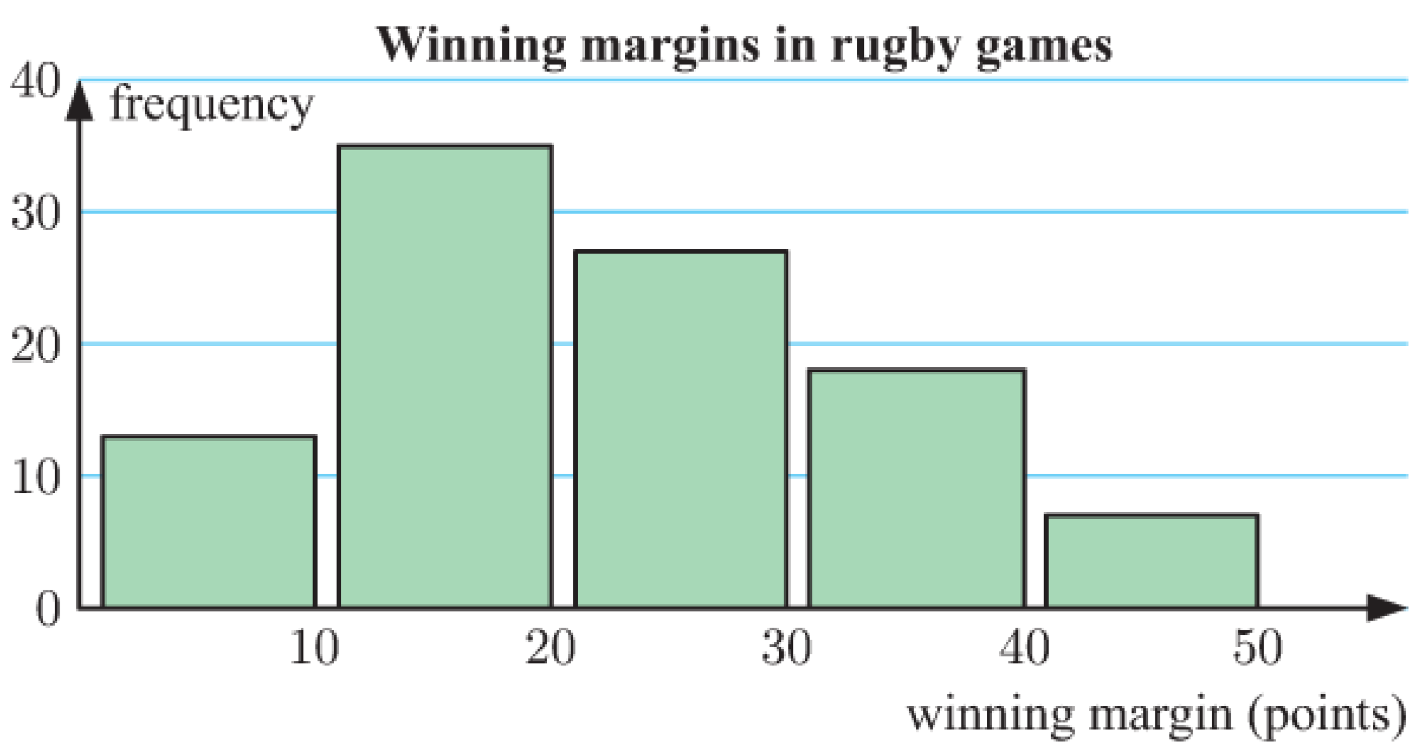
4 Did you learn about our services via:

- friends/family
- the internet
- newspaper
- television
- elsewhere?

5 **a** The tone is not neutral and it is a structured question. The only responses possible are yes or no.

b How would you describe your general behaviour when you were a child?

6



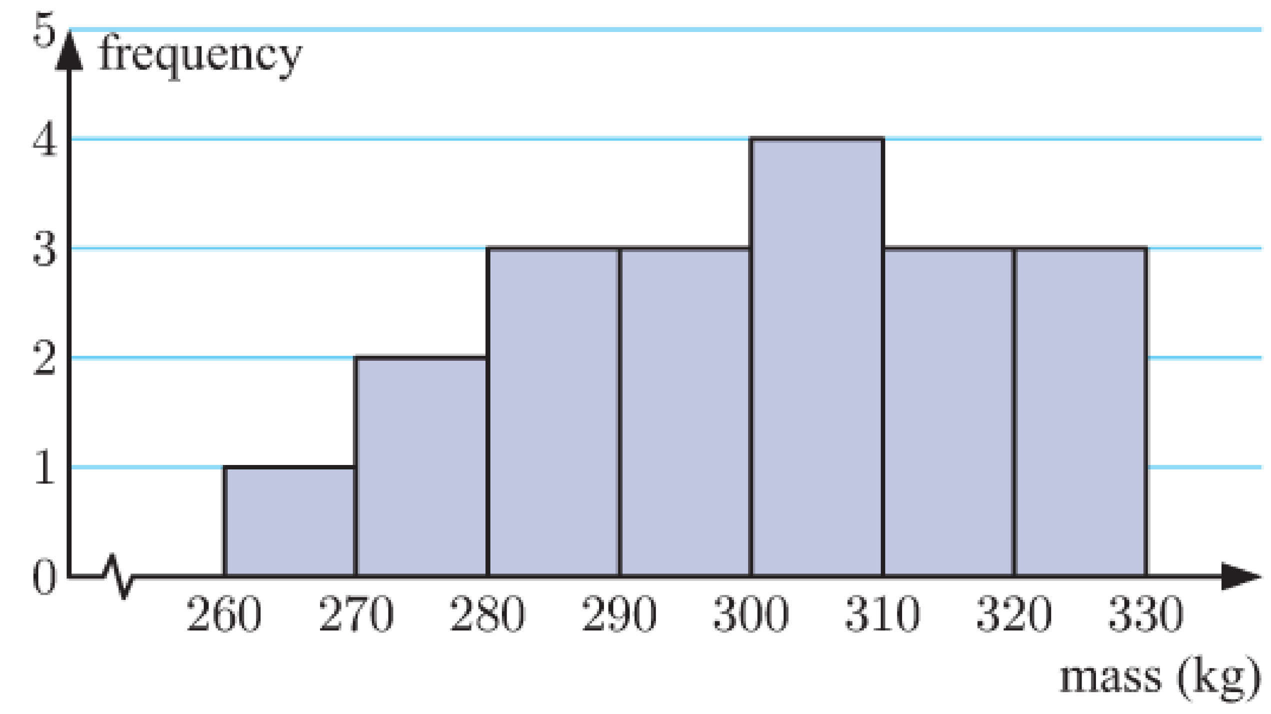
7 **a** Mass is measured on a continuous scale.

b

Mass (m kg)	*Frequency*
$260 \leqslant m < 270$	1
$270 \leqslant m < 280$	2
$280 \leqslant m < 290$	3
$290 \leqslant m < 300$	3
$300 \leqslant m < 310$	4
$310 \leqslant m < 320$	3
$320 \leqslant m < 330$	3

c $300 \leqslant m < 310$ kg

d



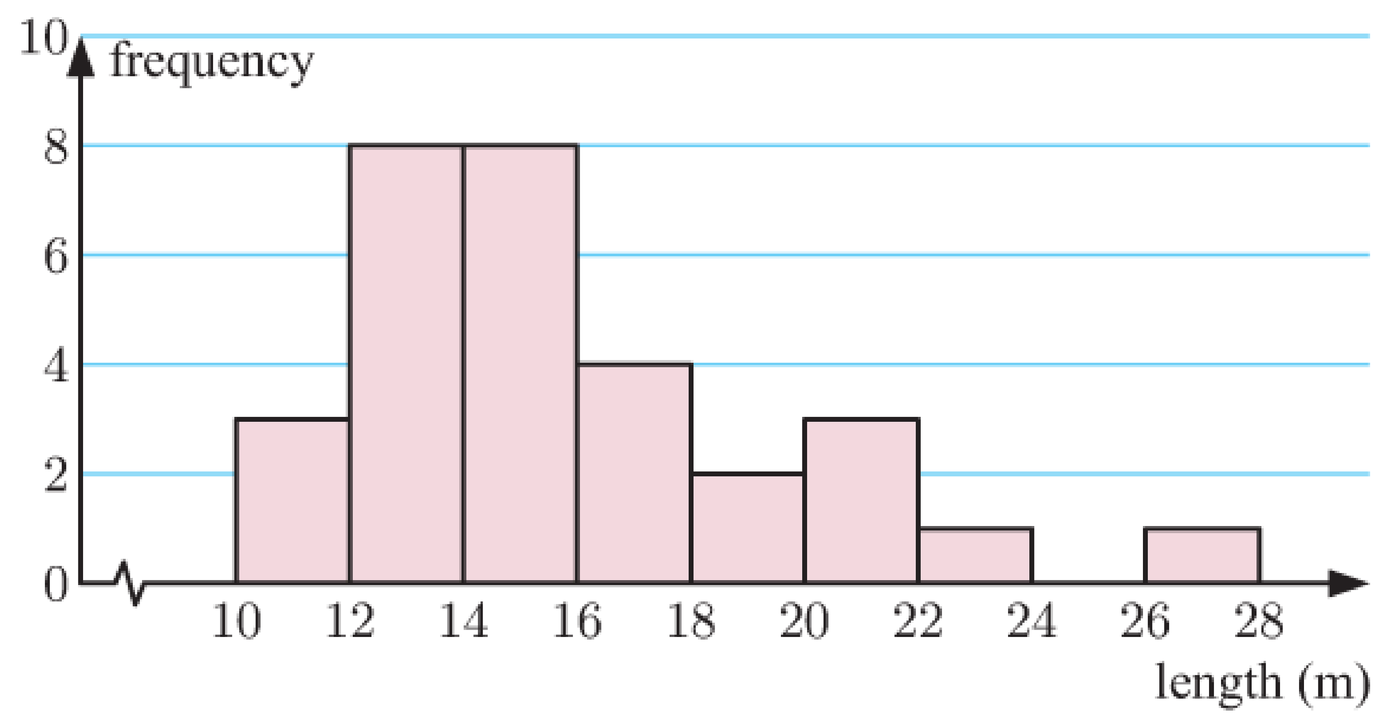
e slightly negatively skewed

8 **a** continuous

b

Length (l m)	*Frequency*
$10 \leqslant l < 12$	3
$12 \leqslant l < 14$	8
$14 \leqslant l < 16$	8
$16 \leqslant l < 18$	4
$18 \leqslant l < 20$	2
$20 \leqslant l < 22$	3
$22 \leqslant l < 24$	1
$24 \leqslant l < 26$	0
$26 \leqslant l < 28$	1

c



d positively skewed, one outlier (27.4 m)

EXERCISE 13A

1 **a** 1 cup **b** 2 cups **c** 1.8 cups

2 **a** **i** ≈ 5.61 **ii** 6 **iii** 6

b **i** ≈ 16.3 **ii** 17 **iii** 18

c **i** ≈ 24.8 **ii** 24.9 **iii** 23.5

3 9 **4** Ruth

5 **a** *data set A*: ≈ 6.46, *data set B*: ≈ 6.85

b *data set A*: 7, *data set B*: 7

c Data sets A and B differ only by their last value. This affects the mean, but not the median.

6 **a** **i** *motichoor ladoo*: ≈ 67.1, *malai jamun*: ≈ 53.6

ii *motichoor ladoo*: 69, *malai jamun*: 52

b The mean and median were much higher for the motichoor ladoo, so the motichoor ladoo were more popular.

7 **a** *Bus*: mean $= 39.7$, median $= 40.5$
Tram: mean ≈ 49.1, median $= 49$

b The tram data has a higher mean and median, but since there are more bus trips per day and more people travel by bus in total, the bus is more popular.

8 **a** 44 points **b** 44 points

c **i** Decrease, since 25 is lower than the mean of 44 for the first four matches.

ii 40.2 points

9 €185 604 **10** 116 **11** 17.25 goals per game

12 $x = 15$ **13** $a = 5$ **14** 37 marks

15 ≈ 14.8 **16** 6 and 12

EXERCISE 13B

1 **a** mean = $363 770, median = $347 200
The mean has been affected by the extreme values (the two values greater than $400 000).
b **i** the mean **ii** the median

2 **a** mode = $33 000, mean = $39 300, median = $33 500
b The mode is the lowest value in the data set.
c No, it is too close to the lower end of the distribution.

3 **a** mean ≈ 3.19 mm, median = 0 mm, mode = 0 mm
b The median is not the most suitable measure of centre as the data is positively skewed.
c The mode is the lowest value.
d 42 mm and 21 mm **e** no

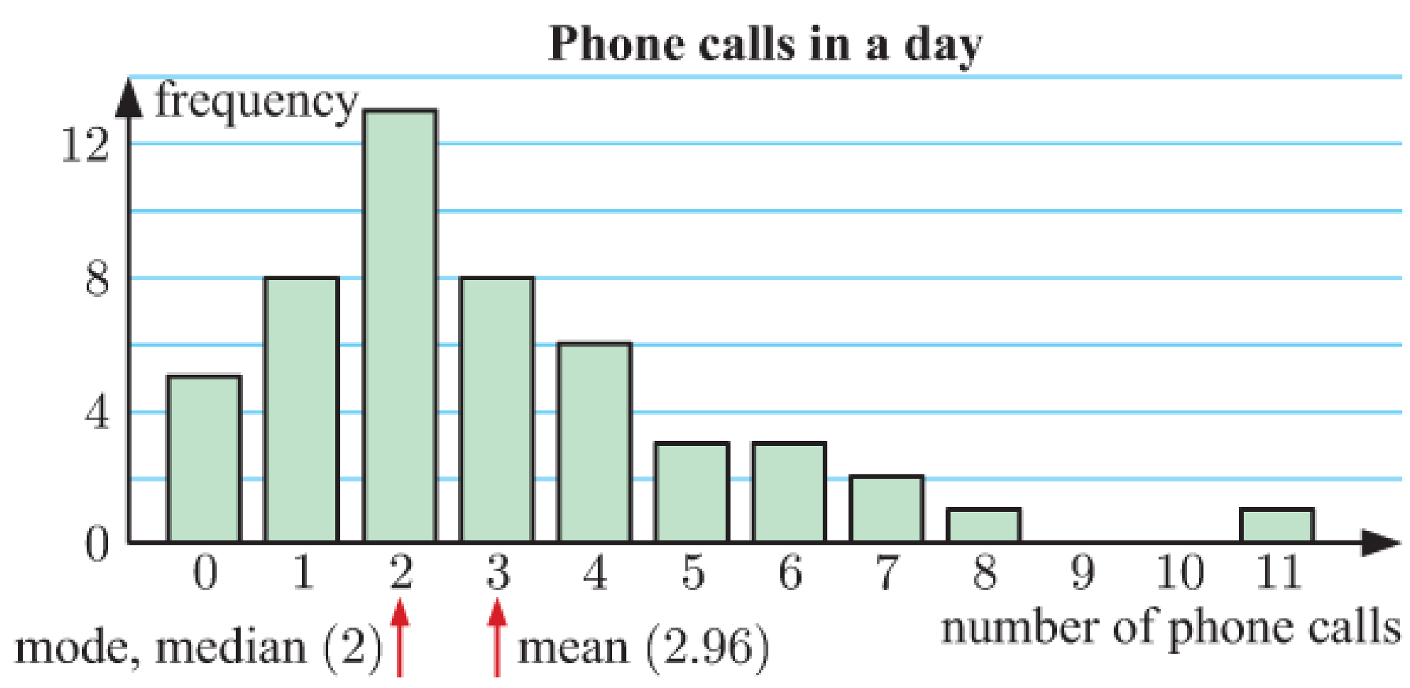
4 **a** mean ≈ 2.03, median = 2, mode = 1 and 2
b Yes, as Esmé can then offer a "family package" to match the most common number of children per family.
c 2 children, since this is one of the modes; it is also the median, and very close to the mean.

EXERCISE 13C

1 **a** 1 person **b** 2 people **c** ≈ 2.03 people

2 **a** **i** 2.96 phone calls **ii** 2 phone calls
iii 2 phone calls

b



c positively skewed
d The mean takes into account the larger numbers of phone calls.
e the mean

3 **a** **i** ≈ 2.61 children **ii** 2 children **iii** 2 children
b This school has more children per family than the average British family.
c positively skewed
d The values at the higher end increase the mean more than the median and the mode.

4 **a**

Pocket money (€)	*Frequency*
1	4
2	9
3	2
4	6
5	8

b 29 children
c **i** ≈ €3.17
ii €3
iii €2
d the mode

5 10.1 cm

6 **a** **i** $63 000 **ii** $56 000 **iii** $66 600 **b** the mean

7 **a** $x = 5$ **b** 75%

EXERCISE 13D

1 **a** 40 phone calls **b** ≈ 15 minutes **2** ≈ 31.7

3 **a** 70 service stations **b** ≈ 411 000 litres (≈ 411 kL)
c ≈ 5870 L
d $6000 < P \leqslant 7000$ L. This is the most frequently occurring amount of petrol sales at a service station in one day.

4 **a**

Runs scored	*Tally*	*Frequency*
0 - 9	~~||||~~ ~~||||~~ |	11
10 - 19	~~||||~~ |||	8
20 - 29	~~||||~~ |||	8
30 - 39	||	2
	Total	29

b ≈ 14.8 runs
c ≈ 14.9 runs; the estimate in **b** was very accurate.

5 **a** $p = 24$ **b** ≈ 3.37 minutes **c** ≈ 15.3%

6 **a** 125 people **b** ≈ 119 marks **c** $\frac{3}{25}$ **d** 28%

EXERCISE 13E

1 **a** **i** 13 **ii** $Q_1 = 9$, $Q_3 = 18$ **iii** 16 **iv** 9
b **i** 18.5 **ii** $Q_1 = 13$, $Q_3 = 23$ **iii** 19 **iv** 10
c **i** 26.5 **ii** $Q_1 = 20$, $Q_3 = 35$ **iii** 28 **iv** 15
d **i** 37 **ii** $Q_1 = 28$, $Q_3 = 52$ **iii** 49 **iv** 24

2 **a** *Jane*: mean = $35.50, median = $35.50
Ashley: mean = $30.75, median = $26.00
b *Jane*: range = $18, IQR = $9
Ashley: range = $40, IQR = $14
c Jane **d** Ashley

3 **a** range = 60, IQR = 8.5 **b** '67' is an outlier.
c range = 18, IQR = 8 **d** the range

4 **a** *Derrick*: range = 240 minutes, IQR = 30 minutes
Gareth: range = 170 minutes, IQR = 120 minutes
b **i** Gareth's **ii** Derrick's
c The IQR is most appropriate as it is less affected by outliers.

5 **a** g **b** **i** $m - a$ **ii** $\left(\frac{j+k}{2}\right) - \left(\frac{c+d}{2}\right)$

6

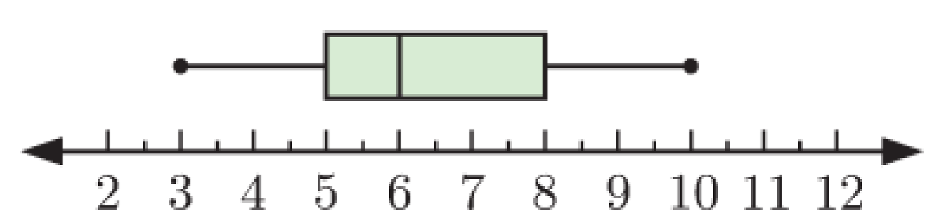
	Measure	median	mode	range	interquartile range
a	Value	11	9	13	6
b		18	14	26	12

EXERCISE 13F

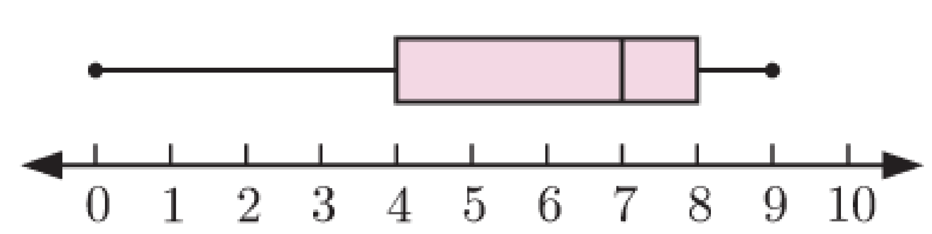
1 **a** 35 points **b** 78 points **c** 13 points **d** 53 points
e 26 points **f** 65 points **g** 27 points

2 **a** **i** 98, 25 marks **ii** 70 marks **iii** 85 marks
iv 55, 85 marks
b range = 73, IQR = 30

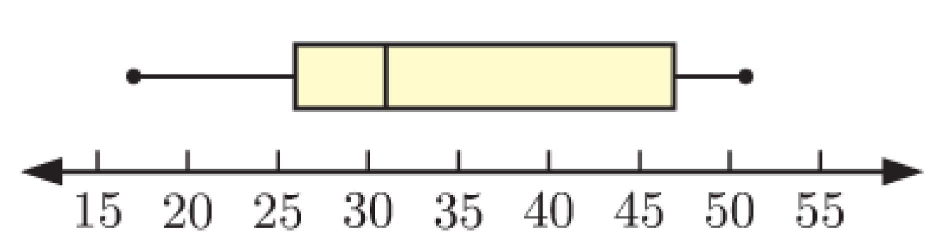
3 **a** **i** min = 3, $Q_1 = 5$, med = 6, $Q_3 = 8$, max = 10
ii



iii range = 7, IQR = 3
b **i** min = 0, $Q_1 = 4$, med = 7, $Q_3 = 8$, max = 9
ii



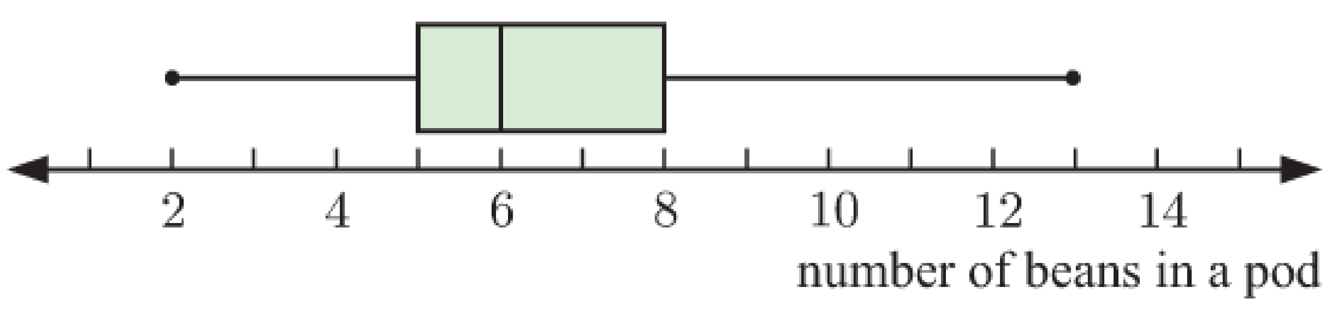
iii range = 9, IQR = 4
c **i** min = 17, $Q_1 = 26$, med = 31, $Q_3 = 47$, max = 51
ii



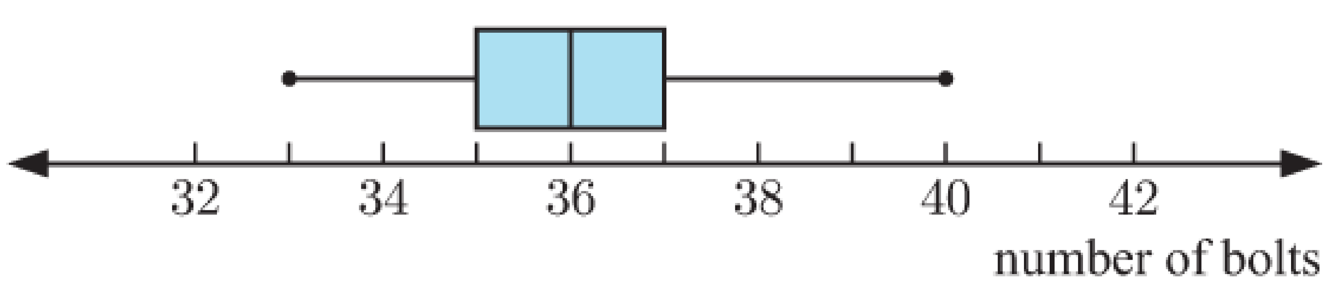
iii range = 34, IQR = 21

4 **a** median = 6, $Q_1 = 5$, $Q_3 = 8$ **b** IQR = 3

c



5 **a**

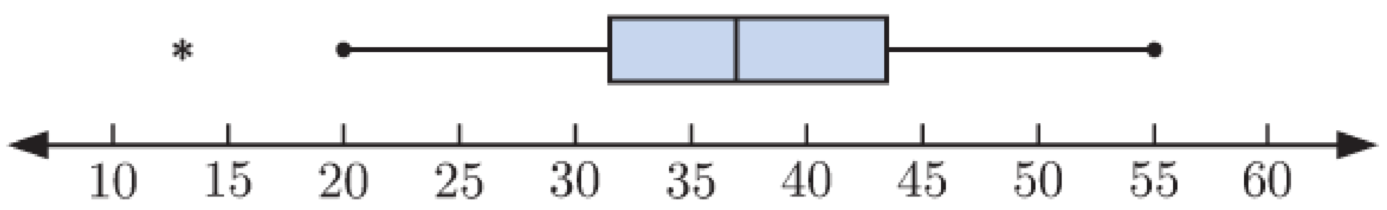


b range = 7, IQR = 2

EXERCISE 13G

1 **a** 12 **b** lower = 13.5, upper = 61.5 **c** 13

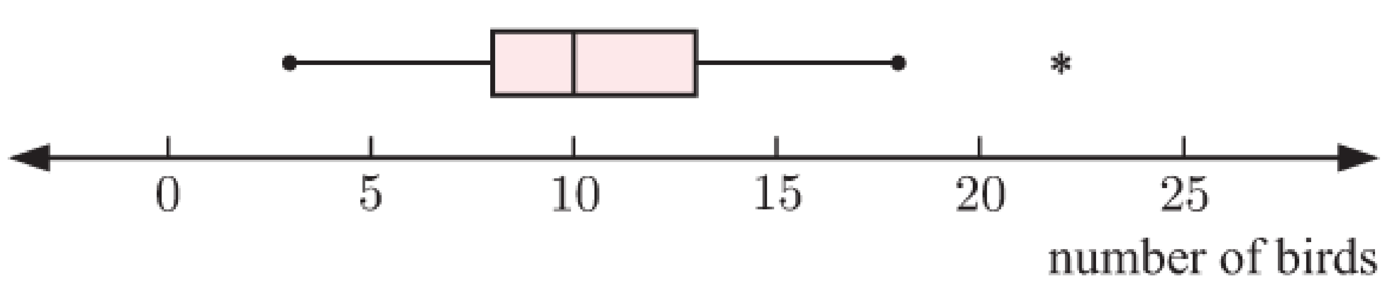
d



2 **a** median = 10, $Q_1 = 8$, $Q_3 = 13$ **b** IQR = 5

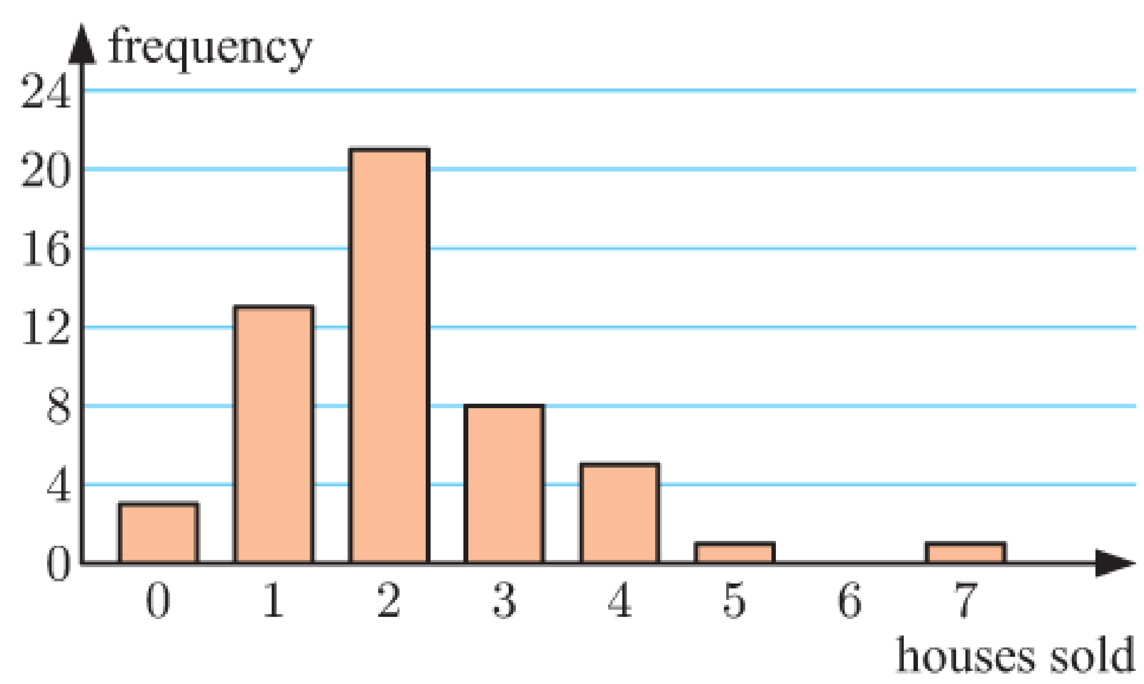
c lower = 0.5, upper = 20.5, 22 is an outlier

d



3 **a** **A** **b** **D** **c** **C** **d** **B**

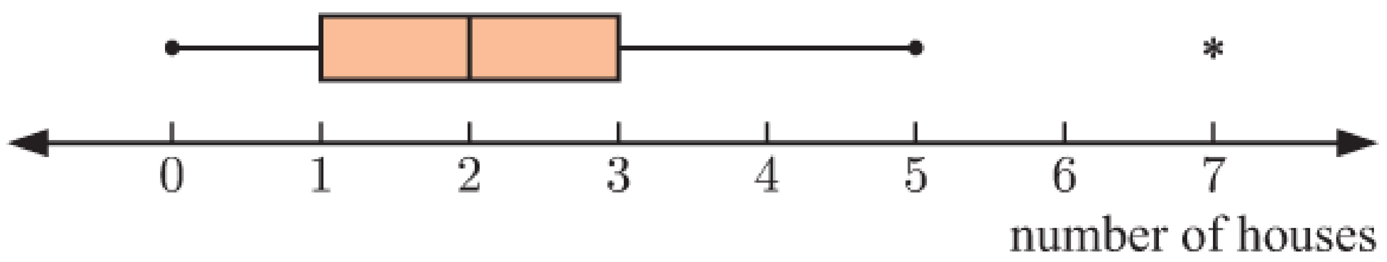
4 **a** **Houses sold by a real estate agent**



b 7 houses appears to be an outlier.

c lower boundary = −2, upper boundary = 6
7 houses is an outlier

d



EXERCISE 13H

1 **a**

Statistic	*Year 9*	*Year 12*
minimum	6	36
Q_1	30	60
median	45	84
Q_3	60	96
maximum	72	105

b **i** Year 9: 66 min
Year 12: 69 min

ii Year 9: 30 min
Year 12: 36 min

c **i** cannot tell **ii** true, since Year 9 Q_1 < Year 12 min

2 **a** Friday: min = €20, Q_1 = €50, med = €70,
Q_3 = €100, max = €180

Saturday: min = €40, Q_1 = €80, med = €100,
Q_3 = €140, max = €200

b **i** Friday: €160, Saturday: €160
ii Friday: €50, Saturday: €60

3 **a** **i** class 1 (96%) **ii** class 1 (37%) **iii** class 1

b 18% **c** 55% **d** **i** 25% **ii** 50%

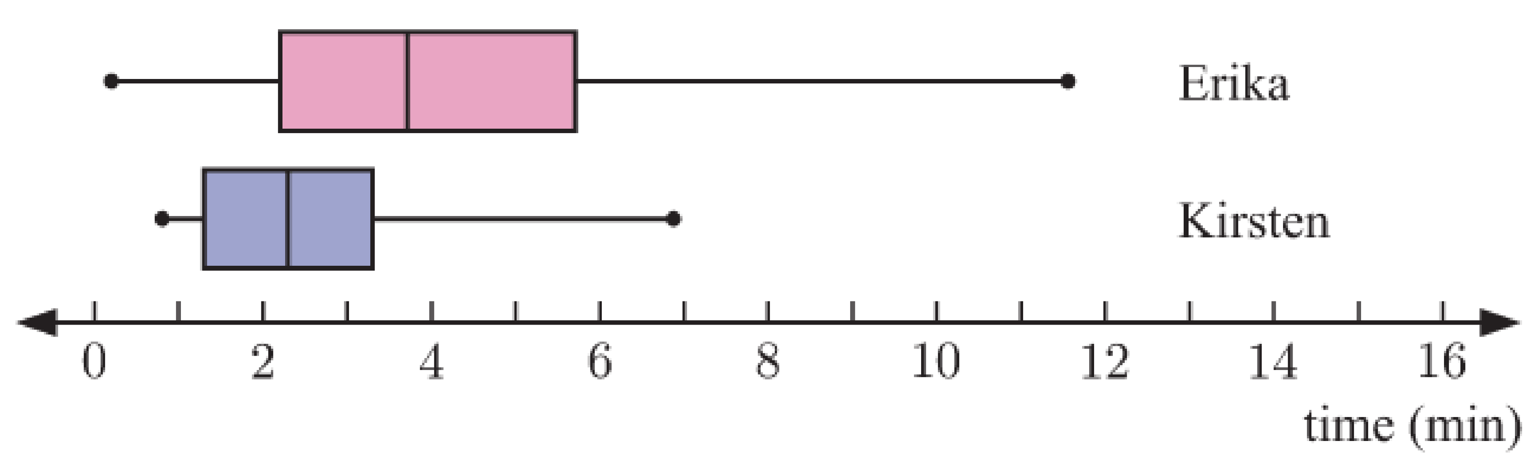
e **i** slightly positively skewed **ii** negatively skewed

f .... class 2, ... class 1

4 **a** Kirsten: min = 0.8 min, Q_1 = 1.3 min, med = 2.3 min,
Q_3 = 3.3 min, max = 6.9 min

Erika: min = 0.2 min, Q_1 = 2.2 min, med = 3.7 min,
Q_3 = 5.7 min, max = 11.5 min

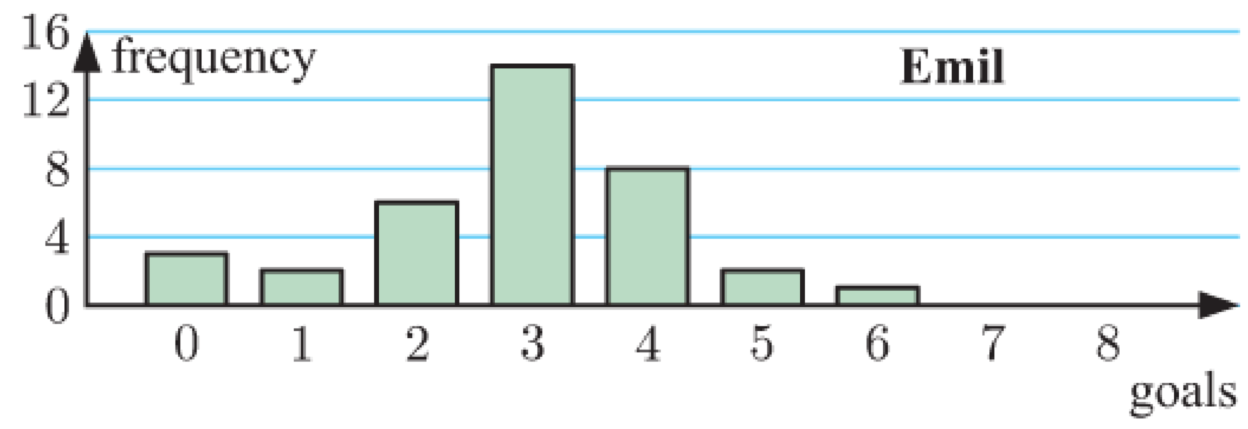
b **Phone call duration**

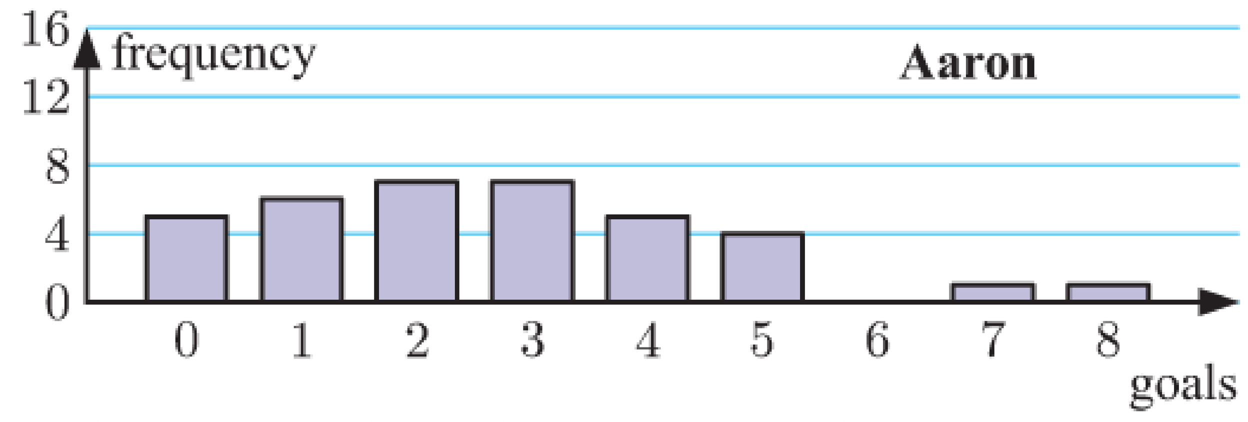


c Both are positively skewed (Erika's more so than Kirsten's).
Erika's phone calls were more varied in duration.

5 **a** discrete

c



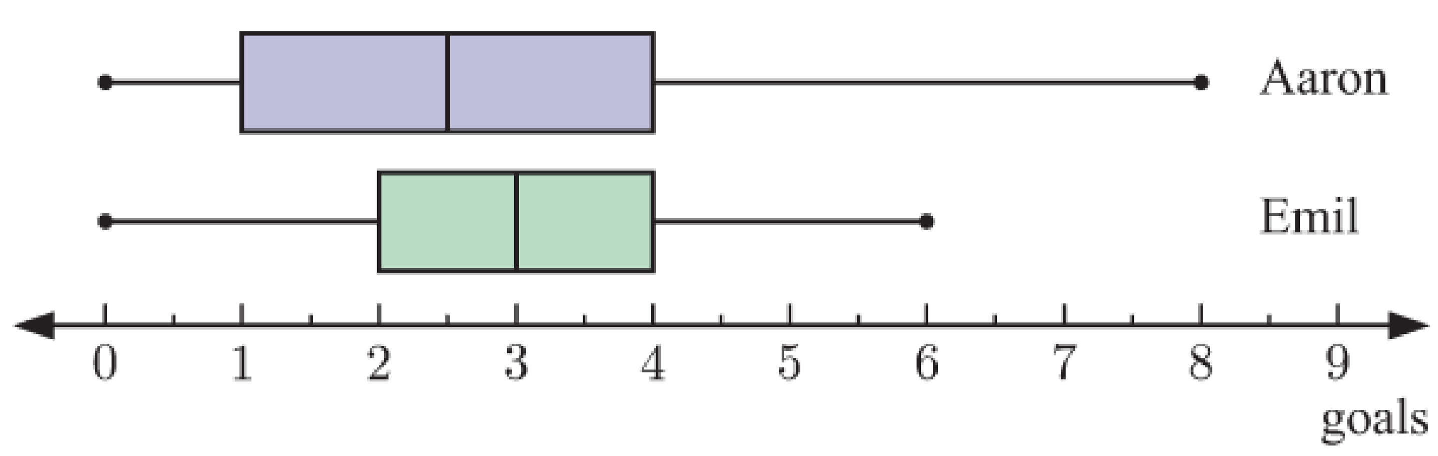


d Emil: approximately symmetrical
Aaron: positively skewed

e Emil: mean ≈ 2.89, median = 3, mode = 3
Aaron: mean ≈ 2.67, median = 2.5, mode = 2, 3
Emil's mean and median are slightly higher than Aaron's.
Emil has a clear mode of 3, whereas Aaron has two modes (2 and 3).

f Emil: range = 6, IQR = 2
Aaron: range = 8, IQR = 3
Emil's data set demonstrates less variability than Aaron's.

g



h Emil is more consistent with his scoring (in terms of goals) than Aaron.

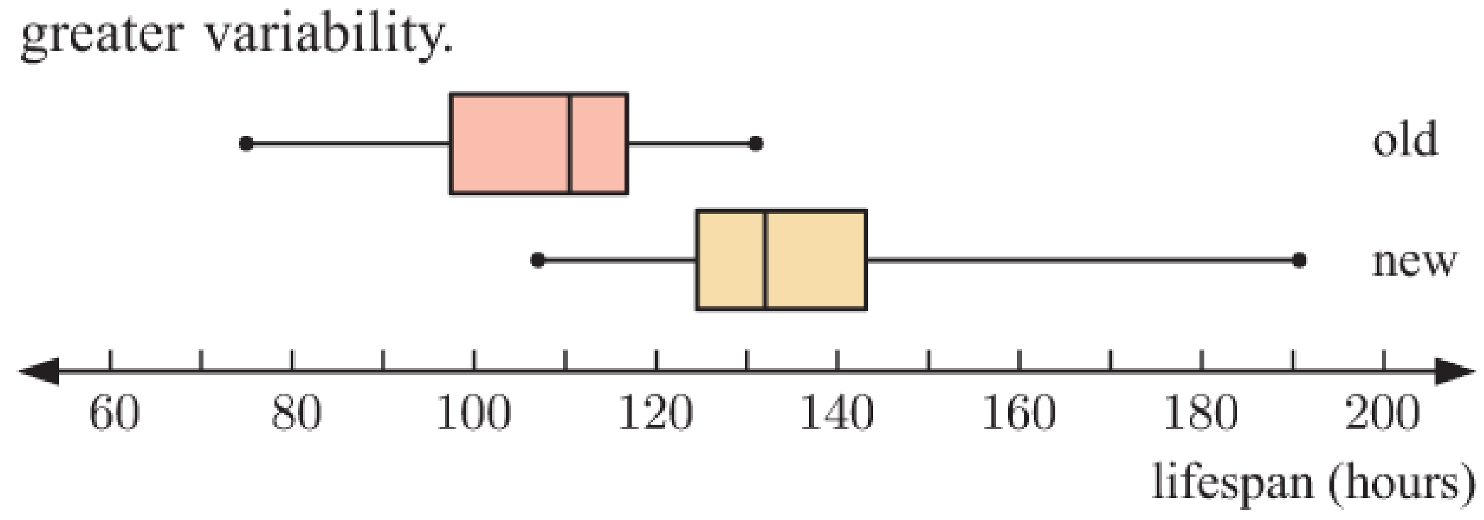
6 **a** continuous (the data is measured)

b Old type: mean = 107 hours, median = 110.5 hours,
range = 56 hours, IQR = 19 hours

New type: mean = 134 hours, median = 132 hours,
range = 84 hours, IQR = 18.5 hours

The "new" type of light globe has a higher mean and median than the "old" type.
The IQR is relatively unchanged going from "old" to "new", however, the range of the "new" type is greater, suggesting greater variability.

c

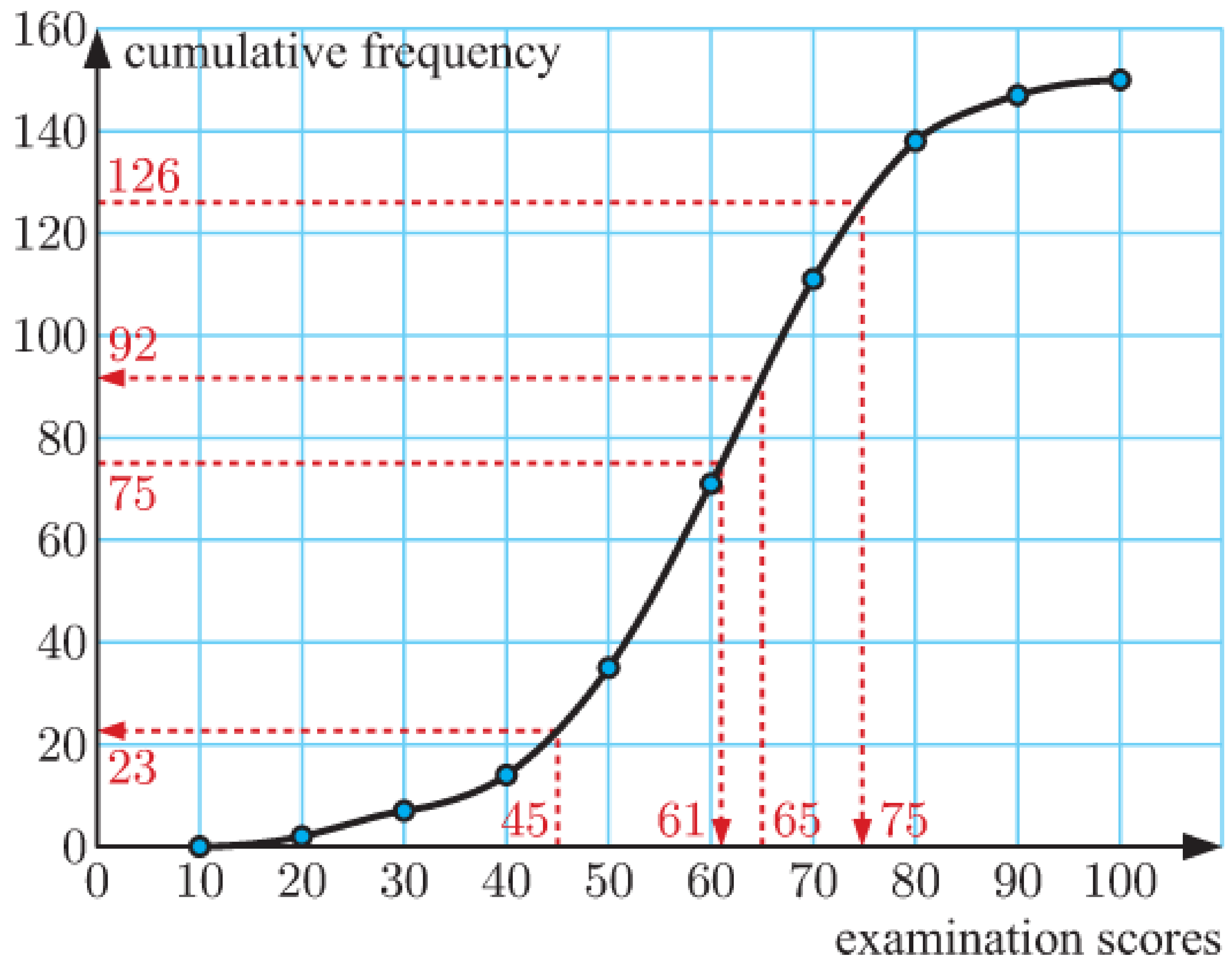


d Old type: negatively skewed
New type: positively skewed

e The "new" type of light globes do last longer than the "old" type. From **c**, both the mean and median for the "new" type are close to 20% greater than that of the "old" type. The manufacturer's claim appears to be valid.

EXERCISE 13I

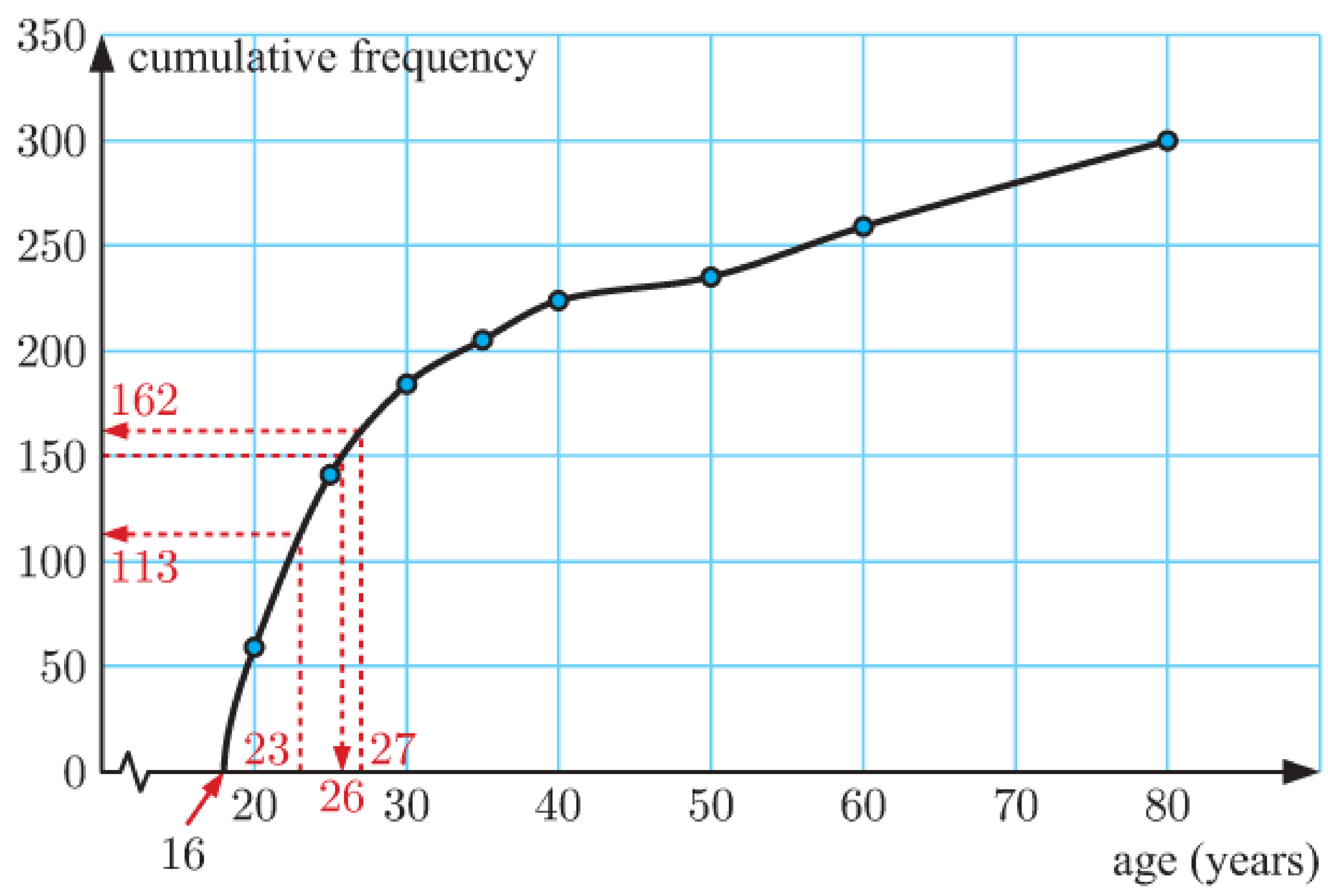
1 **a**



b ≈ 61 marks **c** ≈ 92 students **d** 76 students
e ≈ 23 students **f** ≈ 75 marks

2 **a** ≈ 9 seedlings **b** ≈ 28.3% **c** ≈ 7.1 cm
d ≈ 2.4 cm
e 10 cm, which means that 90% of the seedlings are shorter than 10 cm.

3 **a**

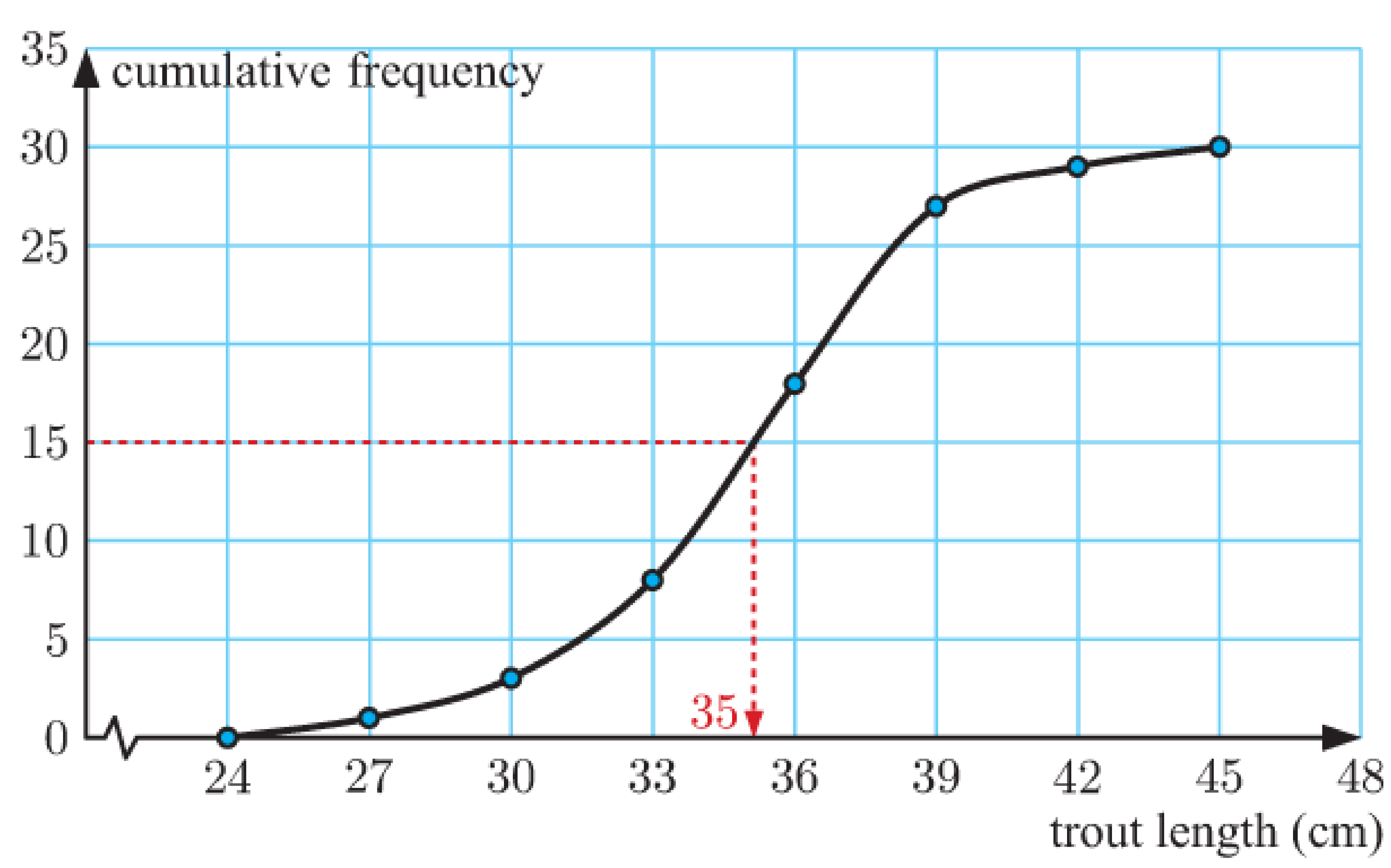


b ≈ 26 years **c** ≈ 37.7% **d** **i** ≈ 0.54 **ii** ≈ 0.04

4 **a**

Length (cm)	*Frequency*	*Cumulative frequency*
$24 \leqslant x < 27$	1	1
$27 \leqslant x < 30$	2	3
$30 \leqslant x < 33$	5	8
$33 \leqslant x < 36$	10	18
$36 \leqslant x < 39$	9	27
$39 \leqslant x < 42$	2	29
$42 \leqslant x < 45$	1	30

b



c median ≈ 35 cm
d median = 34.5 cm; the median found from the graph is a good approximation.

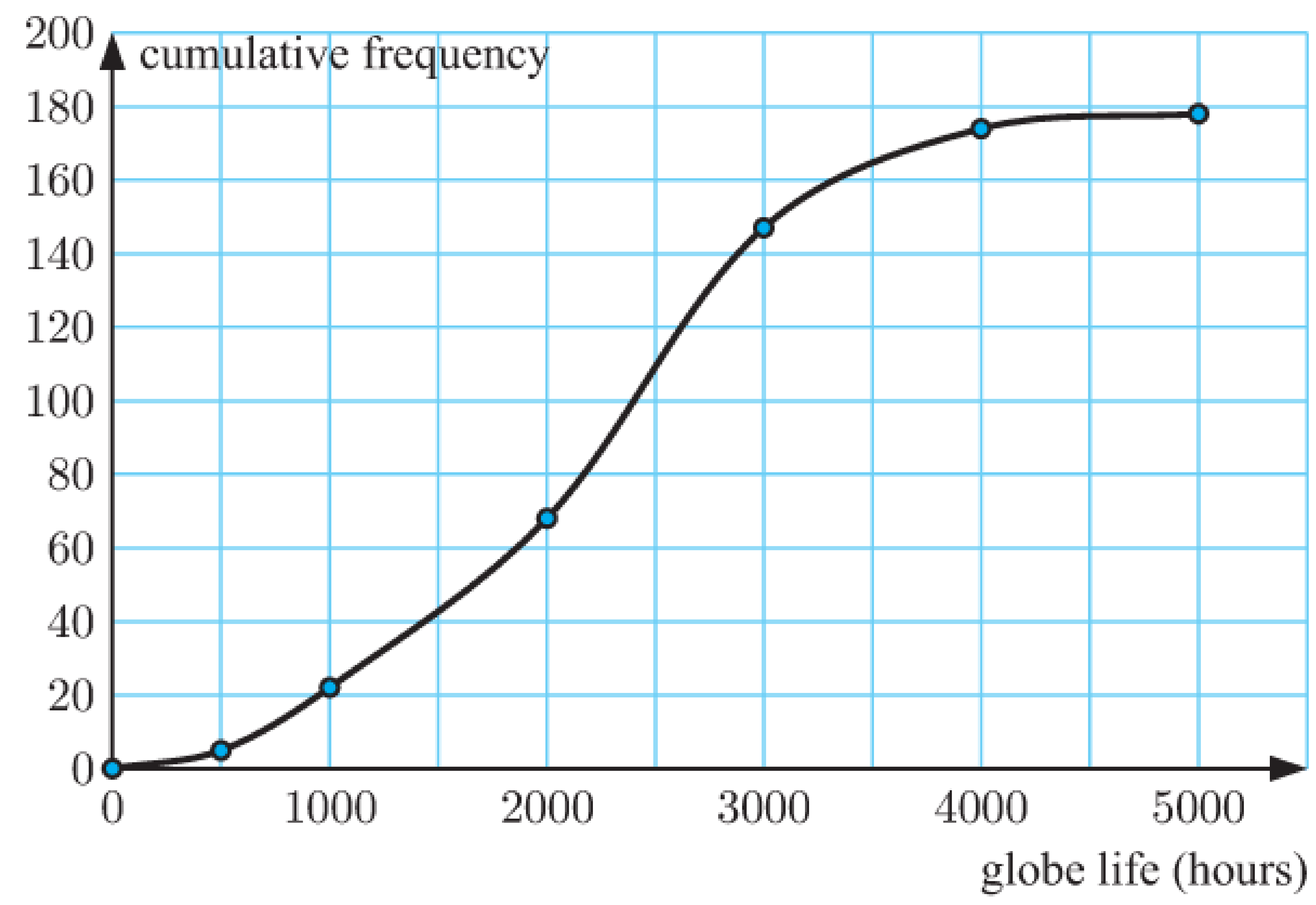
5 **a** ≈ 27 min **b** ≈ 29 min **c** ≈ 31.3 min
d ≈ 4.3 min **e** ≈ 28.2 min

f

Time (t min)	$21 \leqslant t < 24$	$24 \leqslant t < 27$	$27 \leqslant t < 30$
Number of competitors	5	15	30

Time (t min)	$30 \leqslant t < 33$	$33 \leqslant t < 36$
Number of competitors	20	10

6 **a**



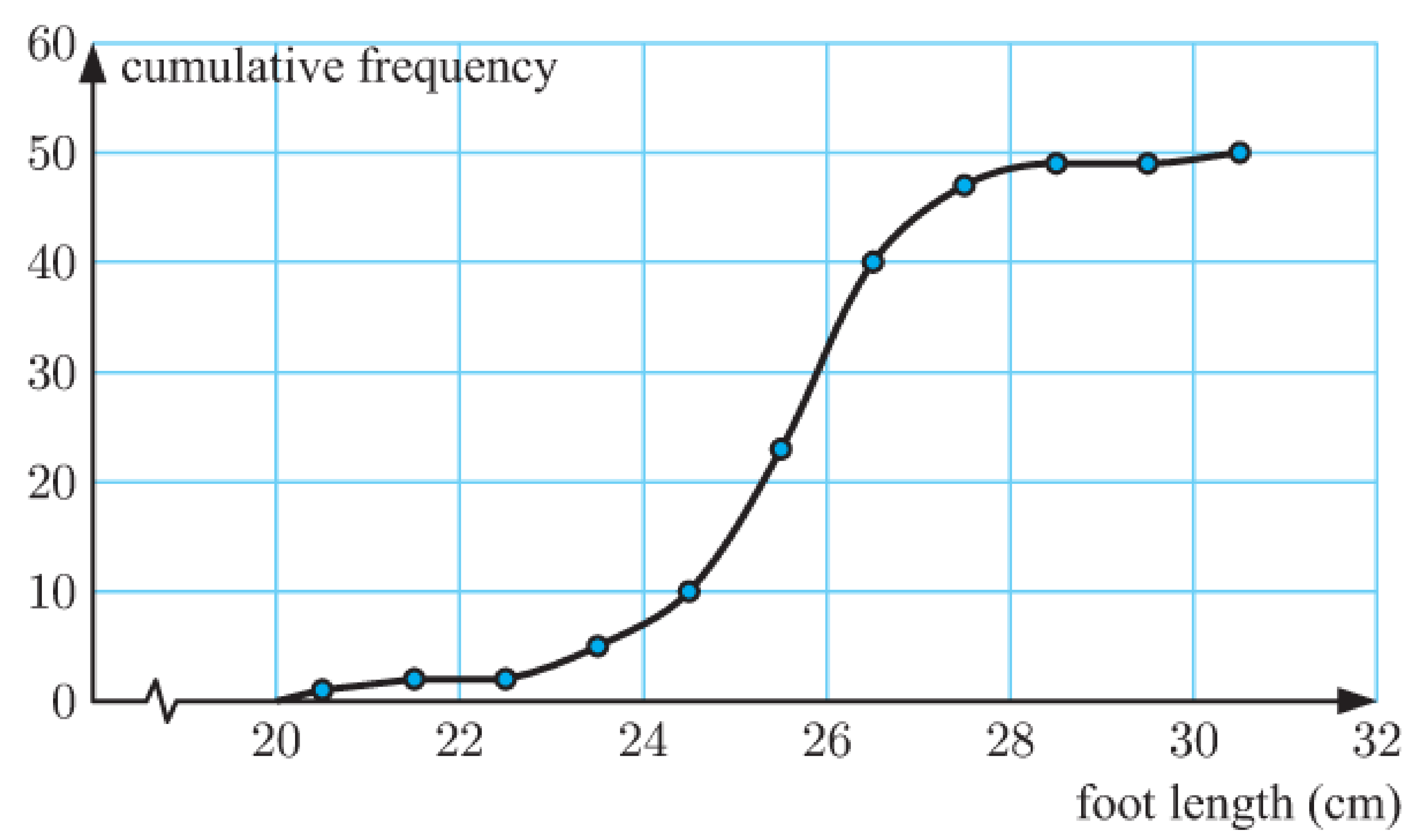
b ≈ 2280 hours **c** ≈ 71% **d** ≈ 67

7 **a** $19.5 \leqslant l < 20.5$ cm

b

Foot length (cm)	*Frequency*	*Cumulative frequency*
$19.5 \leqslant l < 20.5$	1	1
$20.5 \leqslant l < 21.5$	1	2
$21.5 \leqslant l < 22.5$	0	2
$22.5 \leqslant l < 23.5$	3	5
$23.5 \leqslant l < 24.5$	5	10
$24.5 \leqslant l < 25.5$	13	23
$25.5 \leqslant l < 26.5$	17	40
$26.5 \leqslant l < 27.5$	7	47
$27.5 \leqslant l < 28.5$	2	49
$28.5 \leqslant l < 29.5$	0	49
$29.5 \leqslant l < 30.5$	1	50

c



d **i** ≈ 25.2 cm **ii** ≈ 18 people

EXERCISE 13J

1 **a** *Data set A*: mean $= \dfrac{10+7+5+8+10}{5} = 8$

Data set B: mean $= \dfrac{4+12+11+14+1+6}{6} = 8$

b Data set B appears to have a greater spread than data set A, as data set B has more values that are a long way from the mean, such as 1 and 14.

c *Data set A*: $\sigma^2 = 3.6$, $\sigma \approx 1.90$
Data set B: $\sigma^2 \approx 21.7$, $\sigma \approx 4.65$

2 **a** The data is positively skewed. **b** $\sigma \approx 1.59$
c $\sigma^2 \approx 2.54$

3 **a** $\mu = 24.25$, $\sigma \approx 3.07$ **b** $\mu = 28.25$, $\sigma \approx 3.07$
c If each data value is increased or decreased by the same amount, then the mean will also be increased or decreased by that amount, however the population standard deviation will be unchanged.

4 $\sigma \approx 2.64$, $s \approx 2.71$

5 **a** *Danny*: ≈ 3.21 hours; *Jennifer*: 2 hours
b Danny
c *Danny*: $\sigma \approx 0.700$ hours, $s \approx 0.726$ hours;
Jennifer: $\sigma \approx 0.423$ hours, $s \approx 0.439$ hours
d Jennifer

6 **a**

			Standard deviation		
	Mean $\overline{x}$	Median	σ	s	Range
Boys	32.02	31.05	≈ 4.52	≈ 4.77	13.8
Girls	34.77	35.85	≈ 3.76	≈ 3.96	11.7

b **i** boys **ii** boys
c Tyson could increase his sample size.

7 **a** *Rockets*: mean $= 5.7$, range $= 11$
Bullets: mean $= 5.7$, range $= 11$
b We suspect the Rockets, since they twice scored zero runs.
Rockets: $\sigma = 3.9$, $s \approx 4.11$ $\leftarrow$ greater variability
Bullets: $\sigma \approx 3.29$, $s \approx 3.47$
c standard deviation

8 **a** **i** *Museum*: ≈ 934 visitors; *Art gallery*: ≈ 1230 visitors
ii *Museum*: $\sigma \approx 208$ visitors, $s \approx 211$ visitors;
Art gallery: $\sigma \approx 84.6$ visitors, $s \approx 86.0$ visitors
b the museum
c **i** '0' is an outlier.
ii This outlier corresponded to Christmas Day, so the museum was probably closed which meant there were no visitors on that day.
iii Yes, although the outlier is not an error, it is not a true reflection of a visitor count for a particular day.
iv *Museum*: mean ≈ 965 visitors, $\sigma \approx 121$ visitors, $s \approx 123$ visitors
v The outlier had greatly increased the population standard deviation.

9 $s_A > s_B$ does not imply that $\sigma_A > \sigma_B$.
Hint: Find a counter example.

10 $p = 6$, $q = 9$ **11** $a = 8$, $b = 6$ **12** **b** $\mu = \pm 8.7$

13 $\sigma \approx 0.775$ **14** $\mu = 14.48$ years, $\sigma \approx 1.75$ years

15 **a** Data set A **b** *Data set A*: 8, *Data set B*: 8
c *Data set A*: 2, *Data set B*: ≈ 1.06
Data set A does have a wider spread.
d The standard deviation takes all of the data values into account, not just two.

16 **a** The female students' marks are in the range 16 to 20 whereas the male students' marks are in the range 12 to 19.
i the females **ii** the males
b *Females*: $\mu \approx 17.5$, $\sigma \approx 1.02$
Males: $\mu \approx 15.5$, $\sigma \approx 1.65$

17 The results for the mean will differ by 1, but the results for the standard deviation will be the same. Jess' question is worded so that the respondent will not include themselves.

18 **a** $\overline{x} \approx 48.3$ cm **b** $\sigma \approx 2.66$ cm, $s \approx 2.70$ cm

19 **a** $\overline{x} \approx 17.45$ **b** $\sigma \approx 7.87$, $s \approx 7.91$

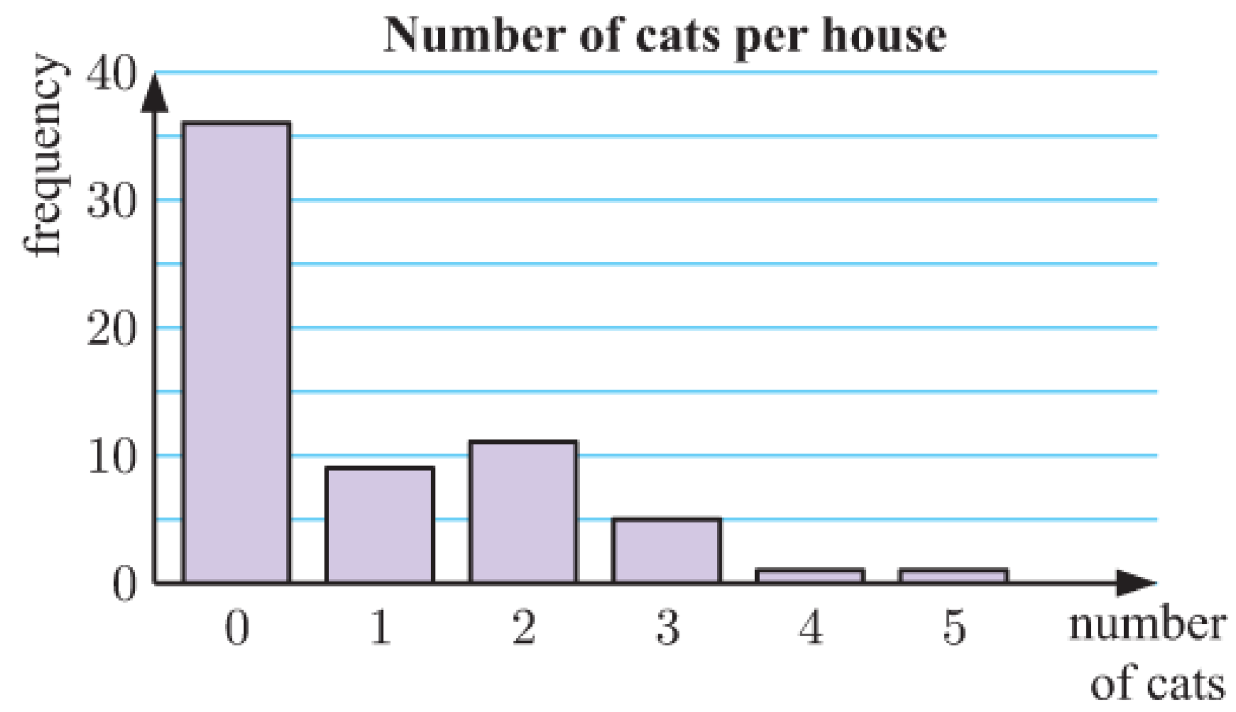
20 **a** $\overline{x} \approx \$780.60$ **b** $\sigma \approx \$31.74$, $s \approx \$31.82$

21 **a** $\overline{x} = 40.35$ hours, $\sigma \approx 4.23$ hours, $s \approx 4.28$ hours
b $\overline{x} = 40.6$ hours, $\sigma \approx 4.10$ hours, $s \approx 4.15$ hours
The mean increases slightly; the standard deviation decreases slightly. These are good approximations.

REVIEW SET 13A

1 **a** **i** ≈ 4.67 **ii** 5 **b** **i** 3.99 **ii** 3.9

2 **a**



b positively skewed
c **i** 0 cats **ii** ≈ 0.873 cats **iii** 0 cats
d The mean, as it suggests that some people have cats. (The mode and median are both 0.)

3 **a**

Distribution	Girls	Boys
median	36 s	34.5 s
mean	36 s	34.45 s
modal class	34.5 - 35.5 s	34.5 - 35.5 s

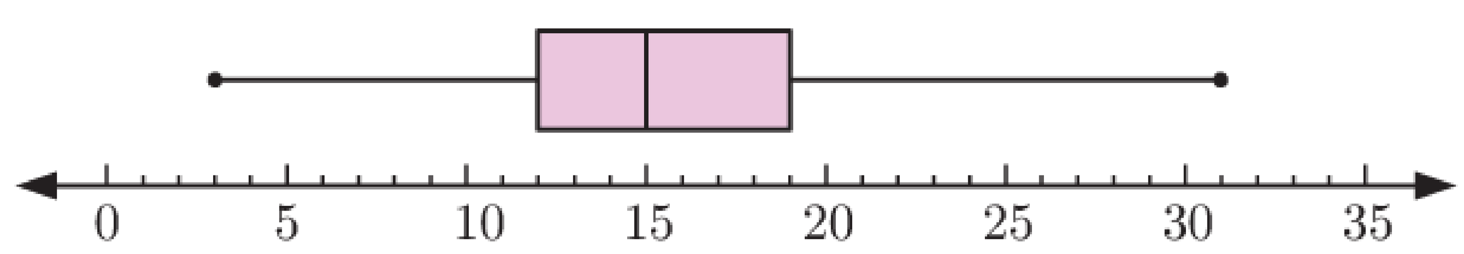
b The girls' distribution is positively skewed and the boys' distribution is approximately symmetrical. The median and mean swim times for boys are both about 1.5 seconds lower than for girls. Despite this, the distributions have the same modal class because of the skewness in the girls' distribution. The analysis supports the conjecture that boys generally swim faster than girls with less spread of times.

4 $a = 8$, $b = 6$

5 **b** $k + 3$

6 **a** We do not know each individual data value, only the intervals they fall in, so we cannot calculate the mean winning margin exactly.
b ≈ 22.6 points

7 **a** min $= 3$, $Q_1 = 12$, med $= 15$, $Q_3 = 19$, max $= 31$
b range $= 28$, IQR $= 7$
c



8 **a** 101.5 **b** 7.5 **c** 100.2 **d** ≈ 7.59

9 **a** A: min $= 11$ s, $Q_1 = 11.6$ s, med $= 12$ s, $Q_3 = 12.6$ s, max $= 13$ s
B: min $= 11.2$ s, $Q_1 = 12$ s, med $= 12.6$ s, $Q_3 = 13.2$ s, max $= 13.8$ s
b A: range $= 2.0$ s, IQR $= 1.0$ s
B: range $= 2.6$ s, IQR $= 1.2$ s
c **i** A, the median time is lower.
ii B, the range and IQR are higher.

10 **a** ≈ 58.5 s **b** ≈ 6 s **c** ≈ 53 s

11 **a** ≈ 88 students **b** $m \approx 24$

c

Time (t min)	*Frequency*
$5 \leqslant t < 10$	20
$10 \leqslant t < 15$	40
$15 \leqslant t < 20$	48
$20 \leqslant t < 25$	42
$25 \leqslant t < 30$	28
$30 \leqslant t < 35$	17
$35 \leqslant t < 40$	5

12 **a** $\sigma^2 \approx 63.0,\ \ \sigma \approx 7.94$ **b** $\sigma^2 \approx 0.969,\ \ \sigma \approx 0.984$

13 **a** $\overline{x} \approx 49.6$ matches, $\sigma \approx 1.60$ matches, $s \approx 1.60$ matches
b The claim is not justified, but a larger sample is needed.

14 **a** $\overline{x} \approx 33.6$ L **b** $\sigma \approx 7.63$ L, $s \approx 7.66$ L

15 **a** No, extreme values have less effect on the standard deviation of a larger population.
b **i** mean **ii** standard deviation
c A low standard deviation means that the weight of biscuits in each packet is, on average, close to 250 g.

REVIEW SET 13B

1 **a**

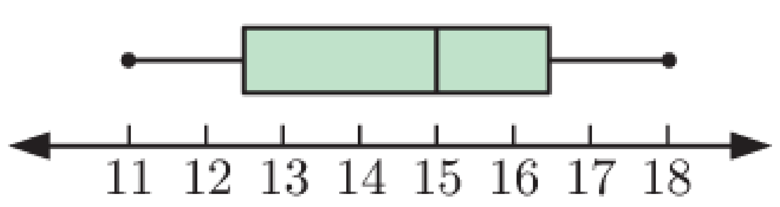
	mean (seconds)	median (seconds)
Week 1	≈ 16.0	16.3
Week 2	≈ 15.1	15.1
Week 3	≈ 14.4	14.3
Week 4	14.0	14.0

b Yes, Heike's mean and median times have gradually decreased each week which indicates that her speed has improved over the 4 week period.

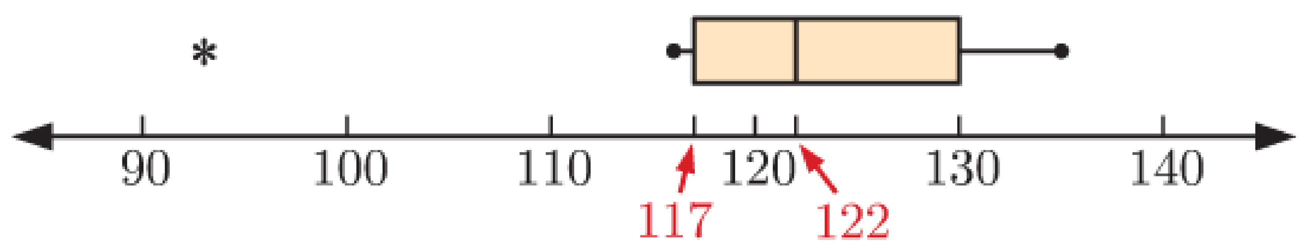
2 **a** 5 **b** 3.52 **c** 3.5 **3** **a** $x = 7$ **b** 6

4 $p = 7,\ \ q = 9$ (or $p = 9,\ \ q = 7$) **5** ≈ 414 patrons

6



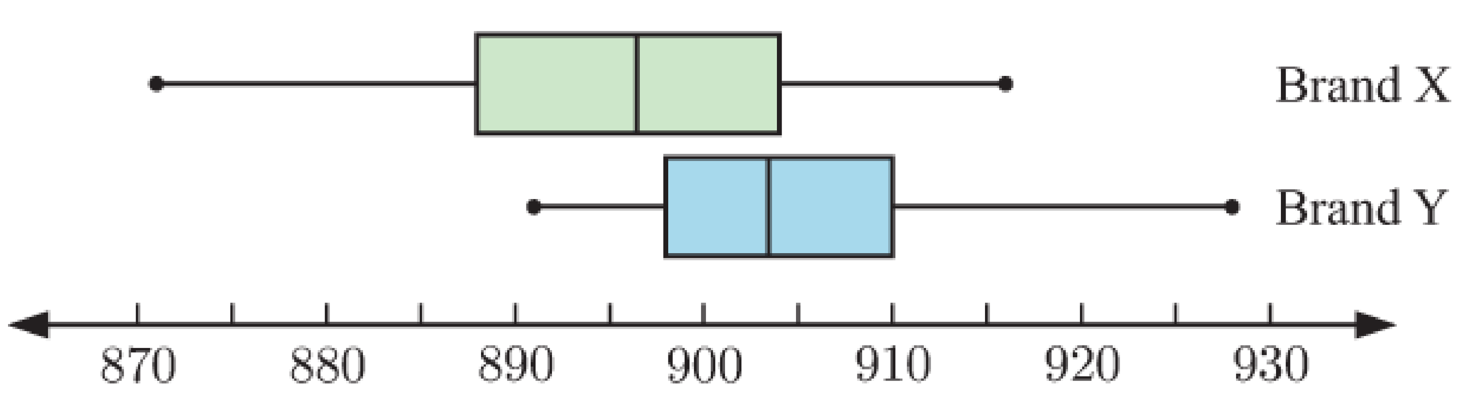
7 **a** $\sigma \approx 11.7,\ \ s \approx 12.4$ **b** $Q_1 = 117,\ \ Q_3 = 130$
c yes, 93
d



8 **a**

	Brand X	*Brand Y*
min	871	891
Q_1	888	898
median	896.5	903.5
Q_3	904	910
max	916	928
IQR	16	12

b



c **i** Brand Y, as the median is higher.
ii Brand Y, as the IQR is lower, so less variation.

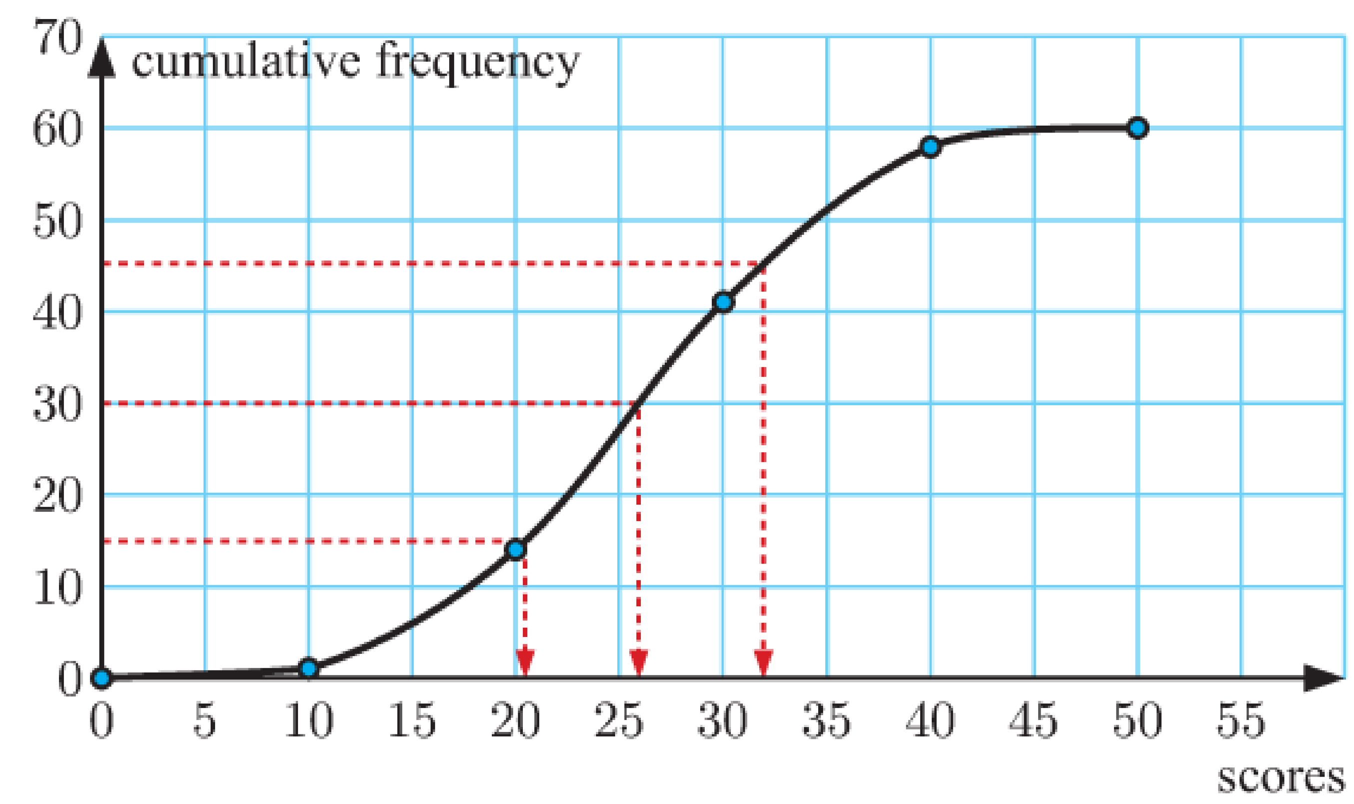
9 **a** $p = 12,\ \ m = 6$ **c** $\overline{x} = \frac{254}{30} = \frac{127}{15}$

b

Measure	*Value*
mode	9
median	9
range	4

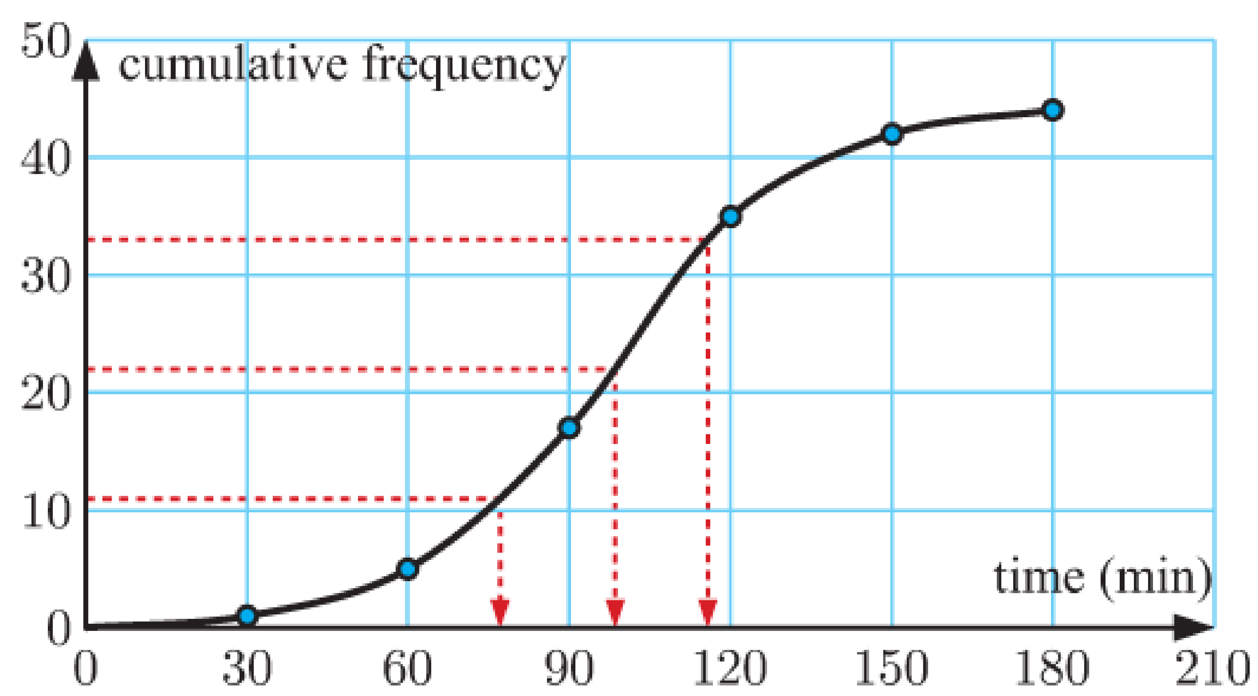
10 **a** ≈ 77 days **b** ≈ 12 days

11 **a**



b **i** median ≈ 26 **ii** IQR ≈ 11.5
iii $\overline{x} \approx 26.0$ **iv** $\sigma \approx 8.31$

12 **a** 44 players **b** $90 \leqslant t < 120$ min
c



d **i** ≈ 98.6 min **ii** ≈ 96.8 min **iii** no
e ".... between 77.2 and 115.7 minutes."

13 **a** $\overline{x} \approx$ €207.02 **b** $\sigma =$ €38.80, $s \approx$ €38.89

14 **a** Kevin: $\overline{x} = 41.2$ min; Felicity: $\overline{x} = 39.5$ min
b Kevin: $\sigma \approx 7.61$ min, $s \approx 7.81$ min;
Felicity: $\sigma \approx 9.22$ min, $s \approx 9.46$ min
c Felicity **d** Kevin

15 10 data values

EXERCISE 14A

1 **a** $y = x^2 - 3x + 1$

x	-2	-1	0	1	2
y	11	5	1	-1	-1

b $y = x^2 + 2x - 5$

x	-2	-1	0	1	2
y	-5	-6	-5	-2	3

c $y = 2x^2 - x + 3$

x	-4	-2	0	2	4
y	39	13	3	9	31

d $y = -3x^2 + 2x + 4$

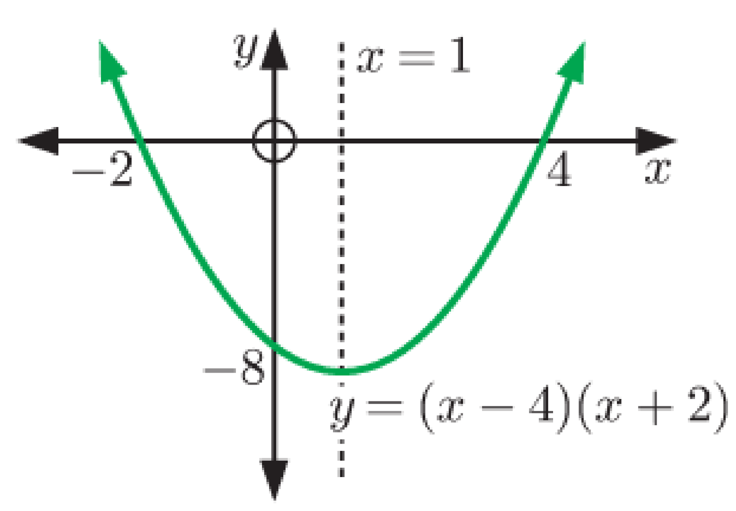
x	-4	-2	0	2	4
y	-52	-12	4	-4	-36

2 **a** no **b** yes **c** yes **d** yes **e** no **f** yes

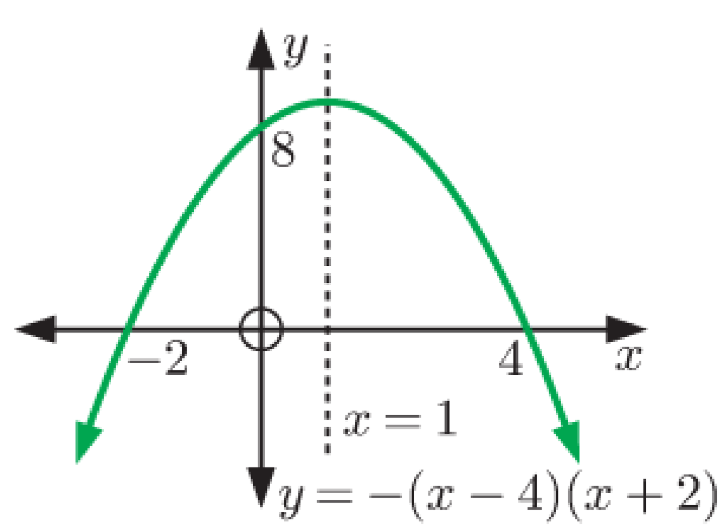
3 **a** $x = -1$ or -2 **b** $x = 2$ **c** $x = 1$ or 5
d $x = -3$ or $\frac{1}{2}$ **e** $x = -6$ or 1 **f** no real solutions

EXERCISE 14B.1

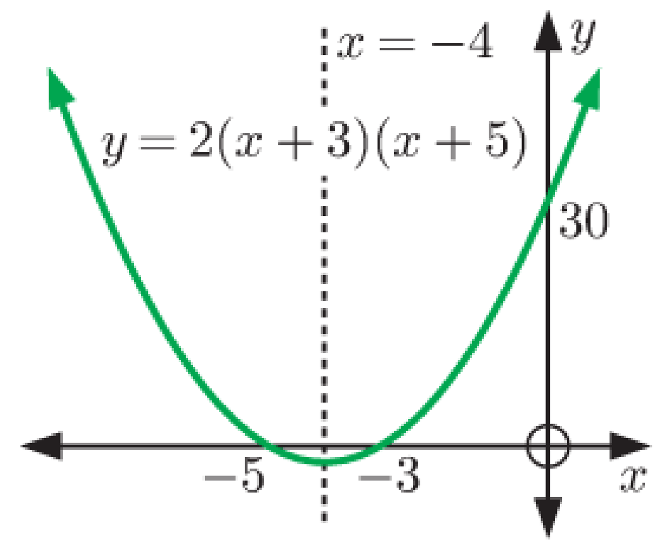
1 **a**



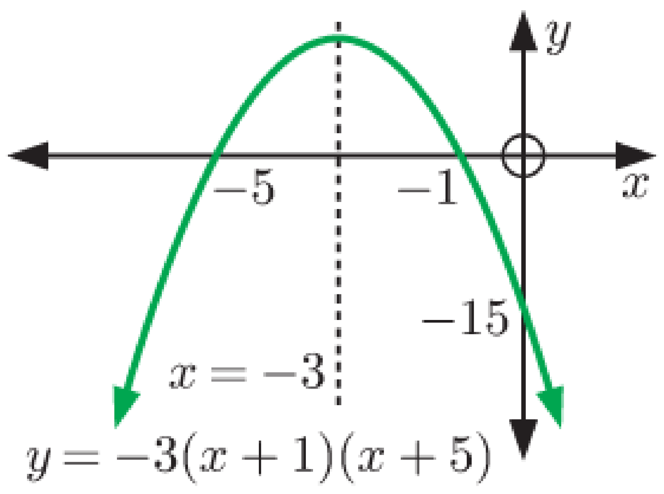
b



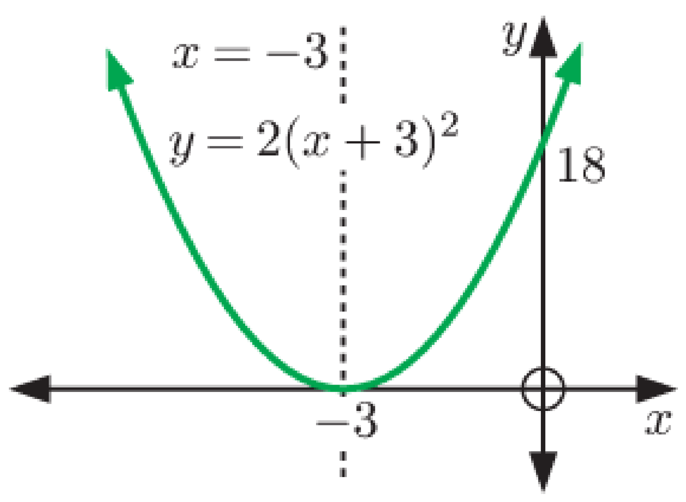
c



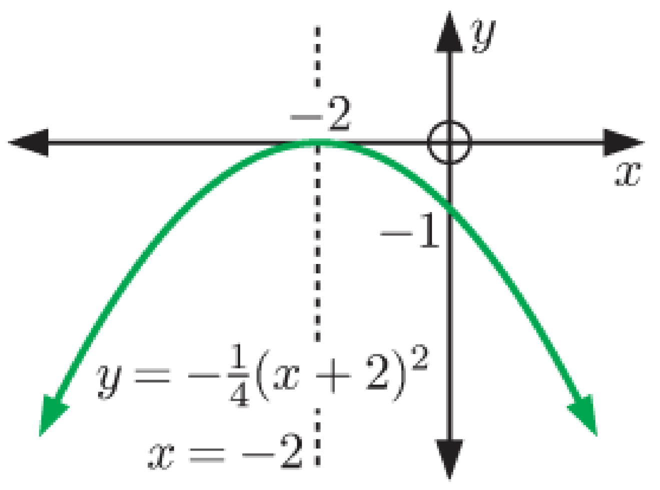
d



e

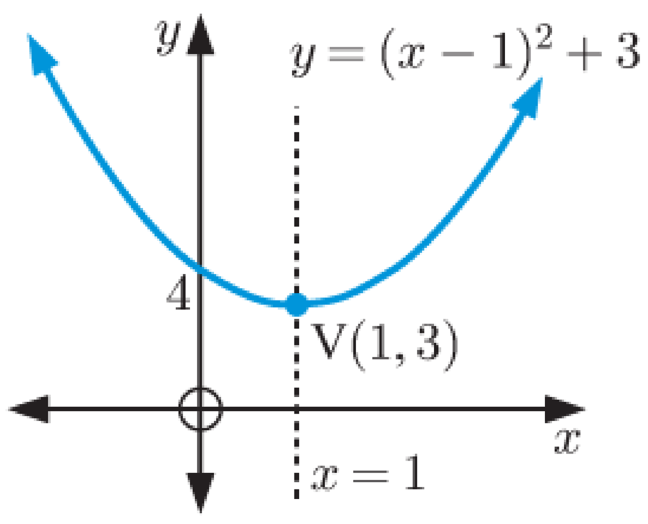


f



2 **a** C **b** E **c** B **d** F **e** G **f** H
g I **h** A **i** D

3 **a**



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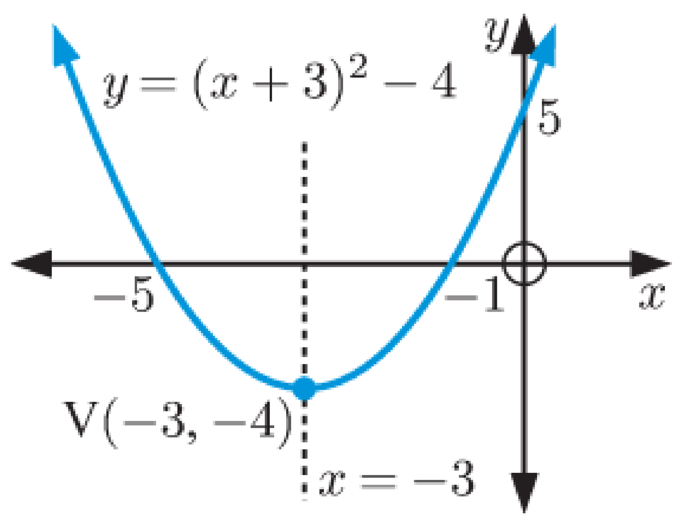

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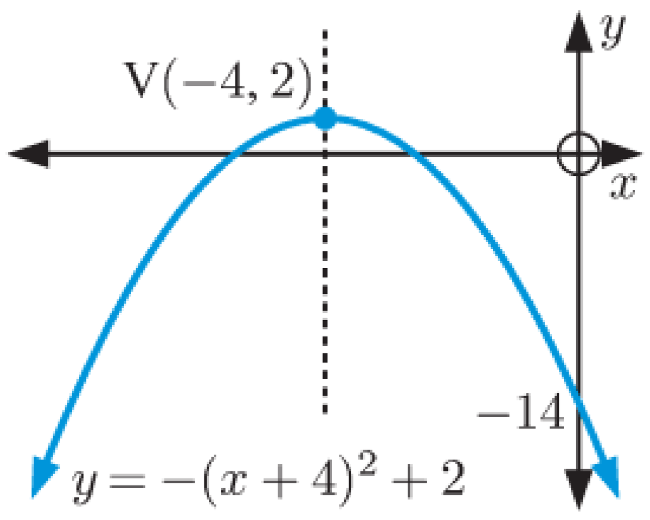

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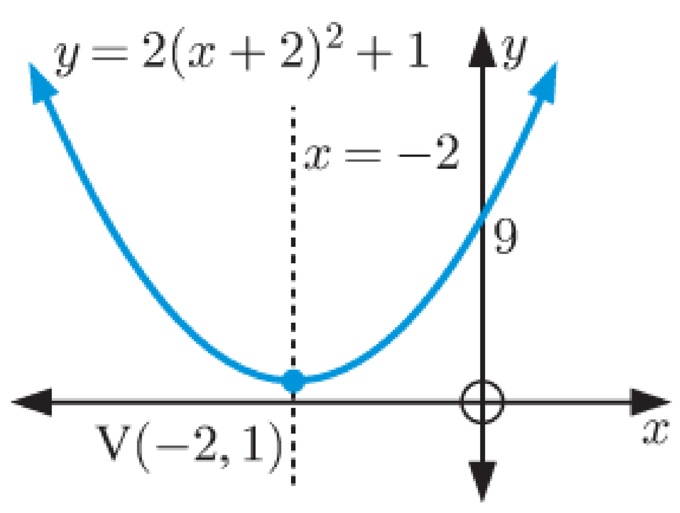

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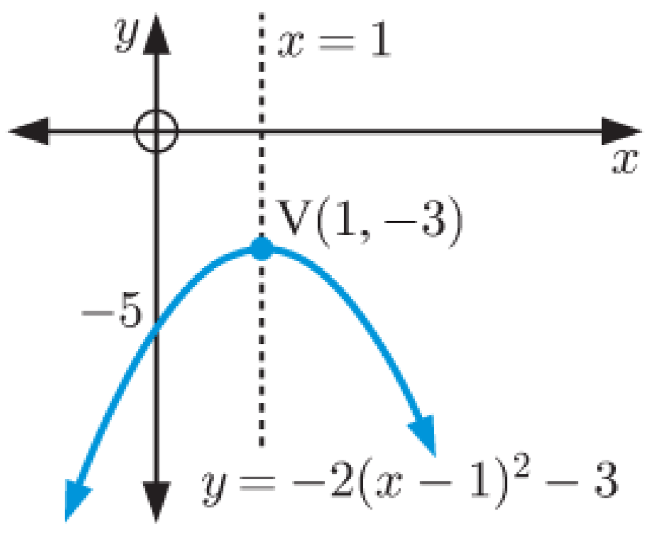

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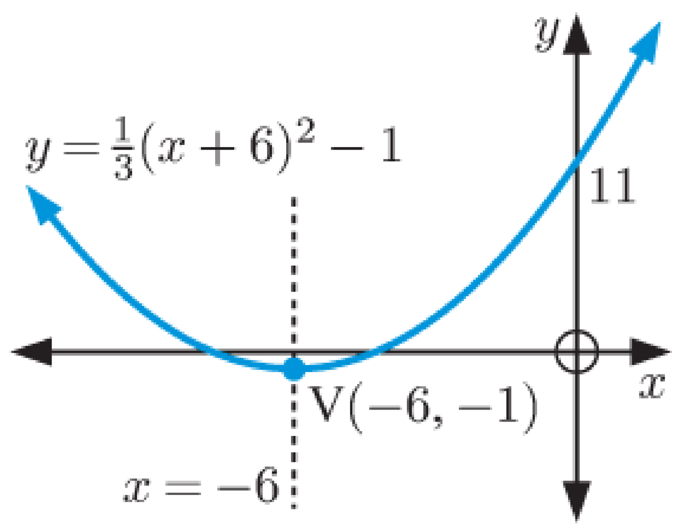

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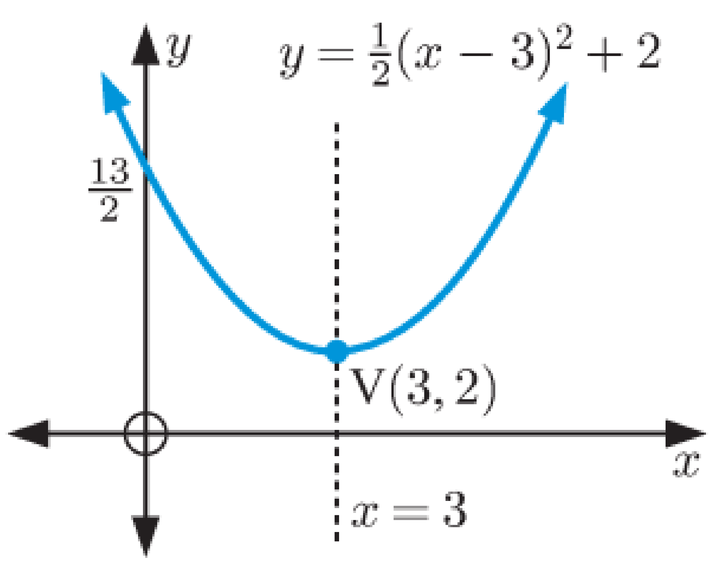

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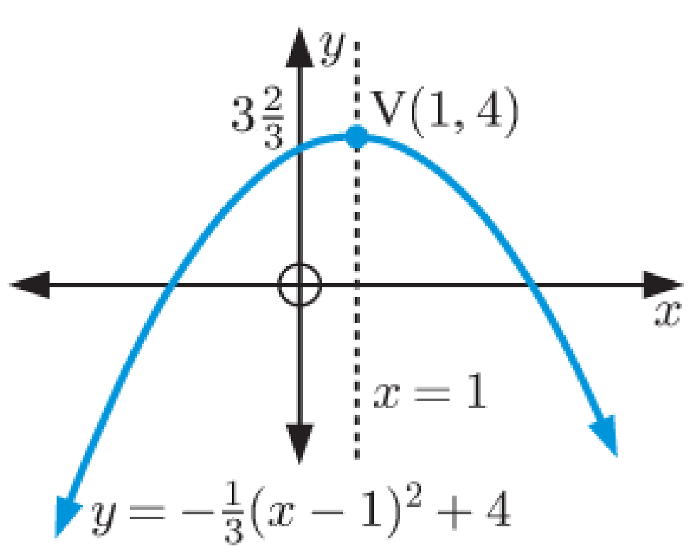
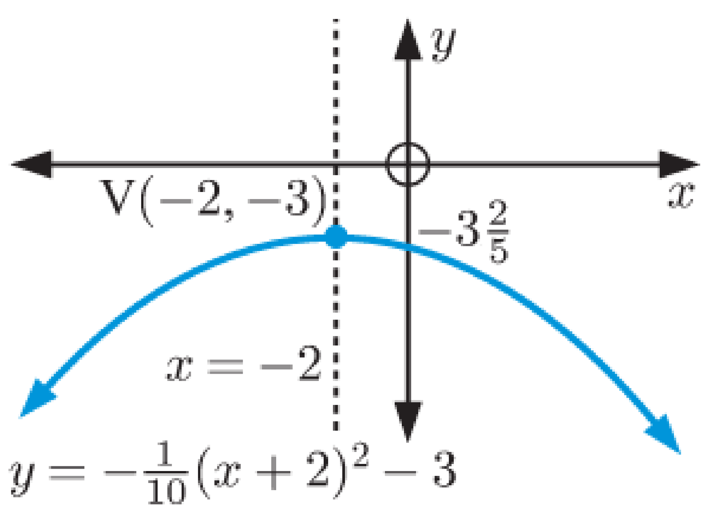
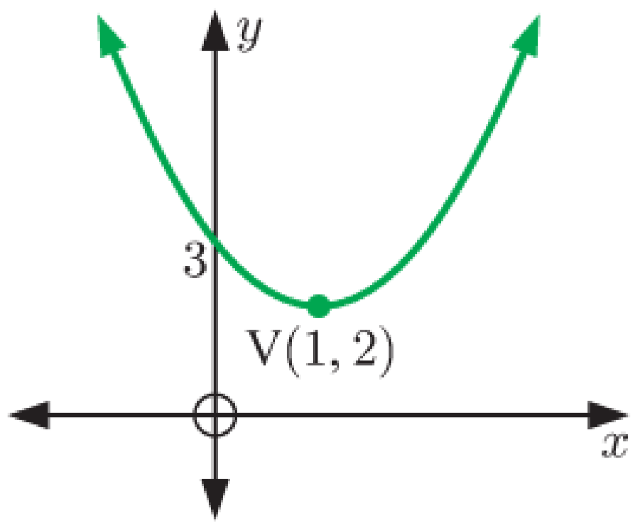


i

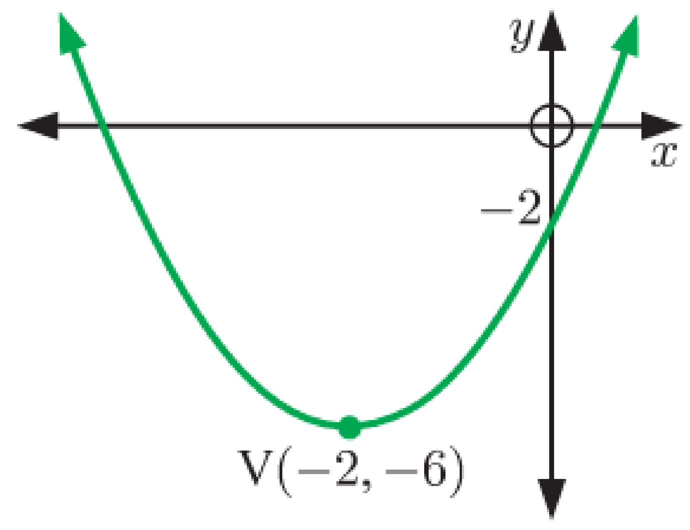


EXERCISE 14B.2

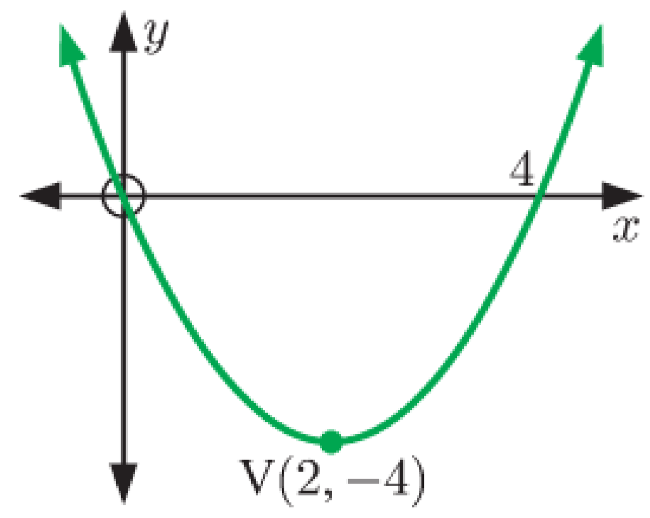
1 **a** $y=(x-1)^2+2$



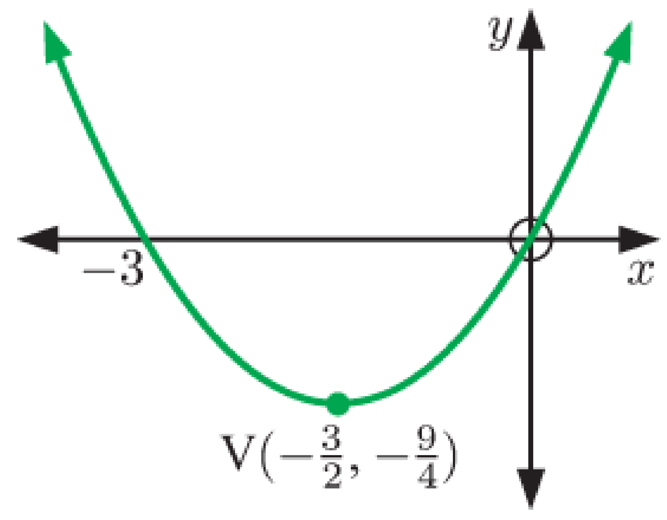
b $y=(x+2)^2-6$



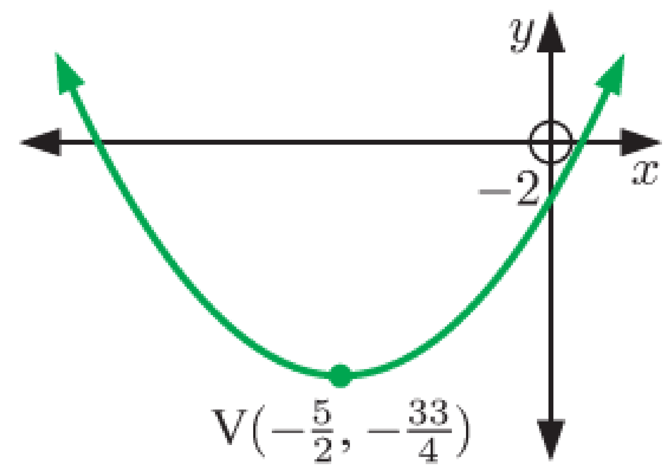
c $y=(x-2)^2-4$



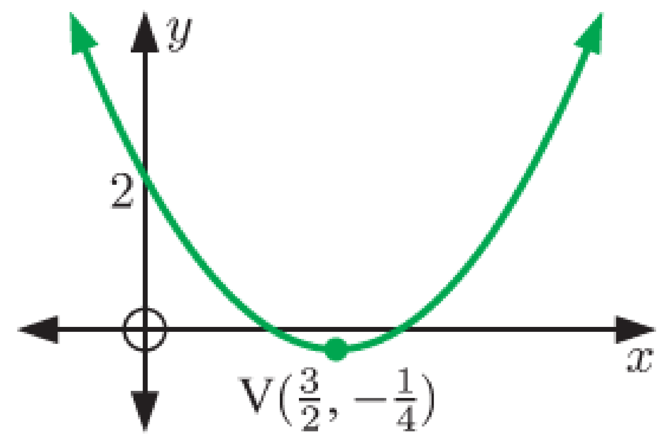
d $y=\left(x+\frac{3}{2}\right)^2-\frac{9}{4}$



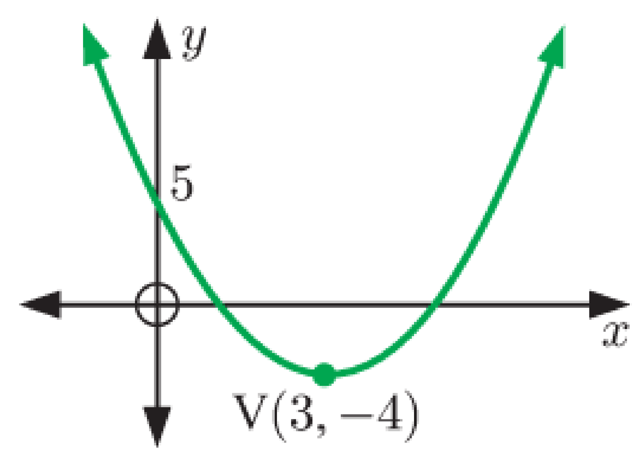
e $y=\left(x+\frac{5}{2}\right)^2-\frac{33}{4}$



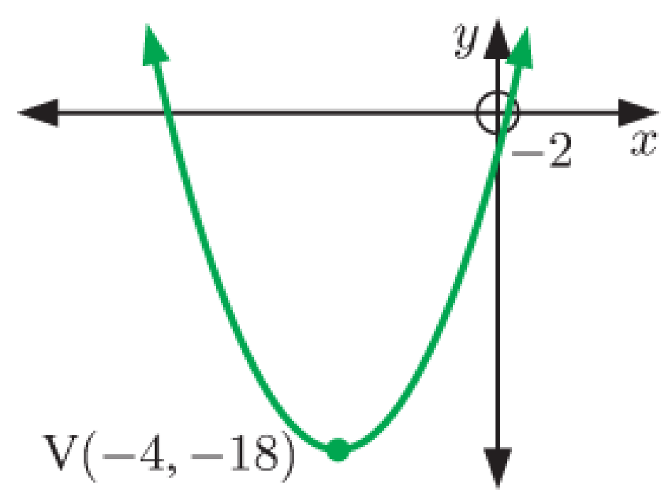
f $y=\left(x-\frac{3}{2}\right)^2-\frac{1}{4}$



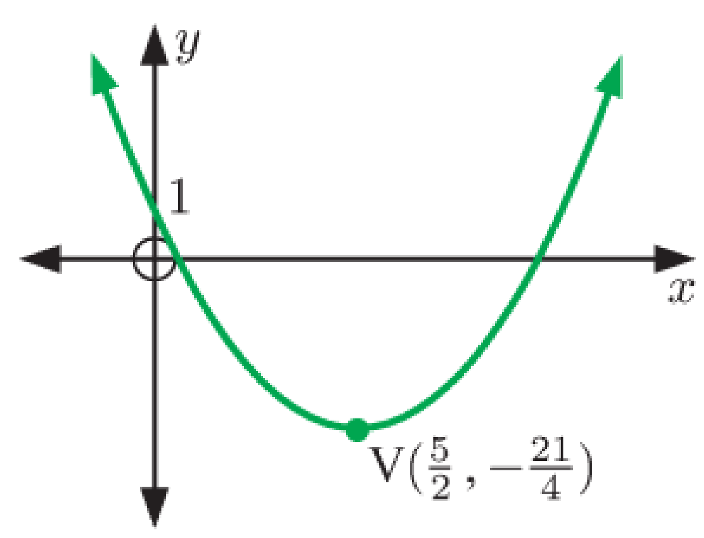
g $y=(x-3)^2-4$



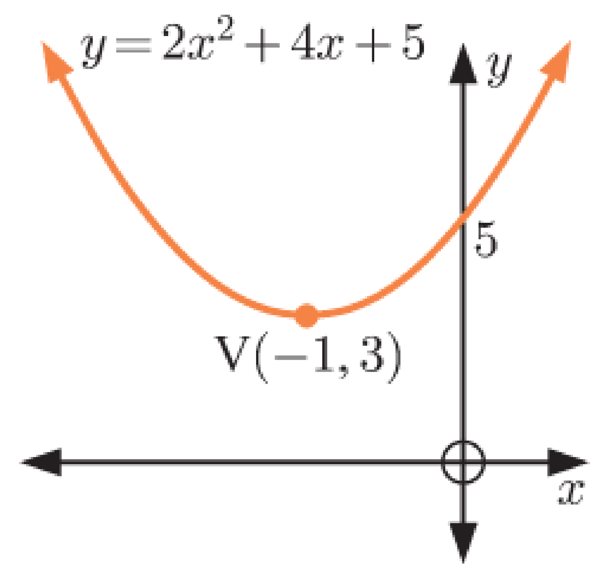
h $y=(x+4)^2-18$



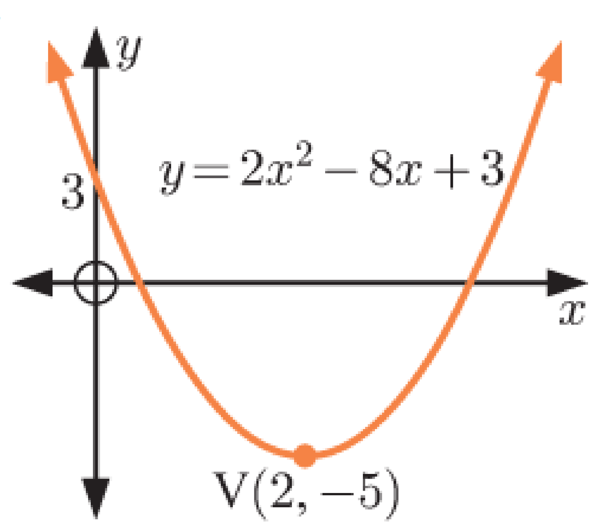
i $y=\left(x-\frac{5}{2}\right)^2-\frac{21}{4}$



2 **a** **i** $y=2(x+1)^2+3$
ii $(-1, 3)$ **iii** 5
iv

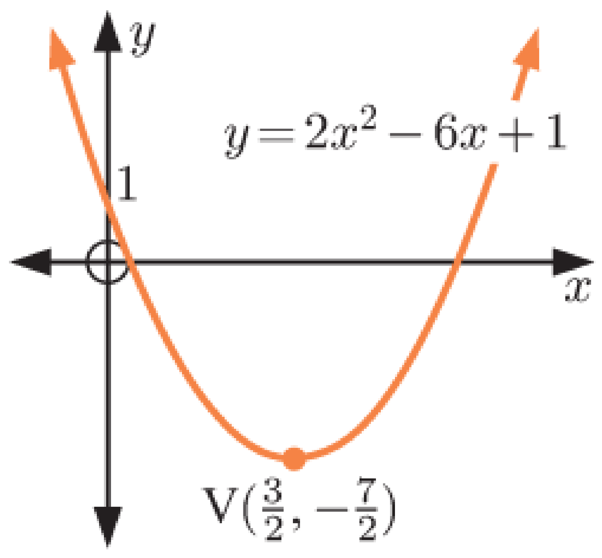


b **i** $y=2(x-2)^2-5$
ii $(2, -5)$ **iii** 3
iv



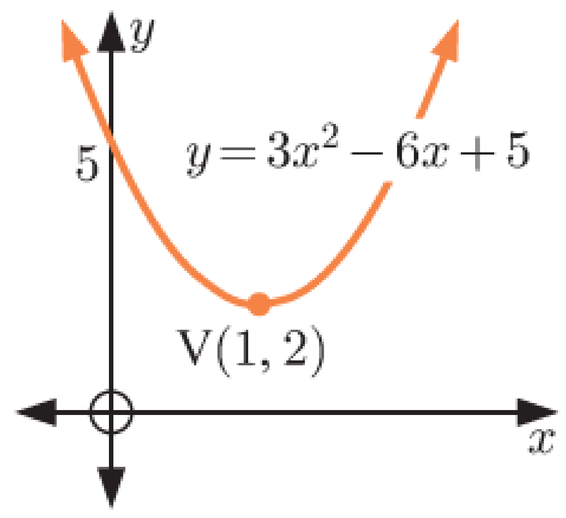
c **i** $y = 2(x - \frac{3}{2})^2 - \frac{7}{2}$ **ii** $(\frac{3}{2}, -\frac{7}{2})$ **iii** 1

iv



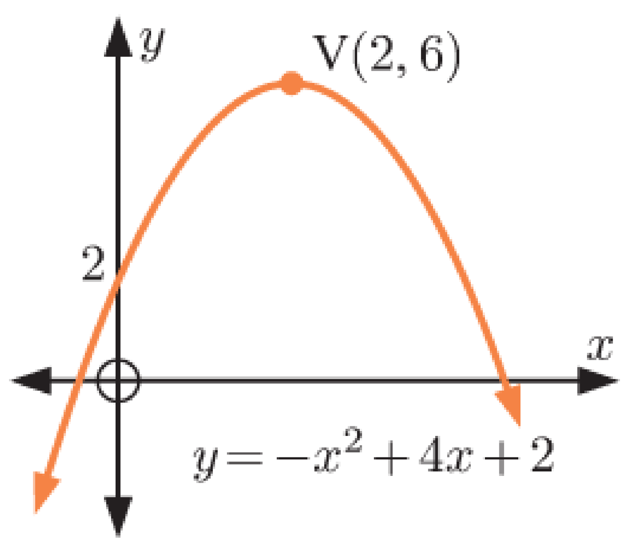
d **i** $y = 3(x - 1)^2 + 2$ **ii** $(1, 2)$ **iii** 5

iv



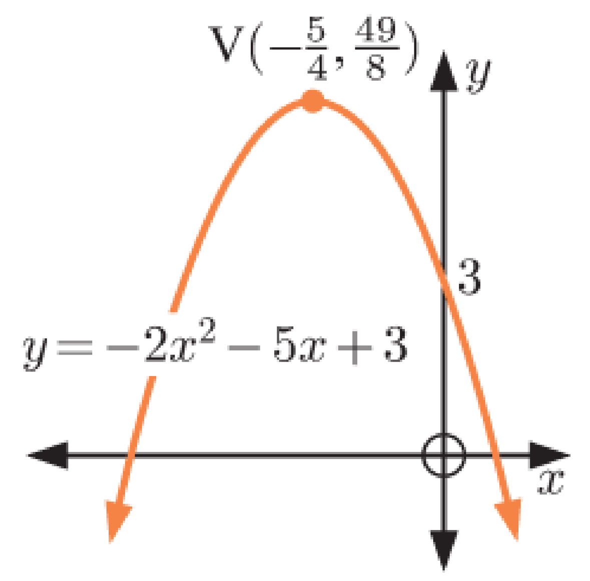
e **i** $y = -(x - 2)^2 + 6$ **ii** $(2, 6)$ **iii** 2

iv



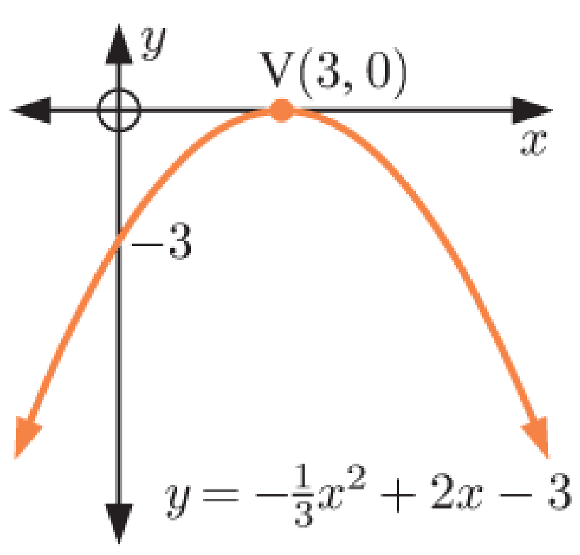
f **i** $y = -2(x + \frac{5}{4})^2 + \frac{49}{8}$ **ii** $(-\frac{5}{4}, \frac{49}{8})$ **iii** 3

iv



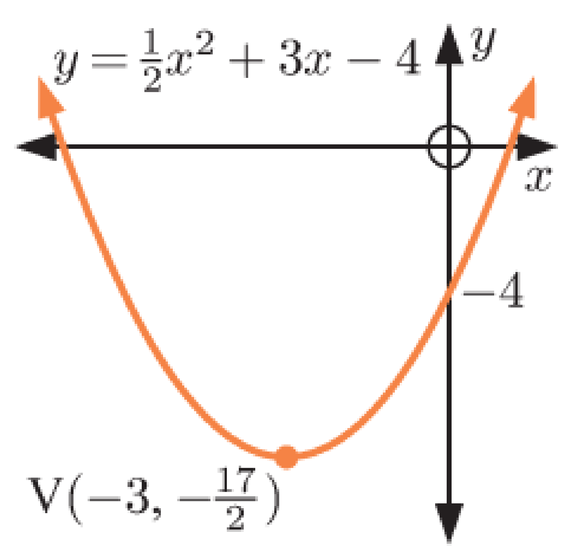
g **i** $y = -\frac{1}{3}(x - 3)^2$ **ii** $(3, 0)$ **iii** -3

iv



h **i** $y = \frac{1}{2}(x + 3)^2 - \frac{17}{2}$ **ii** $(-3, -\frac{17}{2})$ **iii** -4

iv

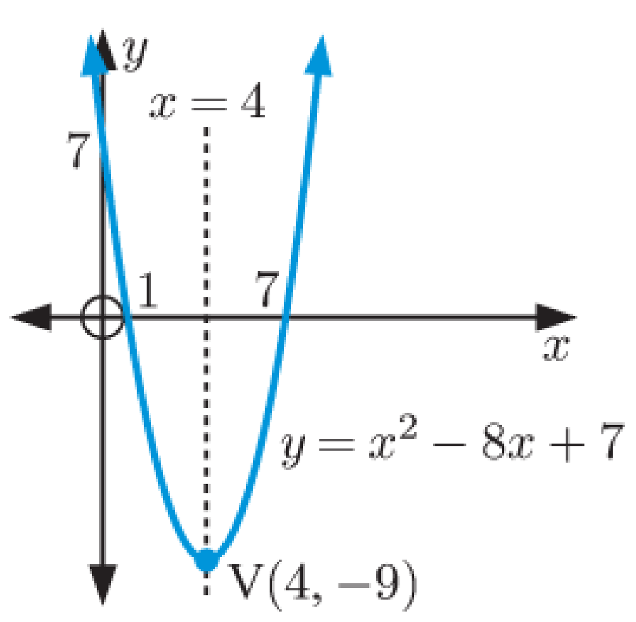


EXERCISE 14B.3

1 **a** **i** $(2, -2)$ **ii** minimum turning point
b **i** $(-1, -4)$ **ii** minimum turning point
c **i** $(0, 4)$ **ii** minimum turning point
d **i** $(0, 1)$ **ii** maximum turning point
e **i** $(-2, -15)$ **ii** minimum turning point
f **i** $(-2, -5)$ **ii** maximum turning point
g **i** $(-\frac{3}{2}, -\frac{11}{2})$ **ii** minimum turning point
h **i** $(\frac{5}{2}, -\frac{19}{2})$ **ii** minimum turning point
i **i** $(1, -\frac{9}{2})$ **ii** maximum turning point
j **i** $(14, -43)$ **ii** minimum turning point

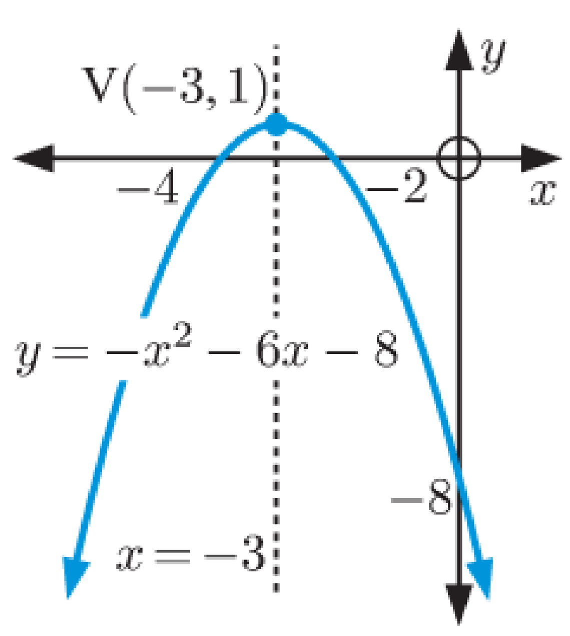
2 **a** **i** $x = 4$ **ii** $(4, -9)$ **iii** x-intercepts 1, 7, y-intercept 7

iv



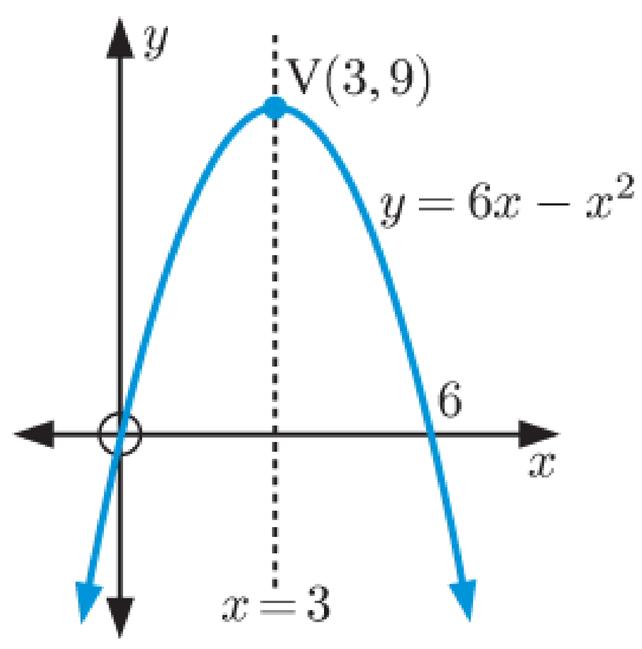
b **i** $x = -3$ **ii** $(-3, 1)$ **iii** x-int. -2, -4, y-intercept -8

iv



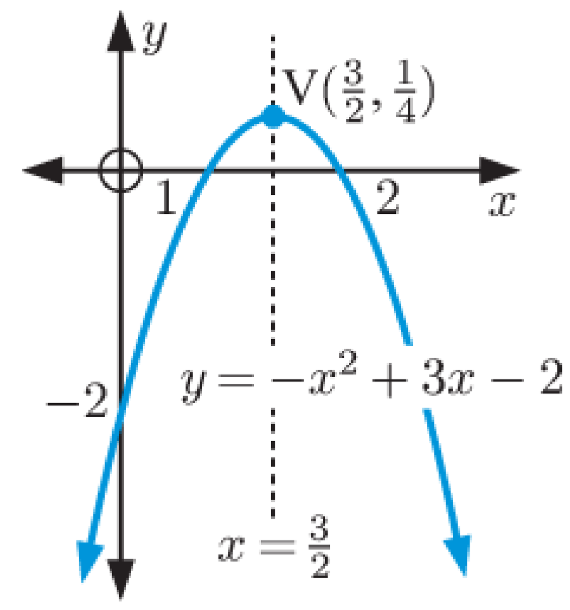
c **i** $x = 3$ **ii** $(3, 9)$ **iii** x-intercepts 0, 6, y-intercept 0

iv



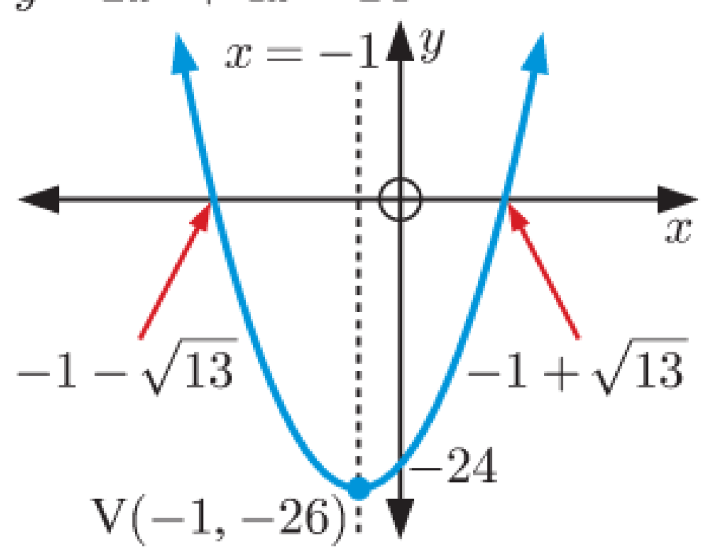
d **i** $x = \frac{3}{2}$ **ii** $(\frac{3}{2}, \frac{1}{4})$ **iii** x-intercepts 1, 2, y-intercept -2

iv



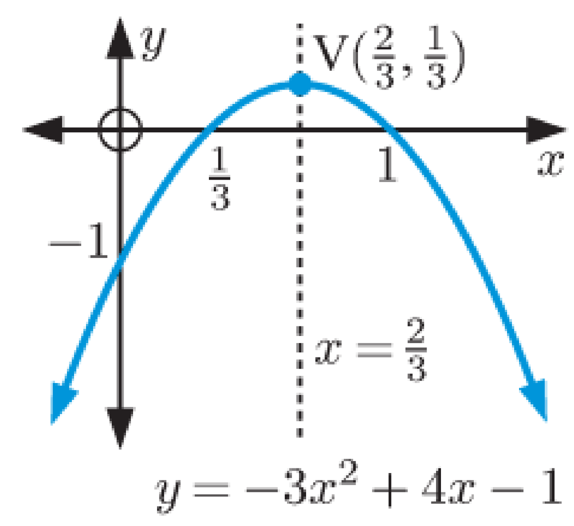
e **i** $x = -1$ **ii** $(-1, -26)$ **iii** x-int. $-1 \pm \sqrt{13}$, y-intercept -24

iv $y = 2x^2 + 4x - 24$



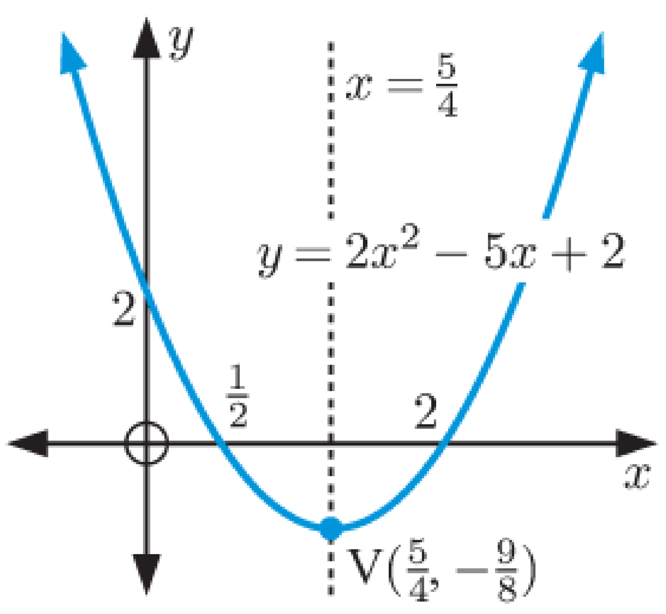
f **i** $x = \frac{2}{3}$ **ii** $(\frac{2}{3}, \frac{1}{3})$ **iii** x-intercepts $\frac{1}{3}$, 1, y-intercept -1

iv



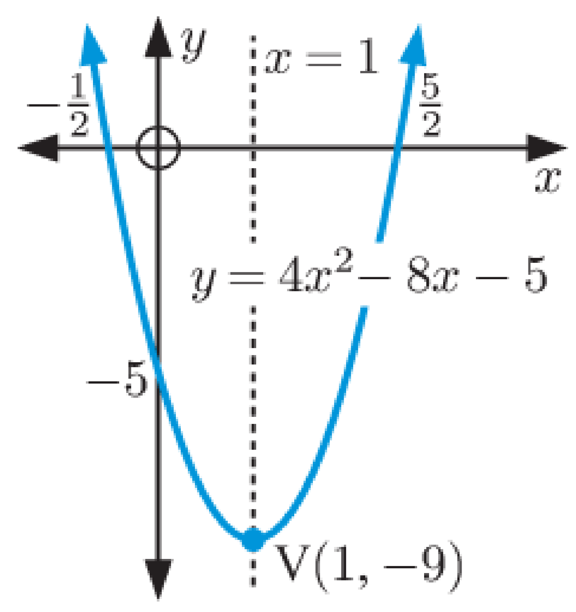
g **i** $x = \frac{5}{4}$ **ii** $(\frac{5}{4}, -\frac{9}{8})$ **iii** x-intercepts $\frac{1}{2}$, 2, y-intercept 2

iv



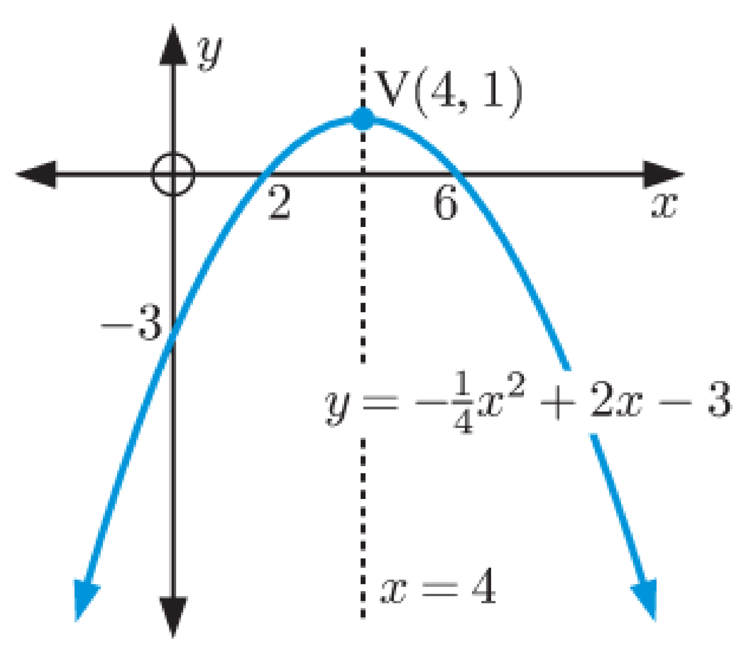
h **i** $x = 1$ **ii** $(1, -9)$ **iii** x-intercepts $-\frac{1}{2}$, $\frac{5}{2}$, y-intercept -5

iv



i **i** $x = 4$ **ii** $(4, 1)$ **iii** x-intercepts 2, 6, y-intercept -3

iv



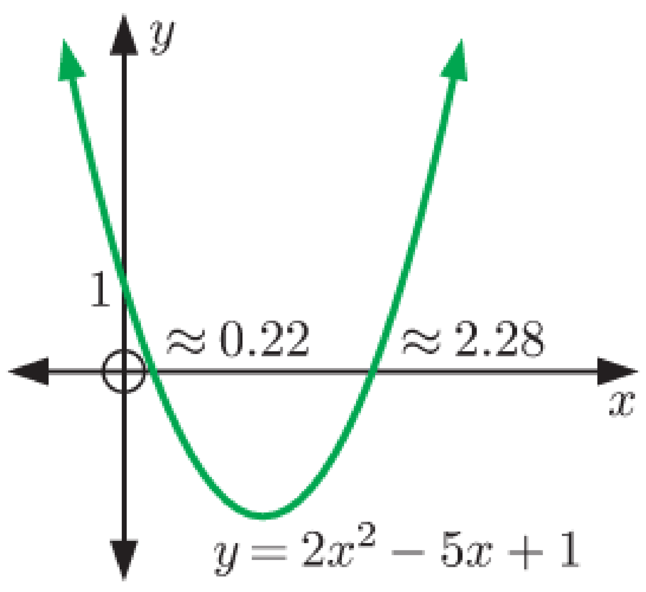
3 **Hint:** $y = ax^2 + bx + c$ has vertex with x-coordinate $-\dfrac{b}{2a}$ and y-coordinate $a\left(-\dfrac{b}{2a}\right)^2 + b\left(-\dfrac{b}{2a}\right) + c$.

EXERCISE 14C

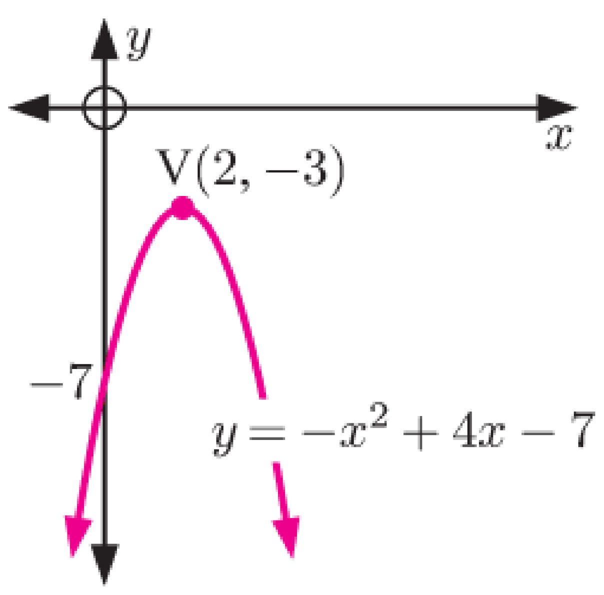
1 **a** $\Delta = 9$ which is > 0, graph cuts x-axis twice; is concave up.
b $\Delta = 12$ which is > 0, graph cuts x-axis twice; is concave up.
c $\Delta = -12$ which is < 0, graph lies entirely below the x-axis; is concave down, negative definite.
d $\Delta = 57$ which is > 0, graph cuts x-axis twice; is concave up.

e $\Delta = 0$, graph touches x-axis; is concave up.
f $\Delta = 17$ which is > 0, graph cuts x-axis twice; is concave down.
g $\Delta = 121$ which is > 0, graph cuts x-axis twice; is concave up.
h $\Delta = 25$ which is > 0, graph cuts x-axis twice; is concave down.
i $\Delta = 0$, graph touches x-axis; is concave up.

2 **a** concave up
b $\Delta = 17$ which is > 0
$\therefore$ cuts x-axis twice
c x-intercepts
≈ 0.22 and 2.28
d y-intercept is 1
e



3 **a** $\Delta = -12$ which is < 0
$\therefore$ does not cut x-axis
b negative definite, since $a < 0$ and $\Delta < 0$
c vertex is $(2, -3)$,
y-intercept is -7
d



4 **a** $a = 2$ which is > 0 and $\Delta = -40$ which is < 0
$\therefore$ positive definite.
b $a = -2$ which is < 0 and $\Delta = -23$ which is < 0
$\therefore$ negative definite.
c $a = 1$ which is > 0 and $\Delta = -15$ which is < 0
$\therefore$ positive definite so $x^2 - 3x + 6 > 0$ for all x.
d $a = -1$ which is < 0 and $\Delta = -8$ which is < 0
$\therefore$ negative definite so $4x - x^2 - 6 < 0$ for all x.

5

Constant	a	b	c	d	e	f	Δ_1	Δ_2
Sign	$+$	$-$	$+$	$-$	$+$	0	$-$	$+$

6 **a** **i** $k < \frac{9}{4}$ **ii** $k = \frac{9}{4}$ **iii** $k > \frac{9}{4}$
b **i** $k < 4$ **ii** $k = 4$ **iii** $k > 4$
c **i** $k > -\frac{4}{3}$ **ii** $k = -\frac{4}{3}$ **iii** $k < -\frac{4}{3}$

7 $a = 3$ which is > 0 and $\Delta = k^2 + 12$ which is always > 0 $\{$as $k^2 \geqslant 0$ for all $k\}$ $\therefore$ cannot be positive definite.

8 $k = -2$, the graph touches the x-axis in this case.

EXERCISE 14D

1 **a** $y = 2(x-1)(x-2)$ **b** $y = 3(x-2)^2$
c $y = (x-1)(x-3)$ **d** $y = -(x-3)(x+1)$
e $y = -3(x-1)^2$ **f** $y = -2(x+2)(x-3)$

2 **a** $y = \frac{3}{2}(x-2)(x-4)$ **b** $y = -\frac{1}{2}(x+4)(x-2)$
c $y = -\frac{4}{3}(x+3)^2$

3 **a** $y = 3x^2 - 18x + 15$ **b** $y = -4x^2 + 6x + 4$
c $y = -x^2 + 6x - 9$ **d** $y = 4x^2 + 16x + 16$
e $y = \frac{3}{2}x^2 - 6x + \frac{9}{2}$ **f** $y = -\frac{1}{3}x^2 + \frac{2}{3}x + 5$

4 **a** $y = -(x-2)^2 + 4$ **b** $y = 2(x-2)^2 - 1$
c $y = \frac{1}{3}(x+3)^2 - 4$ **d** $y = -2(x-3)^2 + 8$
e $y = \frac{2}{3}(x-4)^2 - 6$ **f** $y = -\frac{5}{9}(x+2)^2 + 5$
g $y = -2(x-2)^2 + 3$ **h** $y = \frac{3}{2}(x+4)^2 + 3$
i $y = 2(x - \frac{1}{2})^2 - \frac{3}{2}$

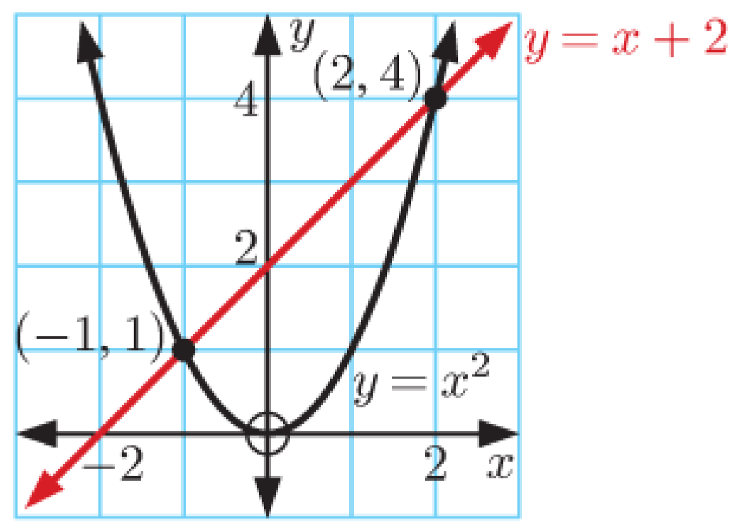
5 $y = 3$

EXERCISE 14E

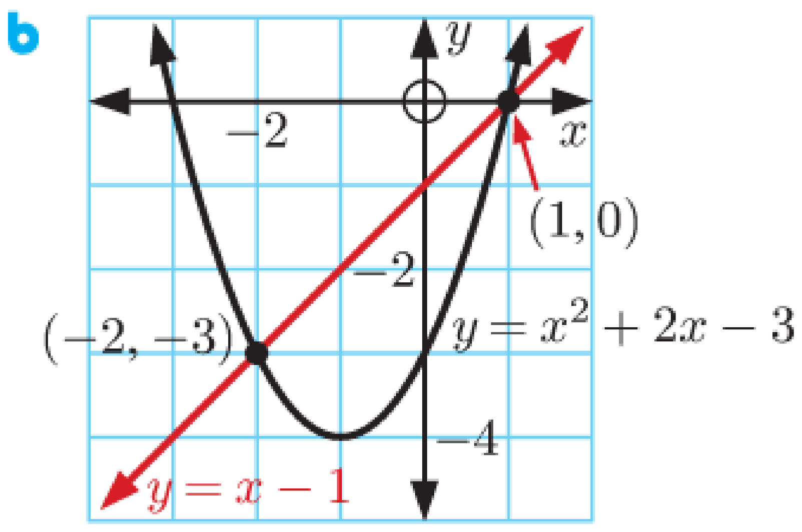
1 **a** $(1, 7)$ and $(2, 8)$ **b** $(4, 5)$ and $(-3, -9)$
c $(3, 0)$ (touching) **d** graphs do not meet

2 **a** $(0.586, 5.59)$ and $(3.41, 8.41)$
b $(3, -4)$ (touching) **c** graphs do not meet
d $(-2.56, -18.8)$ and $(1.56, 1.81)$

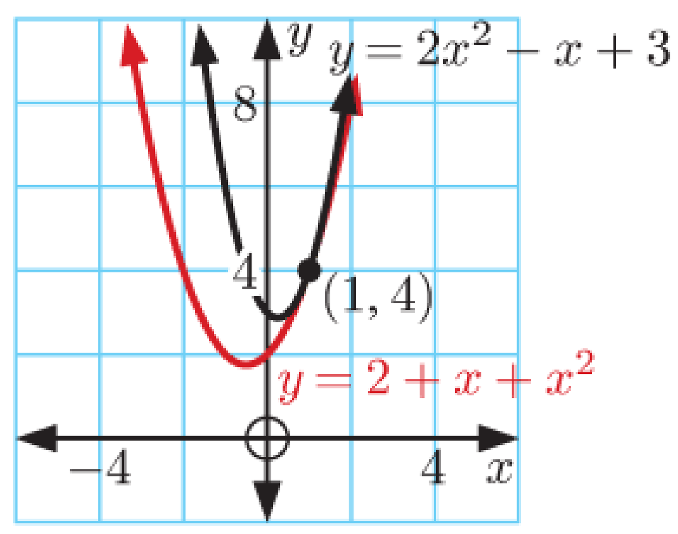
3 **a** $(-1, 1)$ and $(2, 4)$
c $x < -1$ or $x > 2$
b



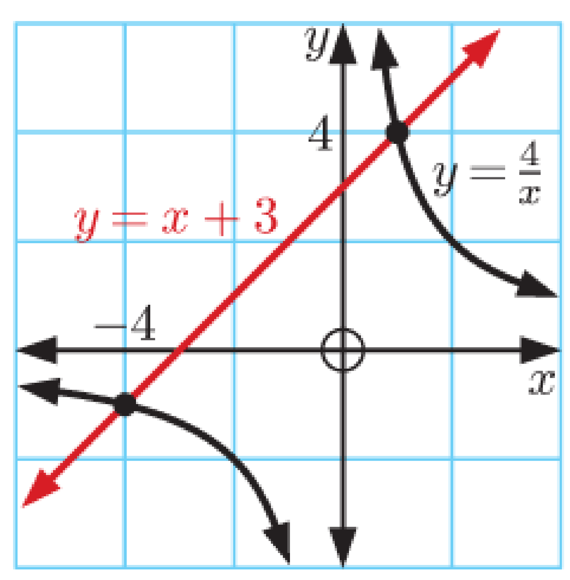
4 **a** $(-2, -3)$ and $(1, 0)$
c $x < -2$ or $x > 1$
b



5 **a** $(1, 4)$
c $x \in \mathbb{R}$, $x \neq 1$
b

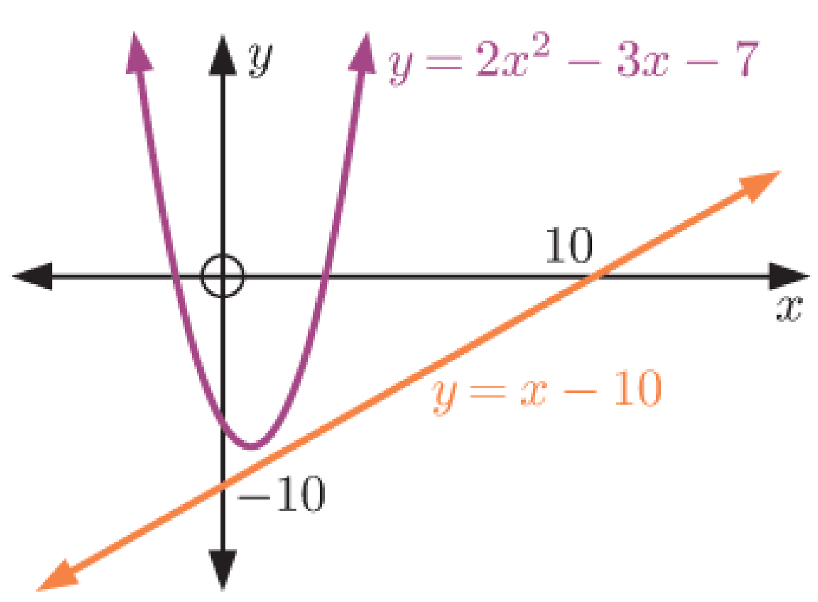


6 **a** $x = -4$ or 1
c $x < -4$ or $0 < x < 1$
b



7 $c = -9$ **8** $m = 0$ or -8 **9** -1 or 11

10 **a** $c < -9$
b example: $c = -10$



12 **a** $c > -2$ **b** $c = -2$ **c** $c < -2$

13 **Hint:** A straight line through $(0, 3)$ will have an equation of the form $y = mx + 3$.

14 $b = 8$, $c = -14$ **15** **a** $c = a^2$, $m \in \mathbb{R}$ **b** $m = 2a$

EXERCISE 14F

1 7 and -5 or -7 and 5 **2** 5 or $\frac{1}{5}$ **3** 14
4 18 and 20 or -18 and -20
5 15 and 17 or -15 and -17 **6** 15 sides **7** ≈ 3.48 cm
8 **b** 6 cm by 6 cm by 7 cm **9** ≈ 11.2 cm square
10 no **12** ≈ 61.8 km h^{-1} **13** 32 elderly citizens

14 **a** $y=-\frac{8}{9}x^2+8$
b No, as the tunnel is only 4.44 m high when it is the same width as the truck.

15 **a** $h=-5(t-2)^2+80$ **b** 75 m **c** 6 seconds

EXERCISE 14G

1 **a** min. -1, when $x=1$ **b** max. 8, when $x=-1$
c max. $8\frac{1}{3}$, when $x=\frac{1}{3}$ **d** min. $-1\frac{1}{8}$, when $x=-\frac{1}{4}$
e min. $4\frac{15}{16}$, when $x=\frac{1}{8}$ **f** max. $6\frac{1}{8}$, when $x=\frac{7}{4}$

2 **a** 40 refrigerators **b** €4000

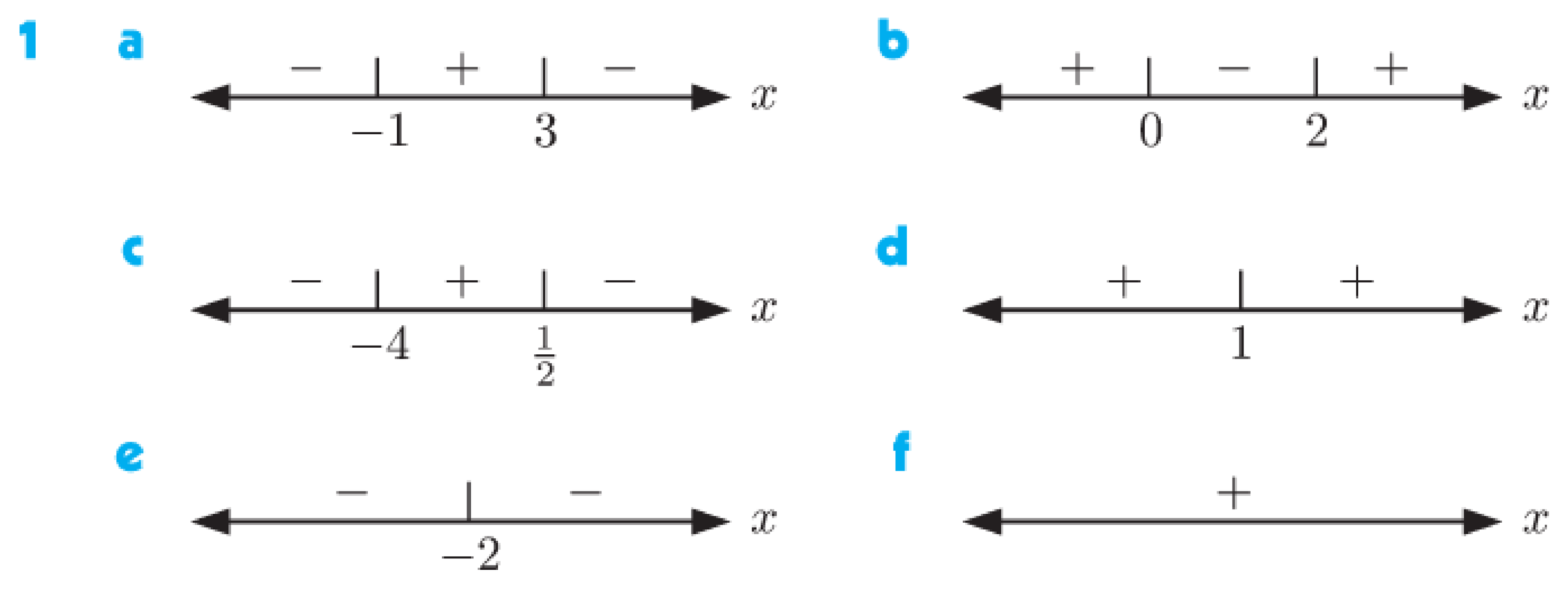
4 500 m by 250 m

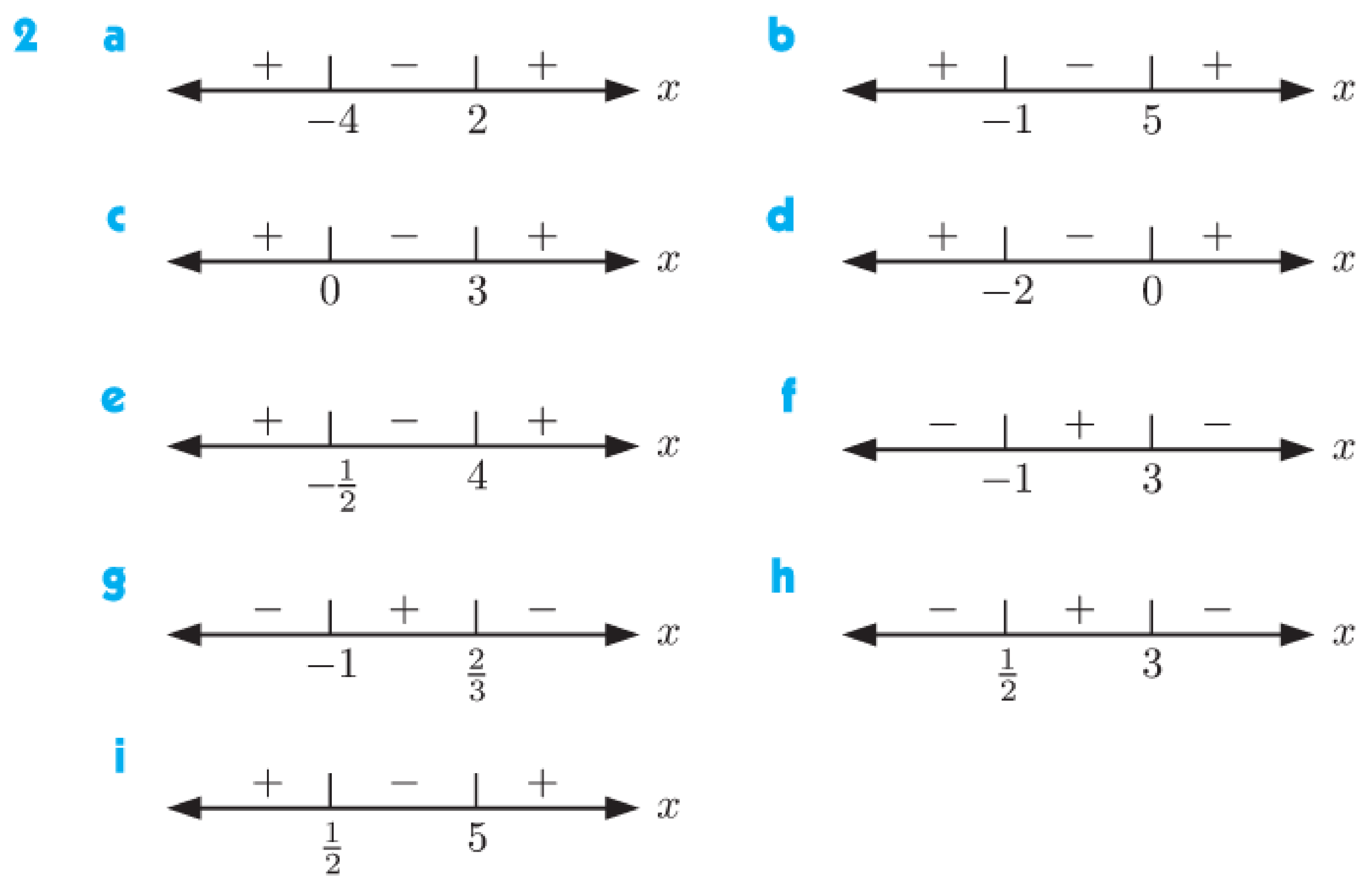
5 **a** $41\frac{2}{3}$ m by $41\frac{2}{3}$ m **b** 50 m by $31\frac{1}{4}$ m

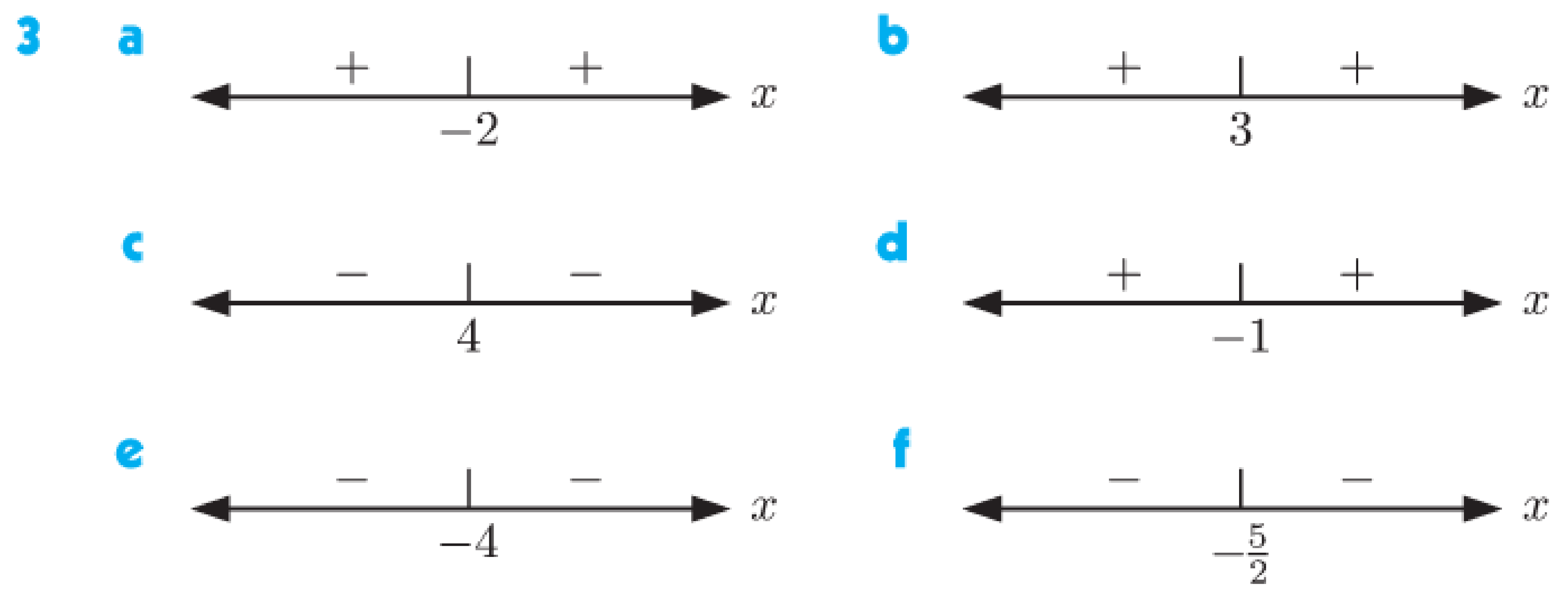
6 **b** $3\frac{1}{8}$ units **7** **a** $y=6-\frac{3}{4}x$ **b** 3 cm by 4 cm

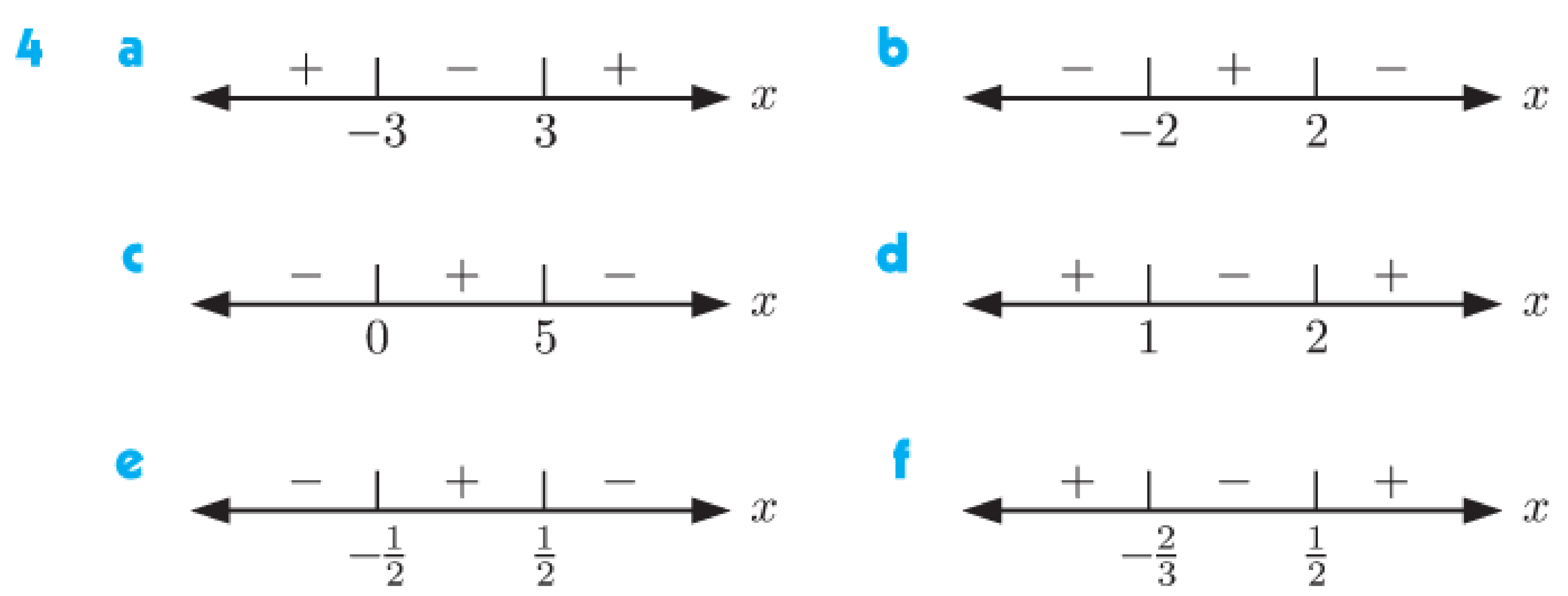
8 $m=\dfrac{\sum_{i=1}^{n} a_ib_i}{\sum_{i=1}^{n} a_i^{\,2}}$ **9** $y=x^4-2(a^2+b^2)x^2+(a^2-b^2)^2$
least value $=-4a^2b^2$

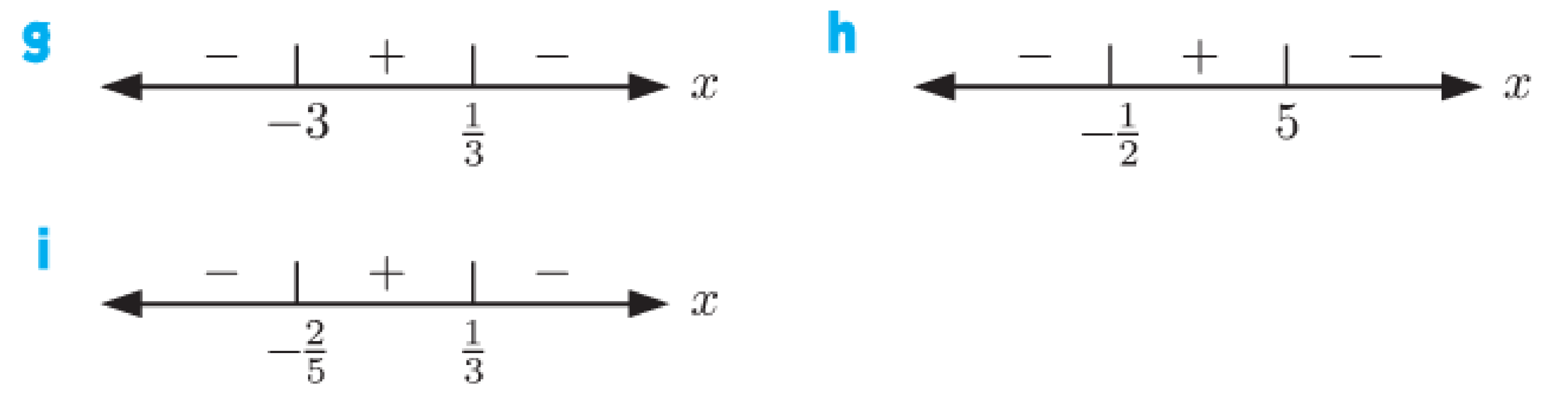
EXERCISE 14H.1

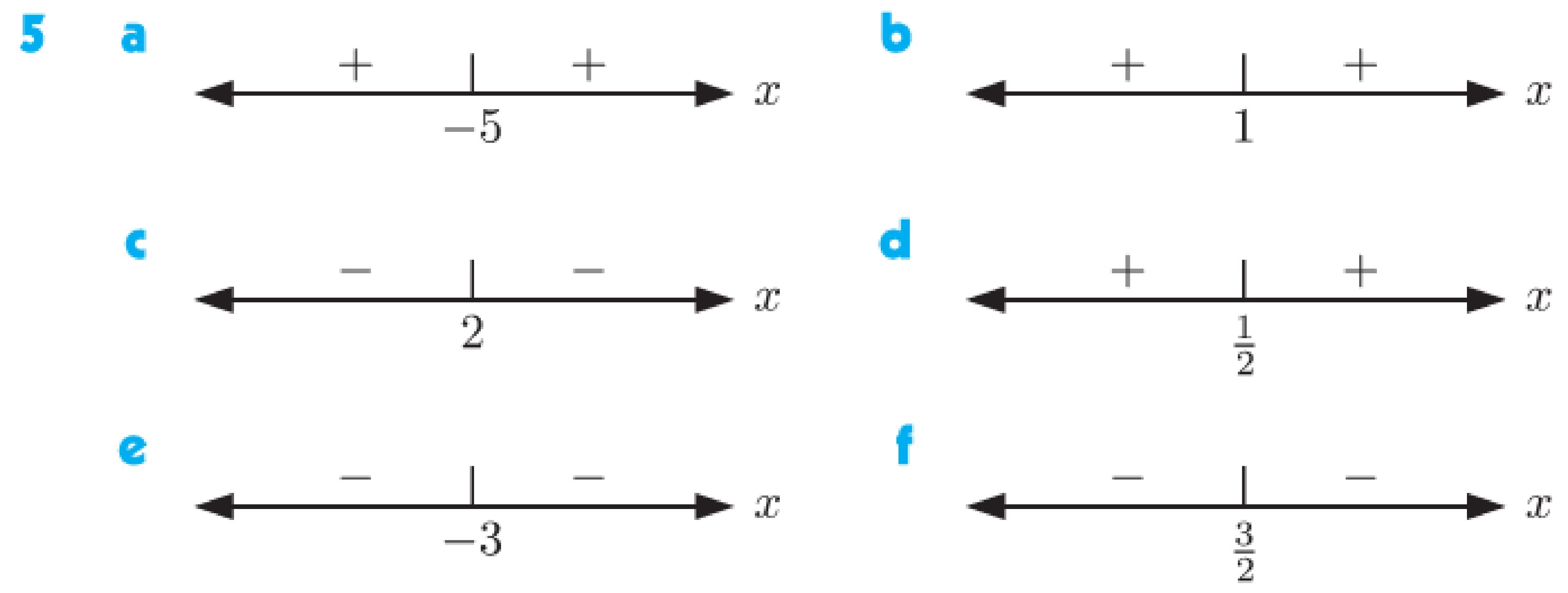












EXERCISE 14H.2

1 **a** $-5\leqslant x\leqslant 2$ **b** $-3\leqslant x\leqslant 2$ **c** no solutions
d all $x\in\mathbb{R}$ **e** $-\frac{1}{2}<x<3$ **f** $-\frac{3}{2}<x<4$

2 **a** $x\leqslant 0$ or $x\geqslant 1$ **b** $-\frac{2}{3}<x<0$ **c** $x\neq -2$
d $-5\leqslant x\leqslant 3$ **e** $x<-2$ or $x>6$ **f** $-4<x<1$

3 **a** $x\leqslant 0$ or $x\geqslant 3$ **b** $-2<x<2$
c $x\leqslant -\sqrt{2}$ or $x\geqslant \sqrt{2}$ **d** $-3\leqslant x\leqslant 7$
e $x<5$ or $x>6$ **f** $x<-6$ or $x>7$
g $x\leqslant -1$ or $x\geqslant \frac{3}{2}$ **h** no solutions
i $-\frac{3}{2}<x<\frac{1}{3}$ **j** $x<-\frac{4}{3}$ or $x>4$
k $x\neq 1$ **l** $\frac{1}{3}\leqslant x\leqslant \frac{1}{2}$ **m** $x<-\frac{1}{6}$ or $x>1$
n $x\leqslant -\frac{1}{4}$ or $x\geqslant \frac{2}{3}$ **o** $x<\frac{3}{2}$ or $x>3$

4 **a** **i** $k<-8$ or $k>0$ **ii** $k=-8$ or 0
iii $-8<k<0$
b **i** $-1<k<1,\ k\neq 0$ **ii** $k=-1$ or 1
iii $k<-1$ or $k>1$
c **i** $k<-6$ or $k>2$ **ii** $k=-6$ or $k=2$
iii $-6<k<2$

5 **a** **i** $k<-2$ or $k>6$ **ii** $k=-2$ or $k=6$
iii $-2<k<6$
b **i** $k<-\frac{13}{9}$ or $k>3$ **ii** $k=-\frac{13}{9}$ or $k=3$
iii $-\frac{13}{9}<k<3$
c **i** $-\frac{4}{3}<k<0,\ k\neq -1$ **ii** $k=-\frac{4}{3}$ or $k=0$
iii $k<-\frac{4}{3}$ or $k>0$

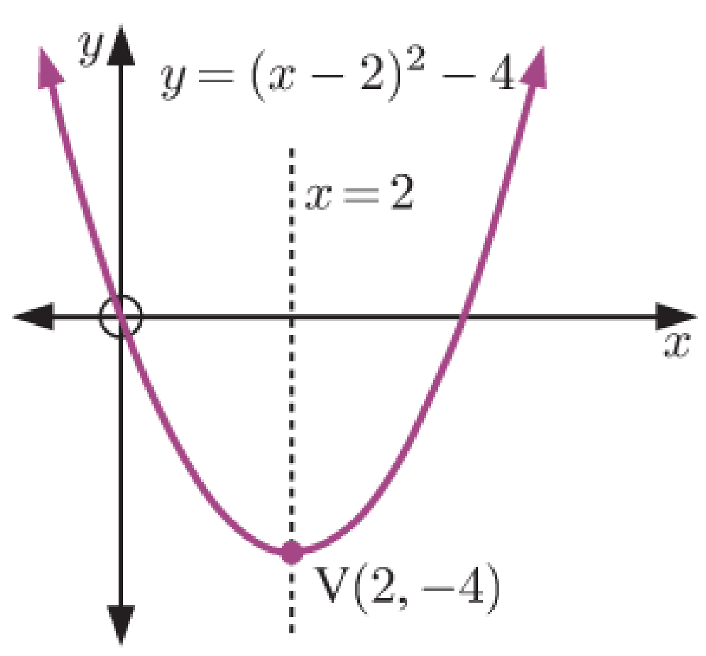
6 **a** $m>3$ **b** $m<-1$

7 **a** $m<-1$ or $m>7$ **b** $m=-1$ or $m=7$
c $-1<m<7$

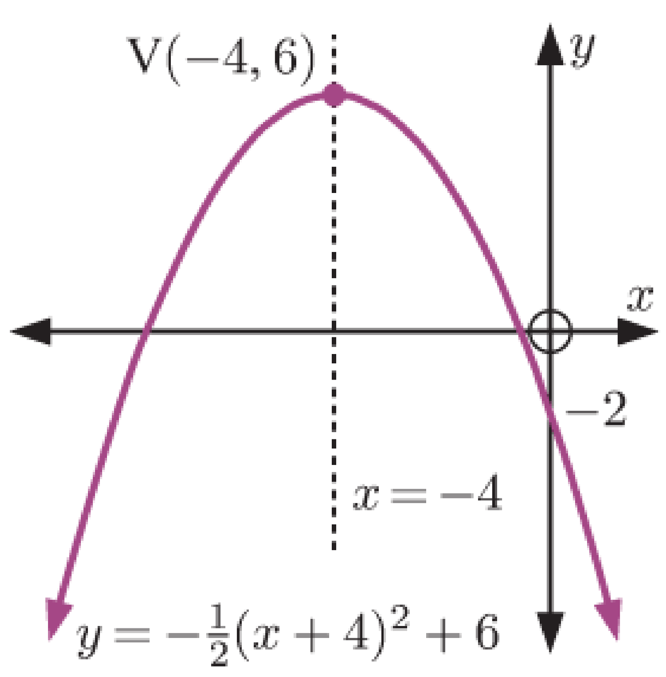
8 **a** $a<6-2\sqrt{10}$ or $a>6+2\sqrt{10}$ **b** $a=6\pm 2\sqrt{10}$
c $6-2\sqrt{10}<a<6+2\sqrt{10}$

REVIEW SET 14A

1 **a**



b

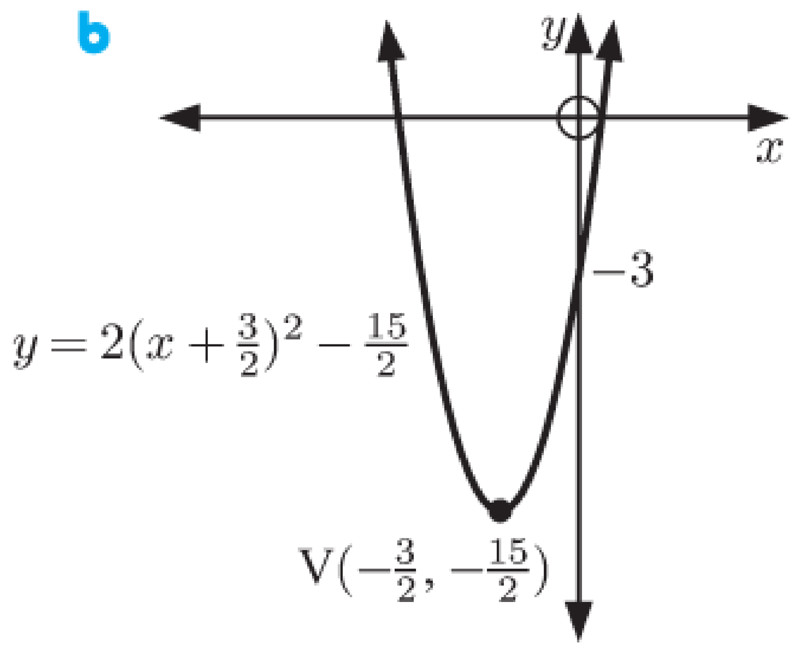


2 (4, 4) and (−3, 18) **3** $k < -3\frac{1}{8}$

4 **a** $m = \frac{9}{8}$ **b** $m < \frac{9}{8}$ **c** $m > \frac{9}{8}$ **5** $\frac{6}{5}$ or $\frac{5}{6}$

6 **Hint:** Let the line have equation $y = mx + 10$.

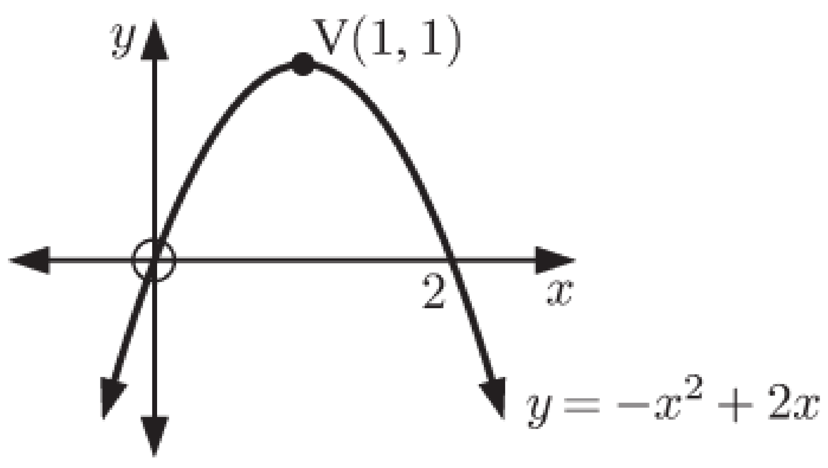
7 **a** $y = 2(x + \frac{3}{2})^2 - \frac{15}{2}$ **b**



8 **a** $y = \frac{20}{9}(x-2)^2 - 20$ **b** $y = -\frac{2}{7}(x-1)(x-7)$

c $y = \frac{2}{9}(x+3)^2$

9



10 $\frac{1}{2}$

11 **a** **i** $\Delta > 0$ **ii** $a < 0$

b **i** $A(-m, 0)$, $B(-n, 0)$ **ii** $x = \dfrac{-m-n}{2}$

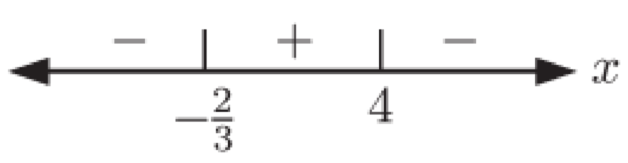
13 $y = -4x^2 + 4x + 24$ **14** $k = \frac{3}{2}$

15 **a** $c = 8$ **b** $3a + b = -3$, $a - b = -5$

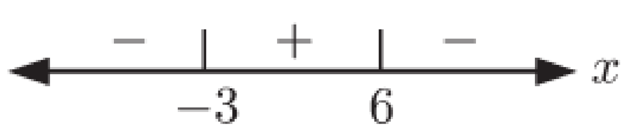
c $a = -2$, $b = 3$, $y = -2x^2 + 3x + 8$

16 $m = -5$ or 19 **17** 21 m

18 **a**



b



19 **a** $x < -2$ or $x > 3$ **b** $-1 \leqslant x \leqslant 5$

c $x < -\frac{5}{2}$ or $x > 2$

20 **a** $k < 6 - 2\sqrt{5}$ or $k > 6 + 2\sqrt{5}$ **b** $k = 6 \pm 2\sqrt{5}$

c $6 - 2\sqrt{5} < k < 6 + 2\sqrt{5}$

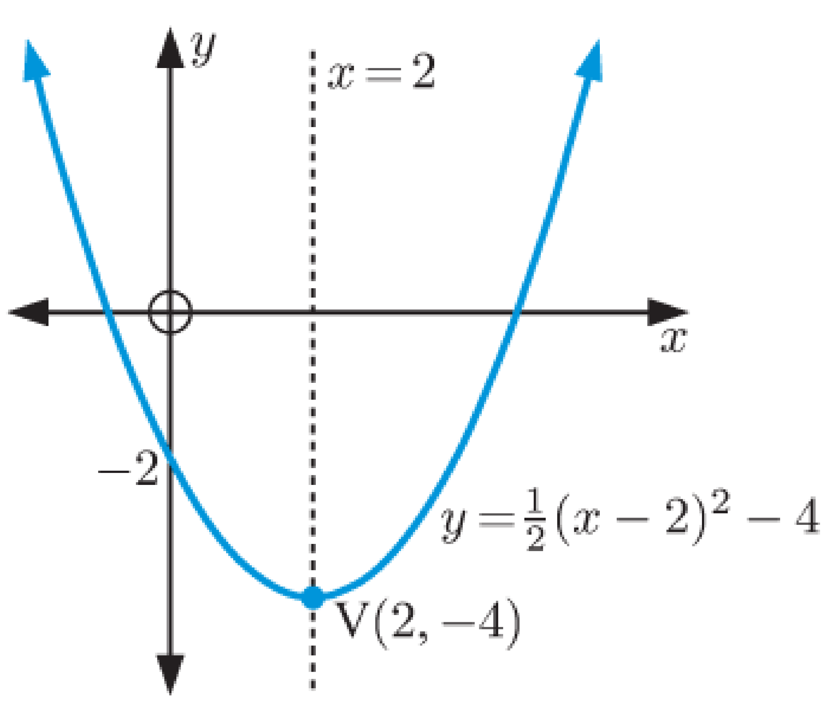
REVIEW SET 14B

1 **a** $x = 2$

b (2, −4)

c −2

d



2 $x = \frac{4}{3}$, $V(1\frac{1}{3}, 12\frac{1}{3})$

3 **a** $\Delta = 65$, the graph cuts the x-axis twice

b $\Delta = 97$, the graph cuts the x-axis twice

4 $y = -6(x-2)^2 + 25$

5 **a** $y = -\frac{2}{5}(x+5)(x-1)$ **b** $(-2, 3\frac{3}{5})$, $x = -2$

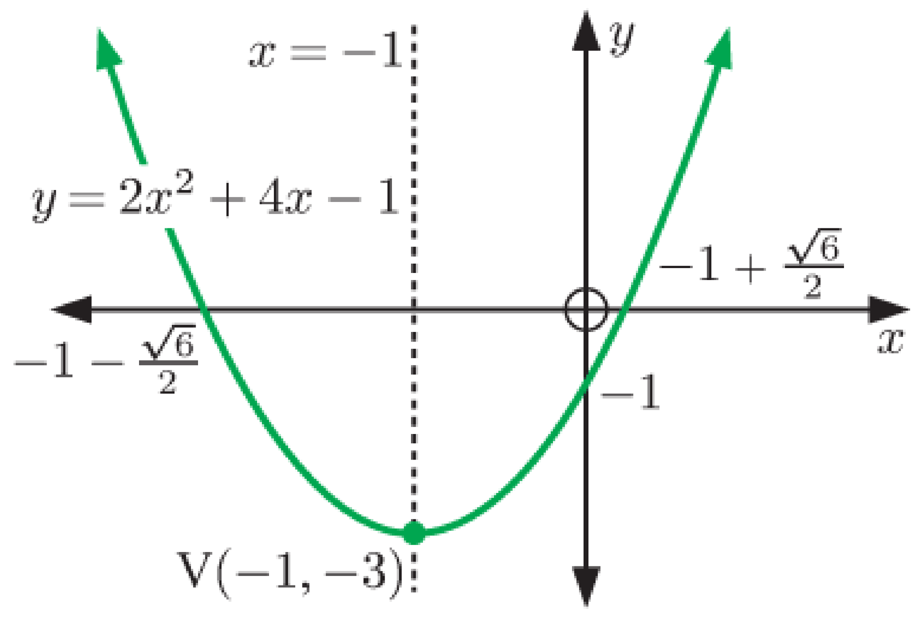
6 **a** $x = -1$

b (−1, −3)

c x-int. $-1 \pm \frac{\sqrt{6}}{2}$

y-intercept −1

d



7 **a** $y = 2x^2 - 12x + 18$ **b** $y = -\frac{1}{2}x^2 + \frac{1}{2}x + 3$

c $y = x^2 + 7x - 3$ **d** $y = -2x^2 + 12x - 3$

8 **a** $c > -6$

b For example, when $c = -2$, points of intersection are (−1, −5) and (3, 7).

9 **a** minimum is $5\frac{2}{3}$ when $x = -\frac{2}{3}$

b maximum is $5\frac{1}{8}$ when $x = -\frac{5}{4}$

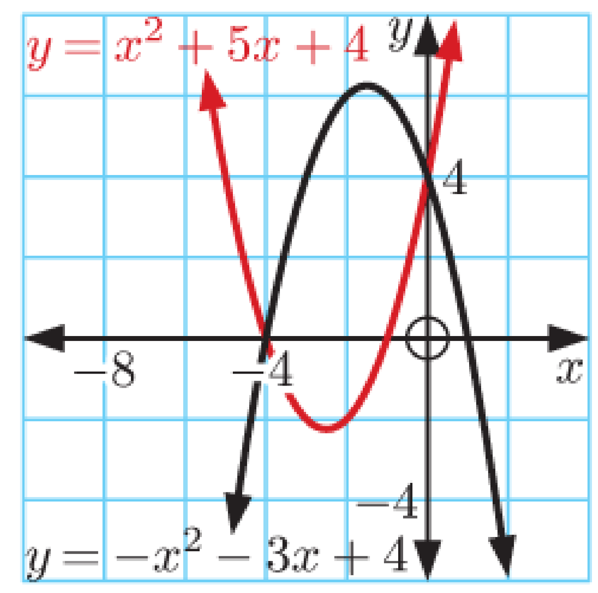
10 **a** $y = 3x^2 - 3x - 18$ **b** −18 **c** $(\frac{1}{2}, -18\frac{3}{4})$

11 **a** $m = -2$, $n = 4$ **b** $k = 7$ **12** ≈ 13.5 cm square

13 **a** $x = -4$ or 0

c $x < -4$ or $x > 0$

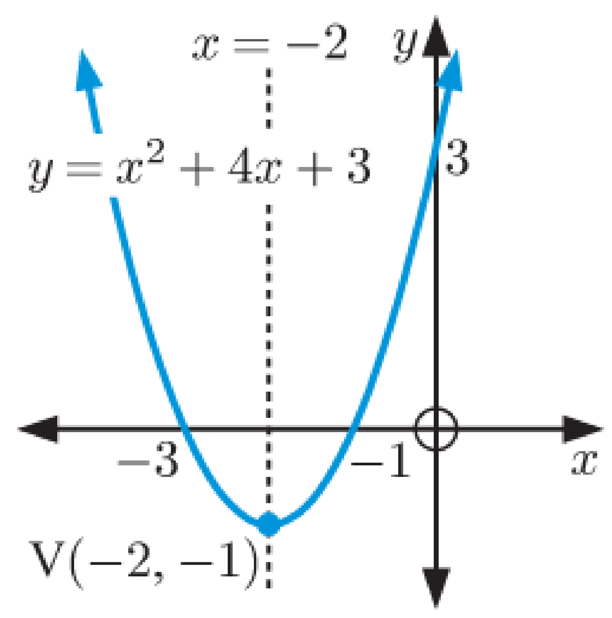
b



14 **a** **i** $y = (x+2)^2 - 1$

ii $y = (x+3)(x+1)$

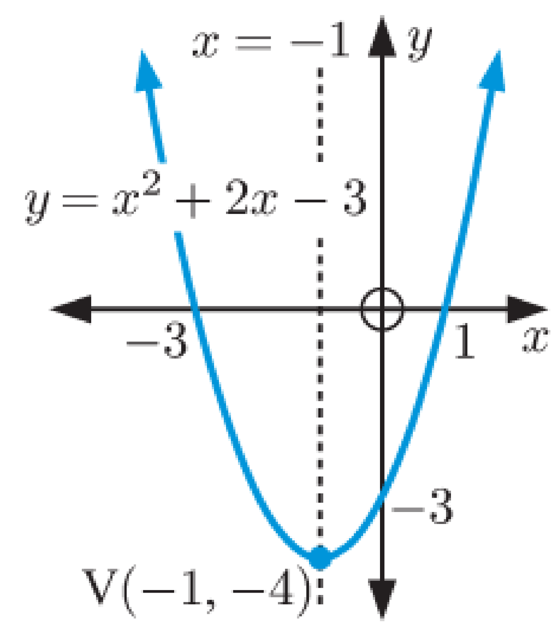
iii



b **i** $y = (x+1)^2 - 4$

ii $y = (x+3)(x-1)$

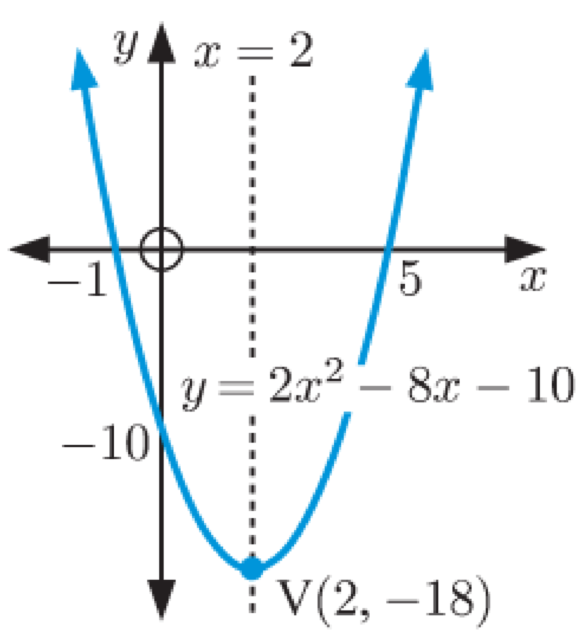
iii



c **i** $y = 2(x-2)^2 - 18$

ii $y = 2(x-5)(x+1)$

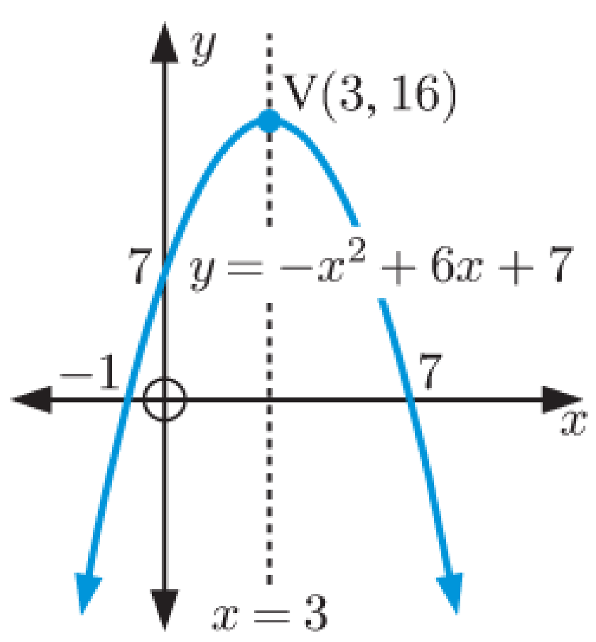
iii



d **i** $y = -(x-3)^2 + 16$

ii $y = -(x-7)(x+1)$

iii

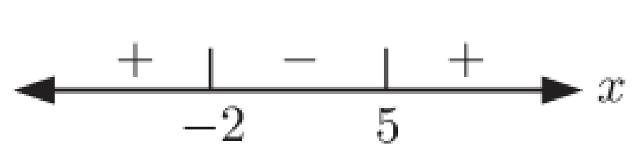


15 **a** $k = \pm 12$ **b** (0, 4)

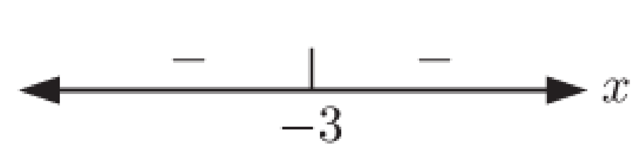
16 **b** $37\frac{1}{2}$ m by $33\frac{1}{3}$ m **c** 1250 m^2

17 **b** \$60, revenue is \$2400 per day

18 **a**



b



19 **a** $0 < x < \frac{3}{4}$ **b** $x \leqslant -1$ or $x \geqslant \frac{5}{2}$

c $x \leqslant \frac{1}{3}$ or $x \geqslant \frac{3}{2}$

20 **a** $-\frac{25}{2} < m < \frac{1}{2},\ m \neq 0$ **b** $m = -\frac{25}{2}$ or $m = \frac{1}{2}$
c $m < -\frac{25}{2}$ or $m > \frac{1}{2}$

EXERCISE 15A

1 **a** Is a function, since for every value of x there is only one corresponding value of y.
b Is not a function. When $x = 2$, $y = 1$ or 0.

2 **a** Is a function, since for any value of x there is at most one value of y.
b Is a function, since for any value of x there is at most one value of y.
c Is not a function. If $x^2 + y^2 = 9$, then $y = \pm\sqrt{9 - x^2}$. So, for example, for $x = 2$, $y = \pm\sqrt{5}$.

3 **a** function **b** not a function **c** function
d not a function

4 Not a function as a 2 year old child could pay \$0 or \$20.

5 No, because a vertical line (the y-axis) would cut the relation more than once.

6 No. A vertical line is not a function. It will not pass the "vertical line" test.

7 **a** $y^2 = x$ is a relation but not a function.
$y = x^2$ is a function (and a relation).
$y^2 = x$ has a horizontal axis of symmetry (the x-axis).
$y = x^2$ has a vertical axis of symmetry (the y-axis).
Both $y^2 = x$ and $y = x^2$ have vertex $(0, 0)$.
$y^2 = x$ is a rotation of $y = x^2$ clockwise through 90° about the origin *or* $y^2 = x$ is a reflection of $y = x^2$ in the line $y = x$.
b **i** The part of $y^2 = x$ in the first quadrant.
ii $y = \sqrt{x}$ is a function as any vertical line cuts the graph at most once.

8 **a** Both curves are functions since any vertical line will cut each curve at most once.
b $y = \sqrt[3]{x}$

EXERCISE 15B

1 **a** 2 **b** 2 **c** -16 **d** -68 **e** $\frac{17}{4}$

2 **a** -3 **b** 3 **c** 3 **d** -3 **e** $\frac{15}{2}$

3 **a** **i** $-\frac{7}{2}$ **ii** $-\frac{3}{4}$ **iii** $-\frac{4}{9}$ **b** $x = 4$ **c** $x = \frac{9}{5}$

4 **a** $7 - 3a$ **b** $7 + 3a$ **c** $-3a - 2$ **d** $7 - 6a$
e $1 - 3x$ **f** $7 - 3x - 3h$

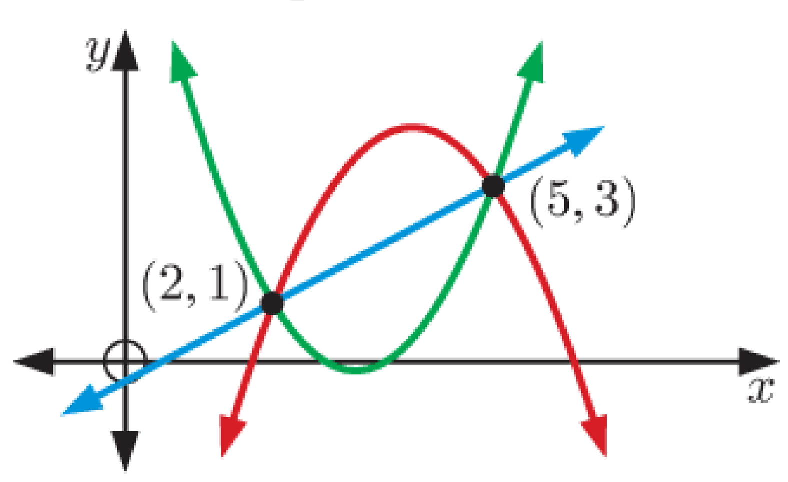
5 **a** $2x^2 + 19x + 43$ **b** $2x^2 - 11x + 13$
c $2x^2 - 3x - 1$ **d** $2x^4 + 3x^2 - 1$
e $18x^2 + 9x - 1$ **f** $2x^2 + (4h + 3)x + 2h^2 + 3h - 1$

6 **a** $9x^2$ **b** $\dfrac{x^2}{4}$ **c** $3x^2$ **d** $2x^2 - 4x + 7$

7 **a** $-\dfrac{1}{x}$ **b** $\dfrac{2}{x}$ **c** $\dfrac{2 + 3x}{x}$ **d** $\dfrac{2x + 1}{x - 1}$

8 f is the function which converts x into $f(x)$ whereas $f(x)$ is the value of the function at any value of x.

9 **Note:** Other answers are possible.



10 $f(x) = -2x + 5$

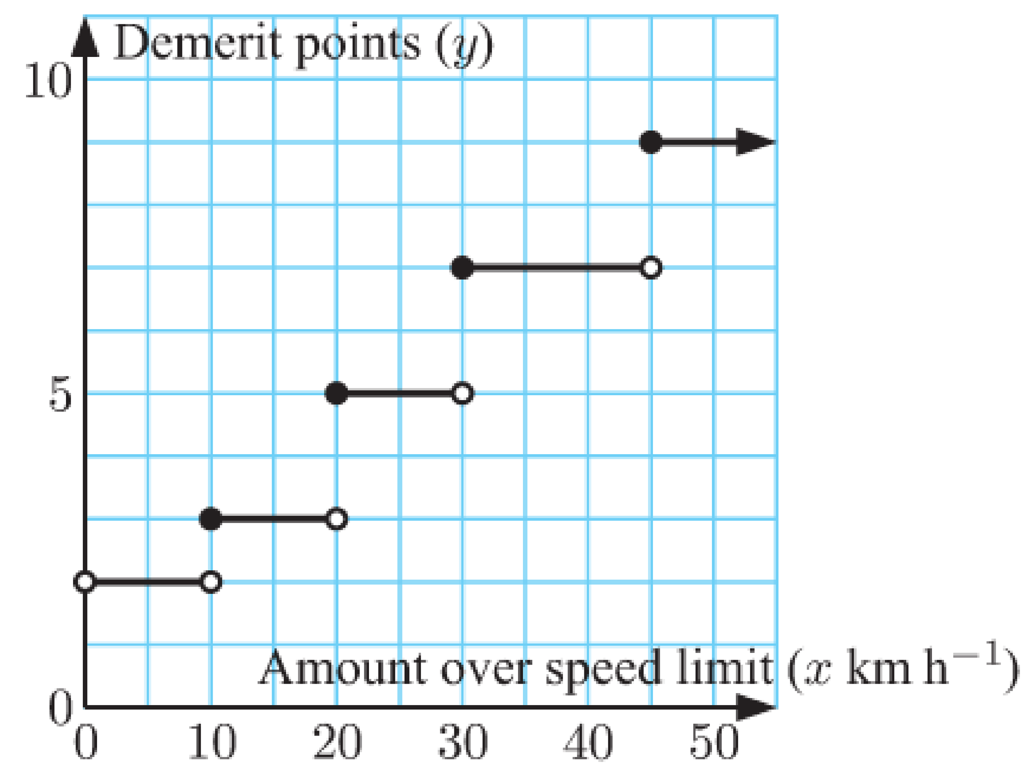
11 **a** $H(30) = 800$. After 30 minutes the balloon is 800 m high.
b $t = 20$ or 70. After 20 minutes and after 70 minutes the balloon is 600 m high.
c $0 \leqslant t \leqslant 80$ **d** 0 m to 900 m

12 $a = 3,\ b = -2$ **13** $a = 3,\ b = -1,\ c = -4$

14 **a** $V(4) = 5400$; $V(4)$ is the value of the photocopier in pounds after 4 years.
b $t = 6$. After 6 years the value of the photocopier is £3600.
c £9000 **d** $0 \leqslant t \leqslant 10$

EXERCISE 15C

1 **a**



b Yes, since for every value of x, there is at most one value of y.
c Domain is $\{x \mid x > 0\}$, Range is $\{2, 3, 5, 7, 9\}$

2 **a** At any moment in time there can be only one temperature, so the graph is a function.
b Domain is $\{t \mid 0 \leqslant t \leqslant 30\}$, Range is $\{T \mid 15 \leqslant T \leqslant 25\}$

3 **a** Domain is $\{x \mid -1 < x \leqslant 5\}$, Range is $\{y \mid 1 < y \leqslant 3\}$
b Domain is $\{x \mid x \neq 2\}$, Range is $\{y \mid y \neq -1\}$
c Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid 0 < y \leqslant 2\}$
d Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid y \leqslant \frac{25}{4}\}$
e Domain is $\{x \mid x \geqslant -4\}$, Range is $\{y \mid y \geqslant -3\}$
f Domain is $\{x \mid x \neq \pm 2\}$,
Range is $\{y \mid y \leqslant -1$ or $y > 0\}$

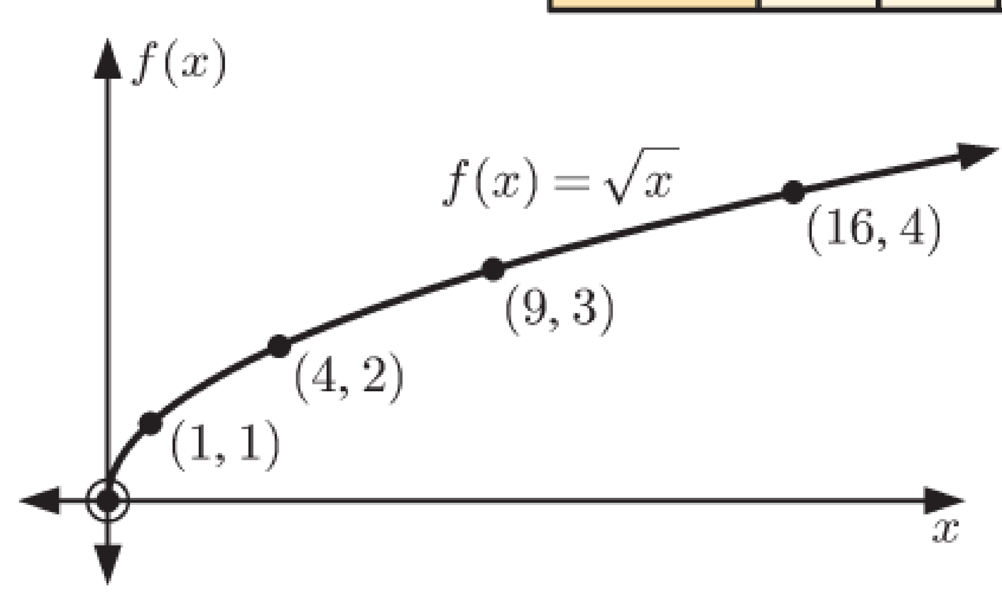
4 **a** true **b** false **c** true **d** true

5 **a** $\{y \mid y \geqslant 0\}$ **b** $\{y \mid y \leqslant 0\}$ **c** $\{y \mid y \geqslant 2\}$
d $\{y \mid y \leqslant 0\}$ **e** $\{y \mid y \leqslant 1\}$ **f** $\{y \mid y \geqslant 3\}$
g $\{y \mid y \geqslant -\frac{9}{4}\}$ **h** $\{y \mid y \leqslant 9\}$ **i** $\{y \mid y \leqslant \frac{25}{12}\}$

6 **a** $\{x \mid x \geqslant 0\}$ **b**

x	0	1	4	9	16
$f(x)$	0	1	2	3	4

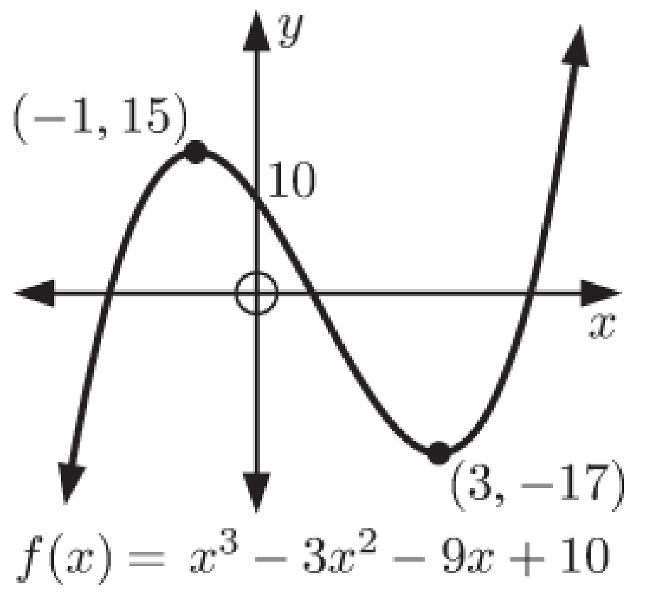
c



d $\{y \mid y \geqslant 0\}$

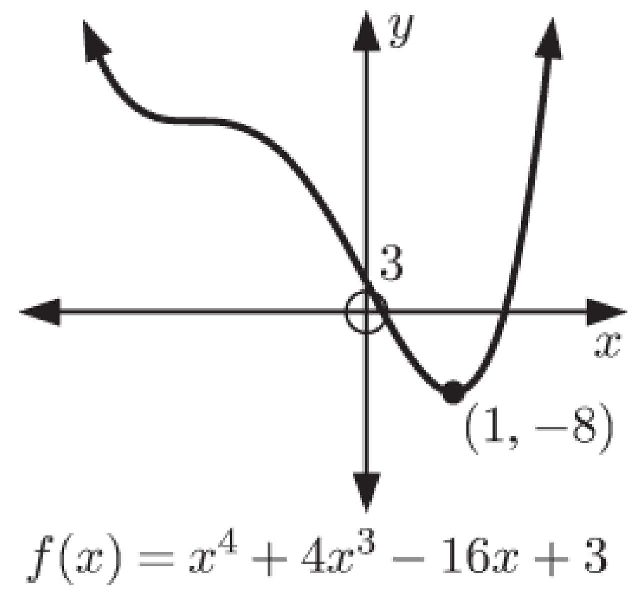
7 **a** Domain is $\{x \mid x \geqslant -6\}$, Range is $\{y \mid y \geqslant 0\}$
b Domain is $\{x \mid x \neq 0\}$, Range is $\{y \mid y > 0\}$
c Domain is $\{x \mid x \neq -1\}$, Range is $\{y \mid y \neq 0\}$
d Domain is $\{x \mid x > 0\}$, Range is $\{y \mid y < 0\}$
e Domain is $\{x \mid x \neq 3\}$, Range is $\{y \mid y \neq 0\}$
f Domain is $\{x \mid x \leqslant 4\}$, Range is $\{y \mid y \geqslant 0\}$

8 **a**



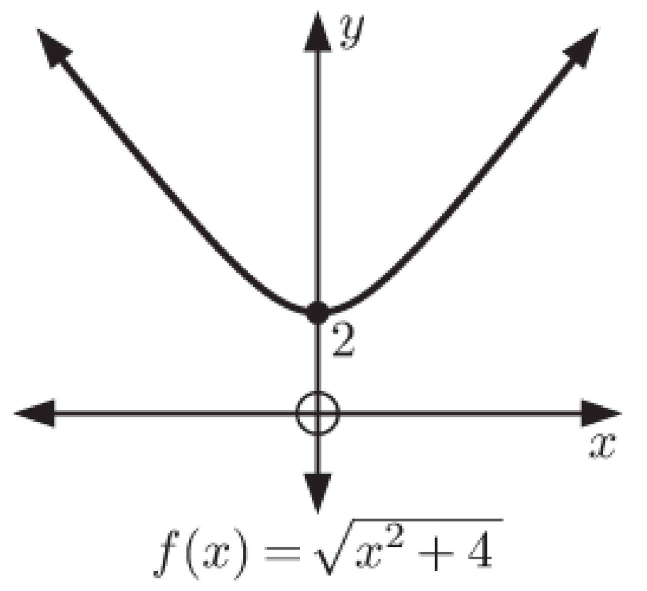
Domain is $\{x \mid x \in \mathbb{R}\}$,
Range is $\{y \mid y \in \mathbb{R}\}$

b



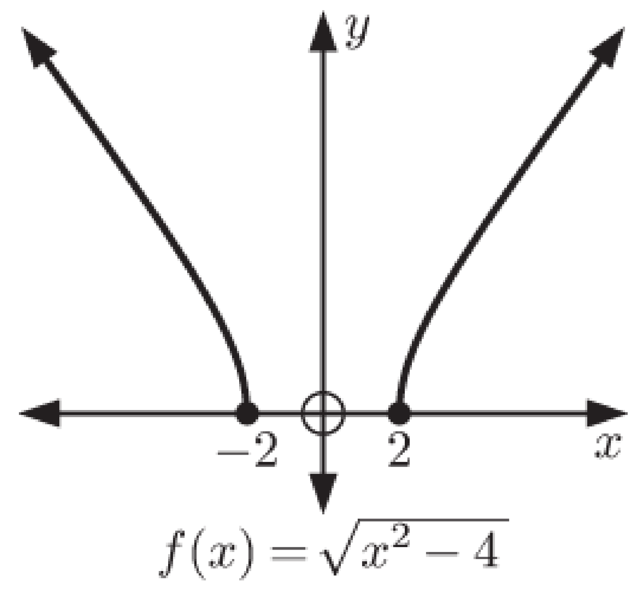
Domain is $\{x \mid x \in \mathbb{R}\}$,
Range is $\{y \mid y \geqslant -8\}$

c



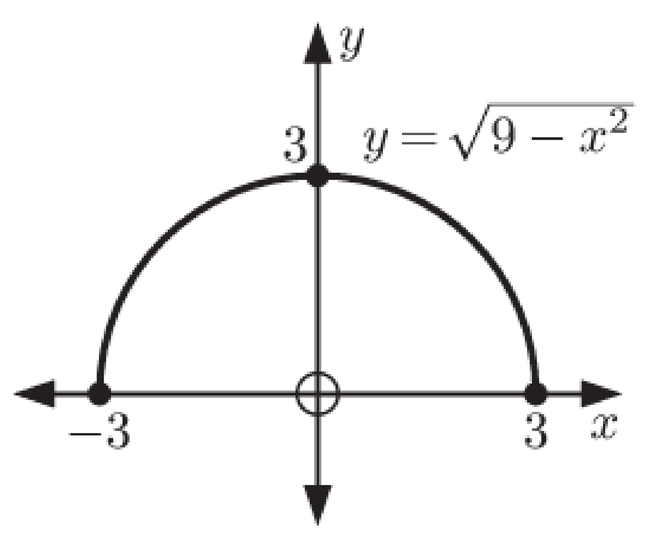
Domain is $\{x \mid x \in \mathbb{R}\}$,
Range is $\{y \mid y \geqslant 2\}$

d



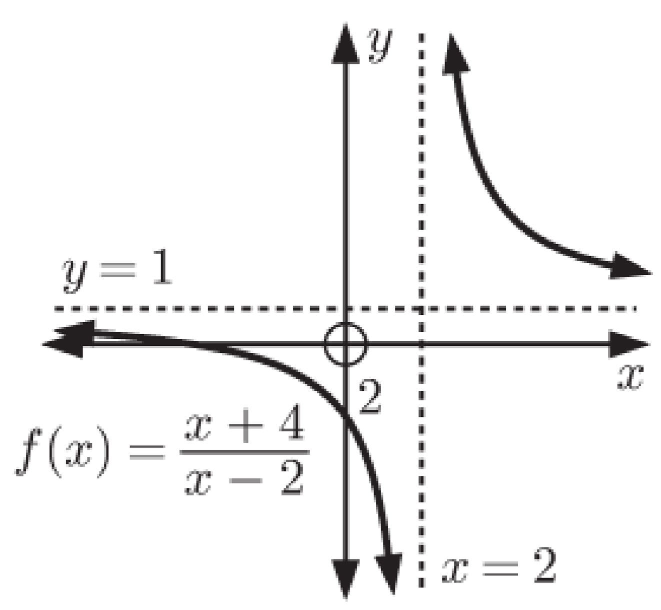
Domain is $\{x \mid x \leqslant -2$ or $x \geqslant 2\}$,
Range is $\{y \mid y \geqslant 0\}$

e



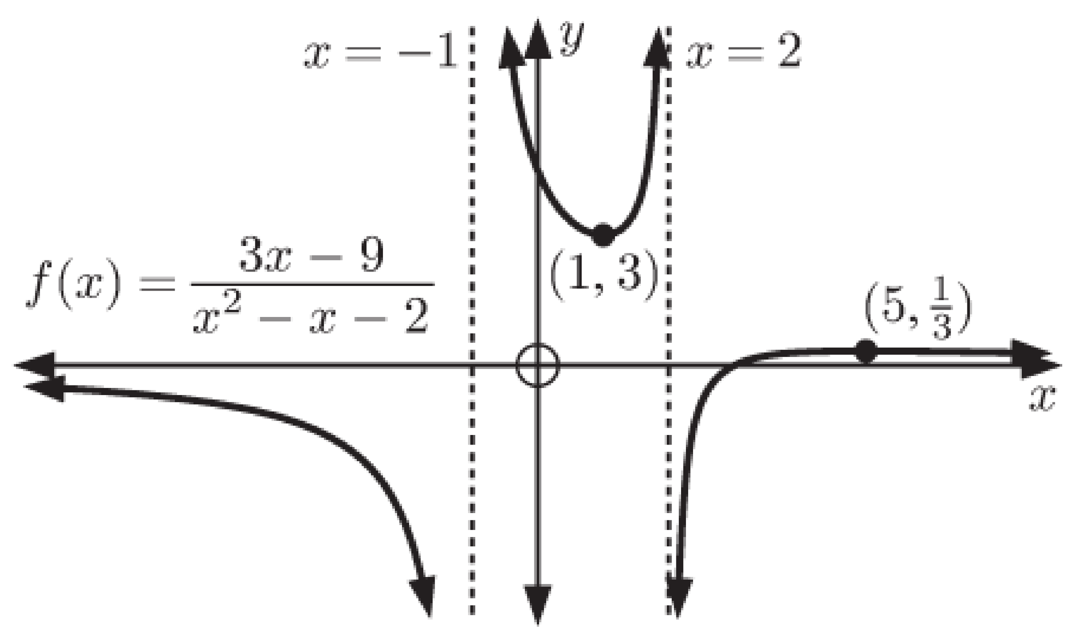
Domain is
$\{x \mid -3 \leqslant x \leqslant 3\}$,
Range is $\{y \mid 0 \leqslant y \leqslant 3\}$

f



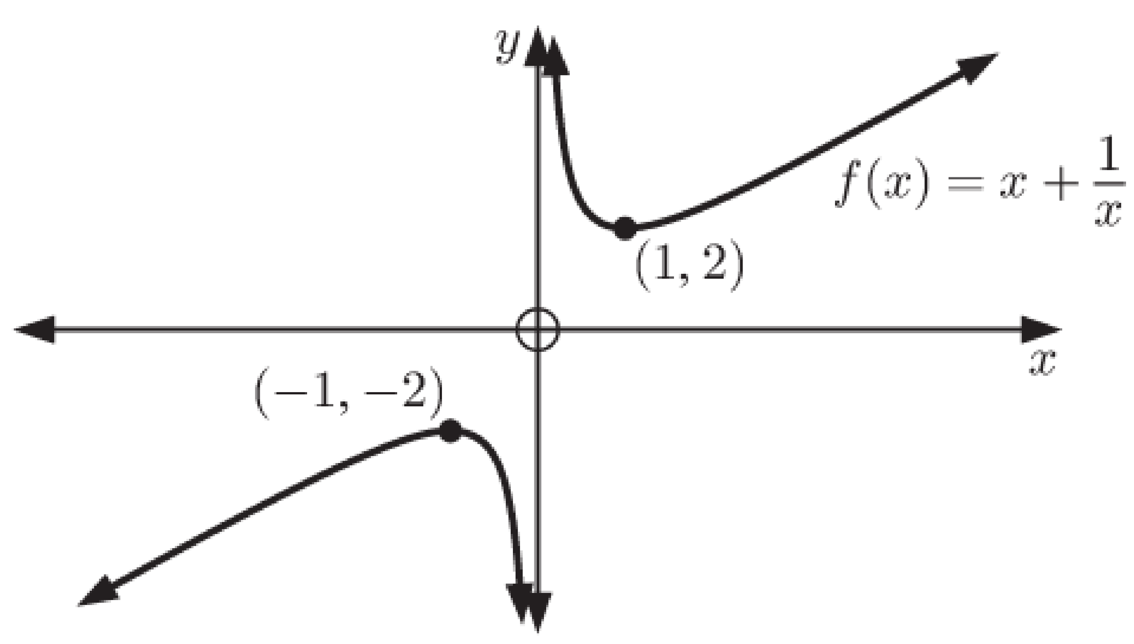
Domain is $\{x \mid x \neq 2\}$,
Range is $\{y \mid y \neq 1\}$

g



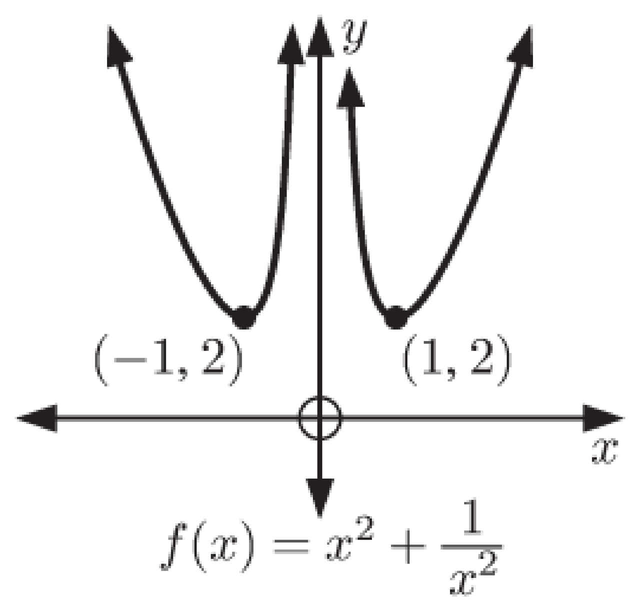
Domain is $\{x \mid x \neq -1$ or $2\}$,
Range is $\{y \mid y \leqslant \frac{1}{3}$ or $y \geqslant 3\}$

h



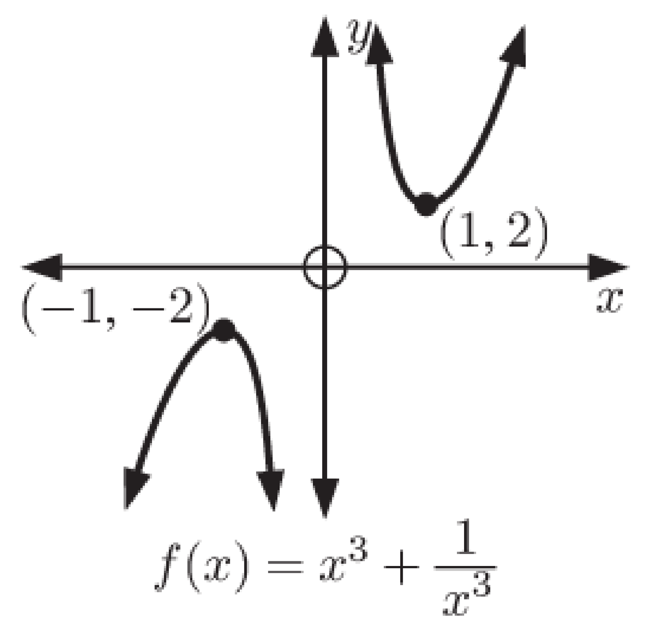
Domain is $\{x \mid x \neq 0\}$,
Range is $\{y \mid y \leqslant -2$ or $y \geqslant 2\}$

i



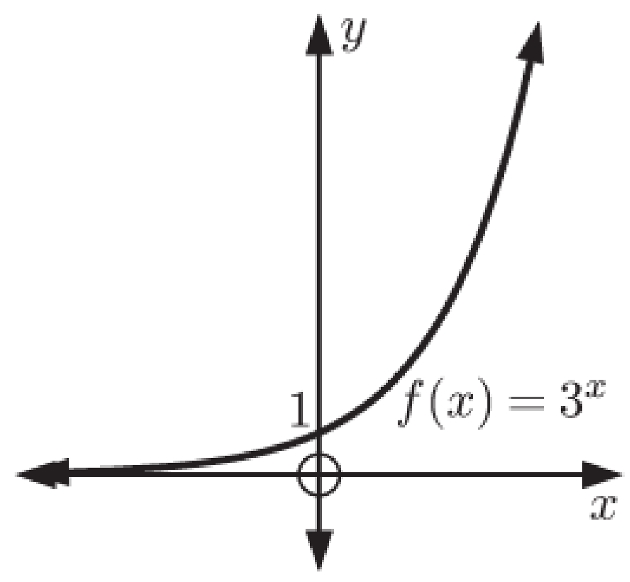
Domain is $\{x \mid x \neq 0\}$,
Range is $\{y \mid y \geqslant 2\}$

j



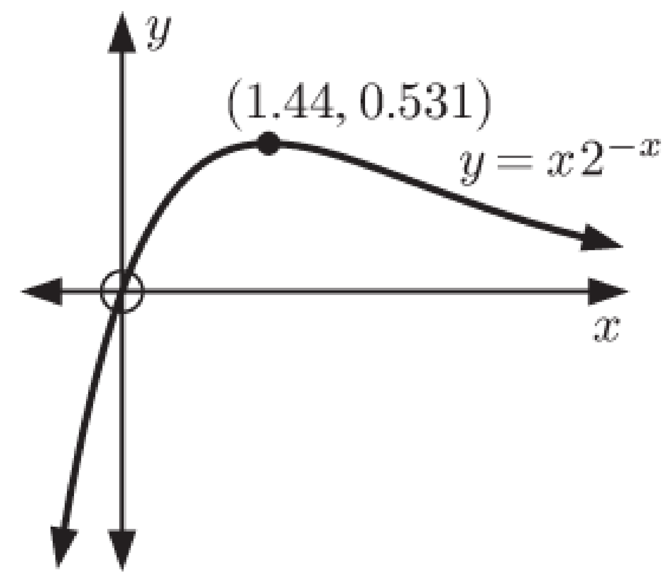
Domain is $\{x \mid x \neq 0\}$,
Range is $\{y \mid y \leqslant -2$ or $y \geqslant 2\}$

k



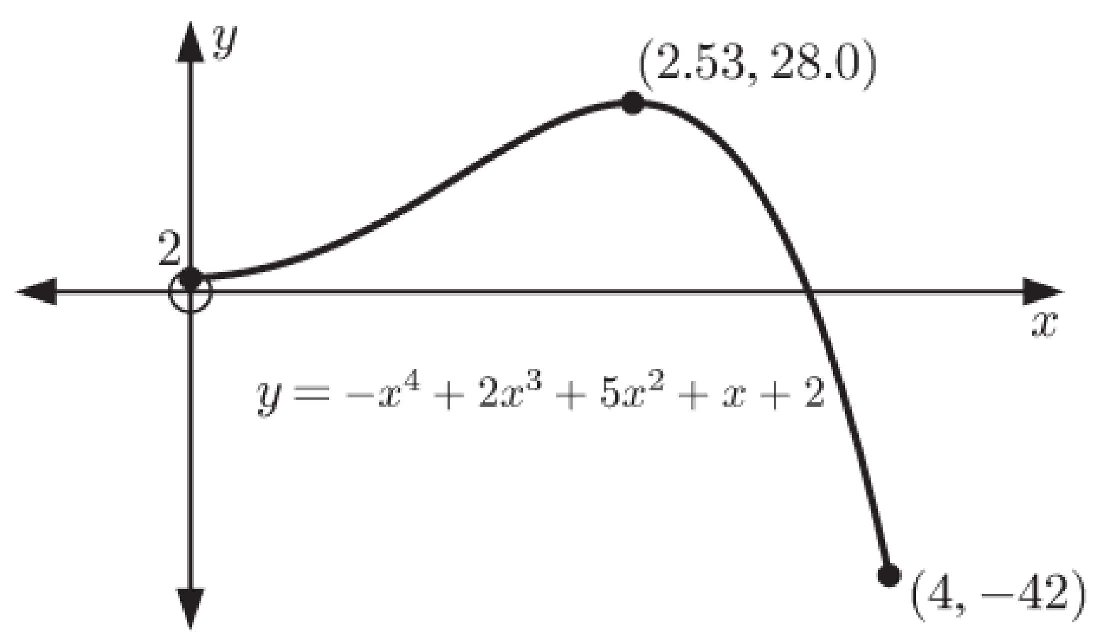
Domain is $\{x \mid x \in \mathbb{R}\}$,
Range is $\{y \mid y > 0\}$

l



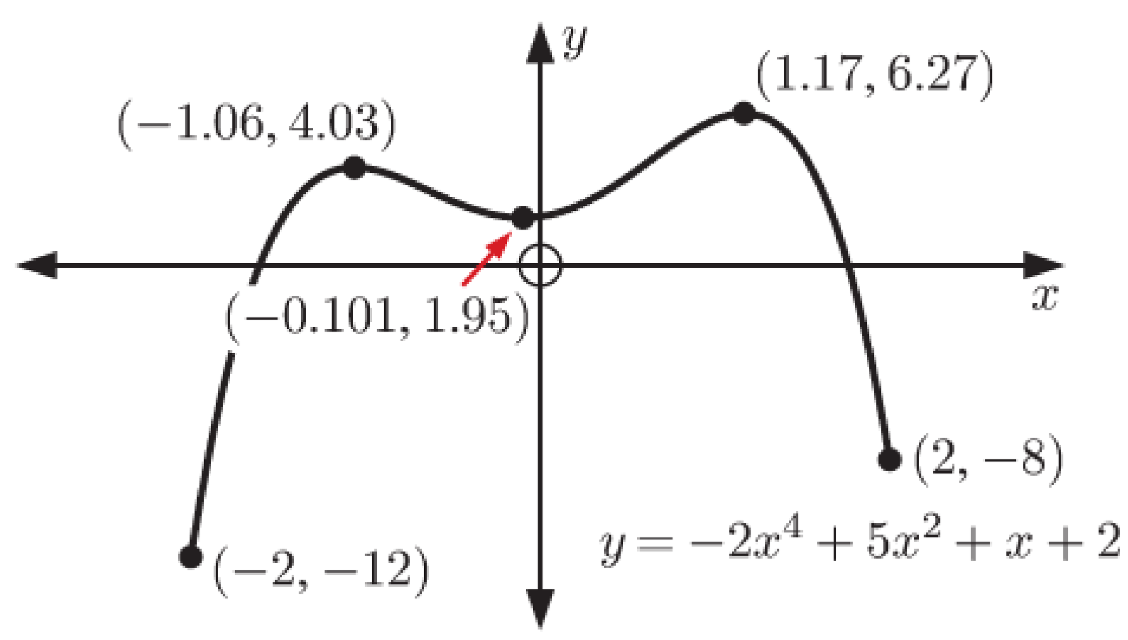
Domain is $\{x \mid x \in \mathbb{R}\}$,
Range is $\{y \mid y \leqslant 0.531\}$

9 **a**



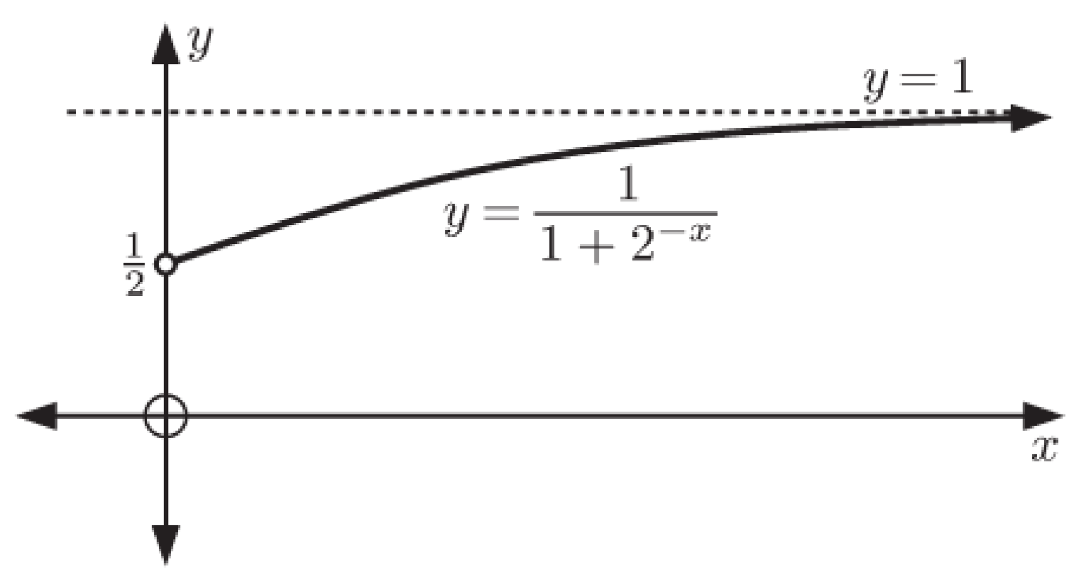
Range is $\{y \mid -42 \leqslant y \leqslant 28.0\}$

b



Range is $\{y \mid -12 \leqslant y \leqslant 6.27\}$

c



Range is $\{y \mid \frac{1}{2} < y < 1\}$

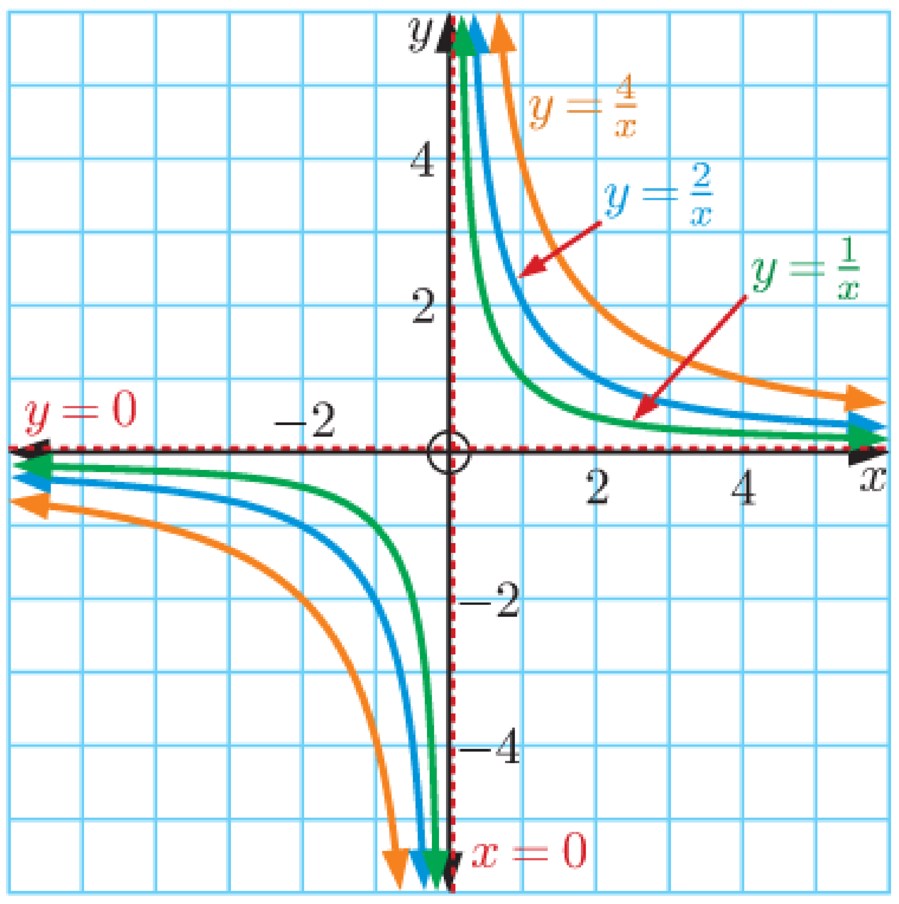
10 **a** $k \geqslant \frac{25}{4}$ **b** Range is $\{y \mid y \geqslant \sqrt{k - \frac{25}{4}}\}$

11 **a** Domain is $\{x \mid -2 \leqslant x \leqslant 2\}$
Range is $\{y \mid -2 \leqslant y \leqslant 2\}$

b Domain is $\{-2, -1, 0, 1, 2\}$
Range is $\{-2, -\sqrt{3}, 0, \sqrt{3}, 2\}$

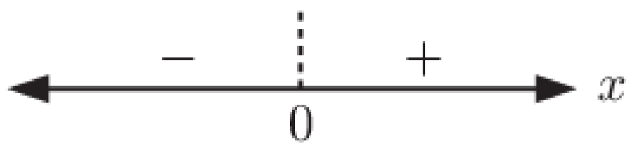
EXERCISE 15D.1

1 **a**

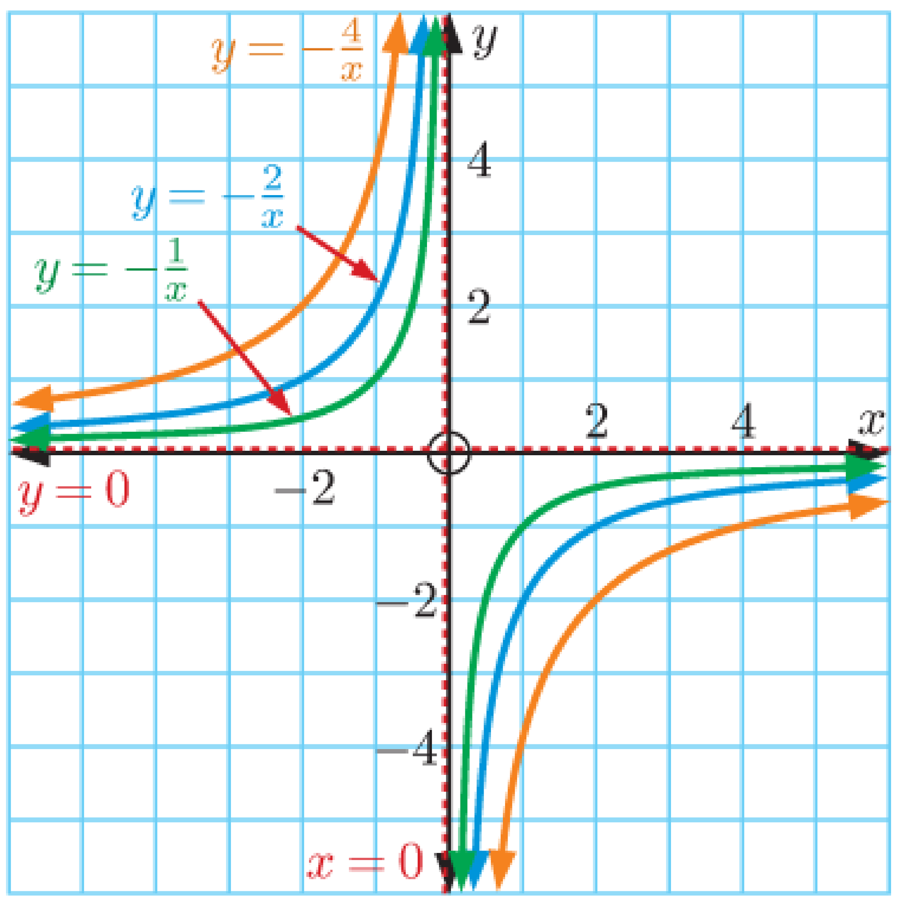


b **i** As k becomes larger the graphs move further from the origin.

ii quadrants 1 and 3 **iii**

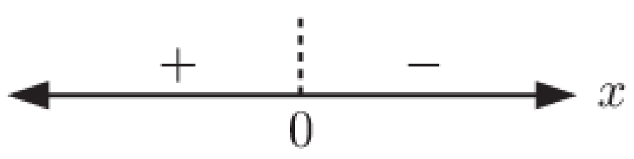


2 **a**



b **i** As $|k|$ becomes larger, the graphs move further from the origin.

ii quadrants 2 and 4 **iii**



3 **a** $\{x \mid x \neq 0\}$ **b** $\{y \mid y \neq 0\}$ **c** $x = 0$ **d** $y = 0$

4 **a** $y = \dfrac{6}{x}$ **b** $y = \dfrac{15}{x}$ **c** $y = -\dfrac{36}{x}$

EXERCISE 15D.2

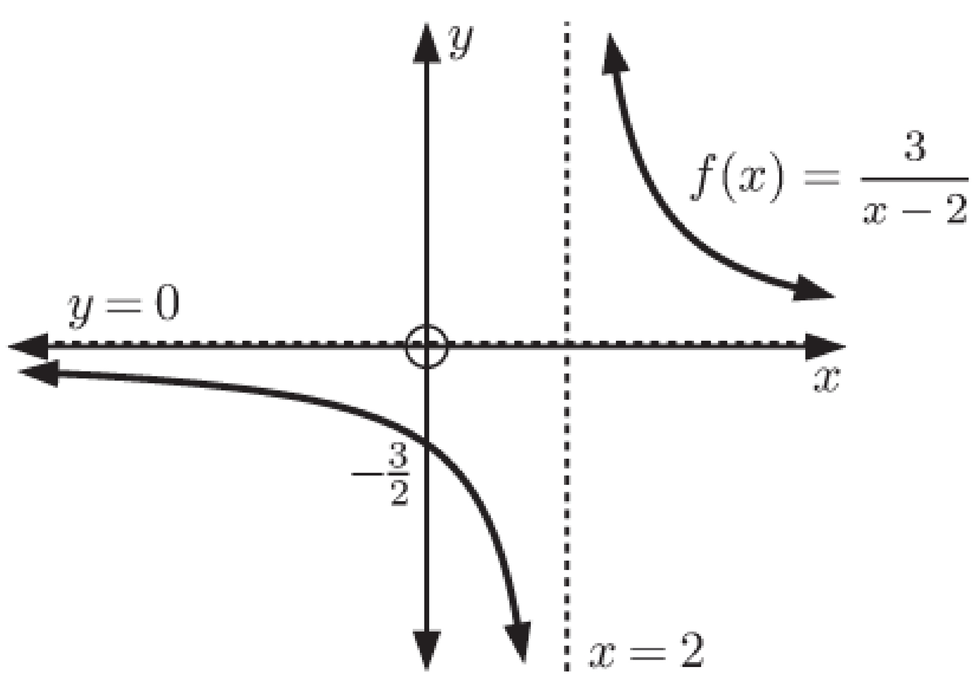
1 **a** **i** vertical asymptote $x = 2$, horizontal asymptote $y = 0$

ii Domain is $\{x \mid x \neq 2\}$, Range is $\{y \mid y \neq 0\}$

iii no x-intercept, y-intercept $-\frac{3}{2}$

iv as $x \to 2^-$, $f(x) \to -\infty$
as $x \to 2^+$, $f(x) \to \infty$
as $x \to -\infty$, $f(x) \to 0^-$
as $x \to \infty$, $f(x) \to 0^+$

v



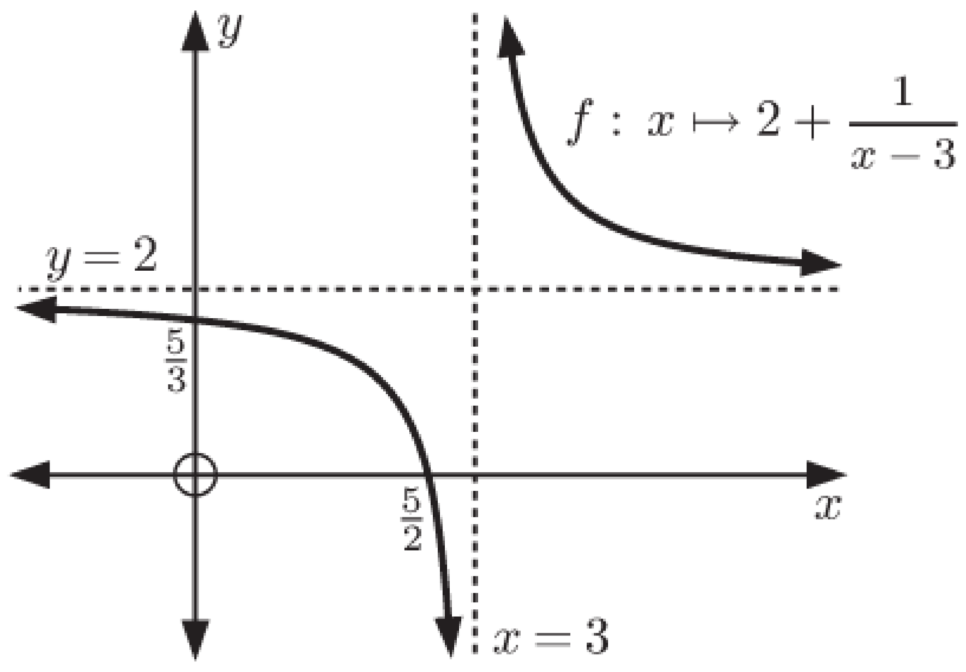
b **i** vertical asymptote $x = 3$, horizontal asymptote $y = 2$

ii Domain is $\{x \mid x \neq 3\}$, Range is $\{y \mid y \neq 2\}$

iii x-intercept $\frac{5}{2}$, y-intercept $\frac{5}{3}$

iv as $x \to 3^-$, $f(x) \to -\infty$
as $x \to 3^+$, $f(x) \to \infty$
as $x \to -\infty$, $f(x) \to 2^-$
as $x \to \infty$, $f(x) \to 2^+$

v



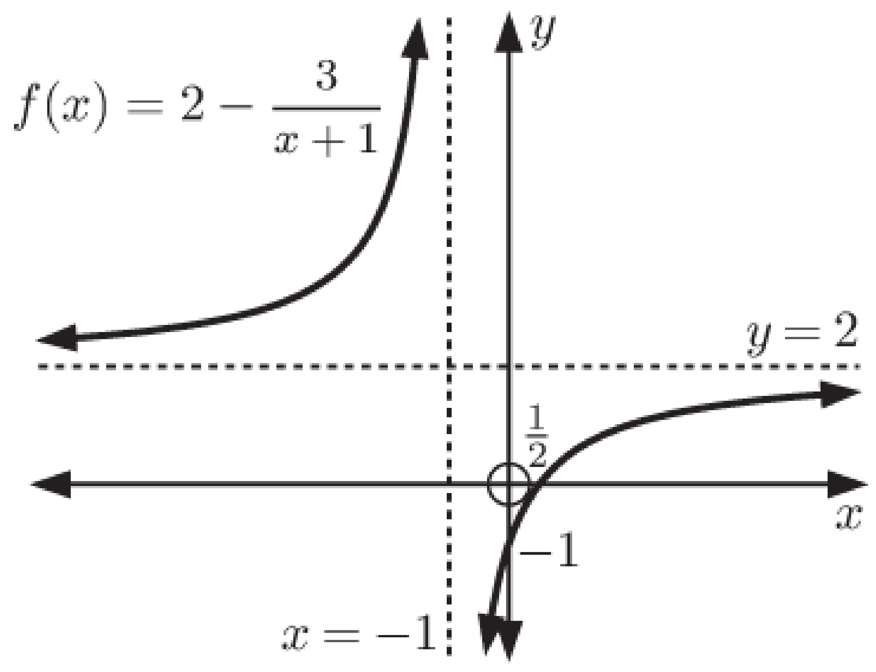
c **i** vertical asymptote $x = -1$, horizontal asymptote $y = 2$

ii Domain is $\{x \mid x \neq -1\}$, Range is $\{y \mid y \neq 2\}$

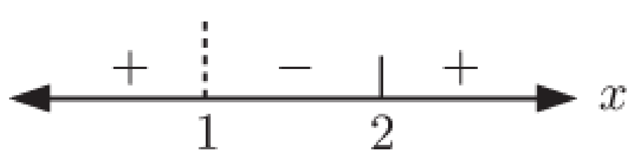
iii x-intercept $\frac{1}{2}$, y-intercept -1

iv as $x \to -1^-$, $f(x) \to \infty$
as $x \to -1^+$, $f(x) \to -\infty$
as $x \to -\infty$, $f(x) \to 2^+$
as $x \to \infty$, $f(x) \to 2^-$

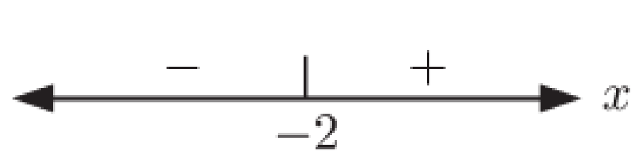
v



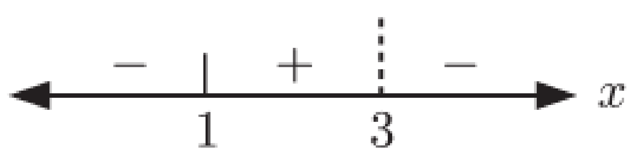
2 **a**



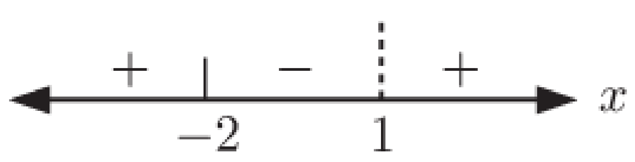
b



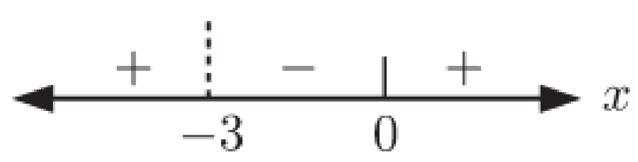
c



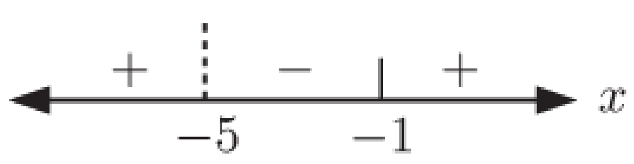
3 **a**



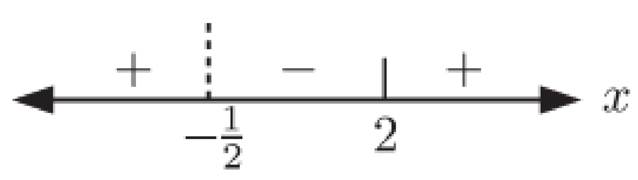
b



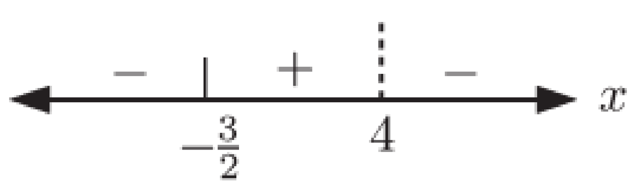
c



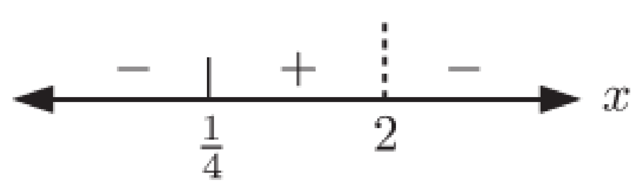
d



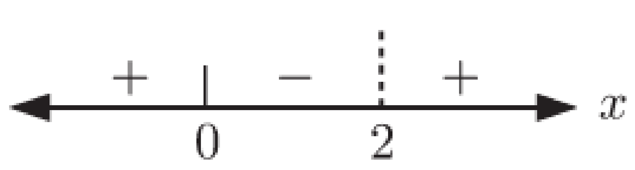
e



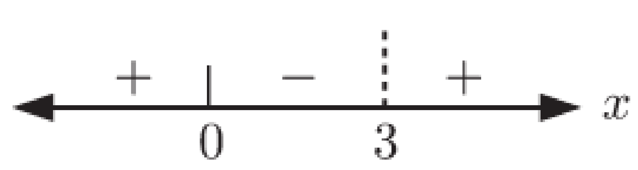
f



g



h

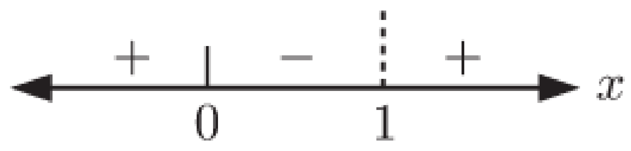


4 **a** **i** vertical asymptote is $x = 1$

ii x-intercept 0, y-intercept 0

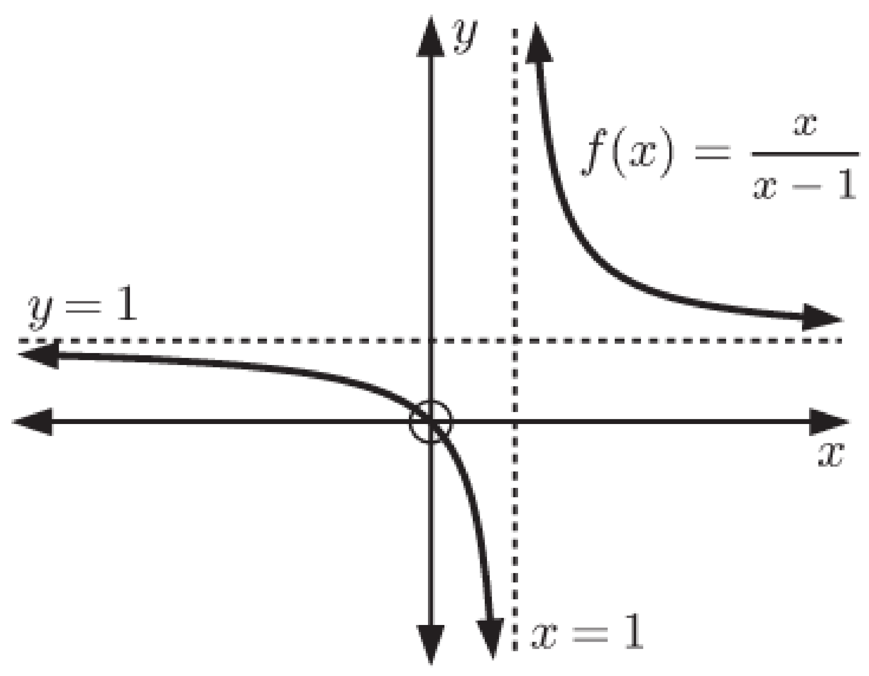
iii $f(x) = 1 + \dfrac{1}{x-1}$, horizontal asymptote is $y = 1$

iv



v as $x \to 1^-$, $f(x) \to -\infty$
as $x \to 1^+$, $f(x) \to \infty$
as $x \to -\infty$, $f(x) \to 1^-$
as $x \to \infty$, $f(x) \to 1^+$

vi

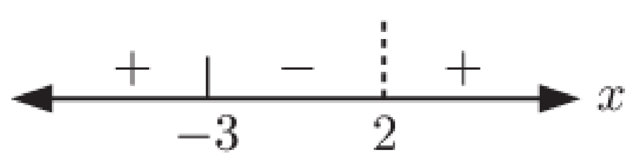


b **i** vertical asymptote is $x = 2$

ii x-intercept -3, y-intercept $-\frac{3}{2}$

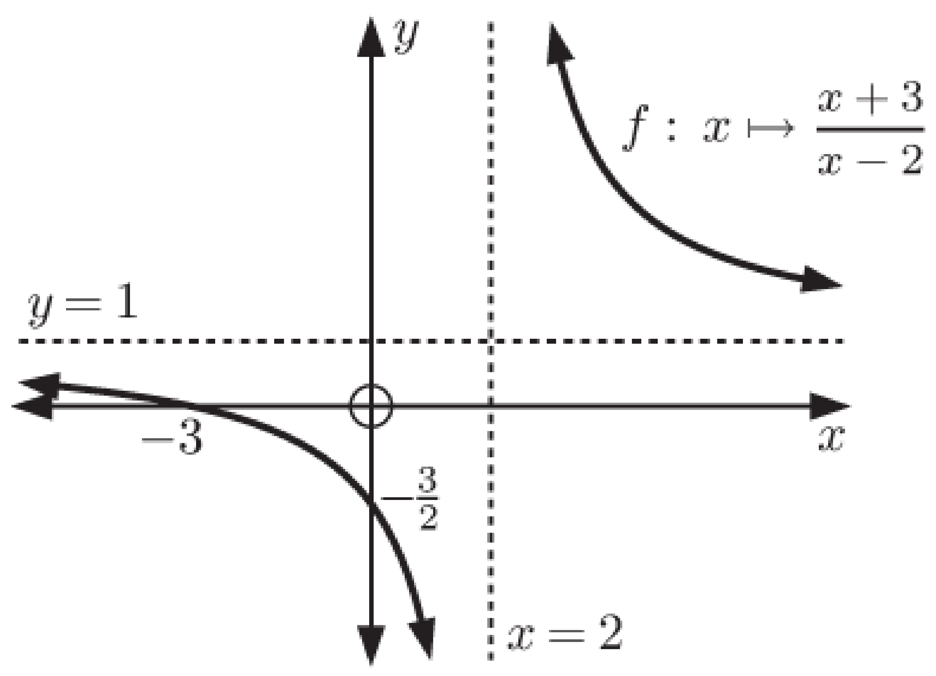
iii $f(x) = 1 + \dfrac{5}{x-2}$, horizontal asymptote is $y = 1$

iv



v as $x \to 2^-$, $f(x) \to -\infty$
as $x \to 2^+$, $f(x) \to \infty$
as $x \to -\infty$, $f(x) \to 1^-$
as $x \to \infty$, $f(x) \to 1^+$

vi

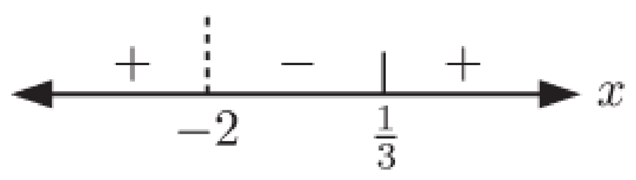


c **i** vertical asymptote is $x = -2$

ii x-intercept $\frac{1}{3}$, y-intercept $-\frac{1}{2}$

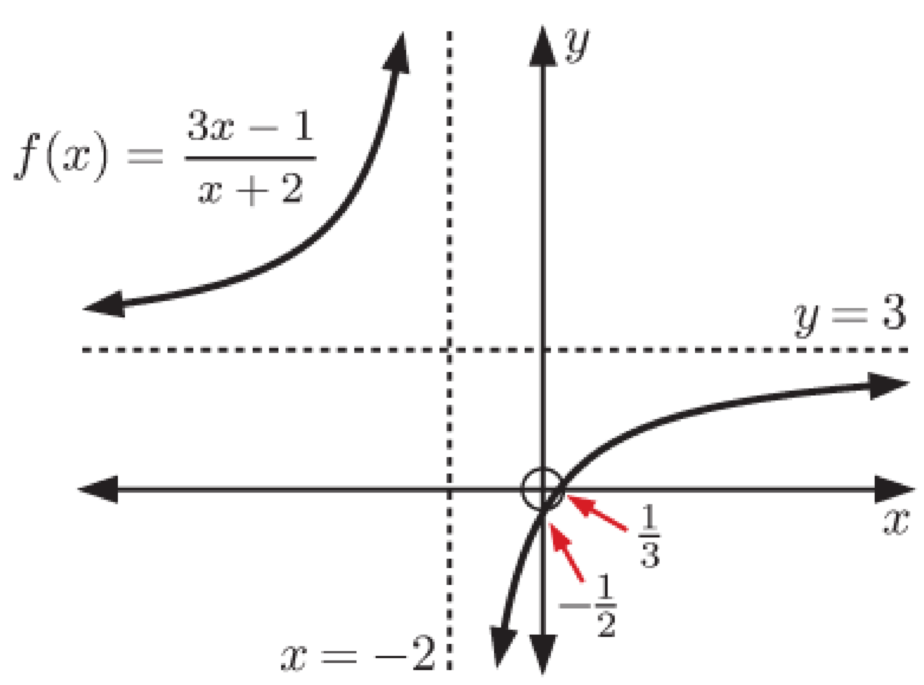
iii $f(x) = 3 - \dfrac{7}{x+2}$, horizontal asymptote is $y = 3$

iv



v as $x \to -2^-$, $f(x) \to \infty$
as $x \to -2^+$, $f(x) \to -\infty$
as $x \to -\infty$, $f(x) \to 3^+$
as $x \to \infty$, $f(x) \to 3^-$

vi

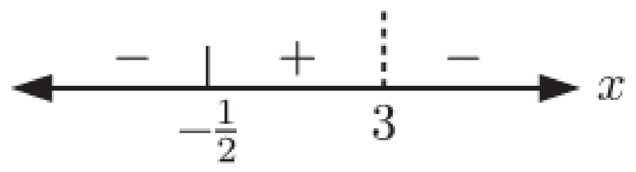


d **i** vertical asymptote is $x = 3$

ii x-intercept $-\frac{1}{2}$, y-intercept $\frac{1}{3}$

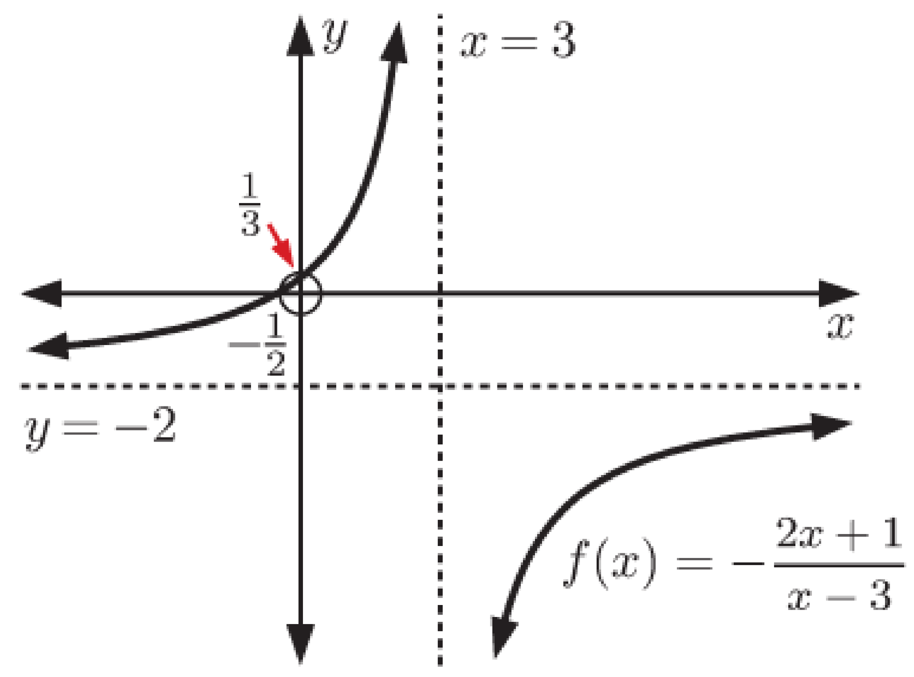
iii $f(x) = -2 - \dfrac{7}{x-3}$, horizontal asymptote is $y = -2$

iv



v as $x \to 3^-$, $f(x) \to \infty$
as $x \to 3^+$, $f(x) \to -\infty$
as $x \to -\infty$, $f(x) \to -2^+$
as $x \to \infty$, $f(x) \to -2^-$

vi

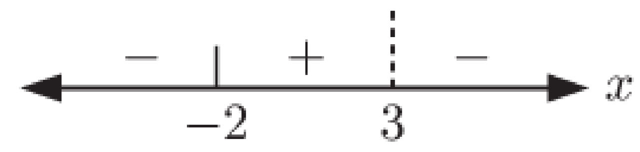


e **i** vertical asymptote is $x = 3$

ii x-intercept -2, y-intercept $\frac{4}{3}$

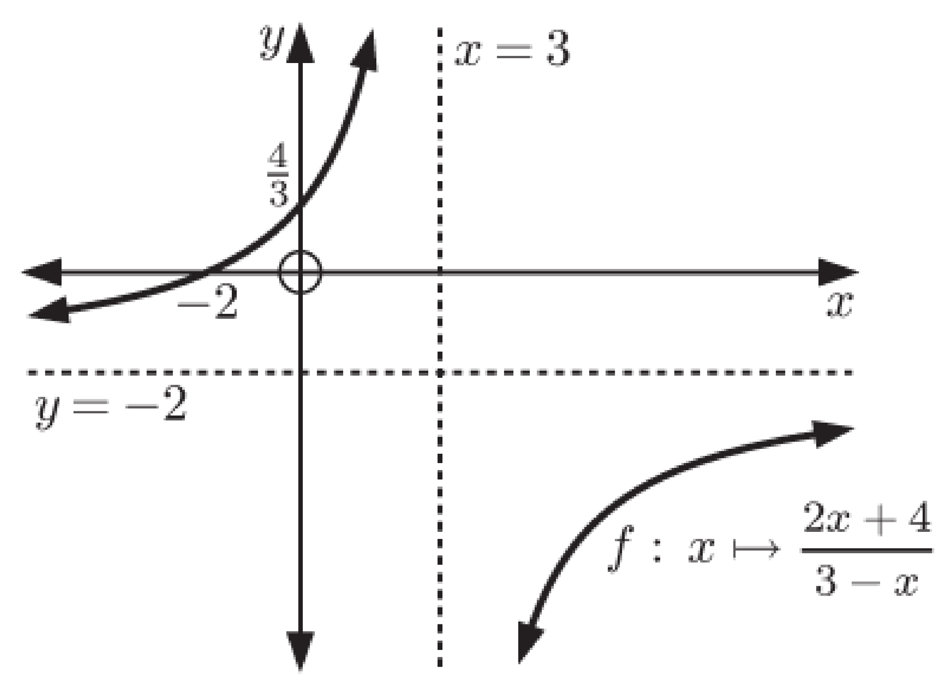
iii $f(x) = -2 + \dfrac{10}{3-x}$, horizontal asymptote is $y = -2$

iv



v as $x \to 3^-$, $f(x) \to \infty$
as $x \to 3^+$, $f(x) \to -\infty$
as $x \to -\infty$, $f(x) \to -2^+$
as $x \to \infty$, $f(x) \to -2^-$

vi

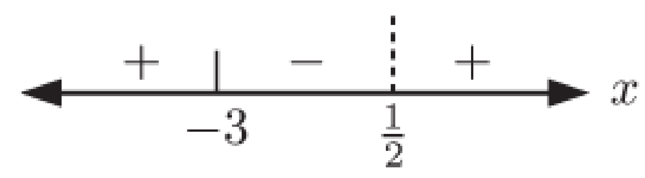


f **i** vertical asymptote is $x = \frac{1}{2}$

ii x-intercept -3, y-intercept -3

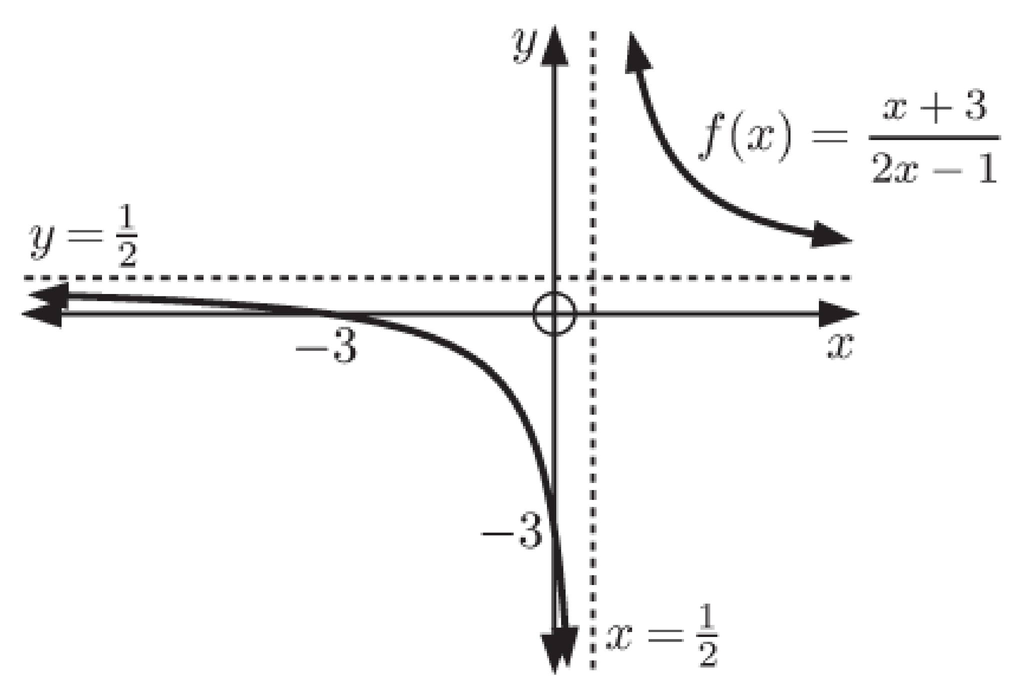
iii $f(x) = \frac{1}{2} + \dfrac{7}{4x-2}$, horizontal asymptote is $y = \frac{1}{2}$

iv

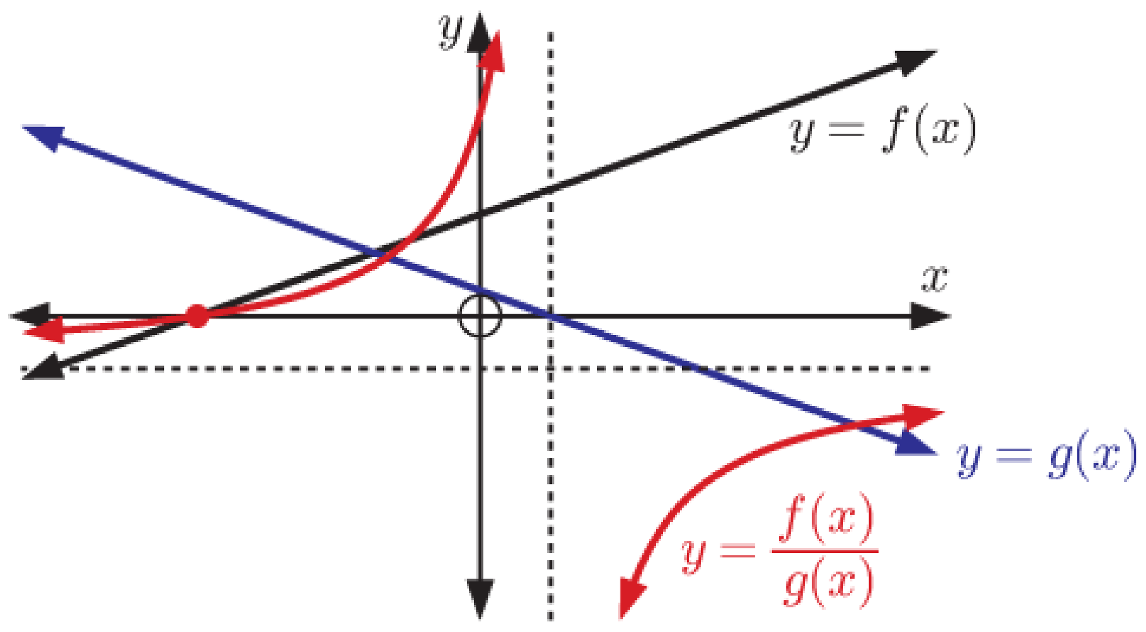


v as $x \to \frac{1}{2}^-$, $f(x) \to -\infty$
as $x \to \frac{1}{2}^+$, $f(x) \to \infty$
as $x \to -\infty$, $f(x) \to \frac{1}{2}^-$
as $x \to \infty$, $f(x) \to \frac{1}{2}^+$

vi



5



6 **a** Domain is $\{x \mid x \neq -\frac{d}{c}\}$

b vertical asymptote is $x = -\frac{d}{c}$

c x-intercept is $-\frac{b}{a}$, y-intercept is $\frac{b}{d}$

d $\dfrac{ax+b}{cx+d} = \dfrac{\frac{a}{c}(cx+d) - \frac{ad}{c} + b}{cx+d}$ and so on

As $x \to \infty$, $\dfrac{b - \frac{ad}{c}}{cx+d} \to 0$.

$\therefore$ the horizontal asymptote is $y = \frac{a}{c}$.

EXERCISE 15E

1 **a** $-2 - 2x^2$ **b** $1 + 4x^2$ **c** -10 **d** -4

2 **a** $-4x^2 - 16x - 13$ **b** $10 - 2x^2$ **c** 14 **d** $-\frac{73}{16}$

3 **a** $25x - 42$ **b** $\sqrt{8}$ **c** -7 **d** 2

4 **a** **i** $x^2 - 6x + 10$ **ii** $2 - x^2$ **b** $x = \pm\frac{1}{\sqrt{2}}$

5 **a** $(f \circ g)(x) = 9 - \sqrt{x^2+4}$

Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid y \leqslant 7\}$

b 53

c $(f \circ f)(x) = 9 - \sqrt{9 - \sqrt{x}}$

Domain is $\{x \mid 0 \leqslant x \leqslant 81\}$, Range is $\{y \mid 6 \leqslant y \leqslant 9\}$

6 **a** $-6x - 9$ **b** $x = -1$

7 **a** **i** $1 - 9x^2$ **ii** $1 + 6x - 3x^2$ **b** $x = -\frac{1}{9}$

8 **a** $(f \circ g)(x) = \dfrac{1}{x-3}$

Domain is $\{x \mid x \neq 3\}$, Range is $\{y \mid y \neq 0\}$

b $(f \circ g)(x) = -\dfrac{1}{x^2+3x+2}$

Domain is $\{x \mid x \neq -1,\ x \neq -2\}$

Range is $\{y \mid y \geqslant 4,\ y < 0\}$

9 **a** $f \circ g = \{(2, 7), (5, 2), (7, 5), (9, 9)\}$

b $g \circ f = \{(0, 2), (1, 0), (2, 1), (3, 3)\}$

10 **a** $(f \circ g)(x) = \dfrac{4x-2}{3x-1}$, Domain is $\{x \mid x \neq \frac{1}{3} \text{ or } 1\}$

b $(g \circ f)(x) = 2x + 5$, Domain is $\{x \mid x \neq -2\}$

c $(g \circ g)(x) = x$, Domain is $\{x \mid x \neq 1\}$

11 **a** Let $x = 0$, $\therefore$ $b = d$ and so

$ax + b = cx + b$

$\therefore$ $ax = cx$ for all x

Let $x = 1$, $\therefore$ $a = c$

b $(f \circ g)(x) = [2a]x + [2b + 3] = 1x + 0$ for all x

$\therefore$ $2a = 1$ and $2b + 3 = 0$

c Yes, $\{(g \circ f)(x) = [2a]x + [3a + b]\}$

12 **a** $(f \circ g)(x) = \sqrt{1 - x^2}$

b Domain is $\{x \mid -1 \leqslant x \leqslant 1\}$, Range is $\{y \mid 0 \leqslant y \leqslant 1\}$

c $(g \circ f)(x) = 1 - x$

d Domain is $\{x \mid x \leqslant 1\}$, Range is $\{y \mid y \geqslant 0\}$

13 **a** $R_g \cap D_f \neq \varnothing$

b Domain is $\{x \mid x \in D_g,\ g(x) \in D_f\}$

14 **a** $V \circ D = 6800 - 400t$

This is the value of Mila's car t years after purchase.

b 4400; the value of Mila's car 6 years after purchase is \$4400.

15 **a** **i** $T \circ S$ **ii** $S \circ T$ **b** €715

16 **a** $V = 2000 - 20t$

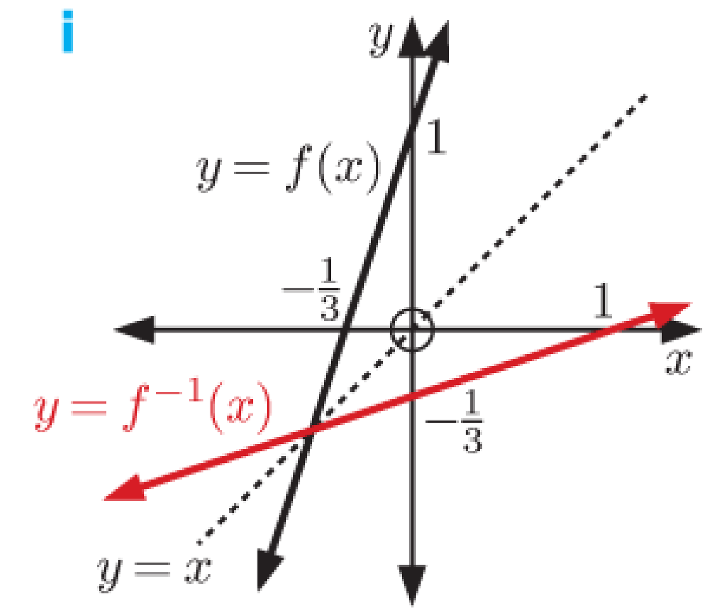
c $H \circ V = \sqrt[3]{\dfrac{24\,000 - 240t}{\pi}}$

This is the height of the solution after t minutes.

d $(H \circ V)(30) \approx 17.5$; the height of the solution after 30 minutes is about 17.5 cm.

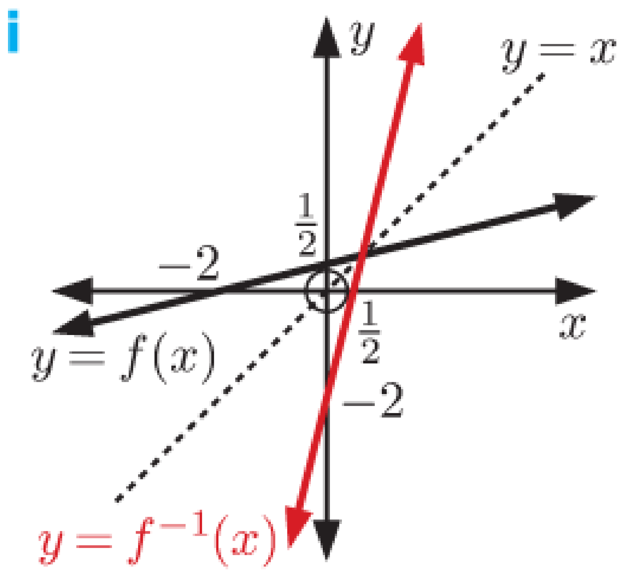
EXERCISE 15F

1 **a** **i**



ii, iii $f^{-1}(x) = \dfrac{x-1}{3}$

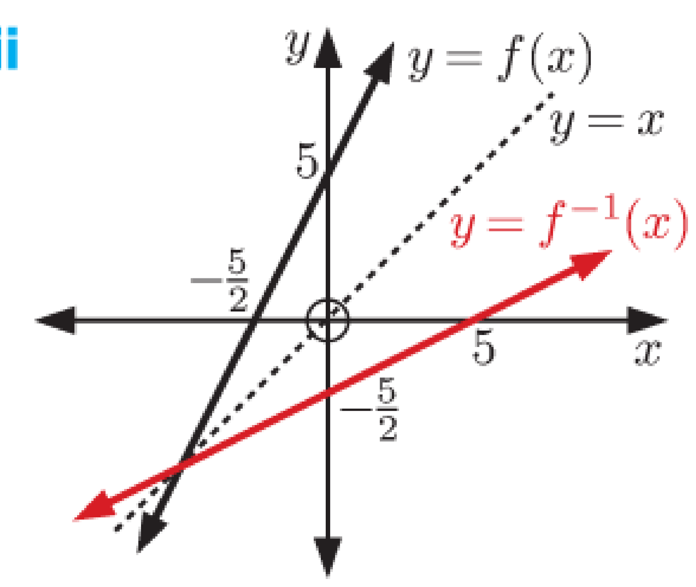
b **i**



ii, iii $f^{-1}(x) = 4x - 2$

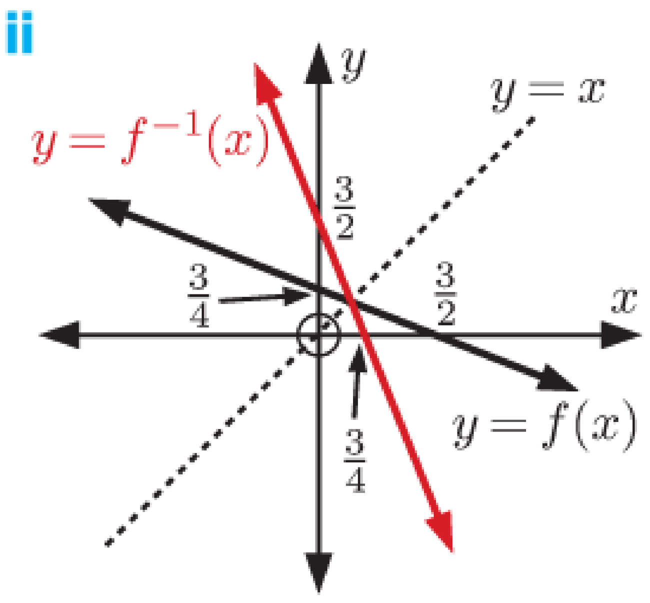
2 **a** **i** $f^{-1}(x) = \dfrac{x-5}{2}$

ii

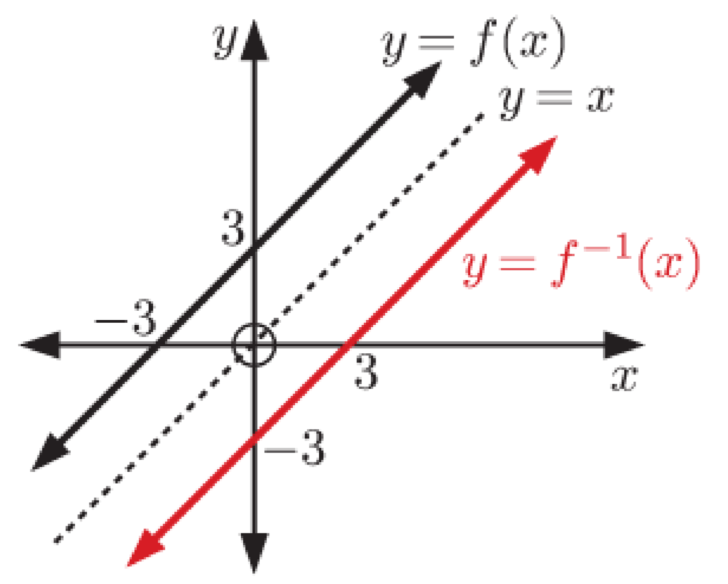


b **i** $f^{-1}(x) = -2x + \frac{3}{2}$

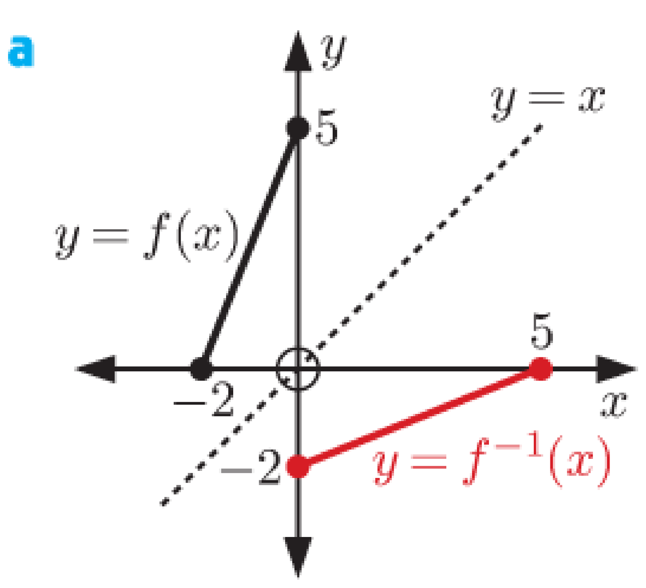
ii



c **i** $f^{-1}(x) = x - 3$ **ii**



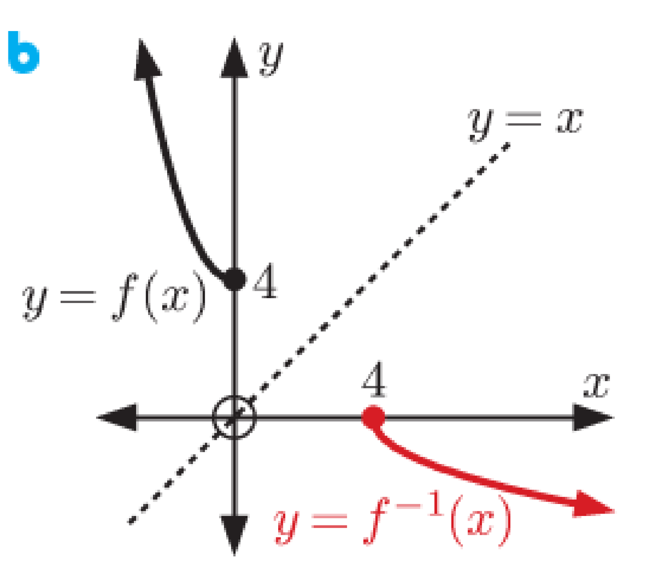
3 **a**



f:
Domain is $\{x \mid -2 \leqslant x \leqslant 0\}$
Range is $\{y \mid 0 \leqslant y \leqslant 5\}$

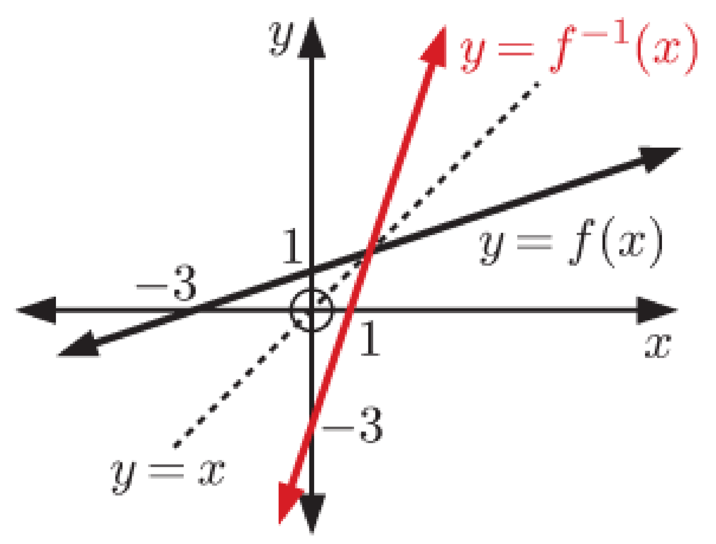
f^{-1}:
Domain is $\{x \mid 0 \leqslant x \leqslant 5\}$
Range is $\{y \mid -2 \leqslant y \leqslant 0\}$

b

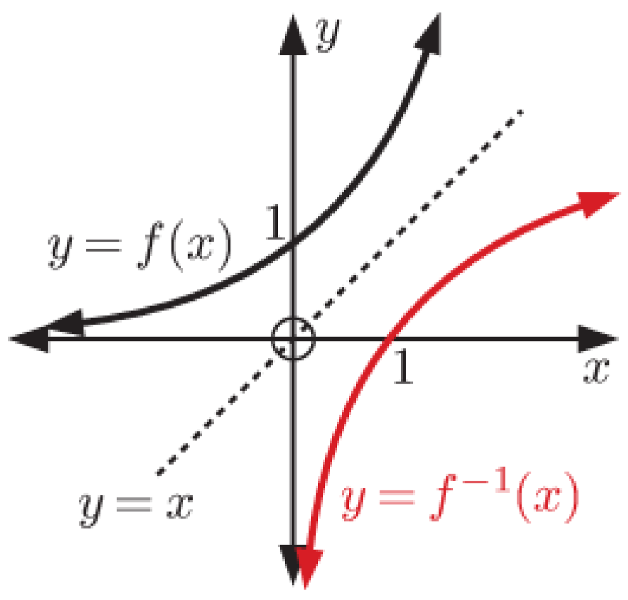


f:
Domain is $\{x \mid x \leqslant 0\}$
Range is $\{y \mid y \geqslant 4\}$

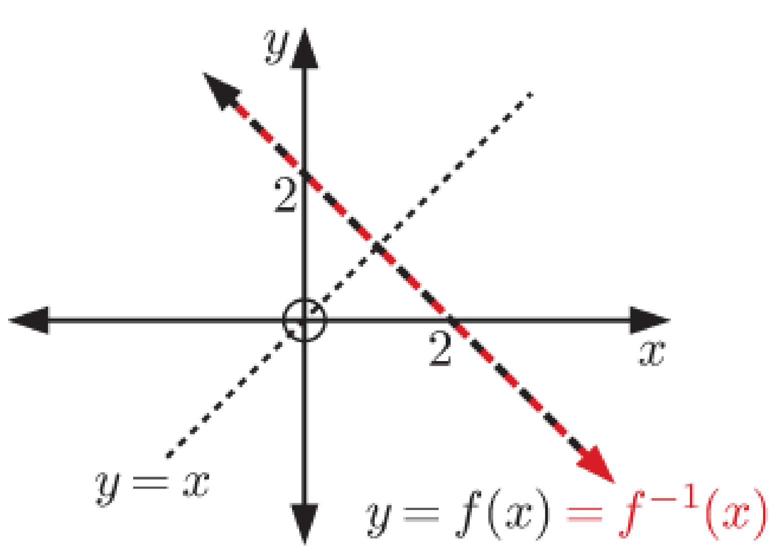
f^{-1}:
Domain is $\{x \mid x \geqslant 4\}$
Range is $\{y \mid y \leqslant 0\}$

c 

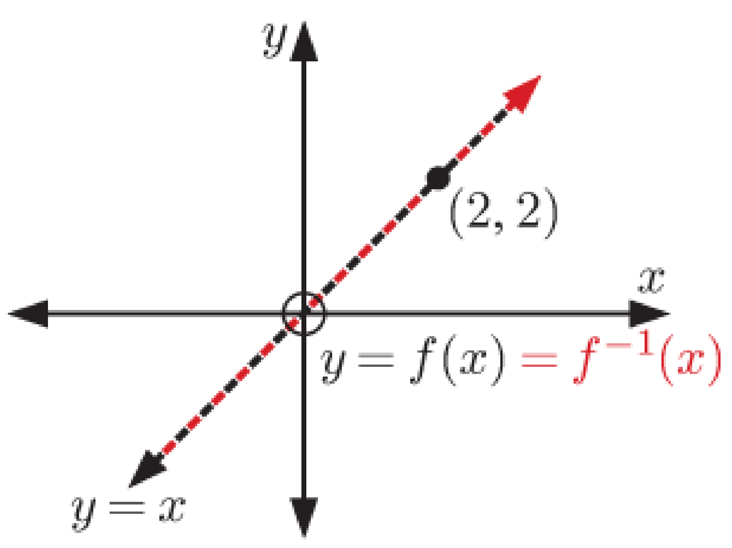
f:
Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y \in \mathbb{R}\}$
f^{-1}:
Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y \in \mathbb{R}\}$

d 

f:
Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y > 0\}$
f^{-1}:
Domain is $\{x \mid x > 0\}$
Range is $\{y \mid y \in \mathbb{R}\}$

e 

f:
Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y \in \mathbb{R}\}$
f^{-1}:
Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y \in \mathbb{R}\}$

f 

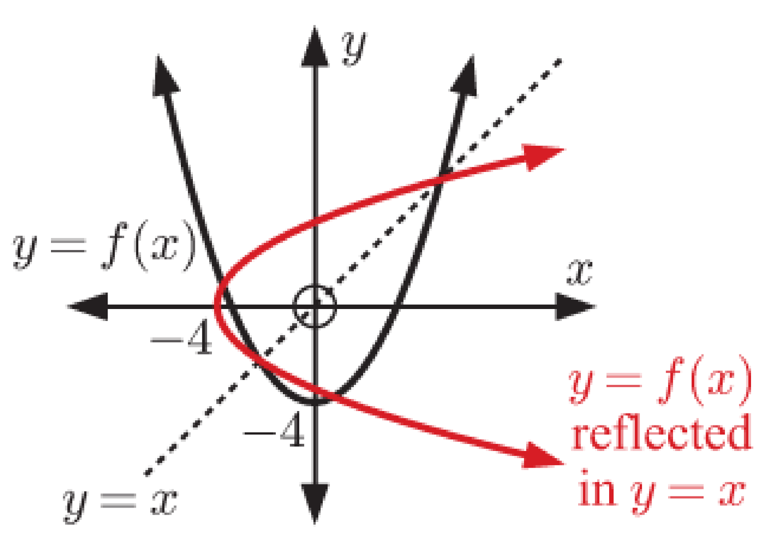
f:
Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y \in \mathbb{R}\}$
f^{-1}:
Domain is $\{x \mid x \in \mathbb{R}\}$
Range is $\{y \mid y \in \mathbb{R}\}$

4 $(f^{-1})^{-1}(x) = 2x - 5 = f(x)$

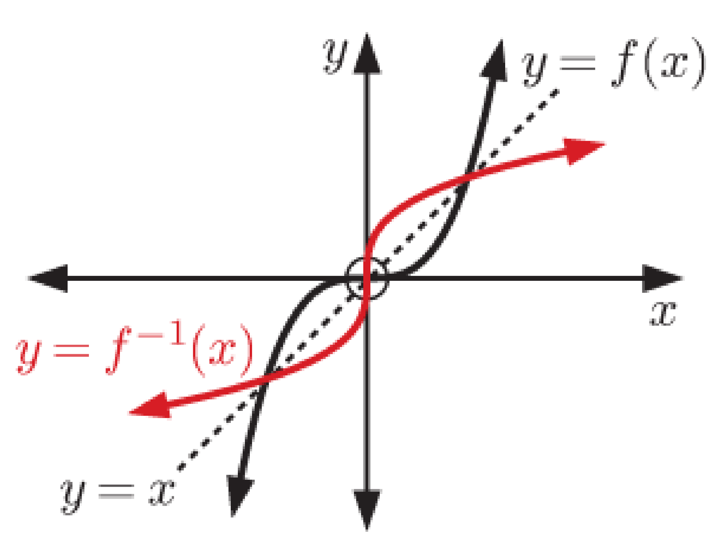
5 **a** $\{(2, 1), (4, 2), (5, 3)\}$ **b** inverse does not exist
c $\{(1, 2), (0, -1), (2, 0), (3, 1)\}$
d $\{(-1, -1), (0, 0), (1, 1)\}$

6 $f(x) = x$ and $f(x) = -x + c,\ c \in \mathbb{R}$

7 Range is $\{y \mid -2 \leqslant y < 3\}$

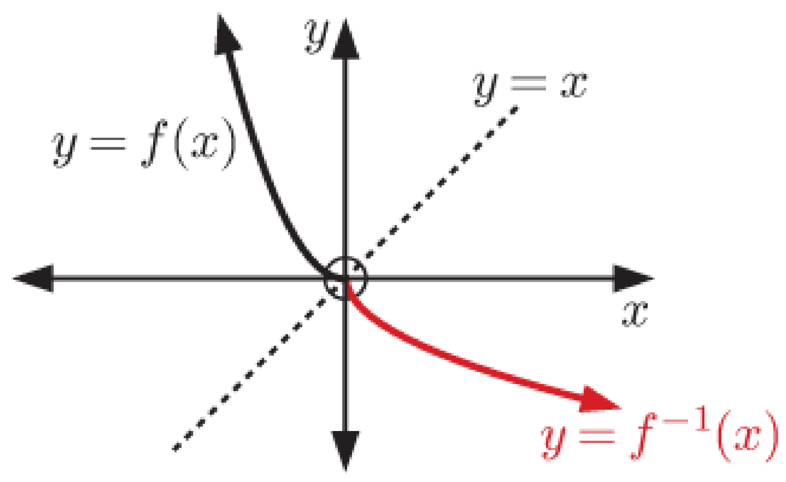
8 **a** 

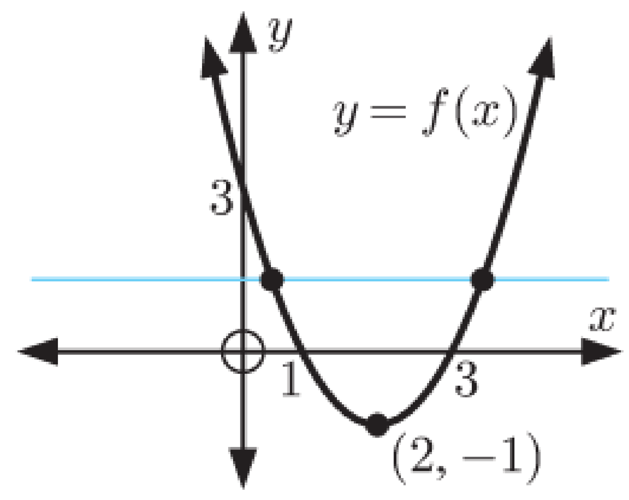
b no **c** yes, it is $f^{-1} : x \mapsto \sqrt{x+4}$

9 

10 f is $y = \dfrac{1}{x},\ x \neq 0$ $\therefore$ f^{-1} is $x = \dfrac{1}{y}$
$\therefore\ y = \dfrac{1}{x}$
$\therefore\ f = f^{-1}$
$\therefore\ f$ is self-inverse.

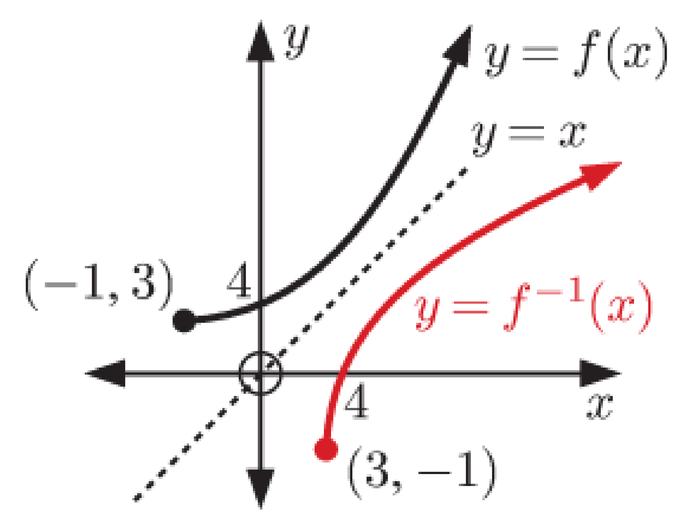
11 **a** The inverse function must also be a function and must therefore satisfy the vertical line test, which it can only do if the original function satisfies the horizontal line test.
b **i** is the only one.
c **i** Domain is $\{x \mid x \geqslant 1\}$ or $\{x \mid x \leqslant 1\}$
ii Domain is $\{x \mid x \geqslant 1\}$ or $\{x \mid x \leqslant -2\}$

12 **a** $f^{-1}(x) = -\sqrt{x}$ **b** 

13 **a** 

Every vertical line cuts the graph once. So, it is a function.
A horizontal line above the vertex cuts the graph **twice**. So, it does not have an inverse.

b For $x \geqslant 2$, all horizontal lines cut the graph at most once.
$\therefore\ g(x)$ has an inverse.
Hint: Inverse is $x = y^2 - 4y + 3$ for $y \geqslant 2$
c g: Domain is $\{x \mid x \geqslant 2\}$, Range is $\{y \mid y \geqslant -1\}$
g^{-1}: Domain is $\{x \mid x \geqslant -1\}$, Range is $\{y \mid y \geqslant 2\}$
d **Hint:** Find $gg^{-1}(x)$ and $g^{-1}g(x)$ and show that they both equal x.

14 **a** $f^{-1}(x) = \sqrt{x-3} - 1,\ x \geqslant 3$
b 

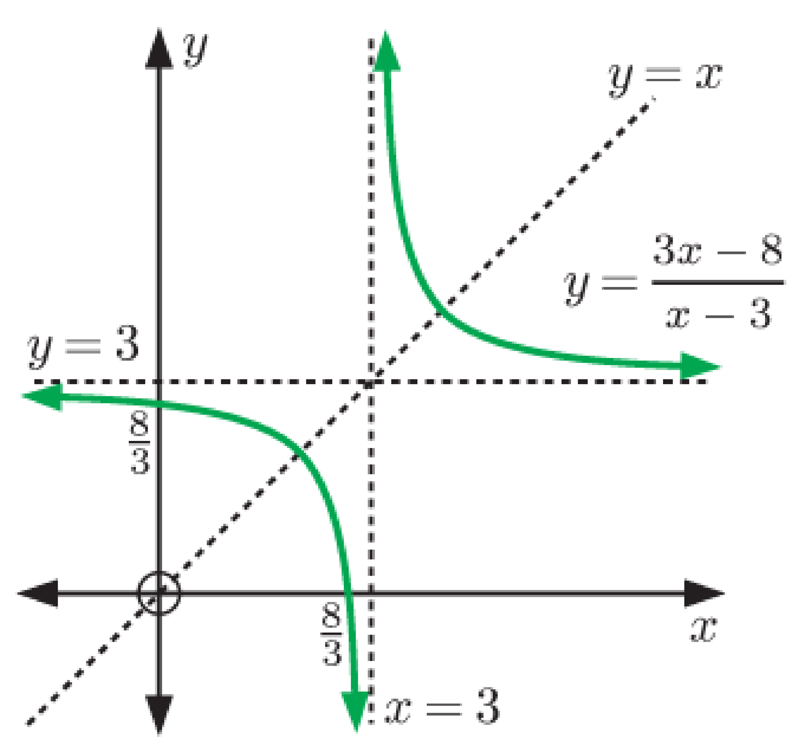
c f:
Domain is $\{x \mid x \geqslant -1\}$
Range is $\{y \mid y \geqslant 3\}$
f^{-1}:
Domain is $\{x \mid x \geqslant 3\}$
Range is $\{y \mid y \geqslant -1\}$

15 **a** $f^{-1}(x) = 3 - \sqrt{13 - x}$
b f: Domain is $\{x \mid x \leqslant 3\}$, Range is $\{y \mid y \leqslant 13\}$
f^{-1}: Domain is $\{x \mid x \leqslant 13\}$, Range is $\{y \mid y \leqslant 3\}$

16 **a** $k = \frac{5}{2}$
b **i** $f^{-1}(x) = \dfrac{5 - \sqrt{2x+13}}{2}$
ii Domain is $\{x \mid x \geqslant -\frac{13}{2}\}$, Range is $\{y \mid y \leqslant \frac{5}{2}\}$

17 **a** $g^{-1}(x) = 8 - 2x$ **b** $x = 10$
c $f^{-1}(-3) - g^{-1}(6) = -4 - (-4) = 0$ **d** $x = 3$

18 **a** $8x - 6$ **b** $k = 10$
c $(f^{-1} \circ g^{-1})(x) = \dfrac{x+3}{8}$ and $(g \circ f)^{-1}(x) = \dfrac{x+3}{8}$

19 **a** 

$y = \dfrac{3x-8}{x-3}$ is symmetrical about $y = x$
$\therefore\ f$ is a self-inverse function.

b $f^{-1}(x)=\dfrac{3x-8}{x-3}$ and $f(x)=\dfrac{3x-8}{x-3}$

$\therefore\ f=f^{-1}\quad\therefore\ f$ is a self-inverse function

20 $d=-a$ **21** **a** $B(f(x),x)$

22 **a** Domain is $\{x \mid x\geqslant 0\}$

b No, as $f(x)$ does not pass the horizontal line test.

c **i** $g^{-1}(x)=(3-\sqrt{x+8})^2$

ii g: Domain is $\{x \mid 0\leqslant x\leqslant 9\}$
Range is $\{y \mid -8\leqslant y\leqslant 1\}$

g^{-1}: Domain is $\{x \mid -8\leqslant x\leqslant 1\}$
Range is $\{y \mid 0\leqslant y\leqslant 9\}$

d **i** $h^{-1}(x)=(3+\sqrt{x+8})^2$

ii h: Domain is $\{x \mid x\geqslant 9\}$
Range is $\{y \mid y\geqslant -8\}$

h^{-1}: Domain is $\{x \mid x\geqslant -8\}$
Range is $\{y \mid y\geqslant 9\}$

iii $x=-8$

REVIEW SET 15A

1 **a** **i** Domain is $\{x \mid x\in\mathbb{R}\}$ **ii** Range is $\{y \mid y>-4\}$

iii yes, it is a function

b **i** Domain is $\{x \mid x\in\mathbb{R}\}$

ii Range is $\{y \mid y\leqslant -1$ or $y\geqslant 1\}$

iii no, not a function

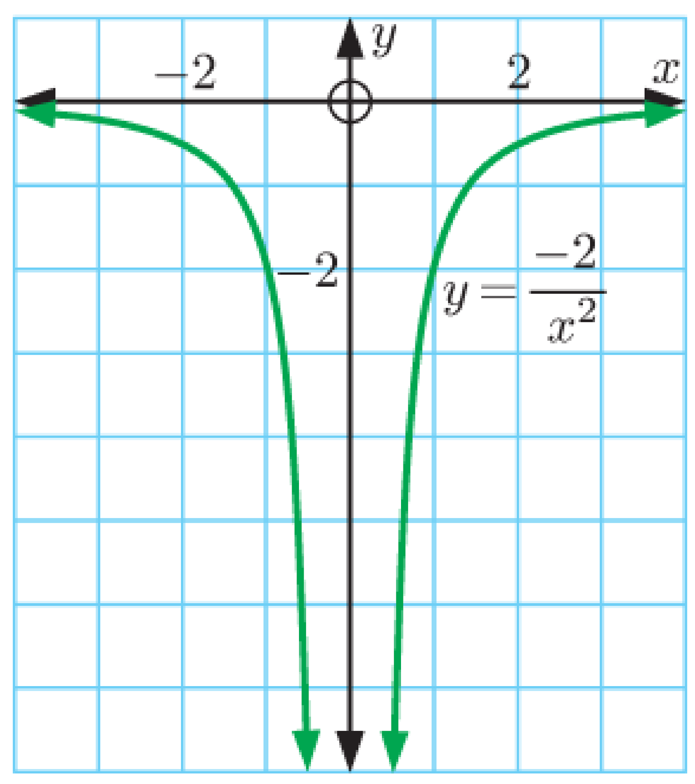
c **i** Domain is $\{x \mid x\in\mathbb{R}\}$

ii Range is $\{y \mid -5\leqslant y\leqslant 5\}$ **iii** yes, it is a function

2 **a** 0 **b** -15 **c** $-\frac{5}{4}$ **3** $a=-6,\ b=13$

4 **a** $x=0$

b



c Domain is $\{x \mid x\neq 0\}$
Range is $\{y \mid y<0\}$

5 **a** $f(-3)=(-3)^2=9,\quad g(-\frac{4}{3})=1-6(-\frac{4}{3})=9$

b $x=-4$

6 **a** Domain is $\{x \mid x\geqslant -4\}$, Range is $\{y \mid y\geqslant 0\}$

b Domain is $\{x \mid x\in\mathbb{R}\}$, Range is $\{y \mid y\leqslant 1\}$

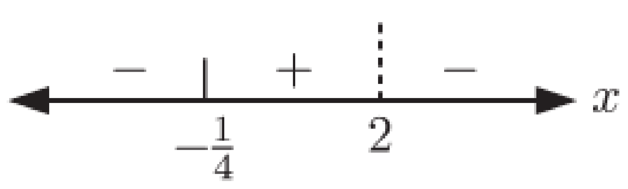
c Domain is $\{x \mid x\in\mathbb{R}\}$, Range is $\{y \mid y\geqslant -\frac{1}{8}\}$

7 **a** $y=-\dfrac{20}{x}$ **b** $y=\dfrac{60}{x}$

8 **a** vertical asymptote $x=2$, horizontal asymptote $y=-4$

b Domain is $\{x \mid x\neq 2\}$, Range is $\{y \mid y\neq -4\}$

c

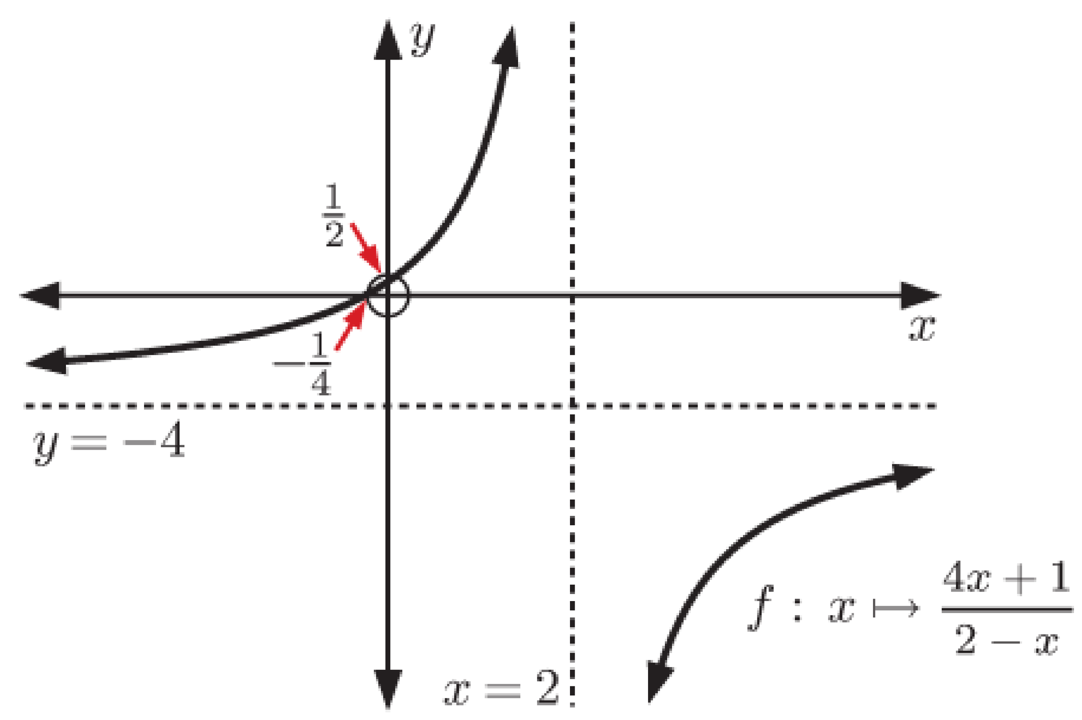


as $x\to 2^-$, $f(x)\to\infty$ as $x\to-\infty$, $f(x)\to -4^+$

as $x\to 2^+$, $f(x)\to-\infty$ as $x\to\infty$, $f(x)\to -4^-$

d x-intercept $-\frac{1}{4}$, y-intercept $\frac{1}{2}$

e



9 **a** $6x-3$ **b** $x=1$

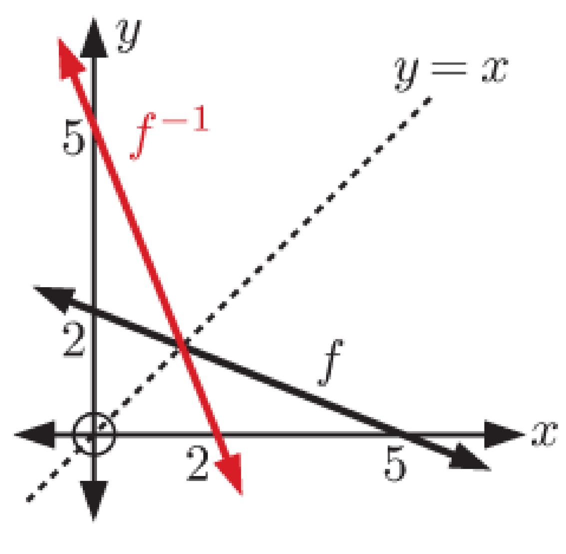
10 **a** $1-2\sqrt{x}$ **b** $\sqrt{1-2x}$ **c** 3

11 **a** $(f\circ g)(x)=\sqrt{x^2-1}$
Domain is $\{x \mid x\leqslant -1$ or $x\geqslant 1\}$
Range is $\{y \mid y\geqslant 0\}$

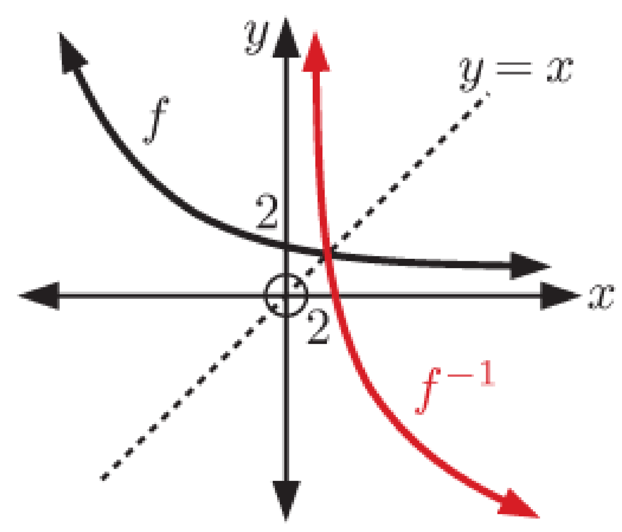
b $(g\circ f)(x)=x-1$
Domain is $\{x \mid x\geqslant -2\}$, Range is $\{y \mid y\geqslant -3\}$

12 $a=1,\ b=-1$

13 **a**

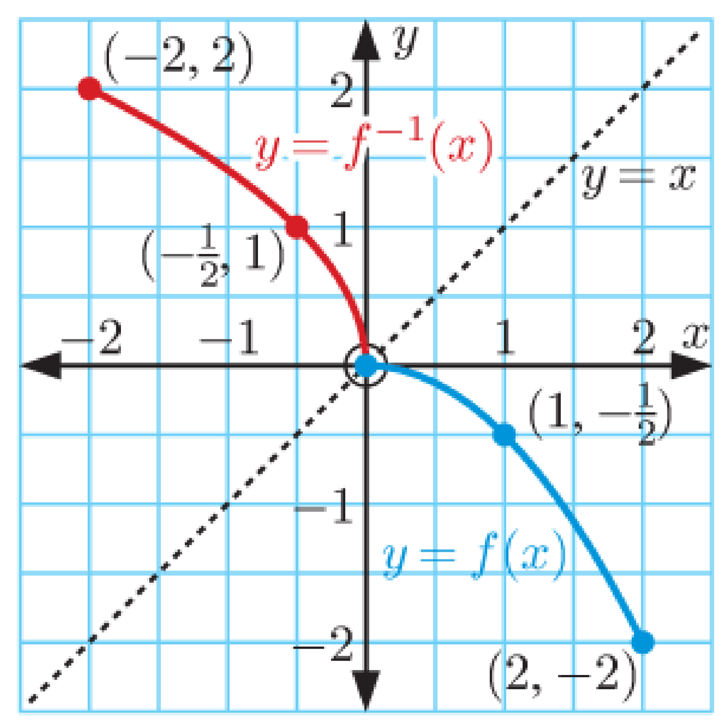


b



14 **a** $f^{-1}(x)=\dfrac{x-2}{4}$ **b** $f^{-1}(x)=\dfrac{3-4x}{5}$

15 **a**

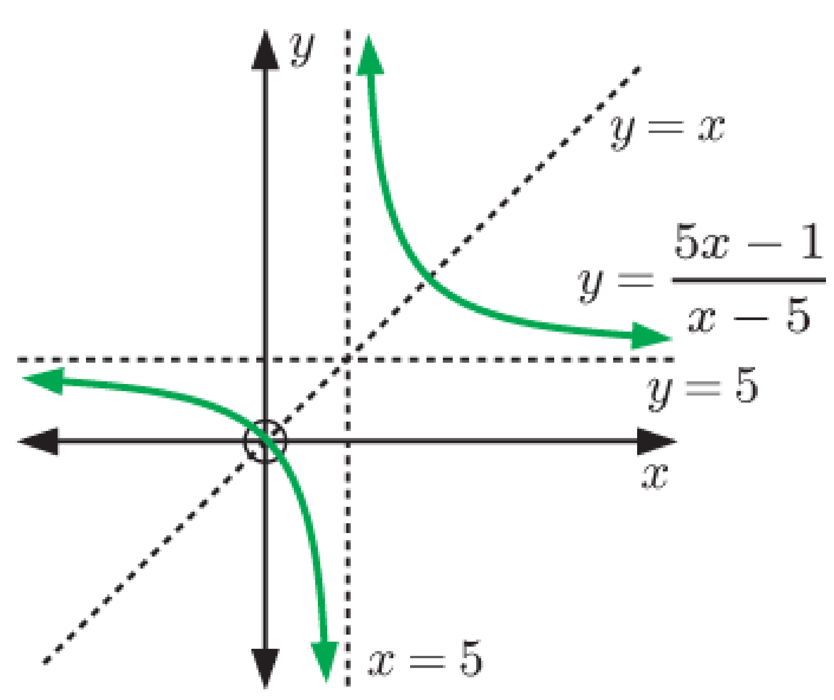


b Range is $\{y \mid 0\leqslant y\leqslant 2\}$

c **i** $x=\sqrt{3}$ **ii** $x=-\frac{1}{2}$

16 $(f^{-1}\circ h^{-1})(x)=(h\circ f)^{-1}(x)=x-2$

17 **a**



$y=\dfrac{5x-1}{x-5}$ is symmetrical about $y=x$

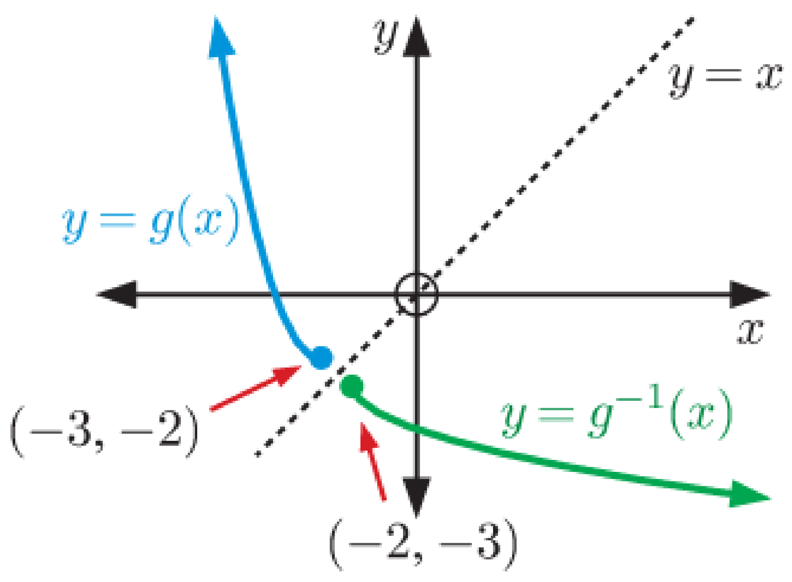
$\therefore\ f$ is a self-inverse function.

b $f^{-1}(x)=\dfrac{5x-1}{x-5}$ and $f(x)=\dfrac{5x-1}{x-5}$

$\therefore\ f=f^{-1}\quad\therefore\ f$ is a self-inverse function.

18 **a** -4 **b** 1

19 **a, d**



b Any horizontal line cuts the graph at most once.

c $g^{-1}(x) = -3 - \sqrt{x+2}, \ x \geqslant -2$

e Range of g is $\{y \mid y \geqslant -2\}$

f Domain of g^{-1} is $\{x \mid x \geqslant -2\}$,
Range of g^{-1} is $\{y \mid y \leqslant -3\}$

REVIEW SET 15B

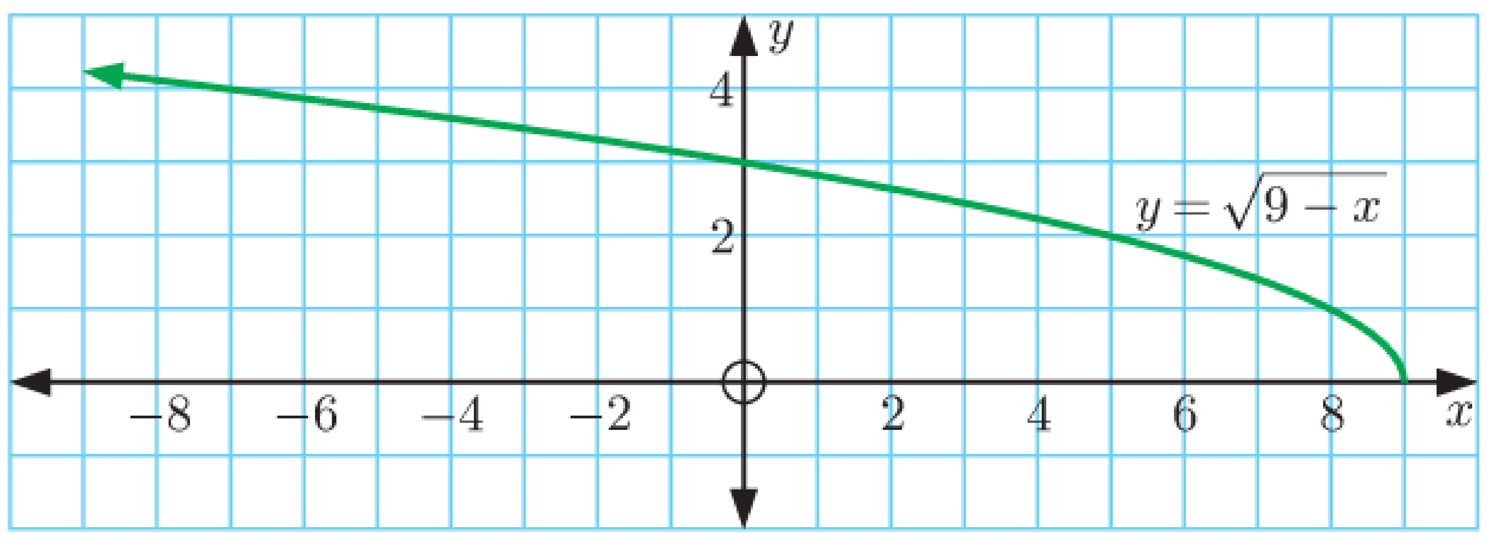
1 **a** Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid y \geqslant -4\}$
b Domain is $\{x \mid x \geqslant -2\}$, Range is $\{y \mid 1 \leqslant y < 3\}$
c Domain is $\{x \mid x \in \mathbb{R}\}$, Range is $\{y \mid y = -1, 1, \text{ or } 2\}$

2 **a** $x^2 - x - 2$ **b** $16x^2 - 12x$

3 **a** is a function **b** is not a function

4 **a** Domain is $\{x \mid x \neq \frac{1}{2}\}$, Range is $\{y \mid y \neq 10\}$
b Domain is $\{x \mid x \geqslant -7\}$, Range is $\{y \mid y \geqslant 0\}$

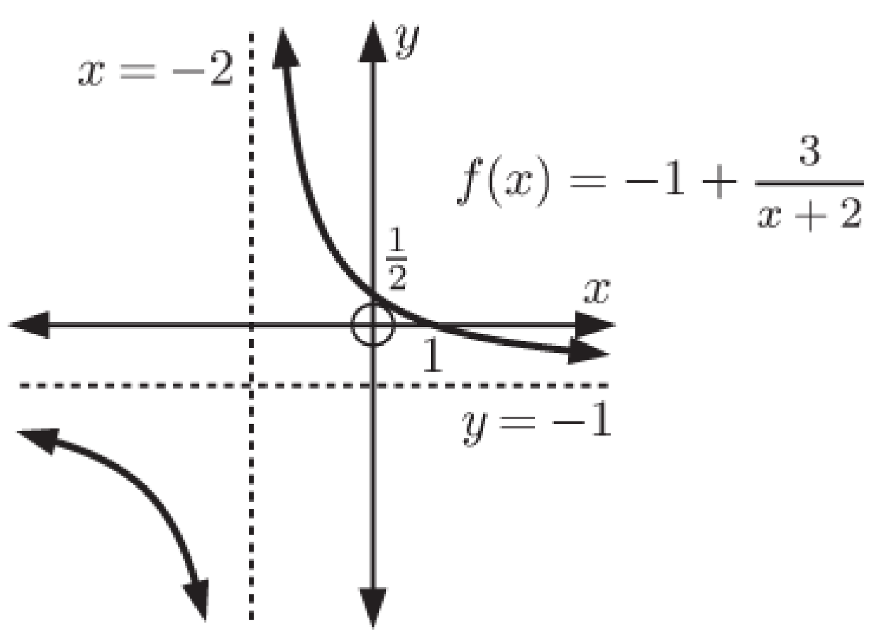
5 **a**



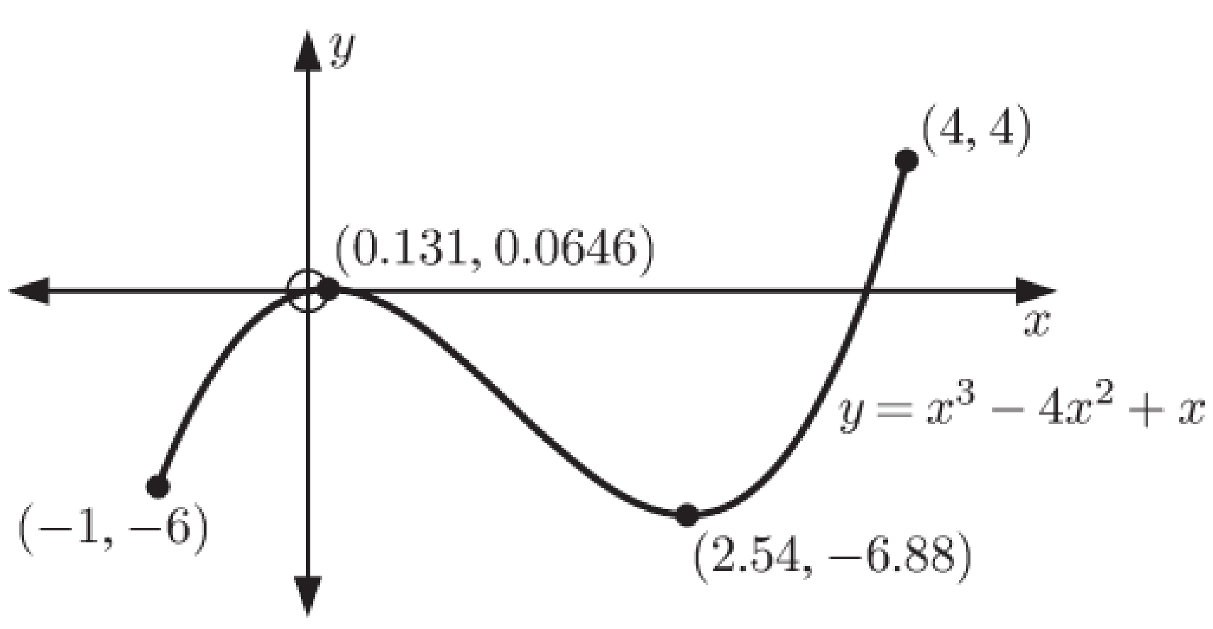
b It is a function.
c Domain is $\{x \mid x \leqslant 9\}$, Range is $\{y \mid y \geqslant 0\}$

6 $a = -2$ **7** $a = 1, \ b = -6, \ c = 5$

8 **a** vertical asymptote is $x = -2$,
horizontal asymptote is $y = -1$
b Domain is $\{x \mid x \neq -2\}$, Range is $\{y \mid y \neq -1\}$
c x-intercept is 1, y-intercept is $\frac{1}{2}$
d as $x \to -2^-$, $f(x) \to -\infty$ as $x \to -\infty$, $f(x) \to -1^-$
as $x \to -2^+$, $f(x) \to \infty$ as $x \to \infty$, $f(x) \to -1^+$
e



9



Range is $\{y \mid -6.88 \leqslant y \leqslant 4\}$

10 **a** $-4x^2 + 4x + 2$ **b** $5 - 2x^2$ **c** 2

11 $(f \circ g)(x) = \dfrac{1}{(x^2 - 4x + 3)^2}$

Domain is $\{x \mid x \neq 3, \ x \neq 1\}$, Range is $\{y \mid y > 0\}$

12 **a** **i** $6x^2 - 3x + 5$ **ii** $18x^2 + 57x + 45$
b $x = -\frac{5}{11}$

13 **a** $D \circ S = 4.9t^2$
This is the distance travelled by the object after t seconds.
b $(D \circ S)(5) = 122.5$ m
The object has travelled 122.5 m after 5 seconds.

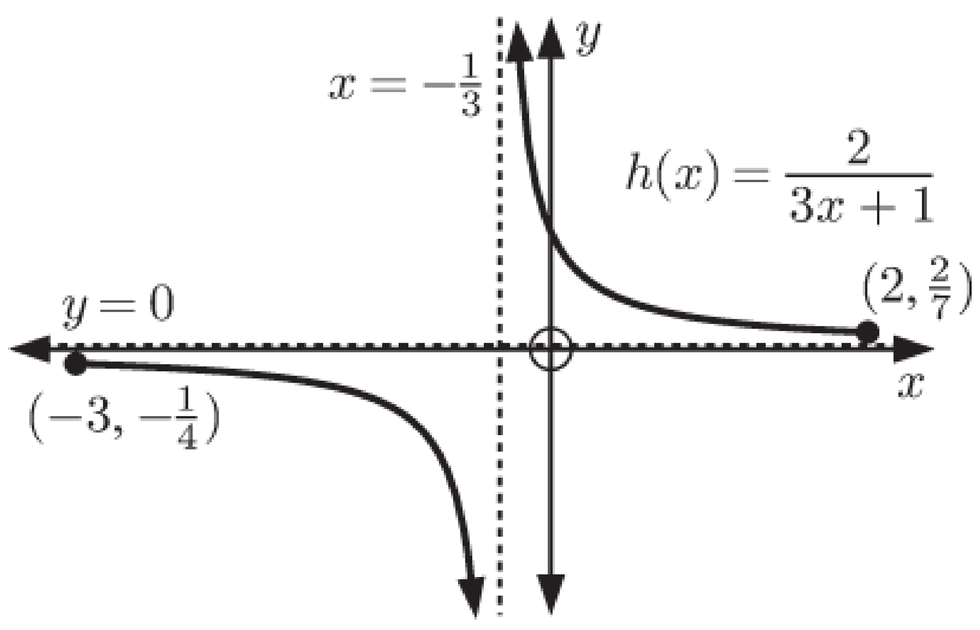
14 $f^{-1}(x) = \dfrac{2x + 29}{5}$

15 $(f^{-1} \circ h^{-1})(x) = (h \circ f)^{-1}(x) = \dfrac{4x+6}{15}$

16 **a** $(g \circ f)(x) = \dfrac{2}{3x+1}$ **b** $x = -\frac{1}{2}$

c **i** vertical asymptote $x = -\frac{1}{3}$,
horizontal asymptote $y = 0$

ii



iii Range is $\{y \mid y \leqslant -\frac{1}{4} \text{ or } y \geqslant \frac{2}{7}\}$

17 16

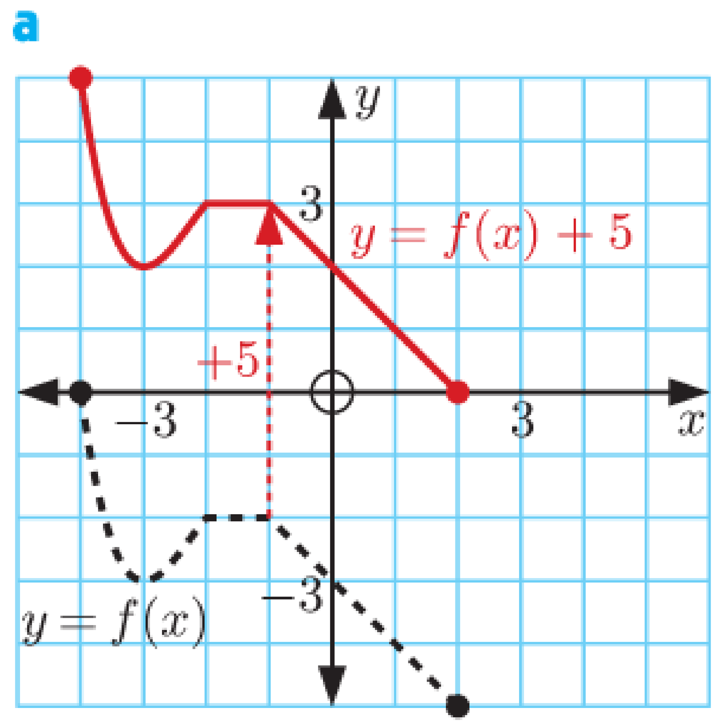
18 **a** $a = 2, \ b = -1$
b Domain is $\{x \mid x \neq 2\}$, Range is $\{y \mid y \neq -1\}$

19 **a** $\dfrac{3x}{x-2}$ **b** $\dfrac{2x+1}{x-1}$

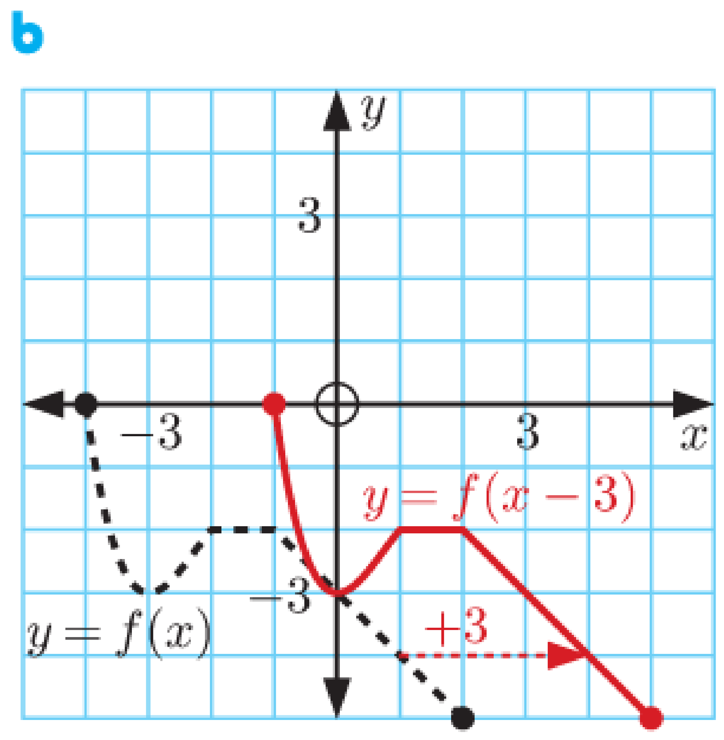
20 **a** $f^{-1}(x) = \sqrt{4 - \sqrt{x+13}}$
Domain is $\{x \mid -13 \leqslant x \leqslant 3\}$,
Range is $\{y \mid 0 \leqslant y \leqslant 2\}$
b $g^{-1}(x) = \sqrt{4 + \sqrt{x+13}}$
Domain is $\{x \mid x \geqslant -13\}$, Range is $\{y \mid y \geqslant 2\}$
c $h^{-1}(x) = -\sqrt{4 - \sqrt{x+13}}$
Domain is $\{x \mid -13 \leqslant x \leqslant 3\}$,
Range is $\{y \mid -2 \leqslant y \leqslant 0\}$
d $j^{-1}(x) = -\sqrt{4 + \sqrt{x+13}}$
Domain is $\{x \mid x \geqslant -13\}$, Range is $\{y \mid y \leqslant -2\}$

EXERCISE 16A

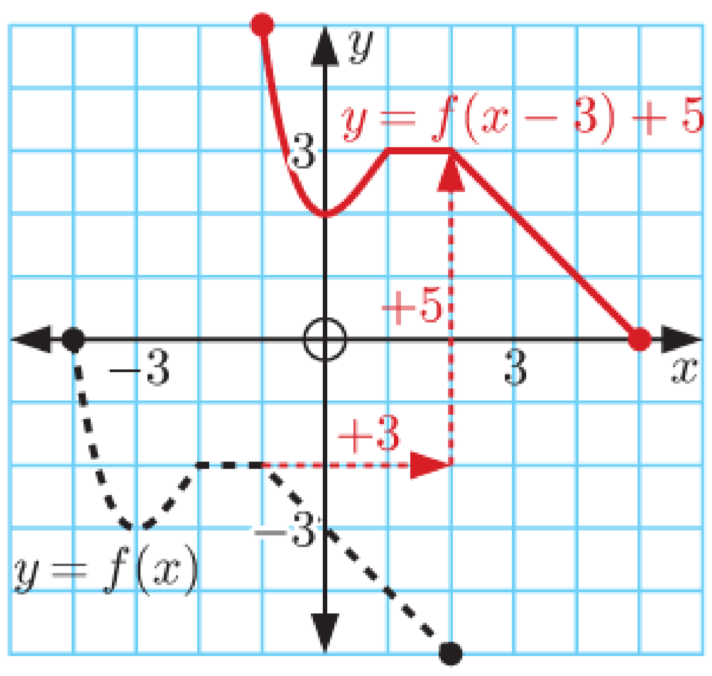
1 **a**



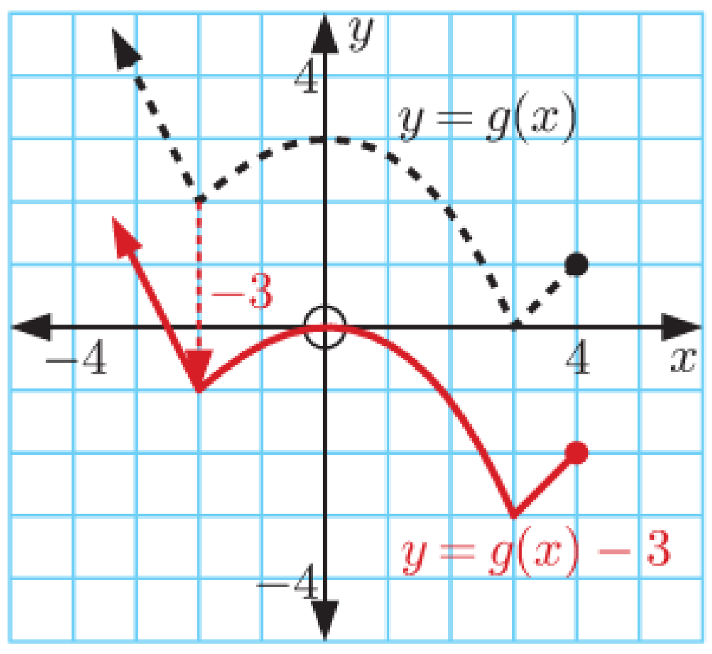
b



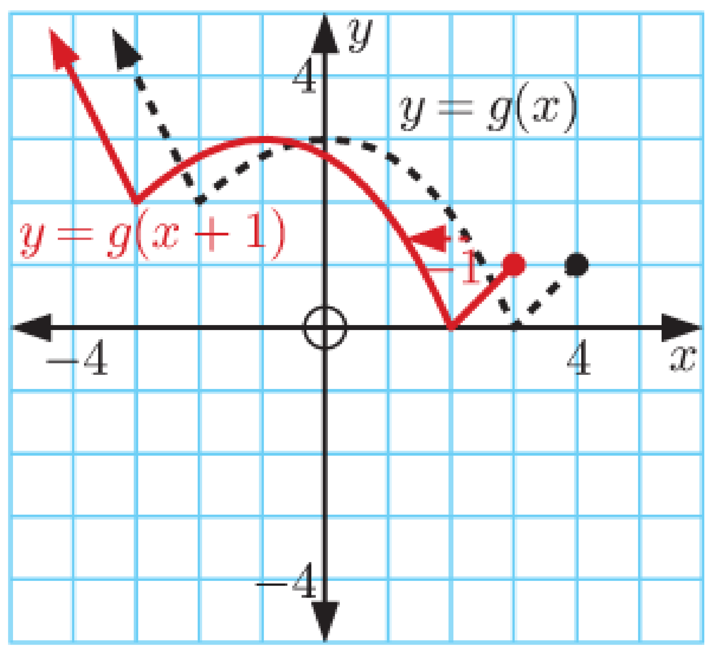
c



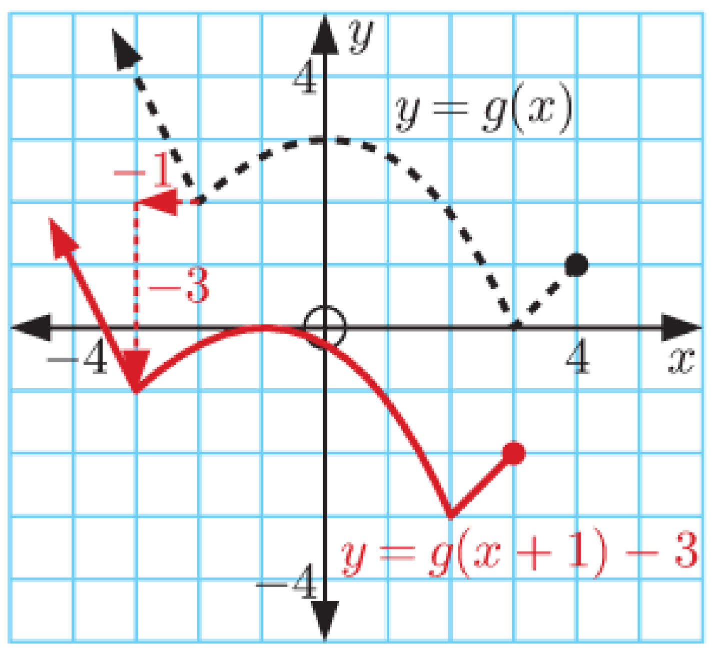
2 **a**



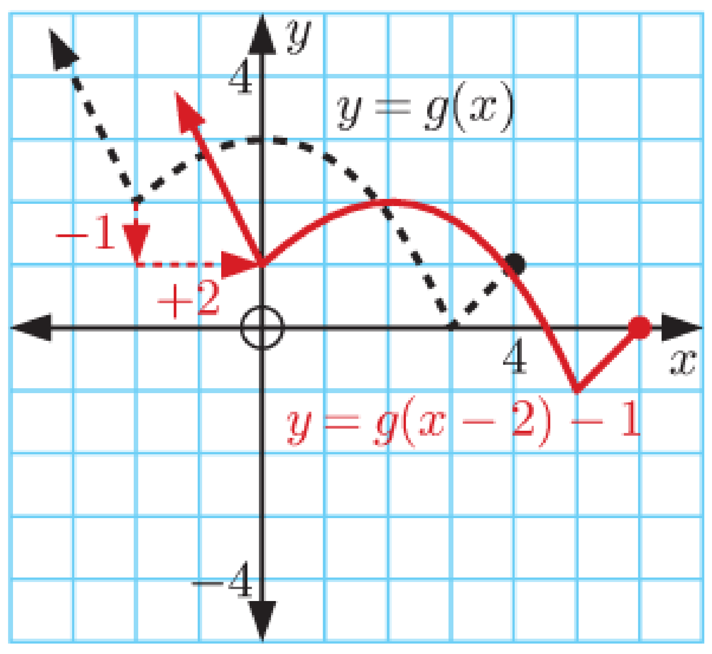
b



c



d

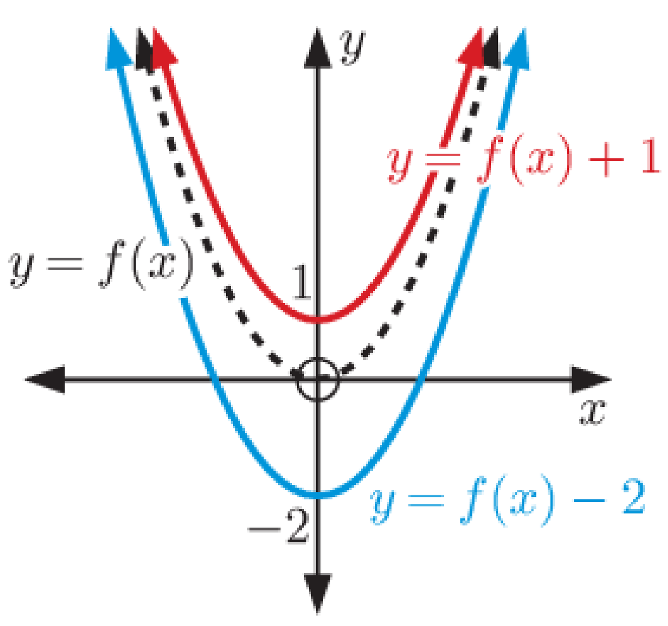


3 **a** $g(x) = f(x - 4)$ **b** $g(x) = f(x + 1) + 3$

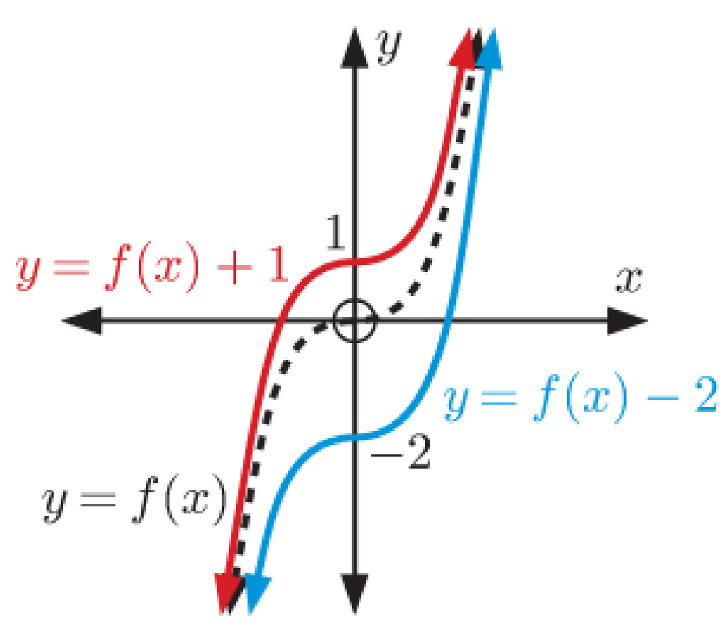
4 **a** $g(x) = 2x - 1$ **b** $g(x) = 3x + 2$

c $g(x) = -x^2 + 5x - 4$ **d** $g(x) = x^2 - 6x + 4$

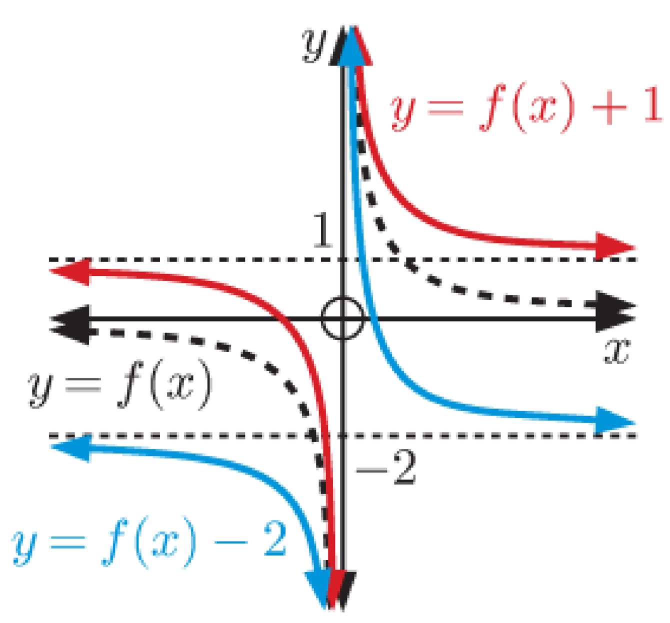
5 **a**



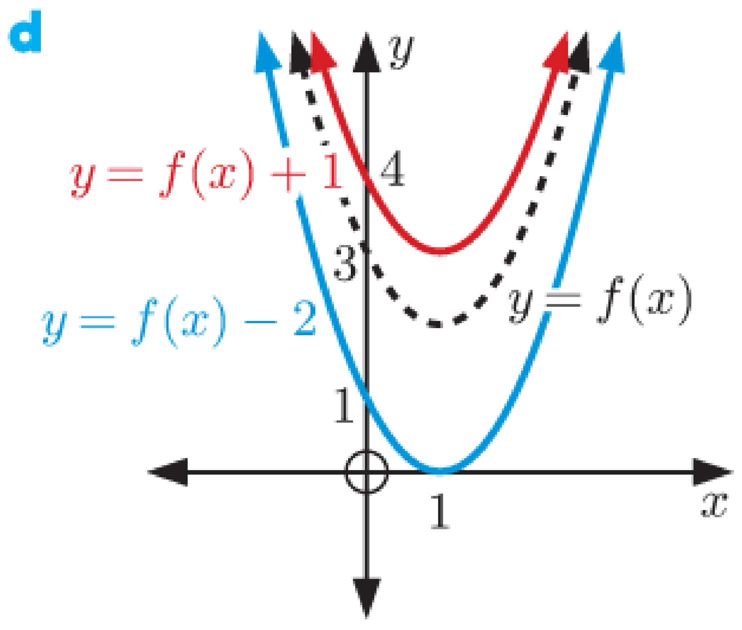
b



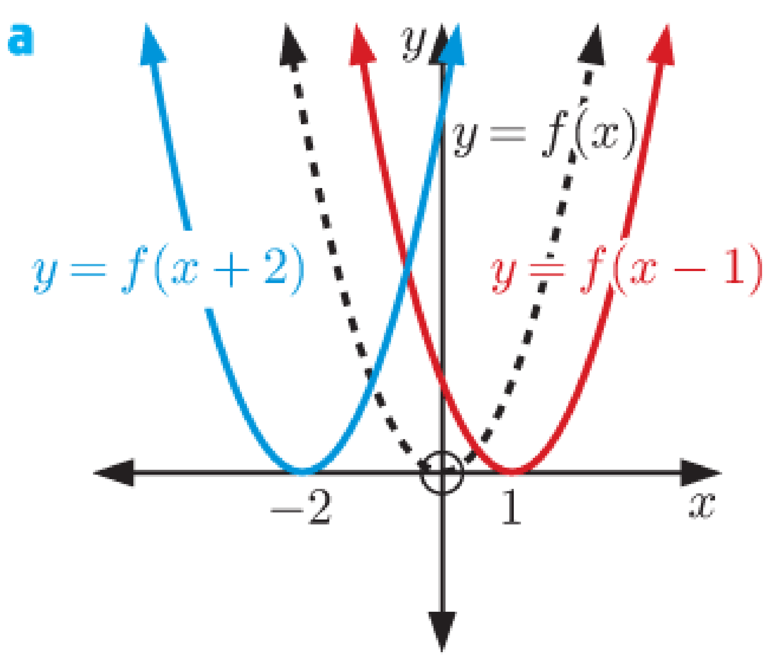
c



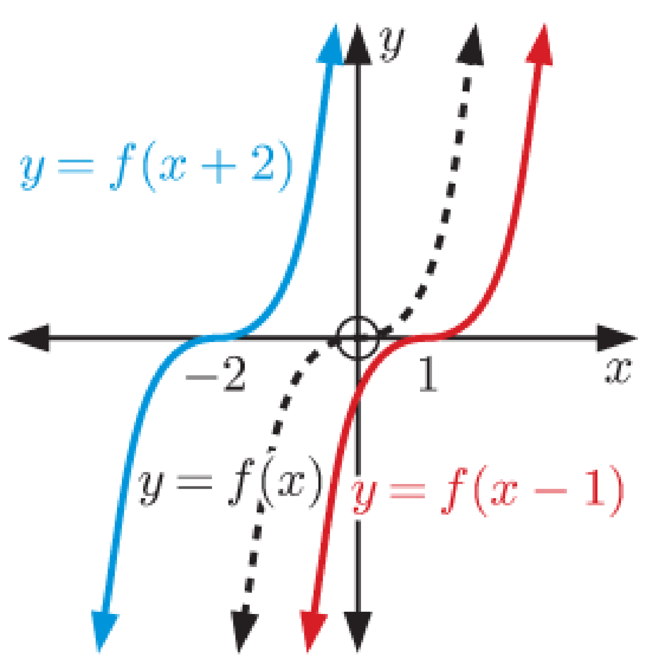
d



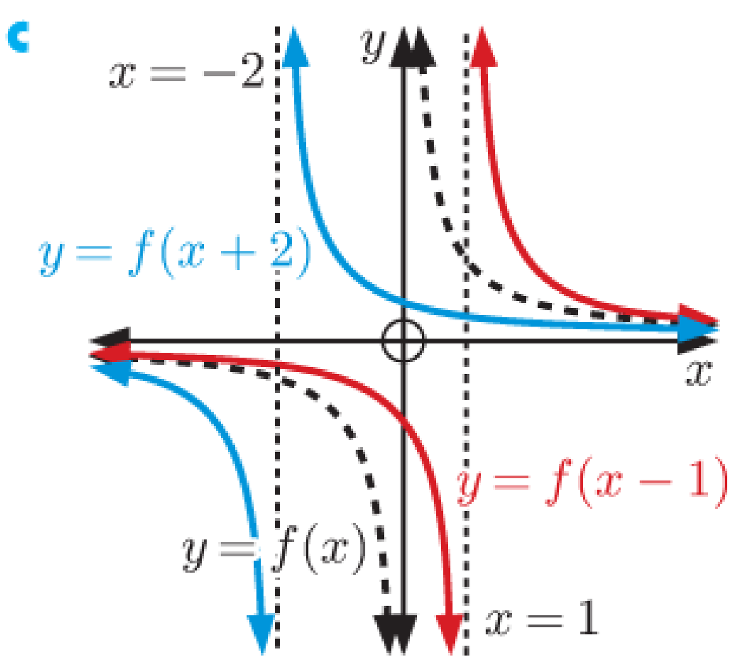
6 **a**



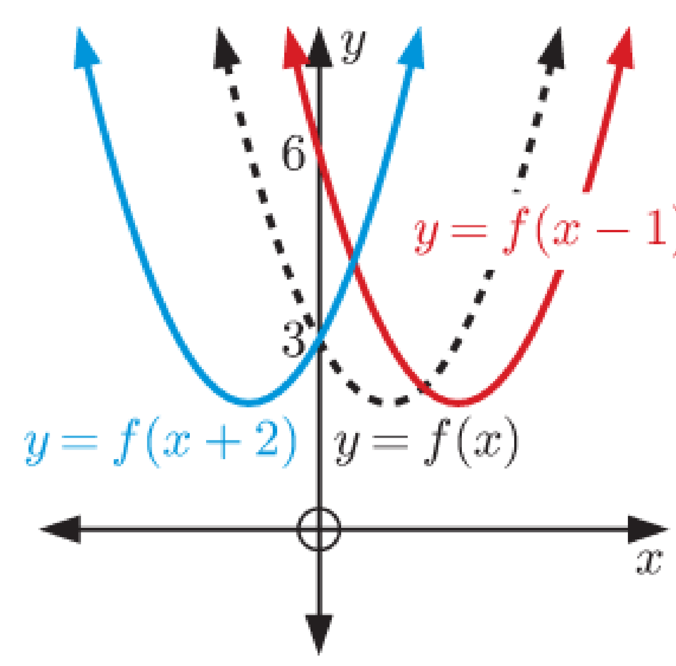
b



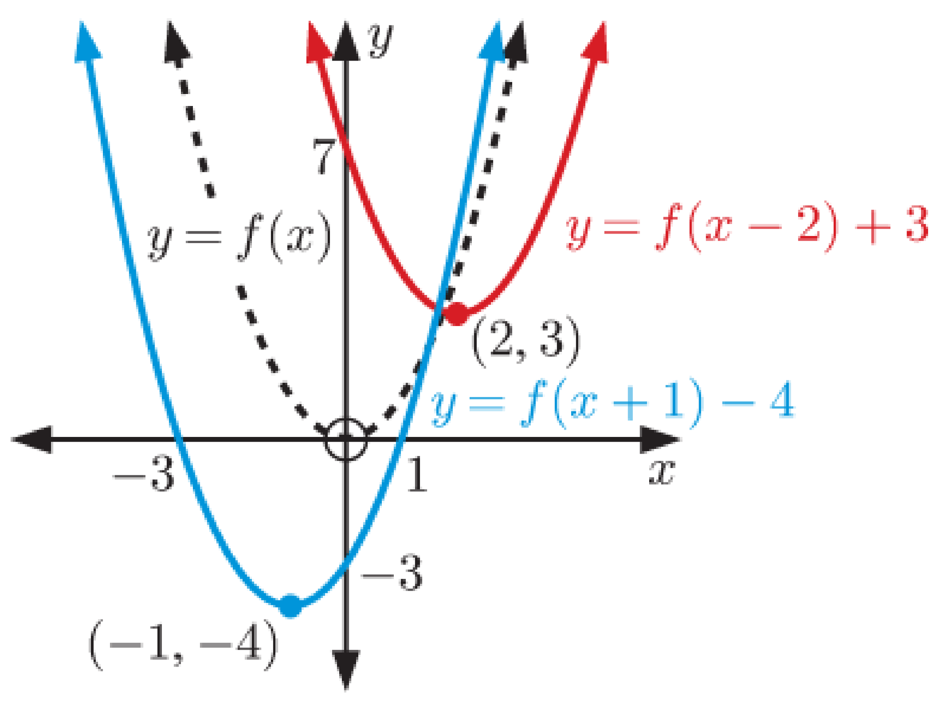
c



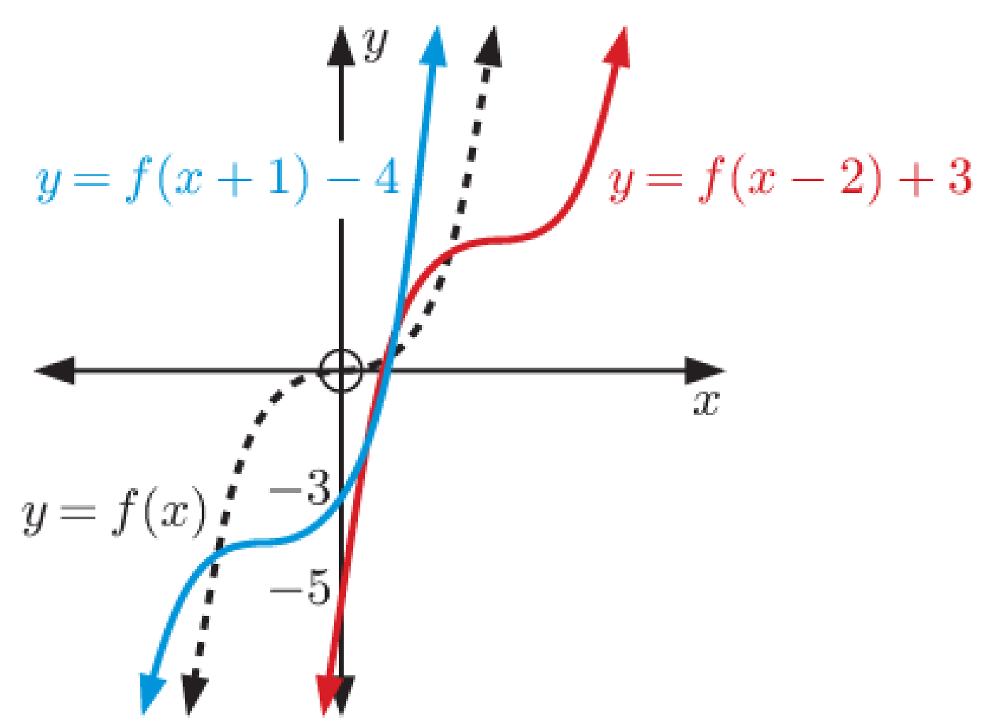
d



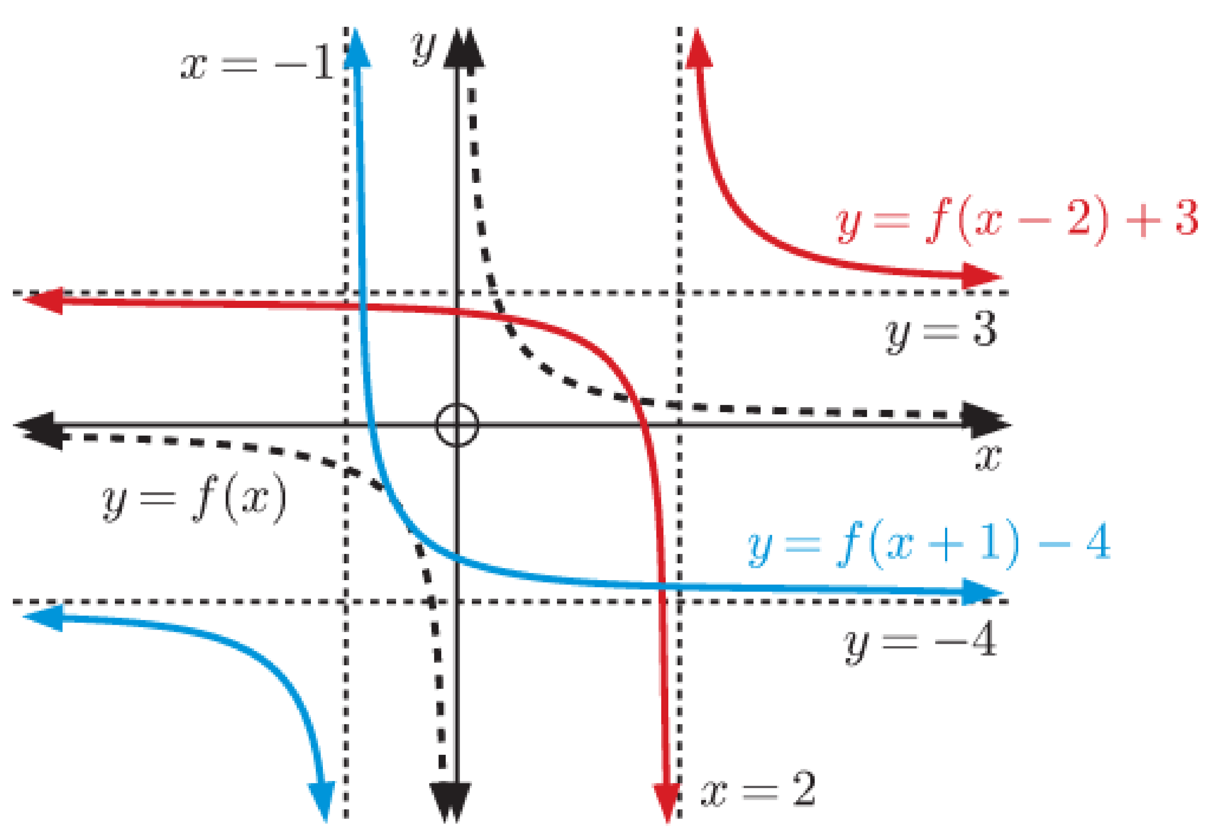
7 **a**



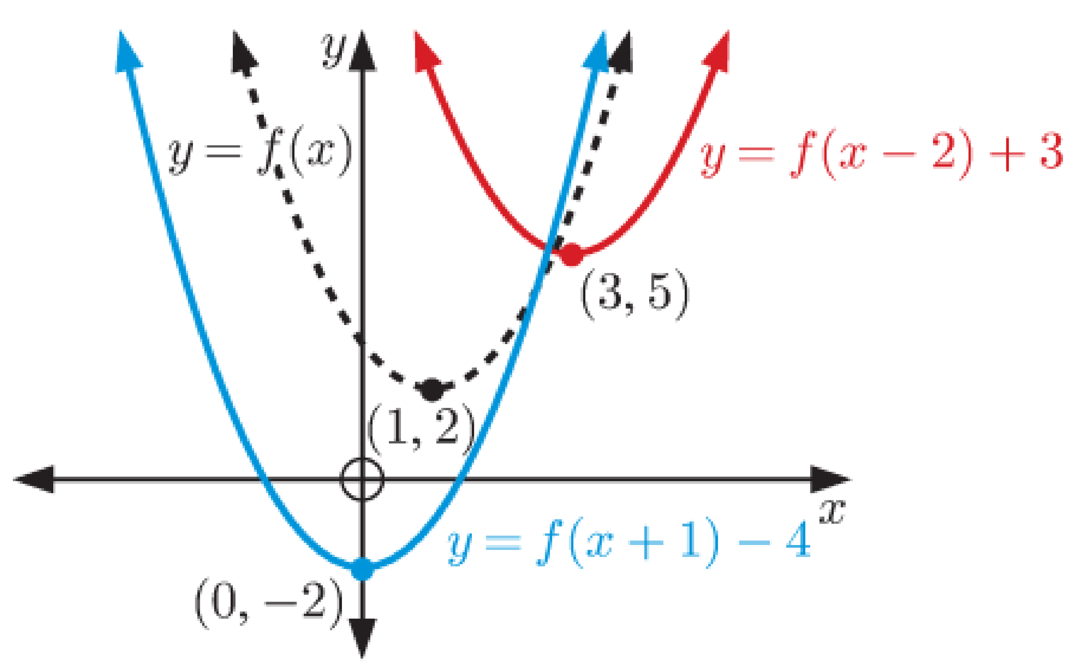
b



c



d



8 $(1, -9)$

9 **a** y-intercept is -1 **b** x-intercepts are -2 and 5

c inconclusive

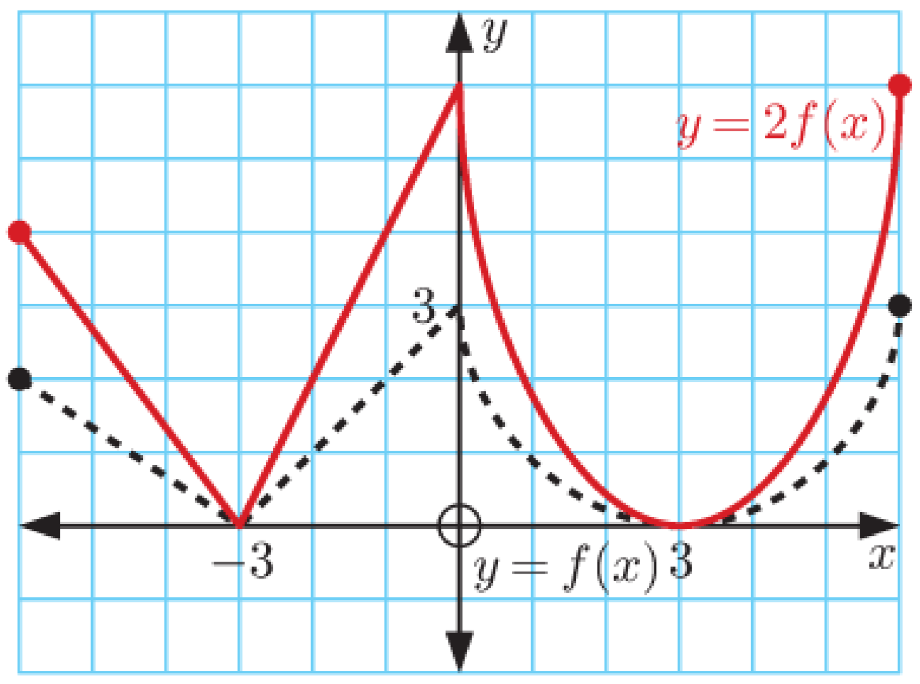
10 $g(x) = x^2 - 8x + 12$ **11** $g(x) = \dfrac{7x + 15}{x + 2}$

12 **a** **i** $(3, 2)$ **ii** $(0, 11)$ **iii** $(5, 6)$

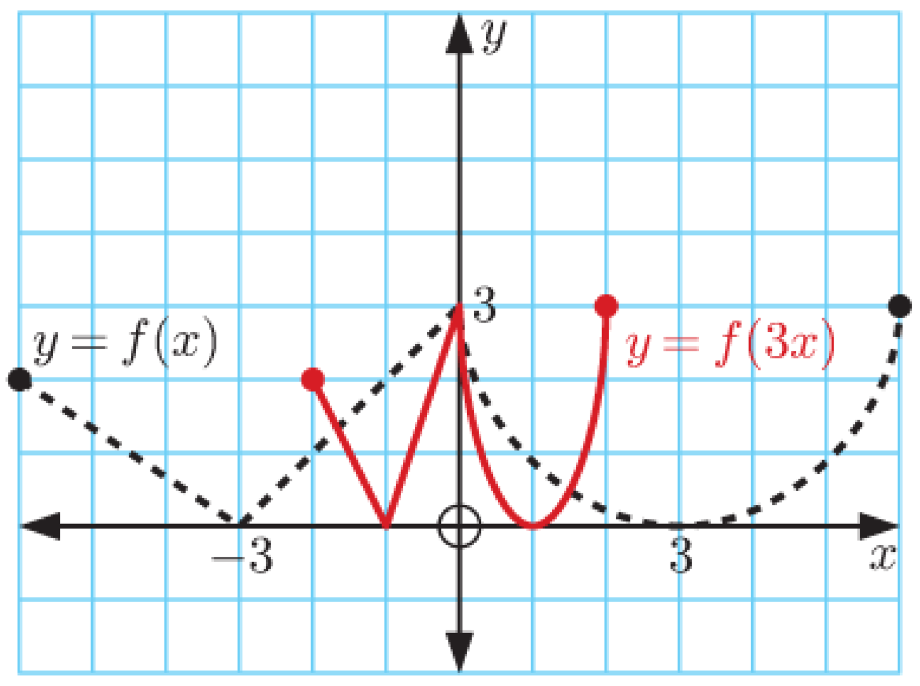
b **i** $(-2, 4)$ **ii** $(-5, 25)$ **iii** $\left(-1\frac{1}{2}, 2\frac{1}{4}\right)$

EXERCISE 16B

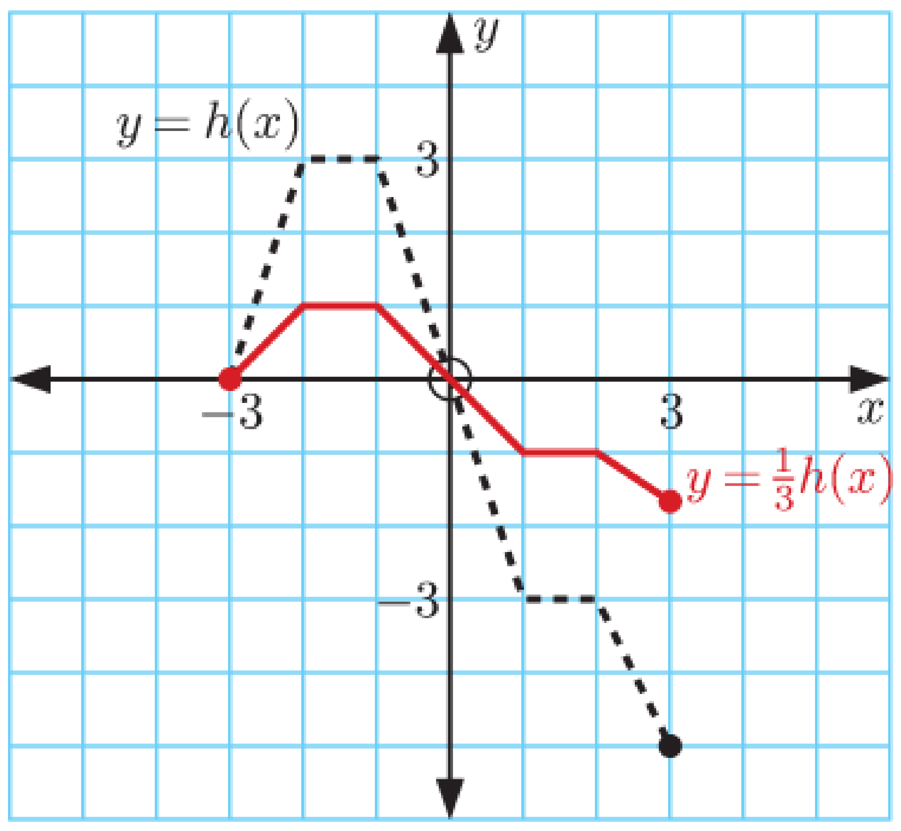
1 **a**



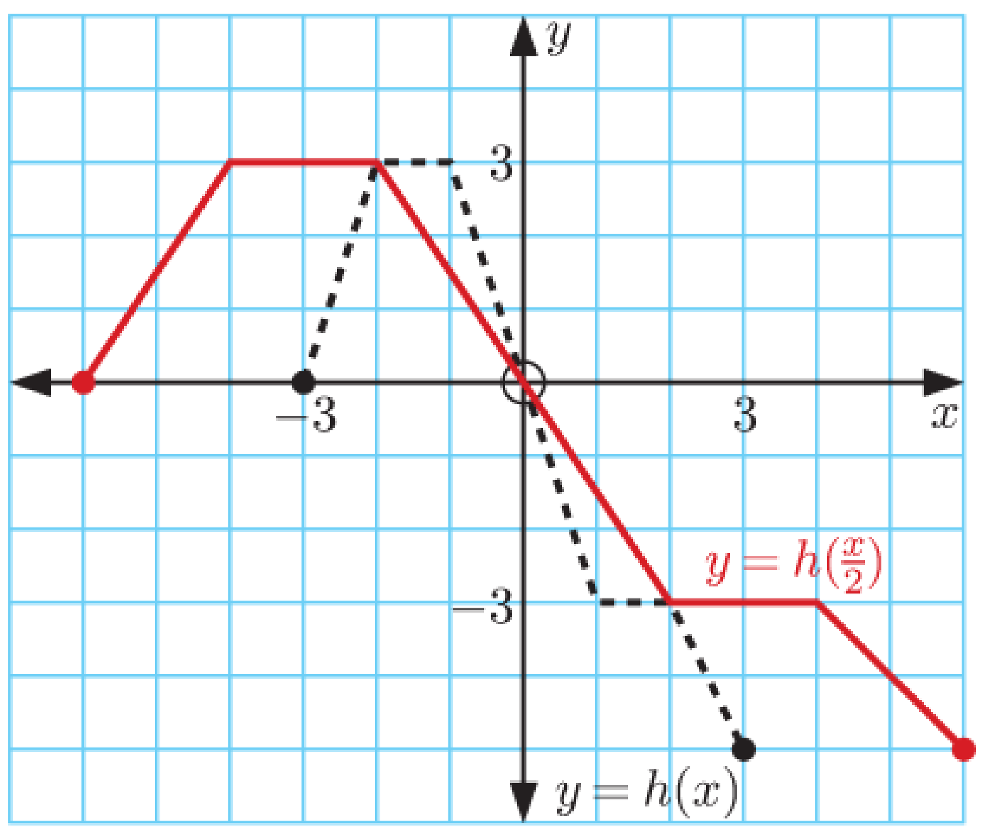
b



2 **a**

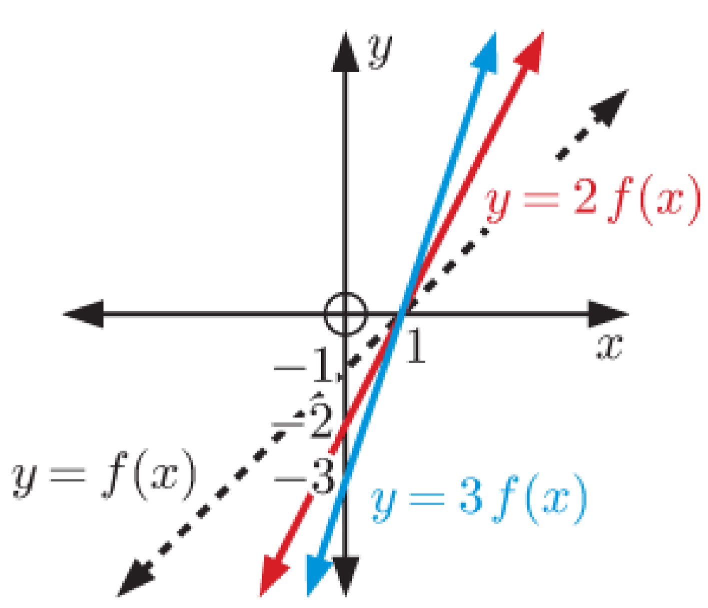


b

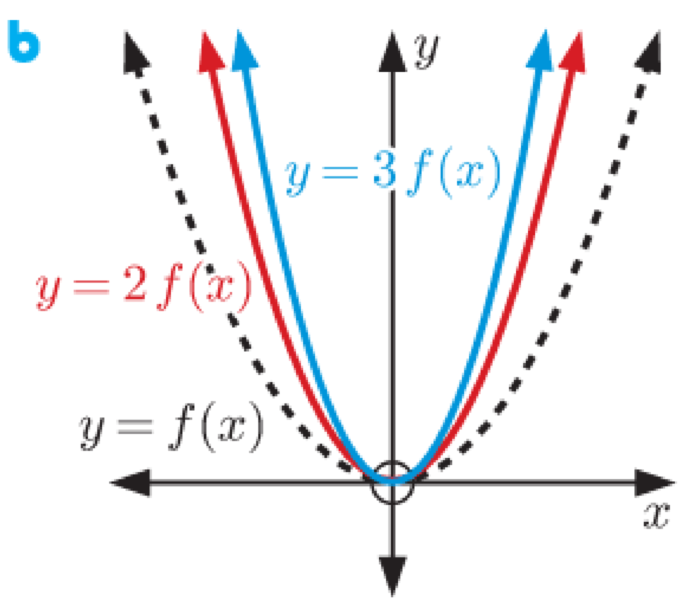


3 **a** $g(x) = 2f(x)$ **b** $g(x) = f\left(\dfrac{x}{3}\right)$ **4** *cm*

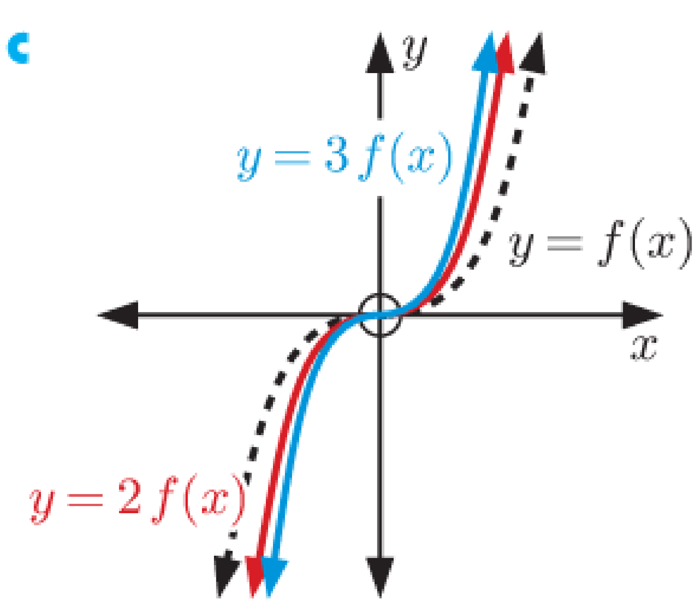
5 **a**



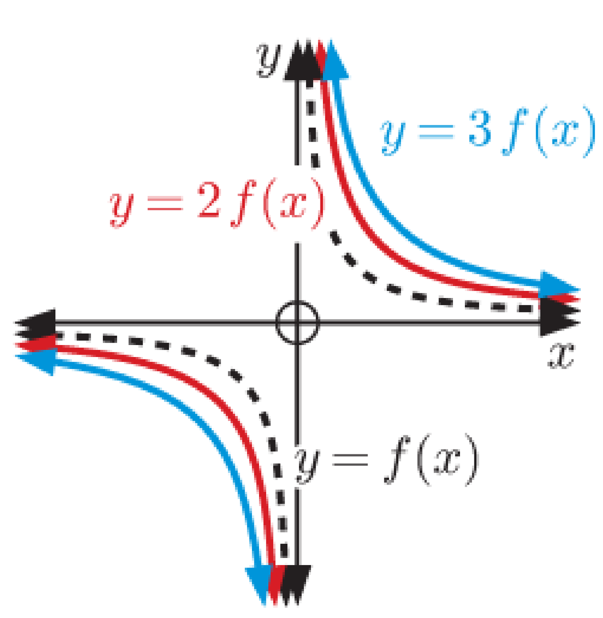
b



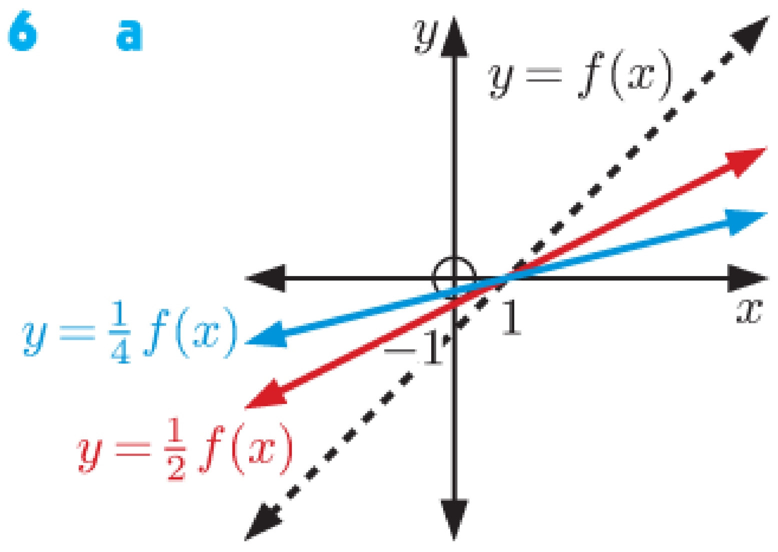
c



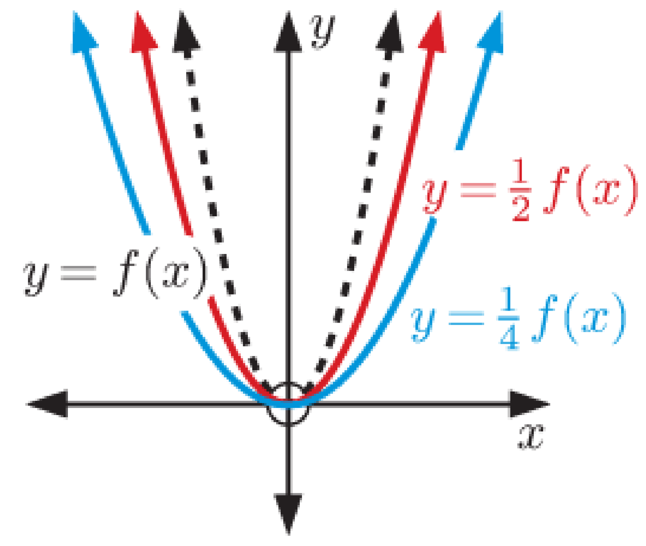
d



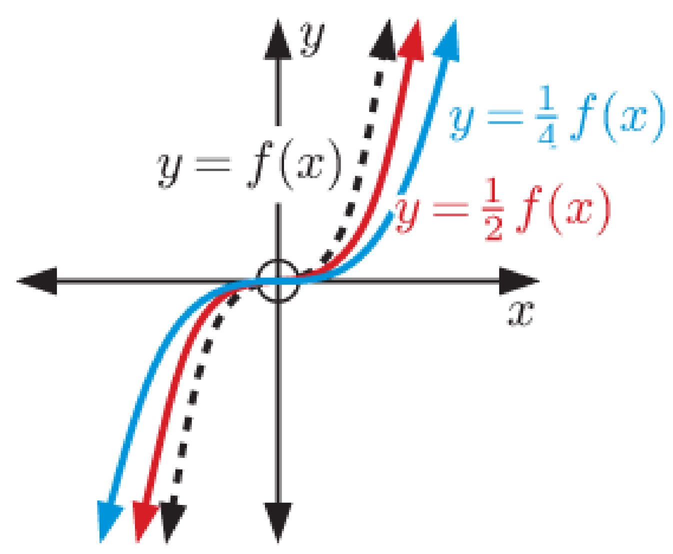
6 **a**



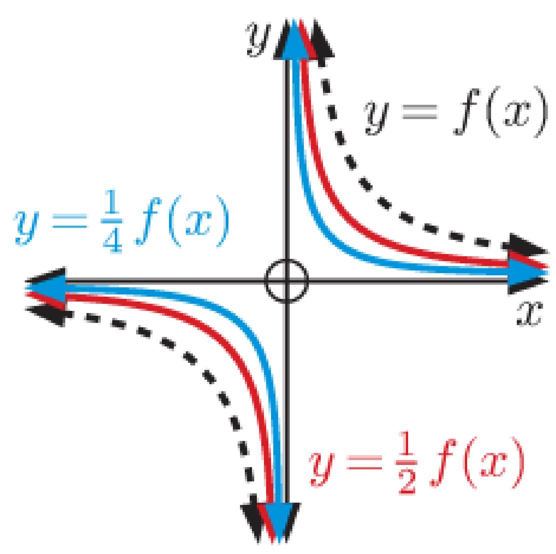
b



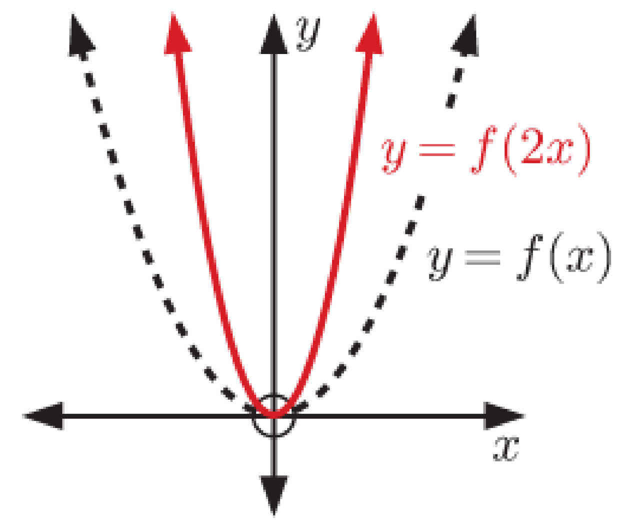
c



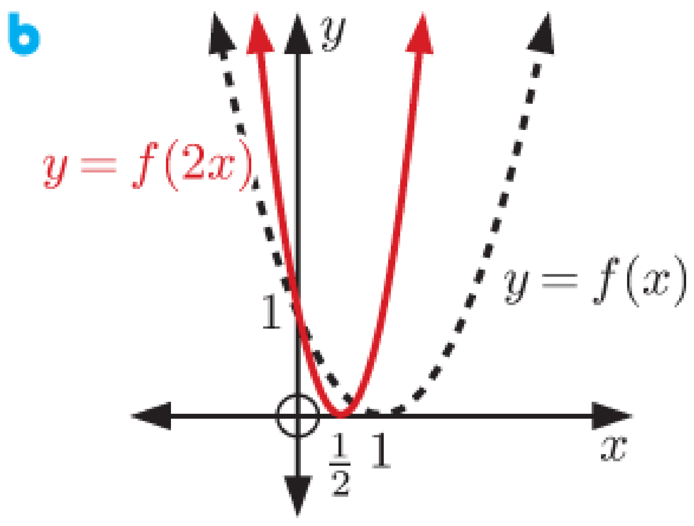
d



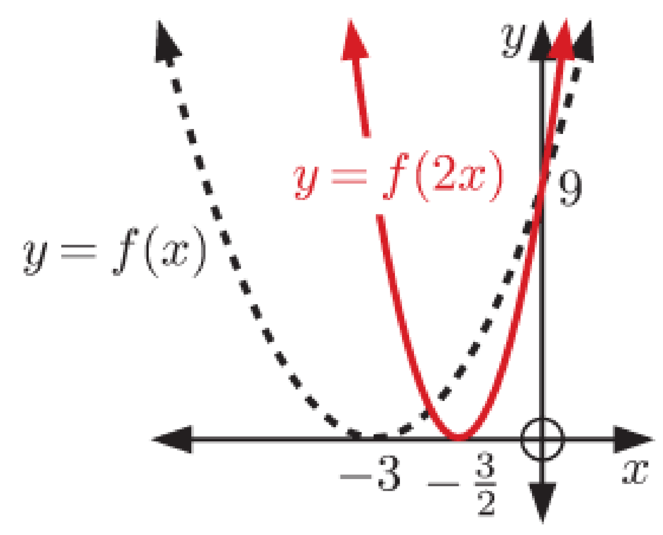
7 **a**



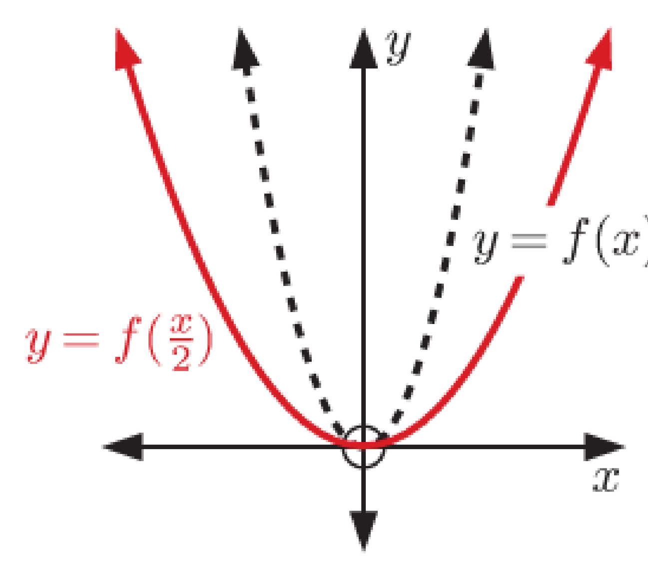
b



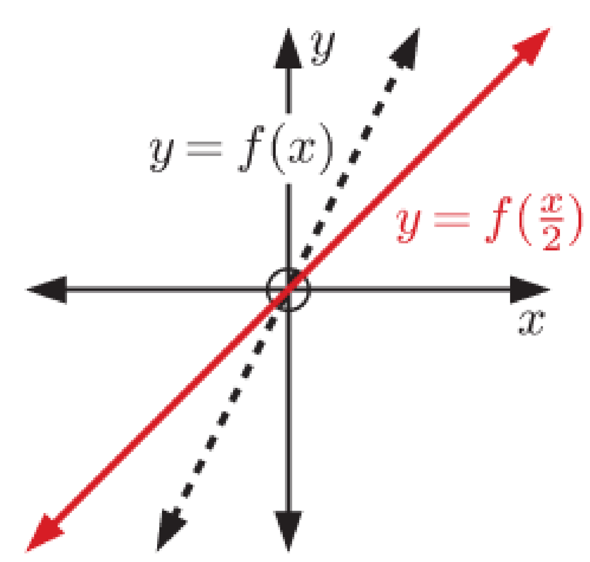
c



8 **a**



b



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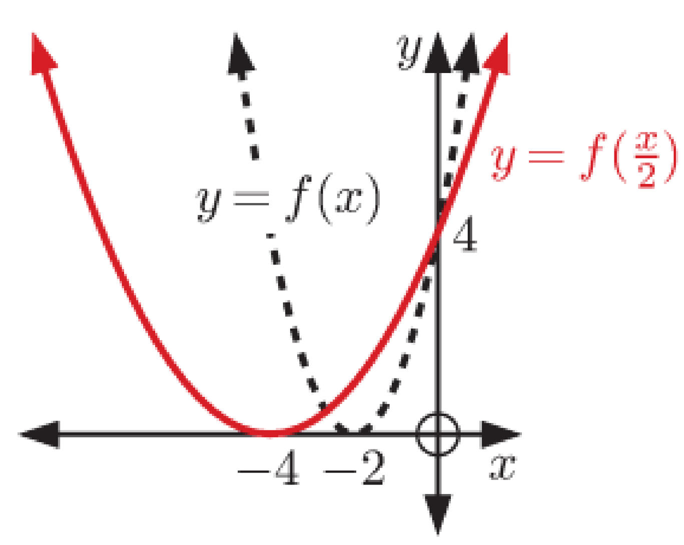
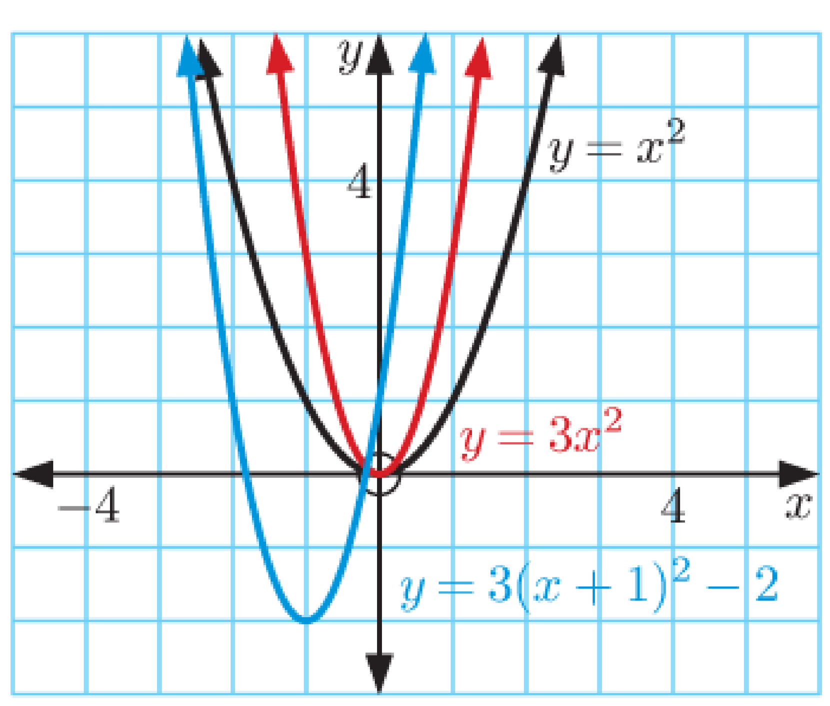


9 **a** $(2, 25)$ **b** $(-25, -15)$

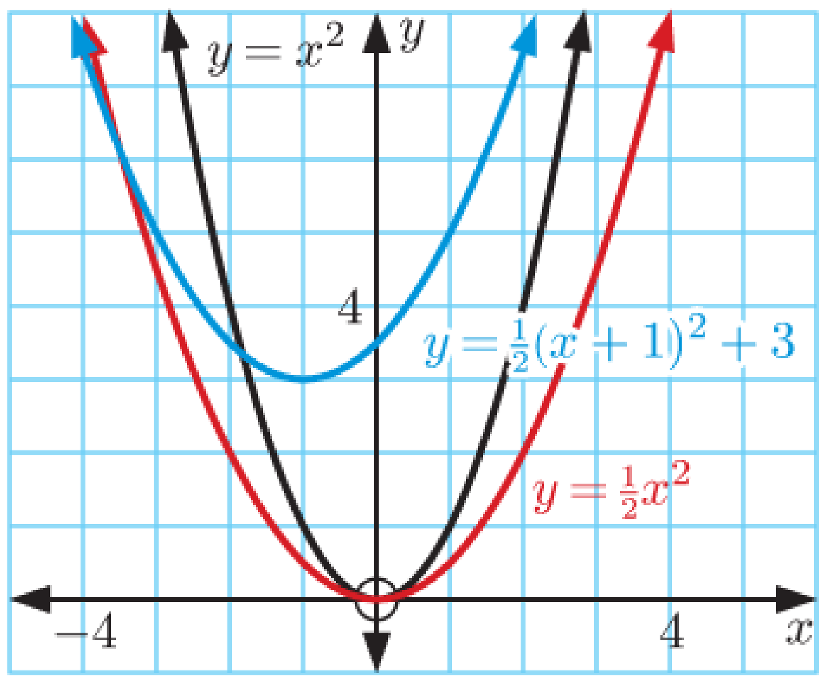
10 **a** $g(x) = 2x^2 + 4$ **b** $g(x) = 5 - x$

c $g(x) = \frac{1}{4}x^3 + 2x^2 - \frac{1}{2}$ **d** $g(x) = 8x^2 + 2x - 3$

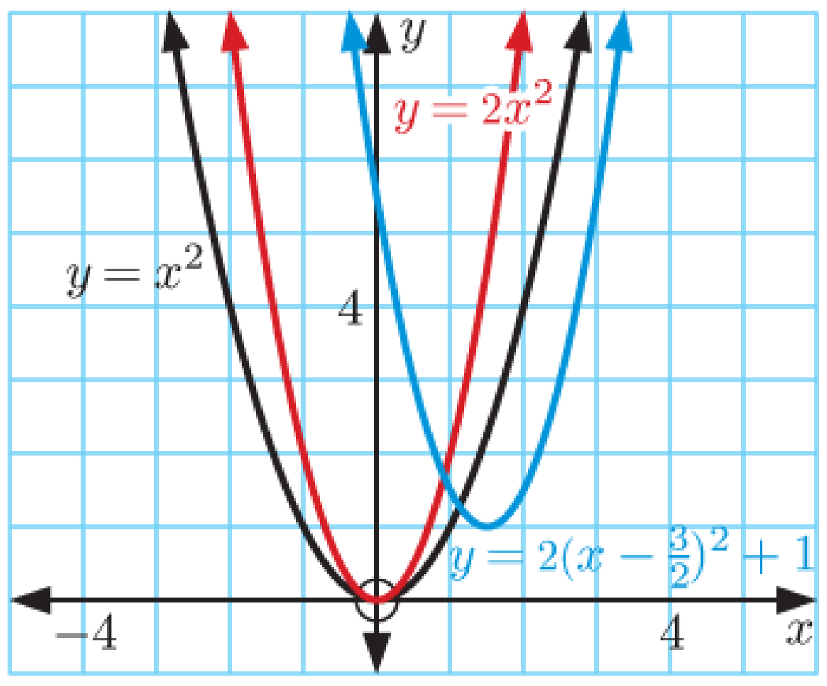
11



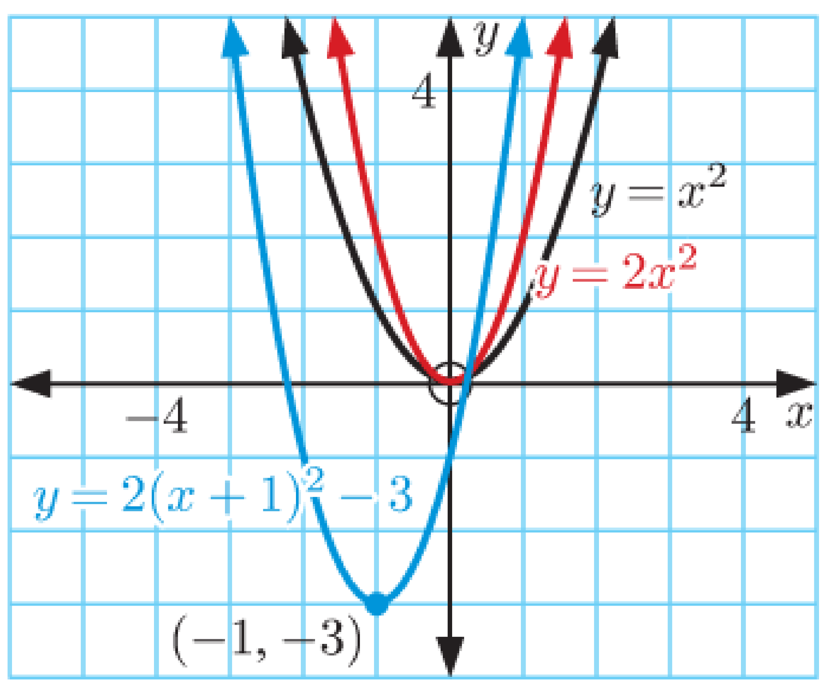
$y = x^2$ is transformed to $y = 3(x+1)^2 - 2$ by vertically stretching with scale factor 3 and then translating through $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$.

12 

$y = x^2$ is transformed to $y = \frac{1}{2}(x+1)^2 + 3$ by vertically stretching with scale factor $\frac{1}{2}$ and then translating through $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

13 

$y = x^2$ is transformed to $y = 2(x - \frac{3}{2})^2 + 1$ by vertically stretching with scale factor 2 and then translating through $\begin{pmatrix} \frac{3}{2} \\ 1 \end{pmatrix}$.

14 

$y = x^2$ is transformed to $y = 2(x+1)^2 - 3$ by vertically stretching with scale factor 2 and then translating through $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$.

15 **a** Horizontally stretching with scale factor $\frac{1}{2}$, then vertically stretching with scale factor 3.

b **i** $(\frac{3}{2}, -15)$ **ii** $(\frac{1}{2}, 6)$ **iii** $(-1, 3)$

c **i** $(4, \frac{1}{3})$ **ii** $(-6, \frac{2}{3})$ **iii** $(-14, 1)$

16 $a = 5,\ b = \sqrt{10}$

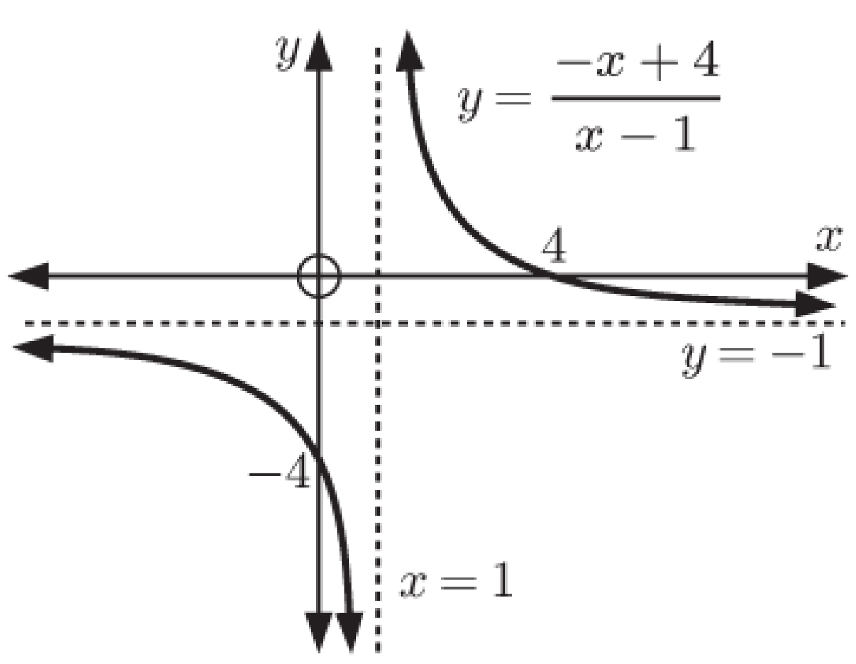
17 **a** $y = \dfrac{1}{2x}$ **b** $y = \dfrac{3}{x}$ **c** $y = \dfrac{1}{x+3}$

d $y = 4 + \dfrac{1}{x} = \dfrac{4x+1}{x}$

18 **a** $g(x) = \dfrac{3}{x-1} - 1 = \dfrac{-x+4}{x-1}$

b vertical asymptote $x = 1$, horizontal asymptote $y = -1$

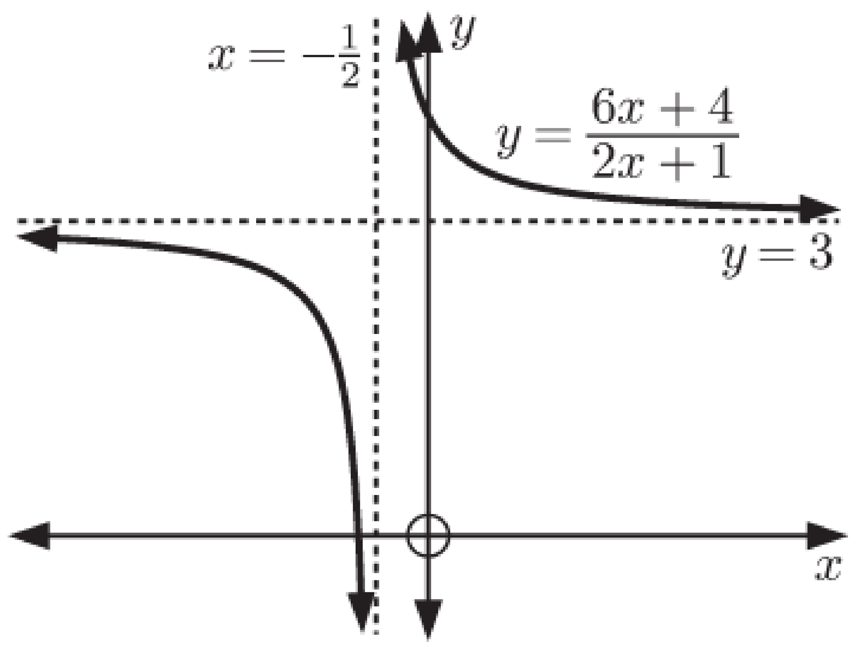
c Domain is $\{x \mid x \neq 1\}$, Range is $\{y \mid y \neq -1\}$

d 

19 **a** $g(x) = \dfrac{6x+4}{2x+1}$

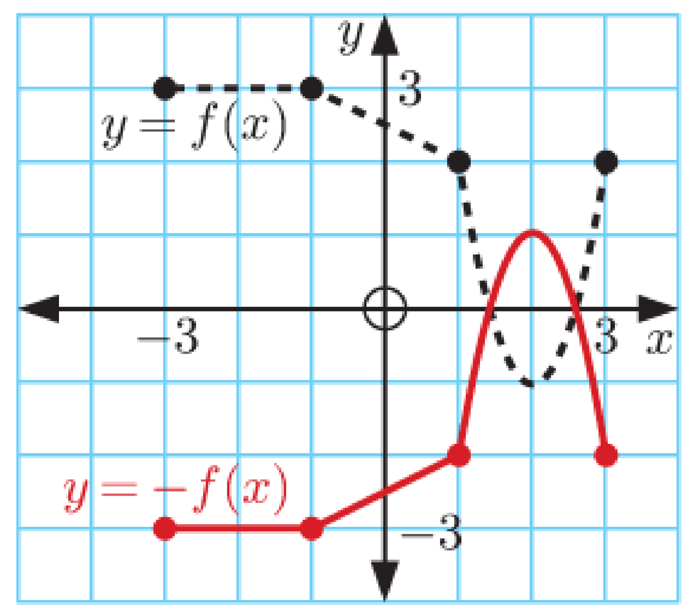
b vertical asymptote $x = -\frac{1}{2}$, horizontal asymptote $y = 3$

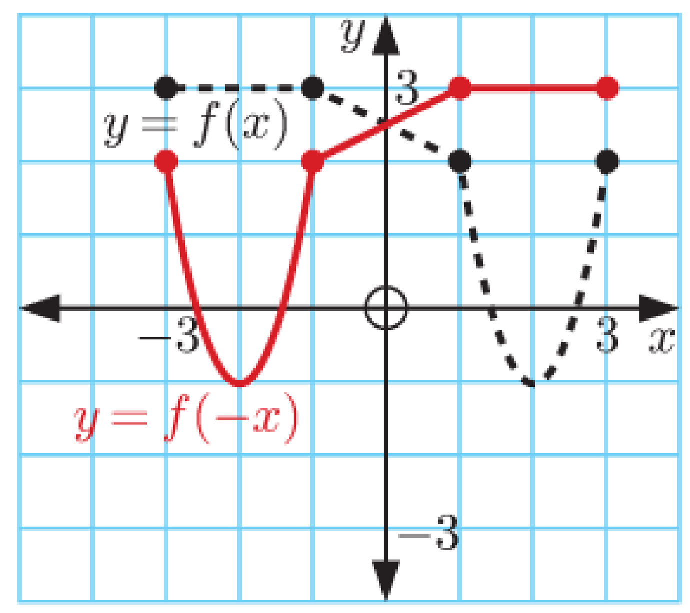
c Domain is $\{x \mid x \neq -\frac{1}{2}\}$, Range is $\{y \mid y \neq 3\}$

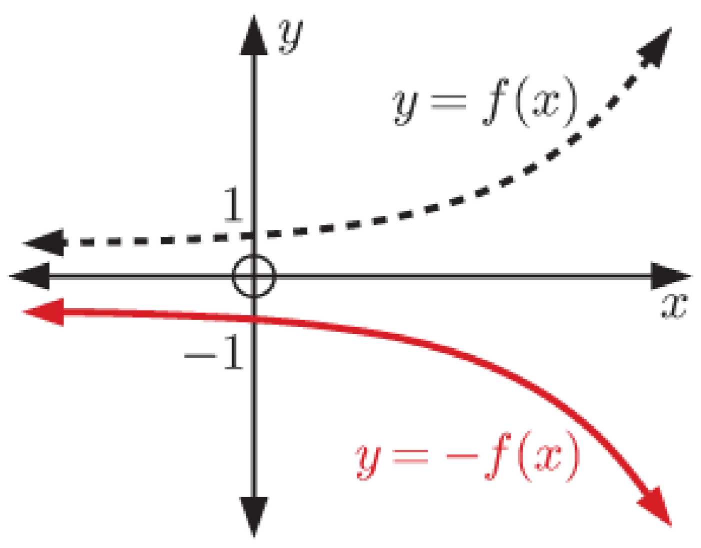
d 

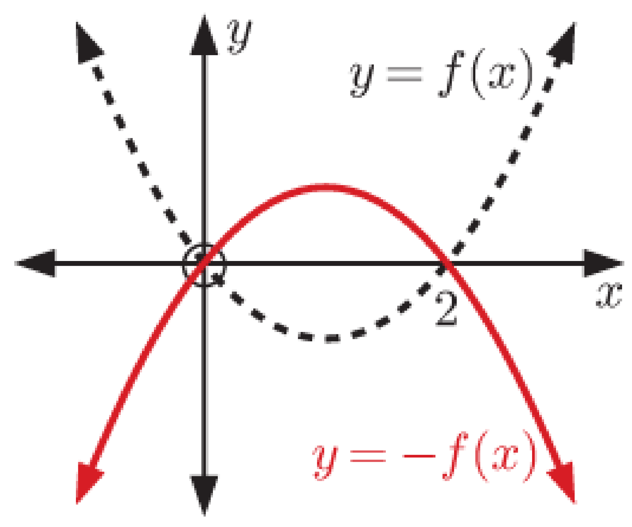
20
- A vertical stretch with scale factor 4 followed by a translation through $\begin{pmatrix} 3 \\ 33 \end{pmatrix}$, *or*
- a translation through $\begin{pmatrix} 3 \\ 8\frac{1}{4} \end{pmatrix}$ followed by a vertical stretch with scale factor 4.

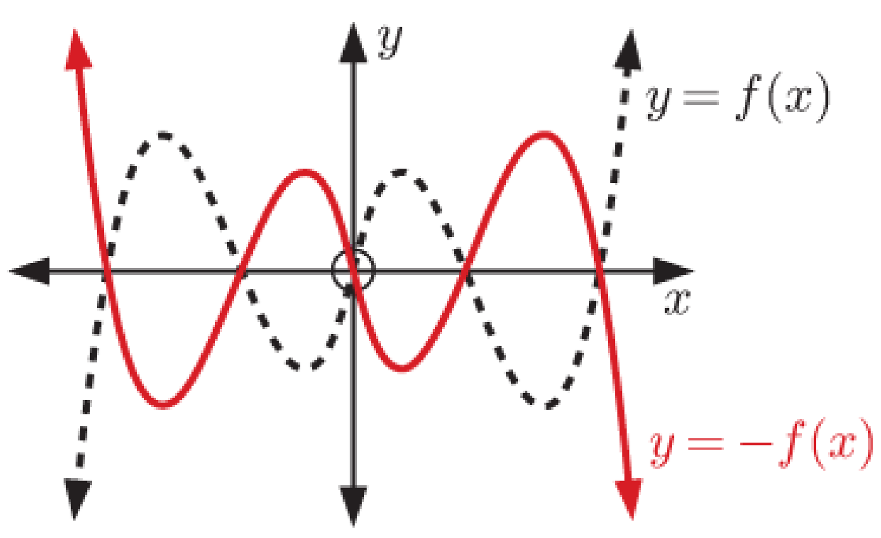
EXERCISE 16C

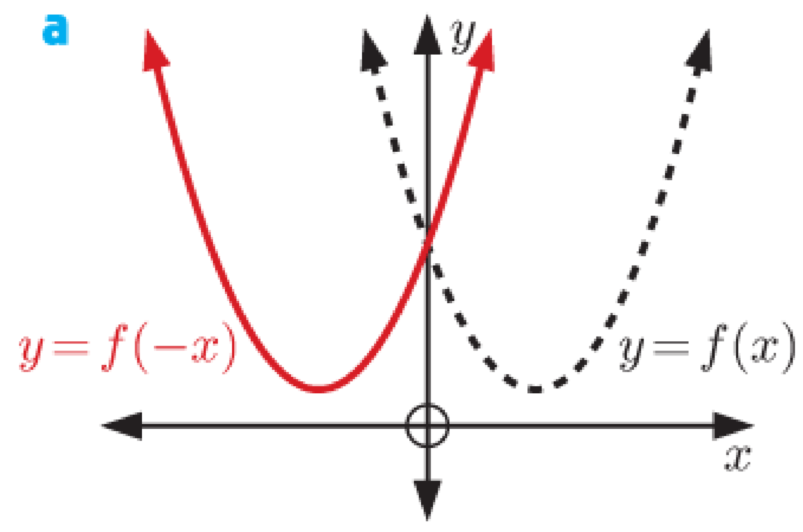
1 **a** 

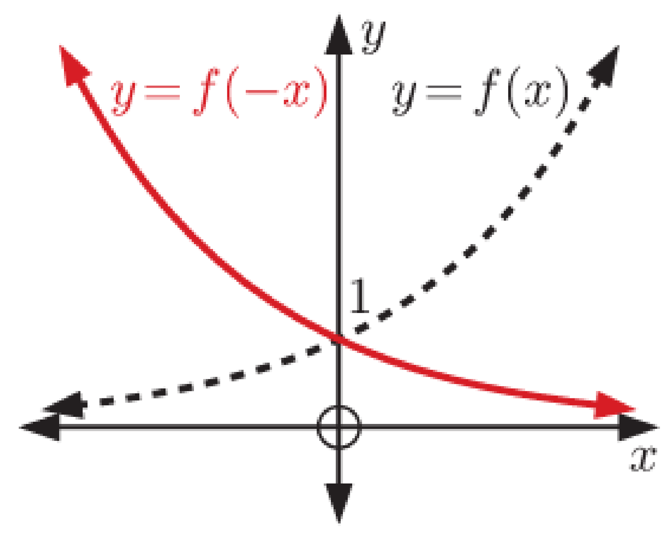
b 

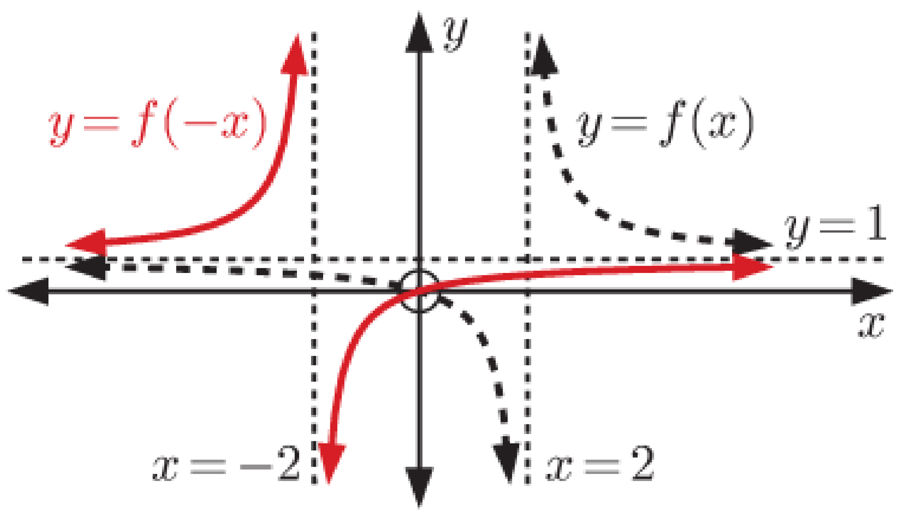
2 **a** 

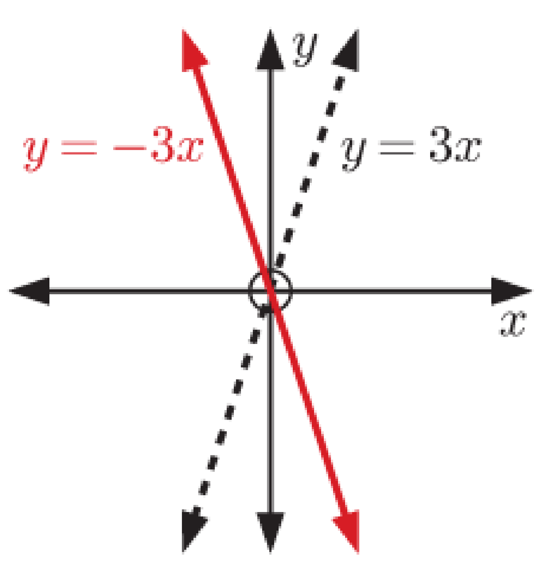
b 

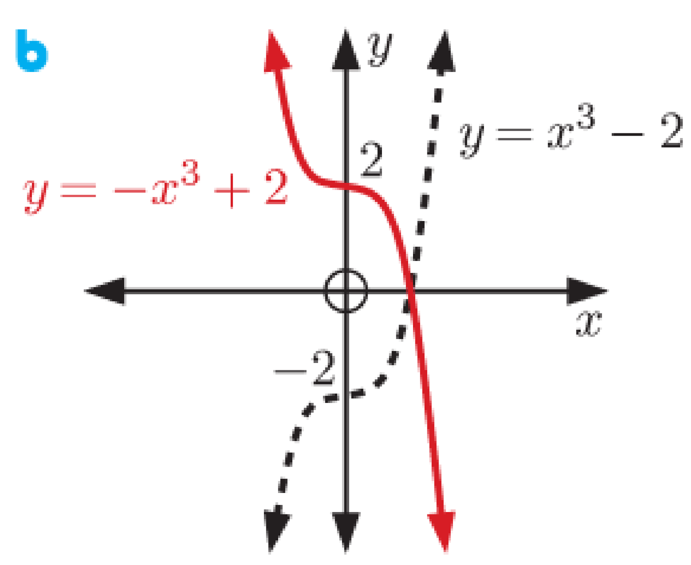
c 

3 **a** 

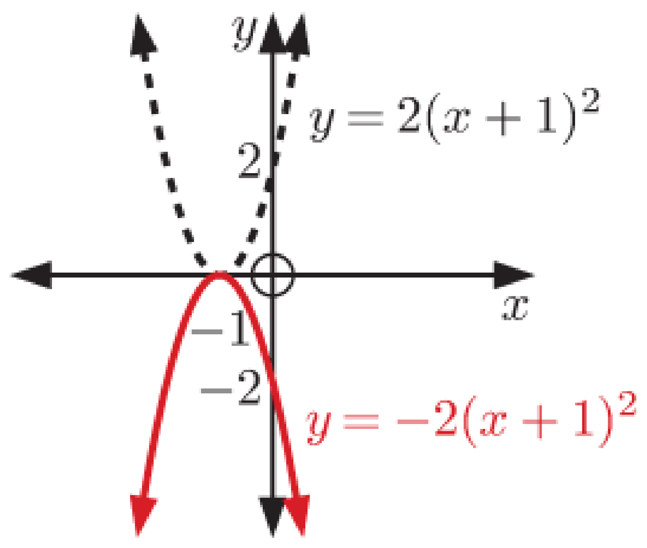
b 

c 

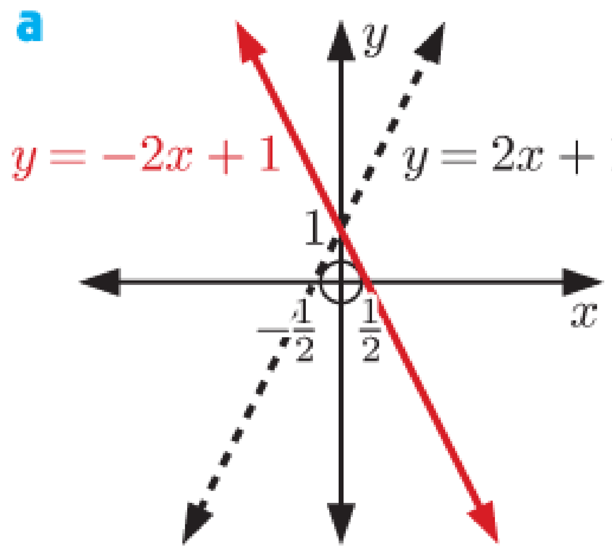
4 **a** 

b 

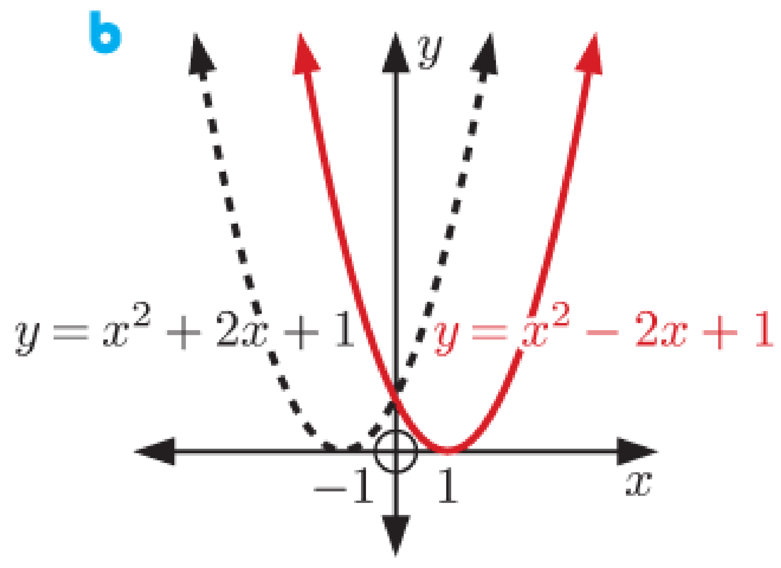
c



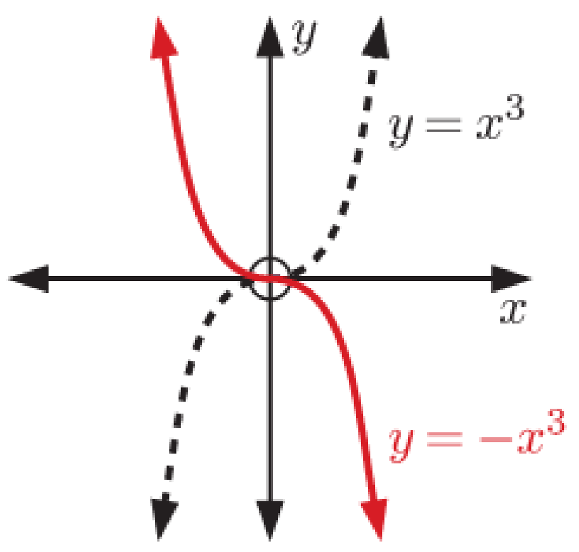
5 **a**



b



c



6 **a** $g(x) = -5x - 7$ **b** $g(x) = 2^{-x}$

c $g(x) = -2x^2 - 1$

d $g(x) = x^4 + 2x^3 - 3x^2 - 5x - 7$

7 **a** **i** $(3, 0)$ **ii** $(2, 1)$ **iii** $(-3, -2)$

b **i** $(7, 1)$ **ii** $(-5, 0)$ **iii** $(-3, 2)$

8 **a** **i** $(-2, -1)$ **ii** $(0, 3)$ **iii** $(1, 2)$

b **i** $(-5, -4)$ **ii** $(0, 3)$ **iii** $(-2, 3)$

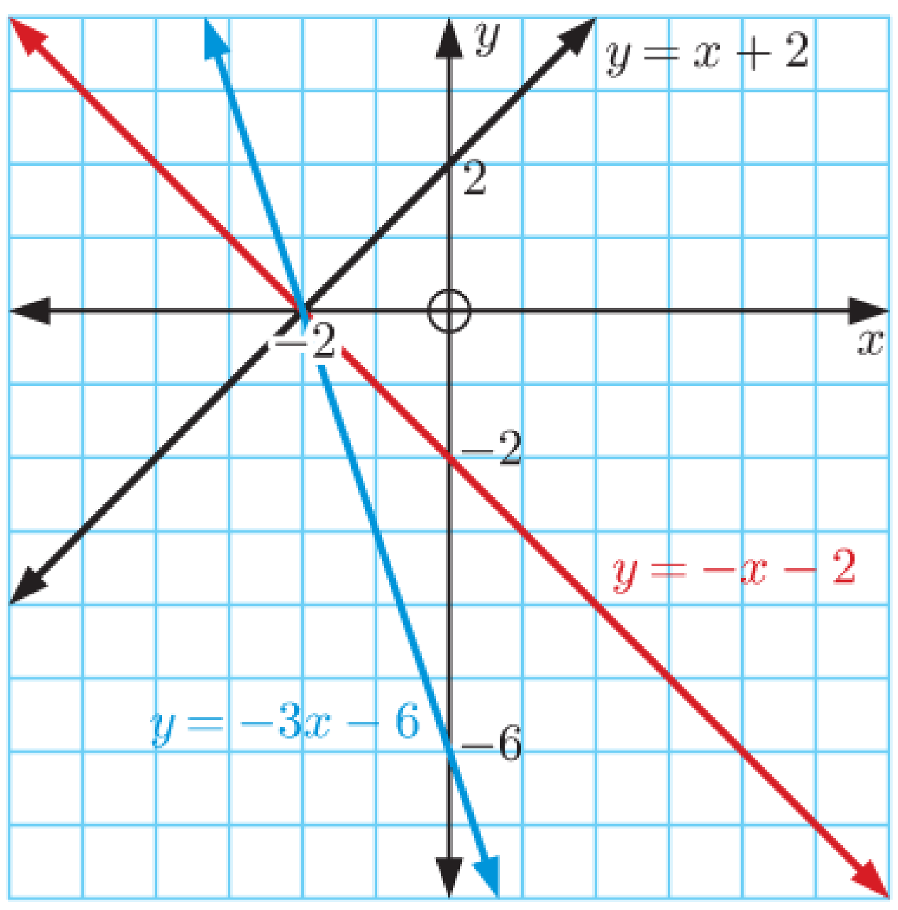
9 **a** A reflection in the y-axis and a reflection in the x-axis.

b $(-3, 7)$ **c** $(5, 1)$

10 **a** A reflection in the x-axis.

b A vertical stretch with scale factor 3.

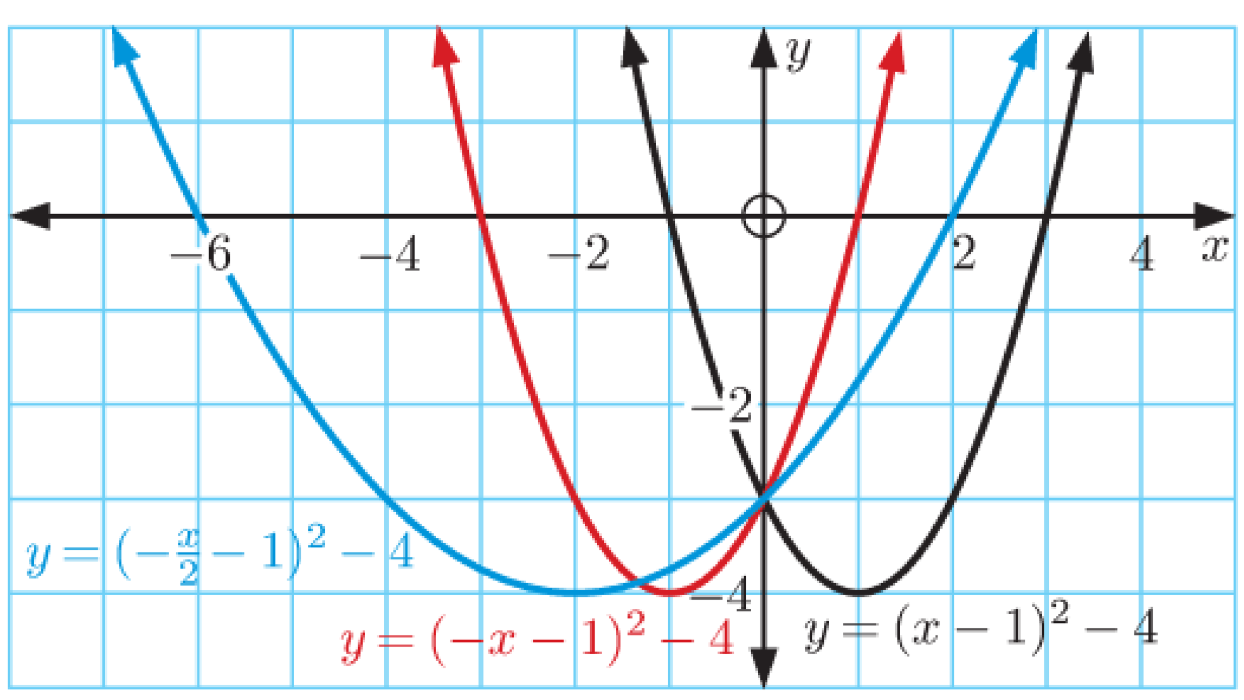
c



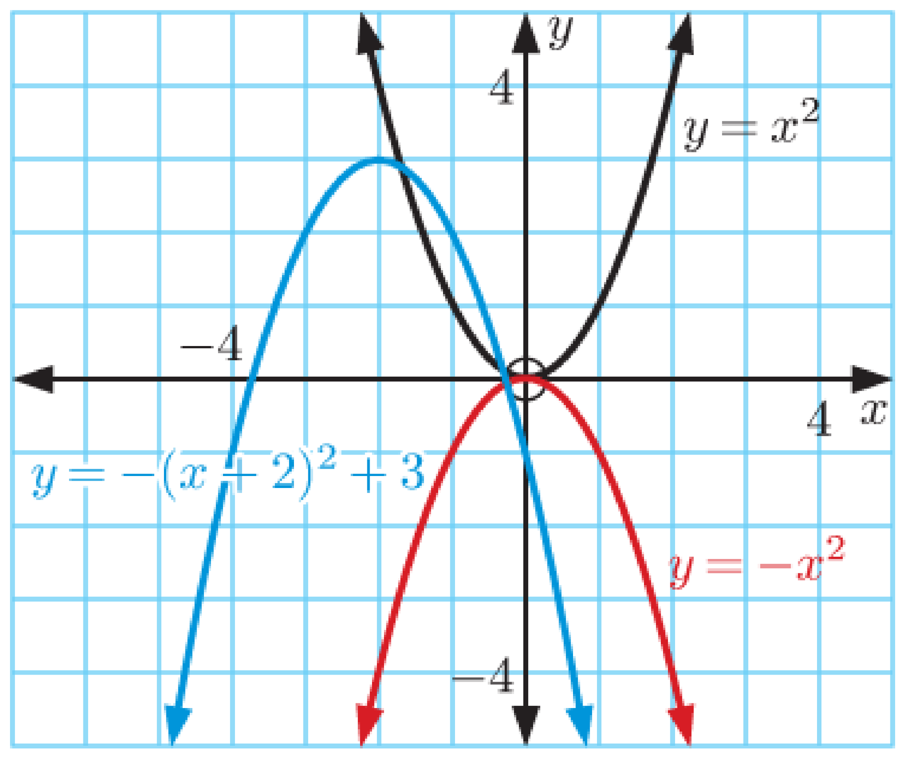
11 **a** A reflection in the y-axis.

b A horizontal stretch with scale factor 2.

c

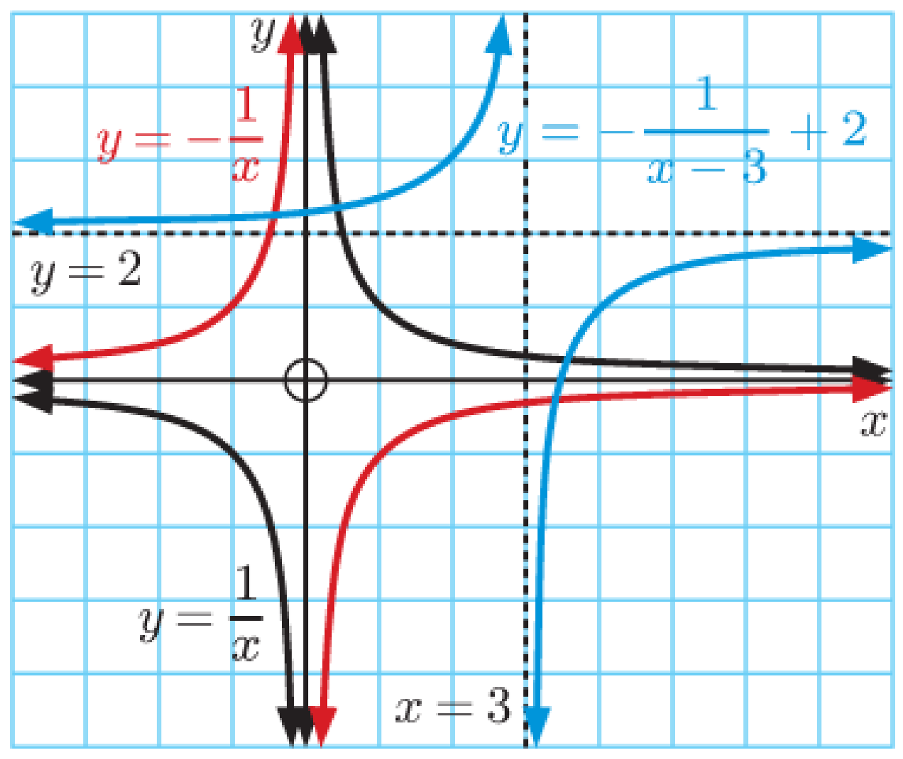


12



$y = x^2$ is transformed to $y = -(x+2)^2 + 3$ by reflecting in the x-axis and then translating through $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$.

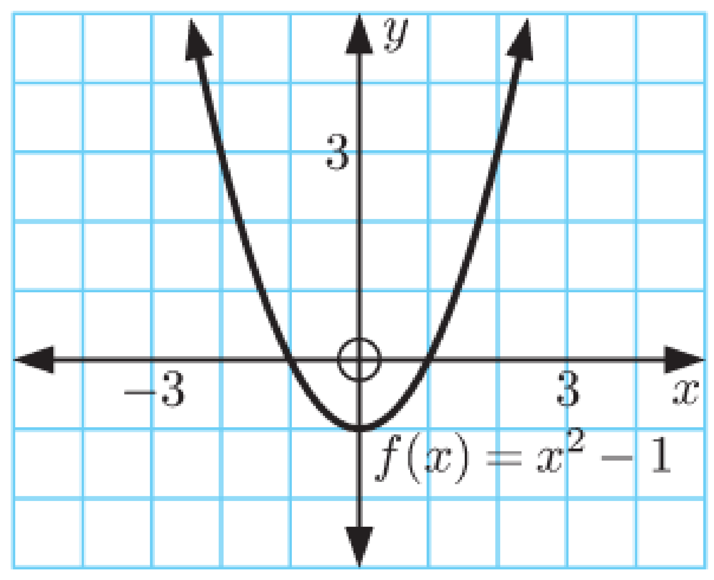
13



$y = \dfrac{1}{x}$ is transformed to $y = -\dfrac{1}{x-3} + 2$ by reflecting in the x-axis and then translating through $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

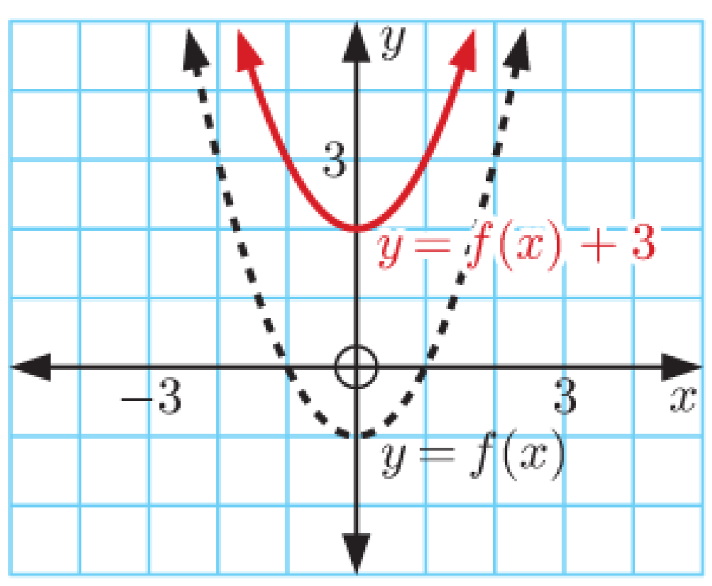
EXERCISE 16D

1 **a**



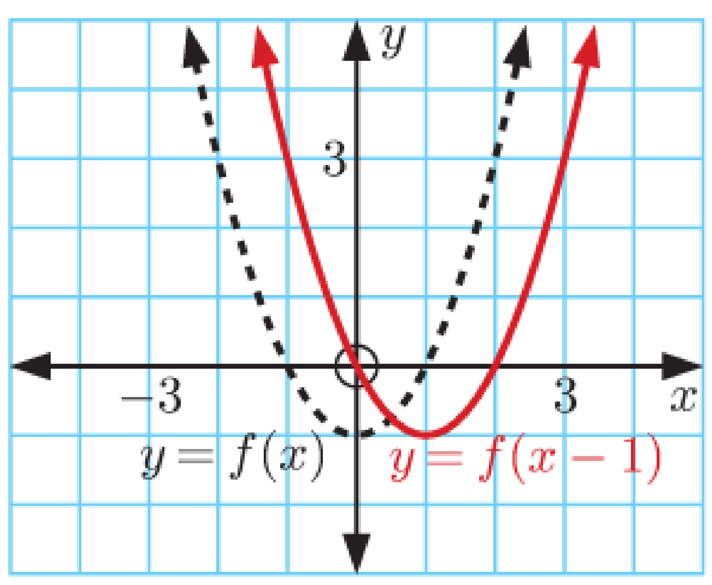
x-intercepts are ± 1, y-intercept is -1

b **i**



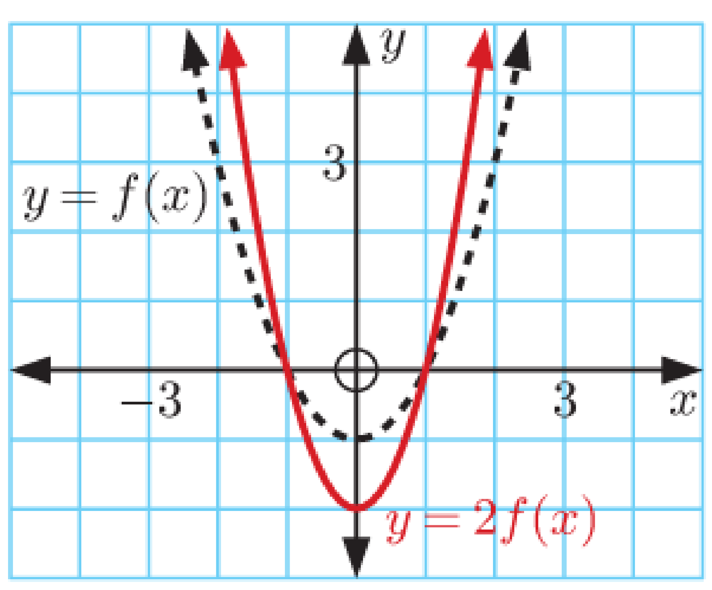
$y = f(x)$ has been translated 3 units upwards.

ii



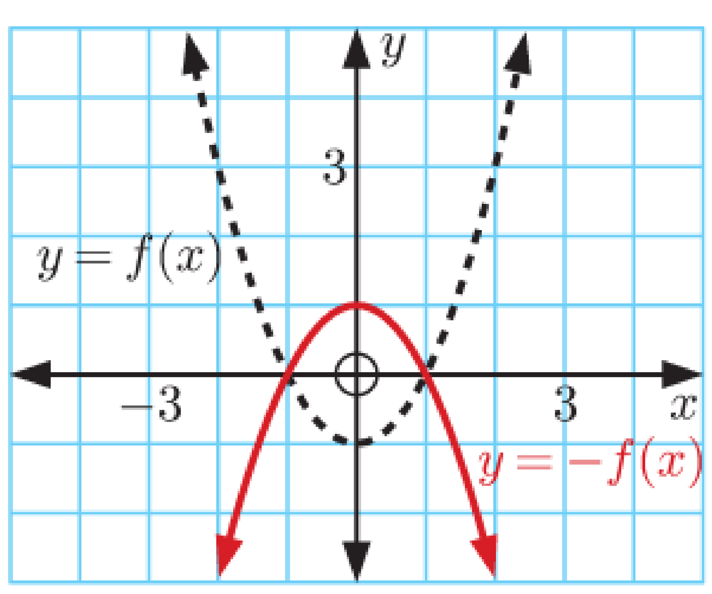
$y = f(x)$ has been translated 1 unit to the right.

iii



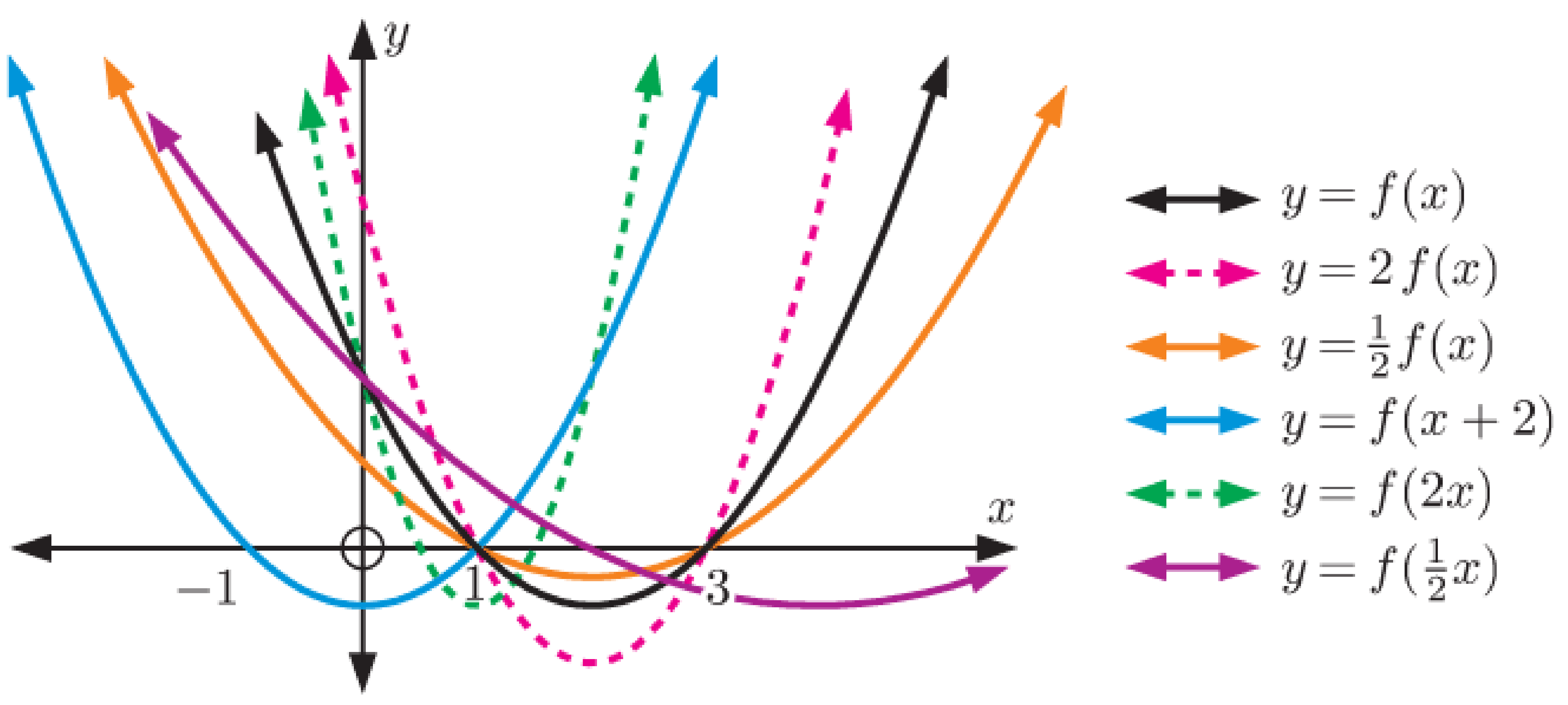
$y = f(x)$ has been vertically stretched with scale factor 2.

iv



$y = f(x)$ has been reflected in the x-axis.

2



3 **a** **i** A vertical translation through $\begin{pmatrix}0\\-2\end{pmatrix}$.

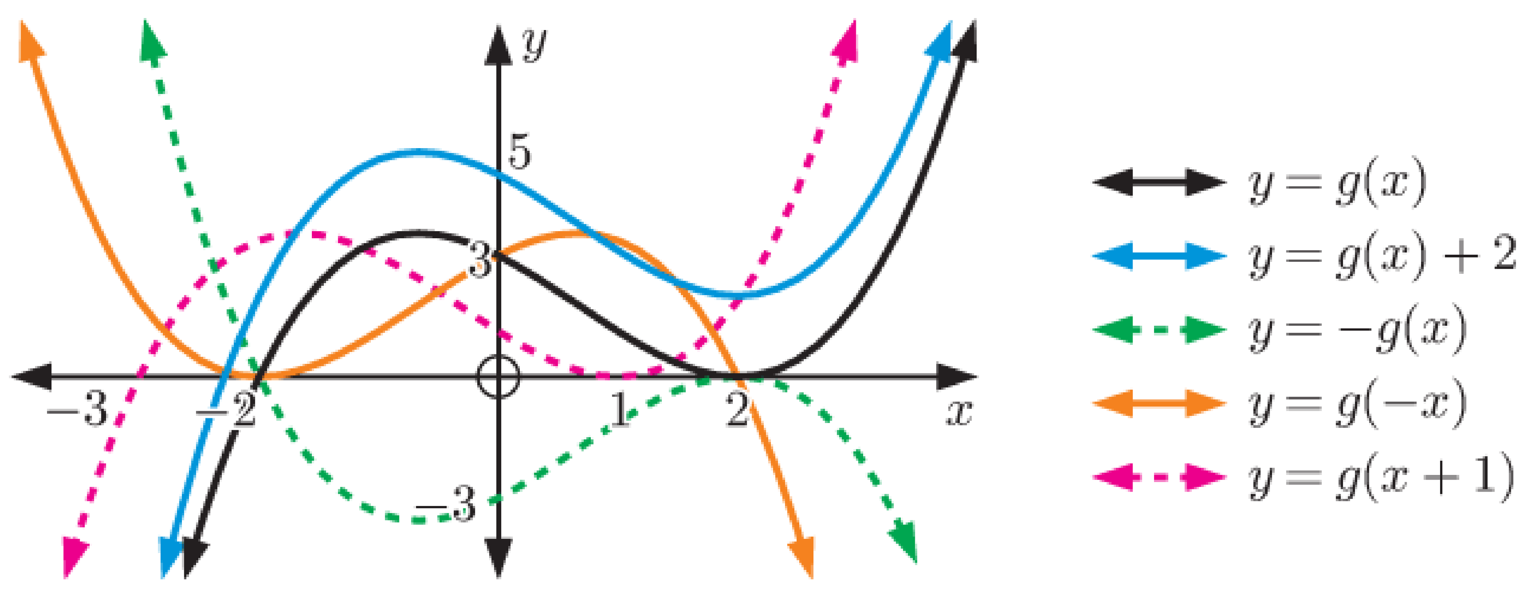
ii $g(x) = f(x) - 2$

b **i** A vertical stretch with scale factor $\frac{1}{2}$.

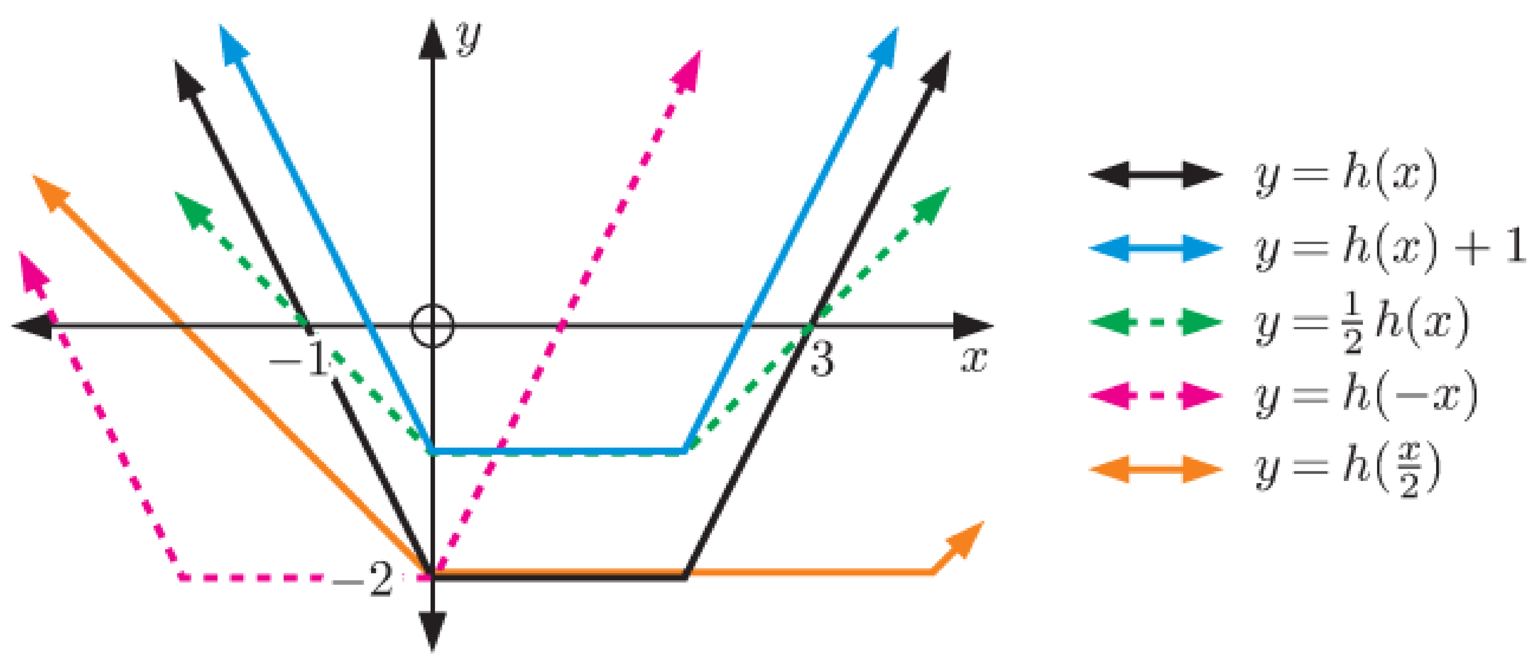
ii $g(x) = \frac{1}{2}f(x)$

c **i** A reflection in the y-axis. **ii** $g(x) = f(-x)$

4

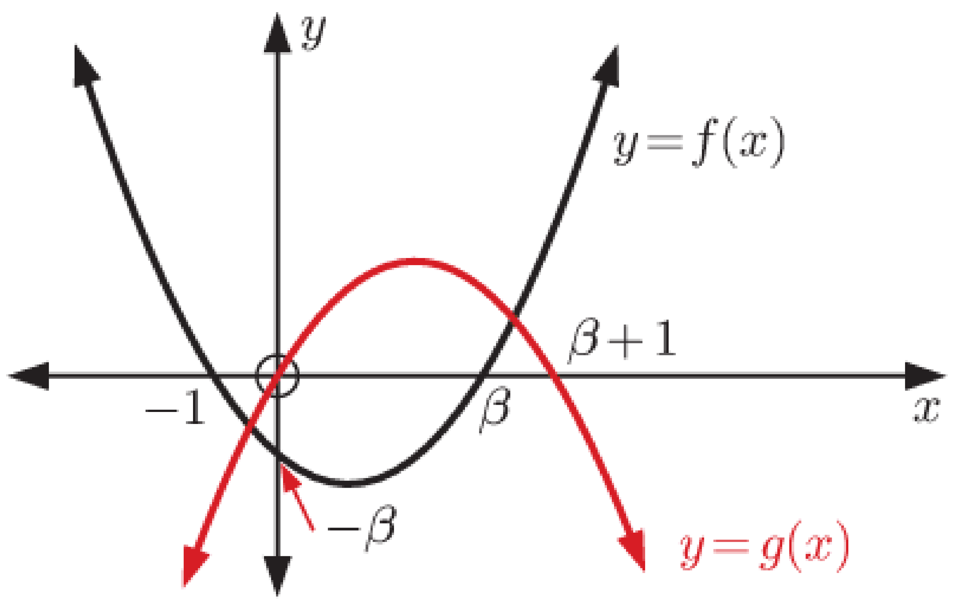


5



6 **a** x-intercepts are -1 and β, y-intercept is $-\beta$

b, c



7 **a** $f(-x-4)-1$ **b** $f(-x+4)-1$

c $\frac{1}{2}f(x+2)+\frac{1}{2}$ **d** $\frac{1}{2}f(x+2)+1$

e $f(\frac{1}{4}x-3)-5$ **f** $f\left(\dfrac{x-3}{4}\right)-5$

8 **a** A reflection in the x-axis, then a translation through $\begin{pmatrix}-1\\3\end{pmatrix}$.

b A horizontal stretch with scale factor 2, then a translation through $\begin{pmatrix}0\\-7\end{pmatrix}$.

c A translation through $\begin{pmatrix}1\\0\end{pmatrix}$, then a horizontal stretch with scale factor $\frac{1}{3}$.

d A vertical stretch with scale factor 2, a translation through $\begin{pmatrix}1\\-1\end{pmatrix}$, then a horizontal stretch with scale factor 4.

e A vertical stretch with scale factor 2, a horizontal stretch with scale factor $\frac{1}{3}$, then a translation through $\begin{pmatrix}1\\5\end{pmatrix}$.

f A reflection in the x-axis, a vertical stretch with scale factor 4, a horizontal stretch with scale factor 2, then a translation through $\begin{pmatrix}-3\\-1\end{pmatrix}$.

9 **a** Domain is $\{x \mid x \geqslant -3\}$, Range is $\{y \mid -3 \leqslant y < 4\}$

b Domain is $\{x \mid x \geqslant \frac{1}{3}\}$, Range is $\{y \mid -10 < y \leqslant 4\}$

c Domain is $\{x \mid x \geqslant 3\}$, Range is $\{y \mid \frac{10}{3} \leqslant y < \frac{17}{3}\}$

10 **a** $5\sqrt{2-x}+15$

Domain is $\{x \mid x \leqslant 2\}$, Range is $\{y \mid y \geqslant 15\}$

b $5\sqrt{2-x}+3$

Domain is $\{x \mid x \leqslant 2\}$, Range is $\{y \mid y \geqslant 3\}$

c $5\sqrt{-x-2}+3$

Domain is $\{x \mid x \leqslant -2\}$, Range is $\{y \mid y \geqslant 3\}$

11 **a** The vertical stretch has scale factor $|\,a\,|$. The reflection in the x-axis occurs if $a < 0$. Each point is then moved h units right and k units up.

b The function has shape ∪ if $a > 0$ and ∩ if $a < 0$.

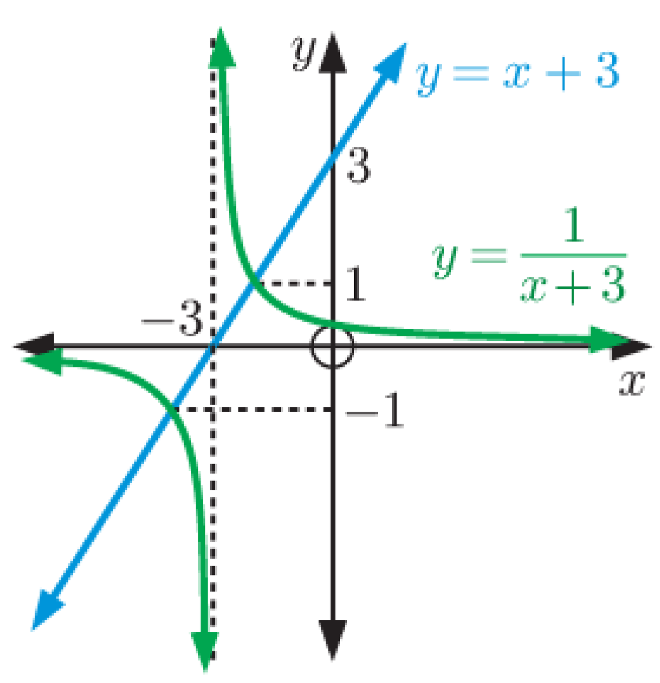
The function has vertex (h, k), and y-intercept $ah^2 + k$.

12 **a** $5 + \dfrac{-4}{2x+3}$

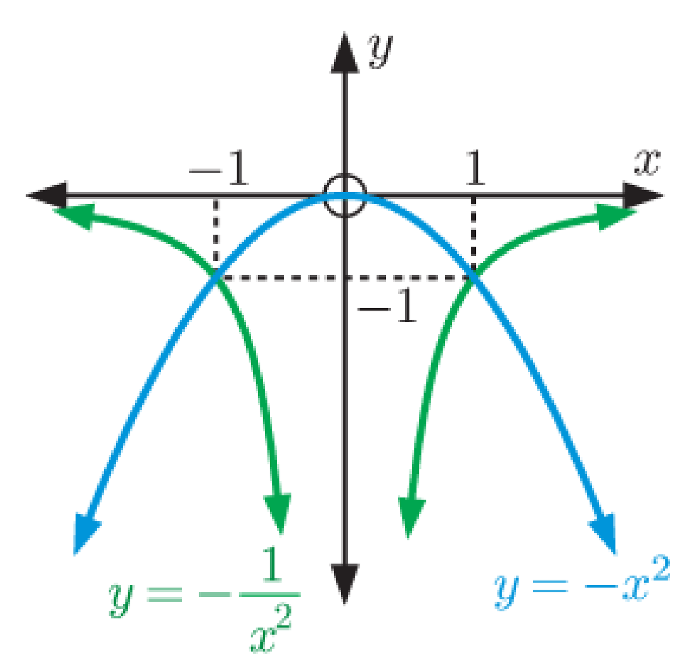
b A reflection in the x-axis, a vertical stretch with scale factor 4, a translation through $\begin{pmatrix}-3\\5\end{pmatrix}$, then a horizontal stretch with scale factor $\frac{1}{2}$.

EXERCISE 16E

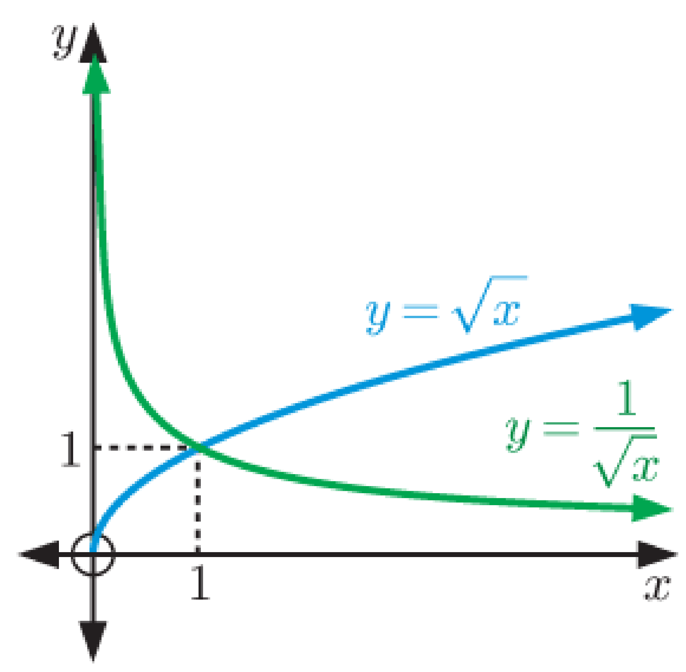
1 **a**



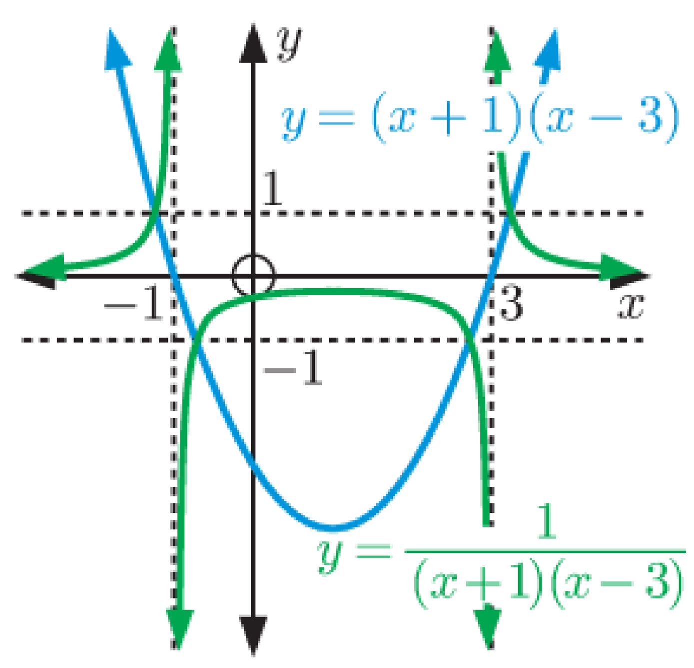
b



c



d



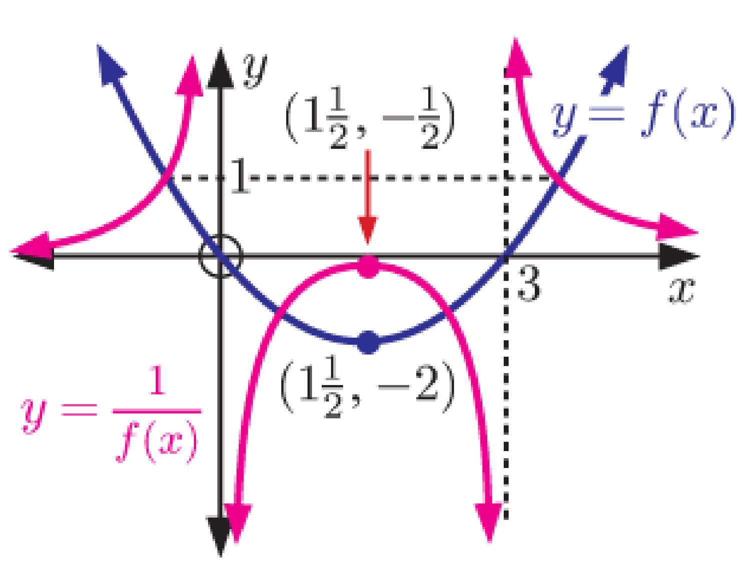
2 **a** invariant points are $(-2, 1)$ and $(-4, -1)$

b invariant points are $(-1, -1)$ and $(1, -1)$

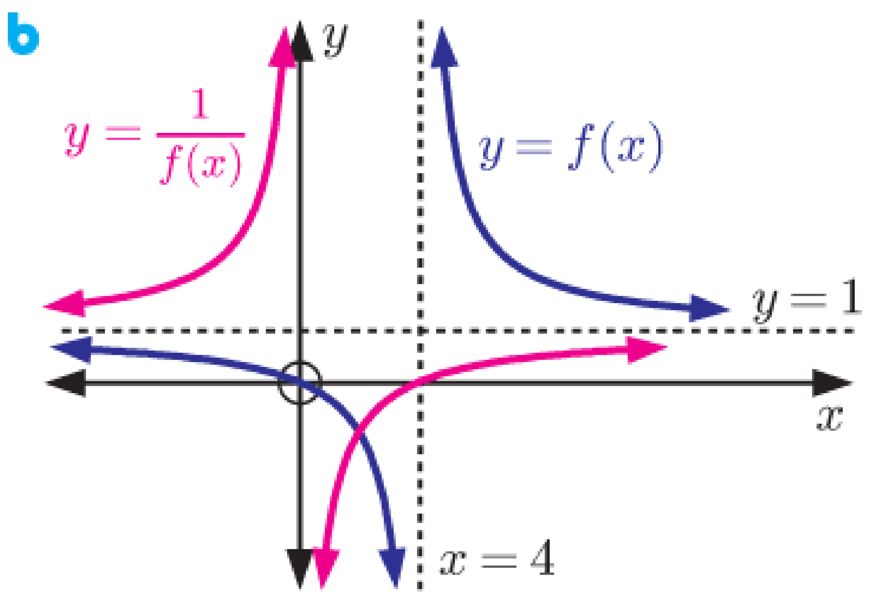
c invariant point is $(1, 1)$

d invariant points are $(\approx -1.24, 1)$, $(\approx -0.732, -1)$, $(\approx 2.73, -1)$, and $(\approx 3.24, 1)$

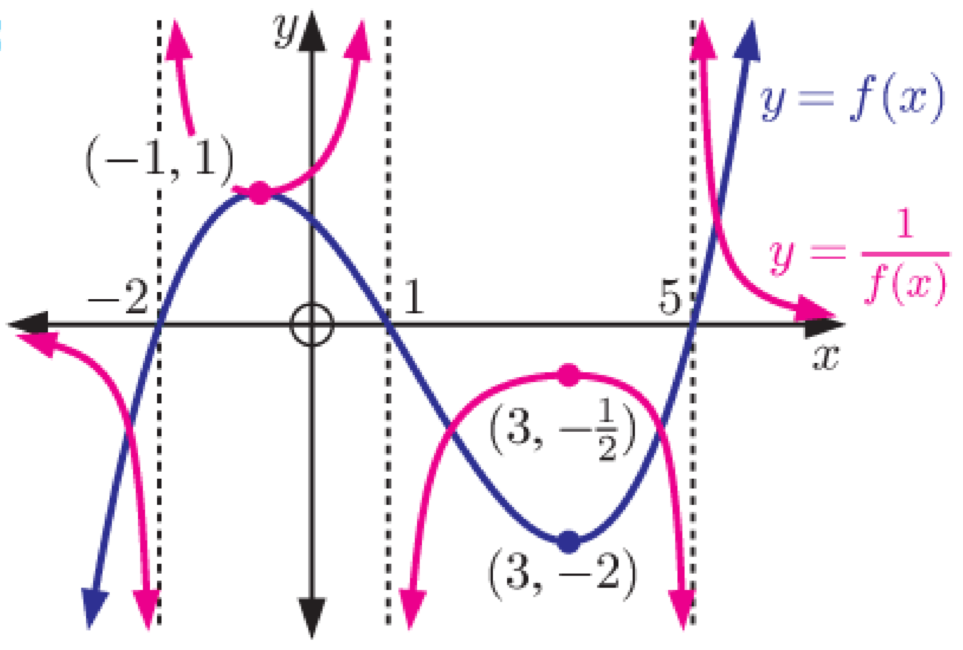
3 **a**



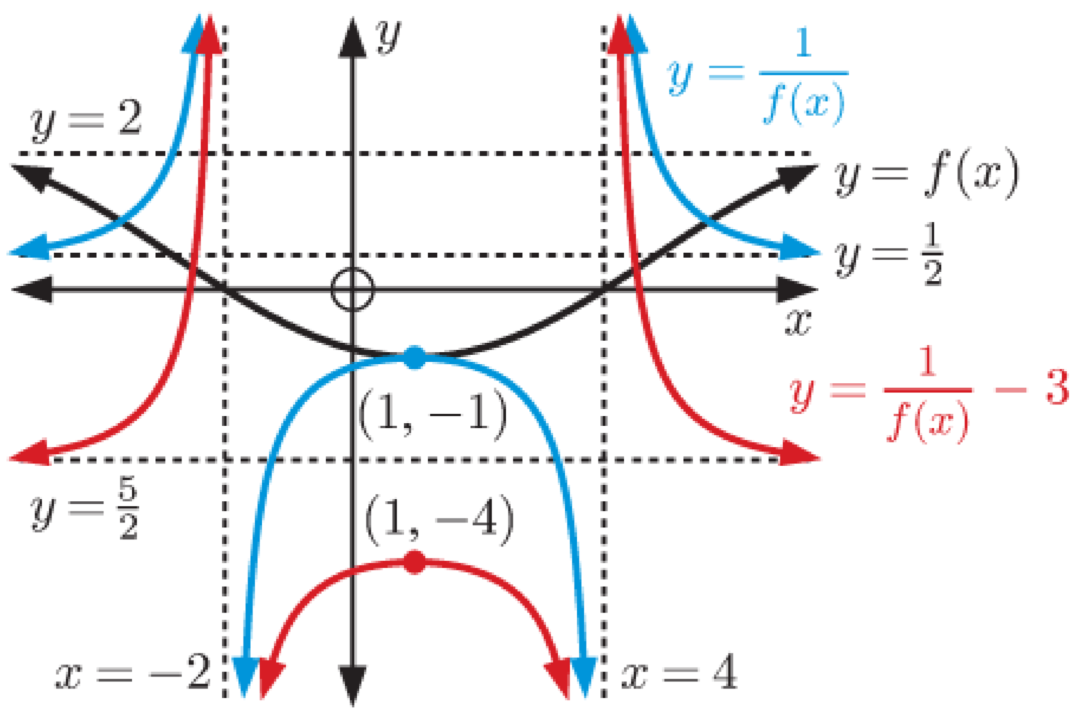
b



c

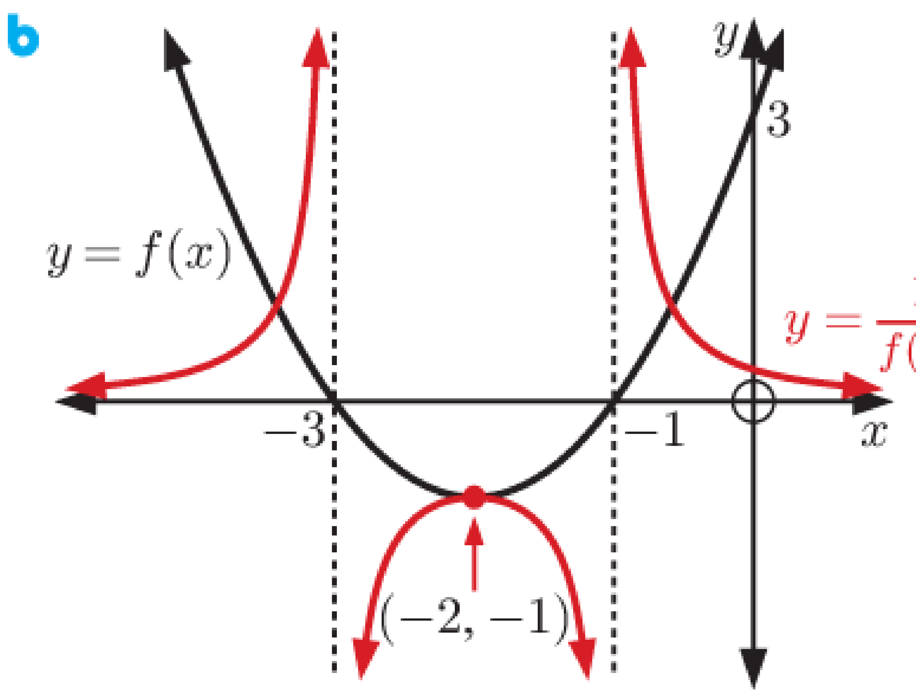


4



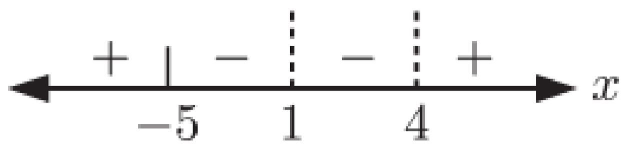
5 **a** x-intercepts -3 and -1, y-intercept 3, vertex $(-2, -1)$

b



c $x = -\frac{9}{2}$ or $\frac{1}{2}$

6

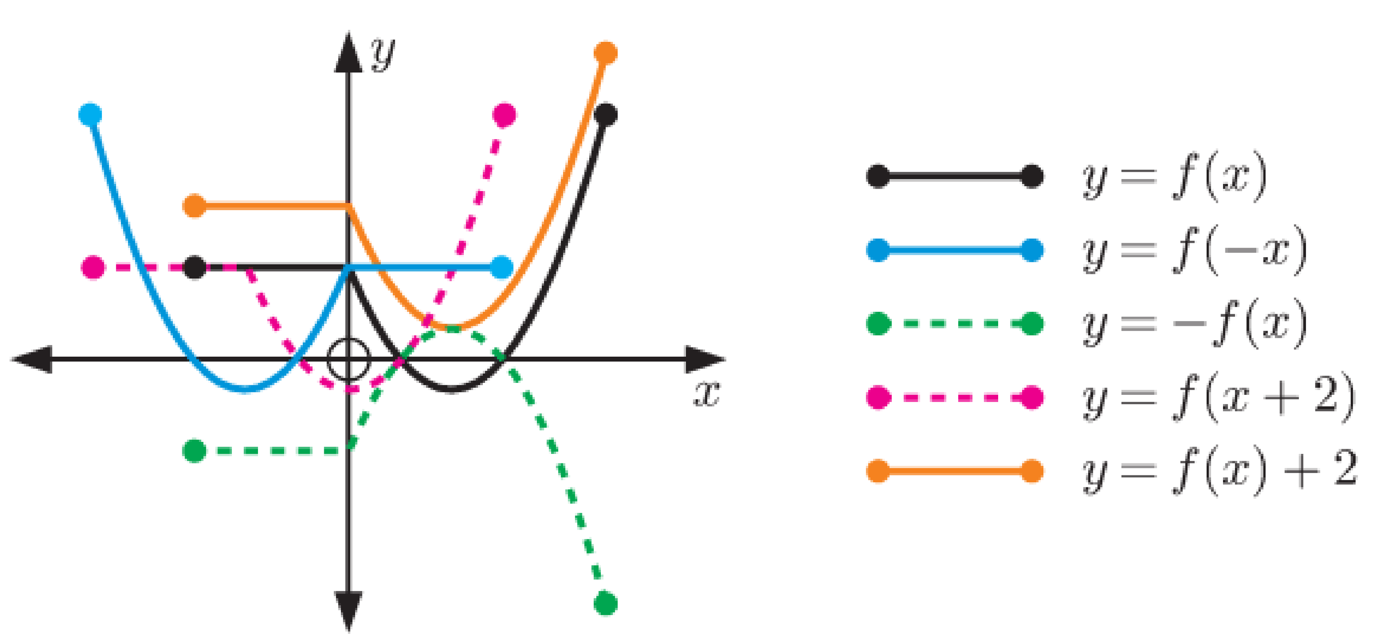


7 **a** Domain is $\{x \mid -1 \leqslant x \leqslant 6\}$, Range is $\{y \mid \frac{1}{5} < y \leqslant \frac{1}{2}\}$

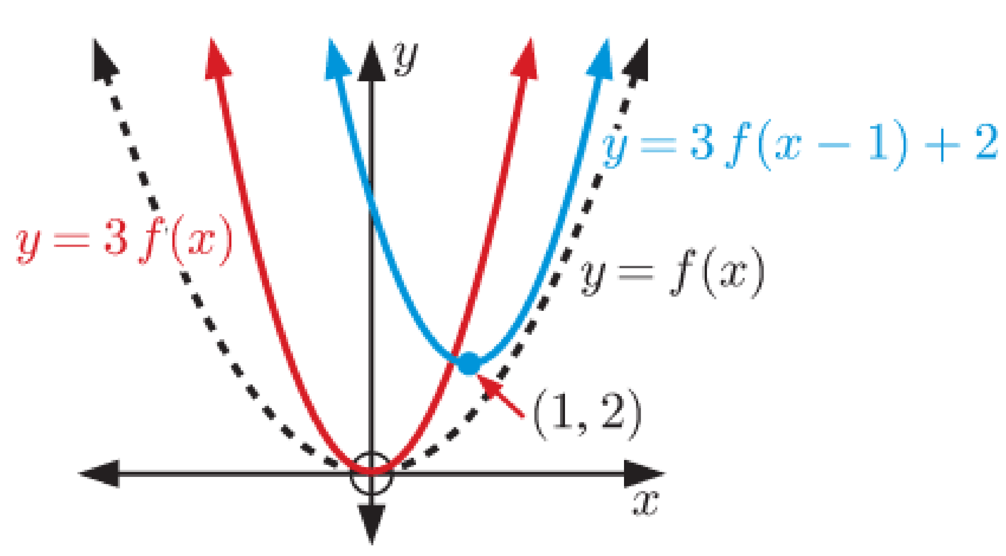
b Range is $\{y \mid y \leqslant -\frac{1}{3}$ or $y \geqslant \frac{1}{3}\}$, cannot say about the domain.

REVIEW SET 16A

1



2

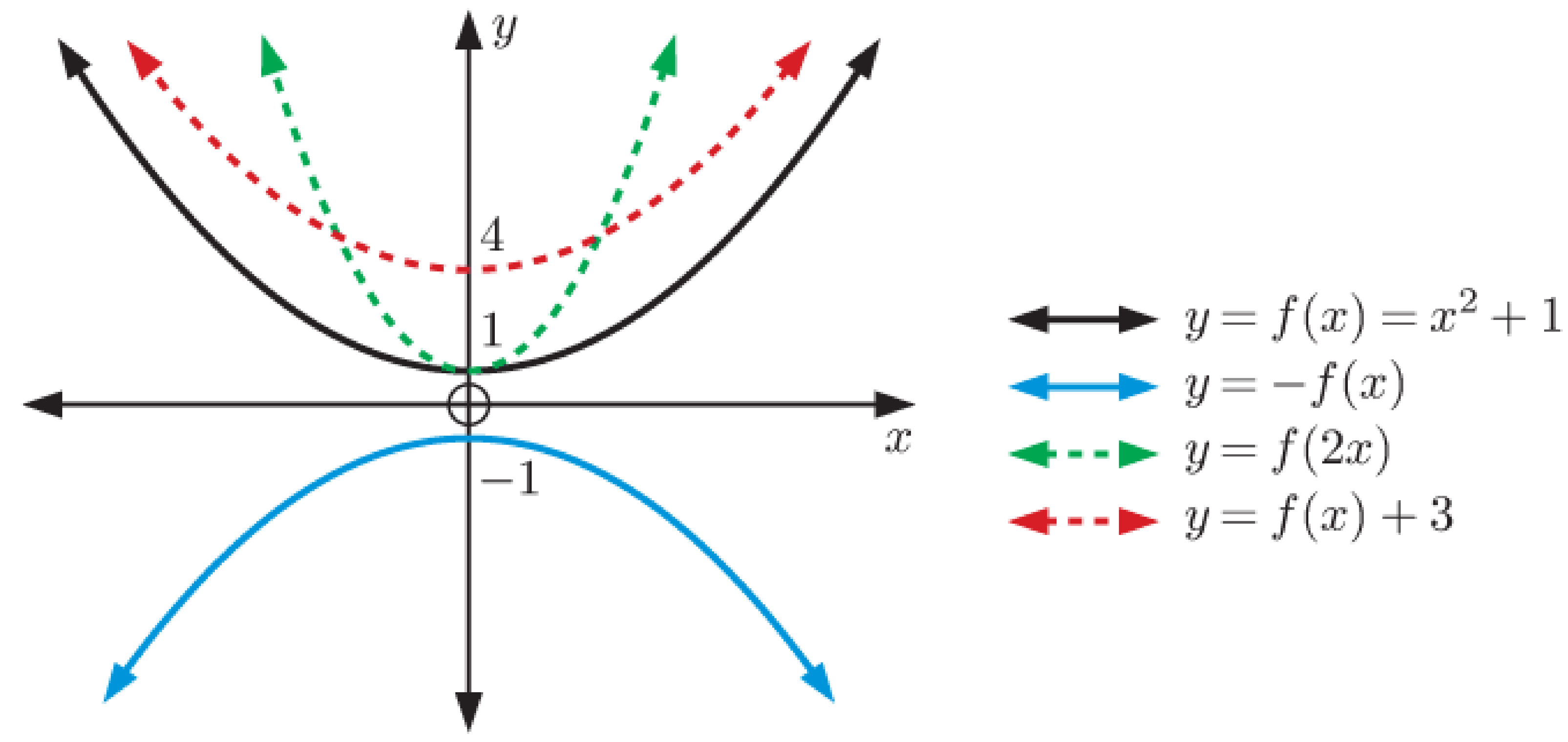


3 **a** $g(x) = 4x - 10$ **b** $g(x) = 5x^2 + 30$

c $g(x) = -3x - 5$ **d** $g(x) = \frac{2}{9}x^2 - \frac{1}{3}x + 4$

e $g(x) = -x^3$

4



5 $g(x)$ is the result of transforming $f(x)$ 3 units to the left and 4 units down.

$\therefore$ domain of $g(x)$ is $\{x \mid -5 \leqslant x \leqslant 0\}$
range of $g(x)$ is $\{y \mid -5 \leqslant y \leqslant 3\}$.

6 **a** $g(x) = (x-1)^2 + 8$

b **i** $\{y \mid y \geqslant 4\}$ **ii** $\{y \mid y \geqslant 8\}$

8 $g(x) = 3x^2 + 5x + 9$

9 **a** $-f(x+2) + 3$ **b** $2f(x-4) - 2$

10 **a** $(0, 4)$ **b** $(0, 6)$ **c** $(\frac{1}{2}, 3)$

11 **a** x-intercepts -9 and -3

b x-intercepts -5 and 1, y-intercept -9

c x-intercepts -10 and 2, y-intercept -3

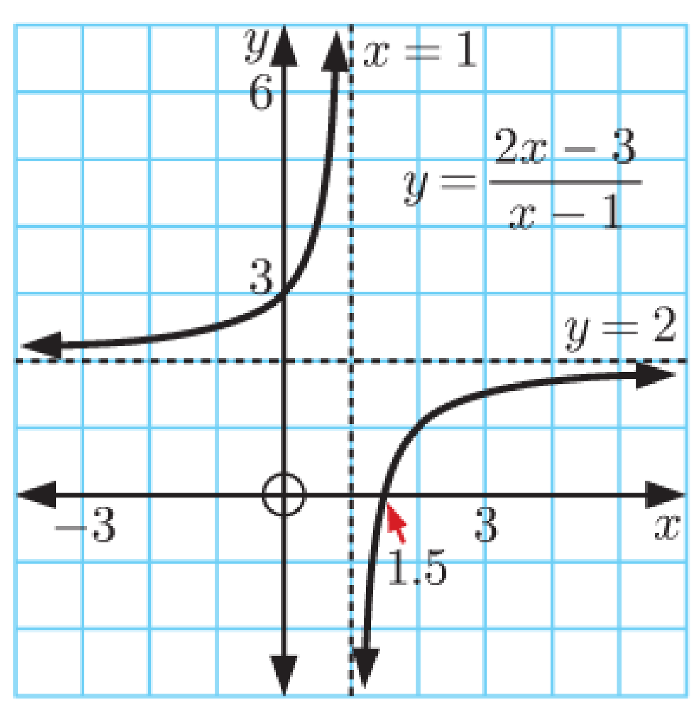
d x-intercepts -5 and 1, y-intercept 3

12 **a** $g(x) = \dfrac{2x-3}{x-1}$

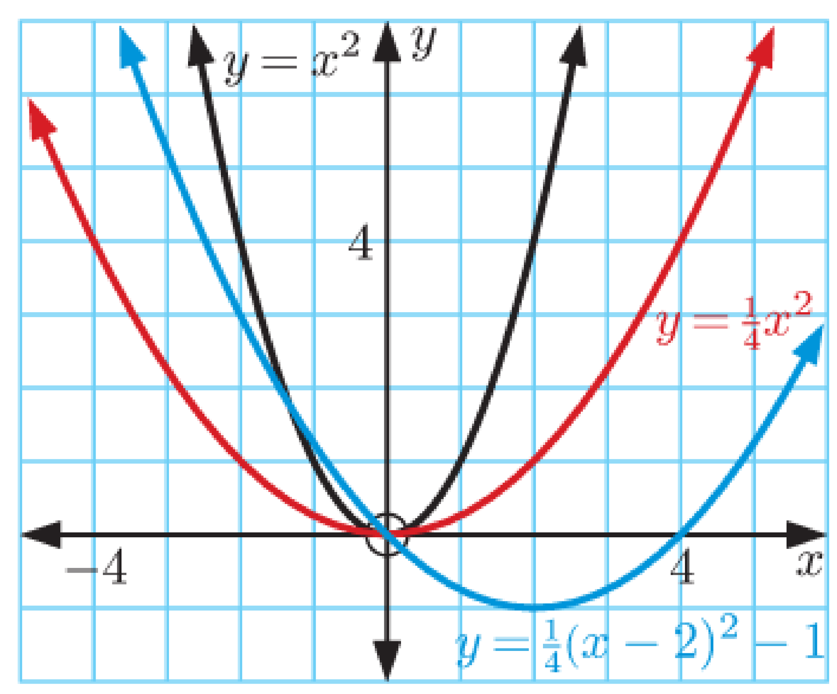
b vertical asymptote $x = 1$, horizontal asymptote $y = 2$

c Domain is $\{x \mid x \neq 1\}$ Range is $\{y \mid y \neq 2\}$

d

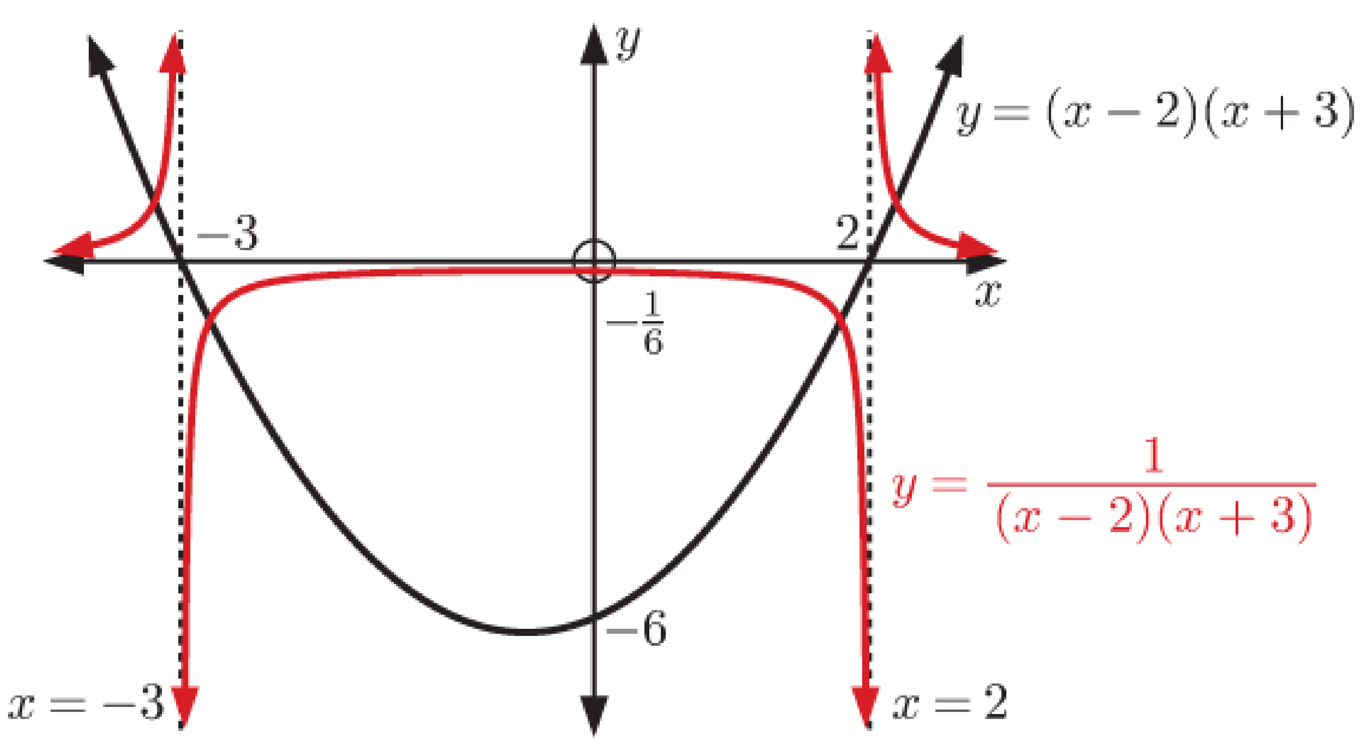


13



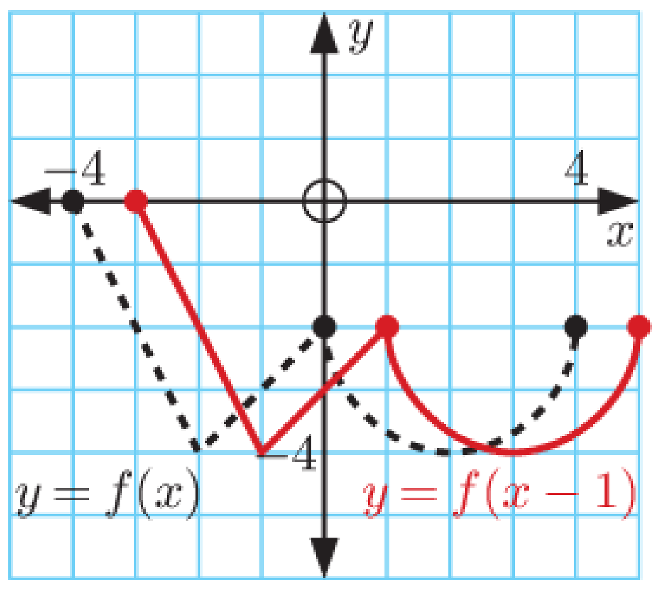
$y = x^2$ is transformed to $y = \frac{1}{4}(x-2)^2 - 1$ by vertically stretching with scale factor $\frac{1}{4}$ and then translating through $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

14

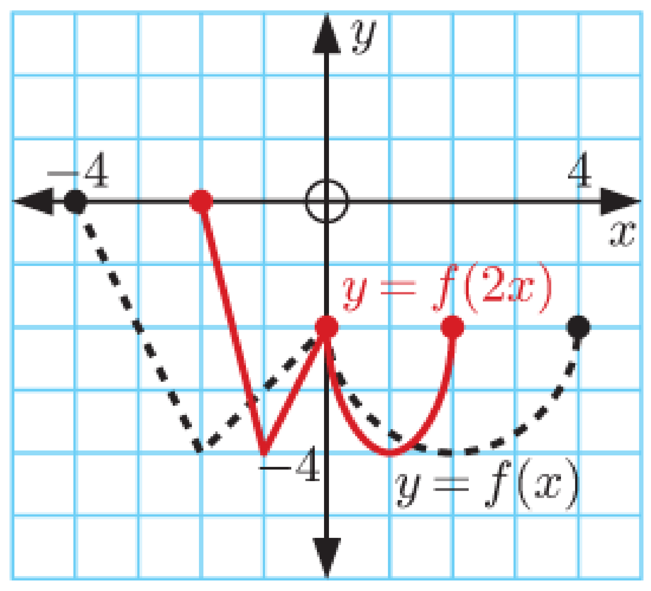


REVIEW SET 16B

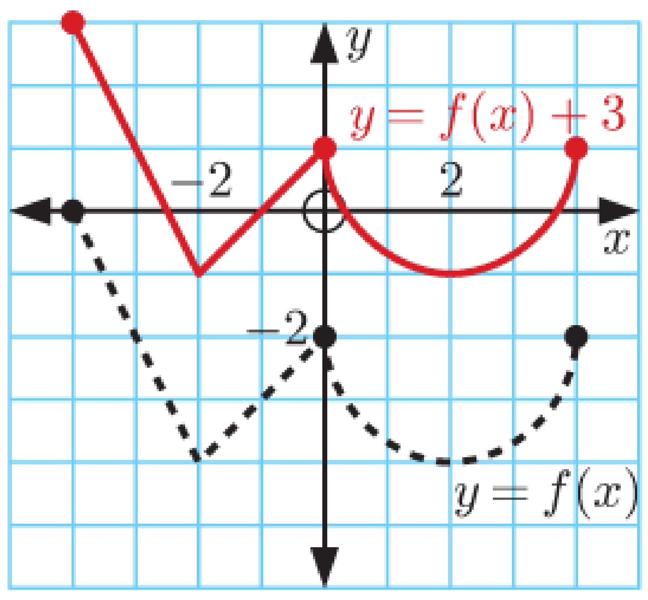
1 **a**



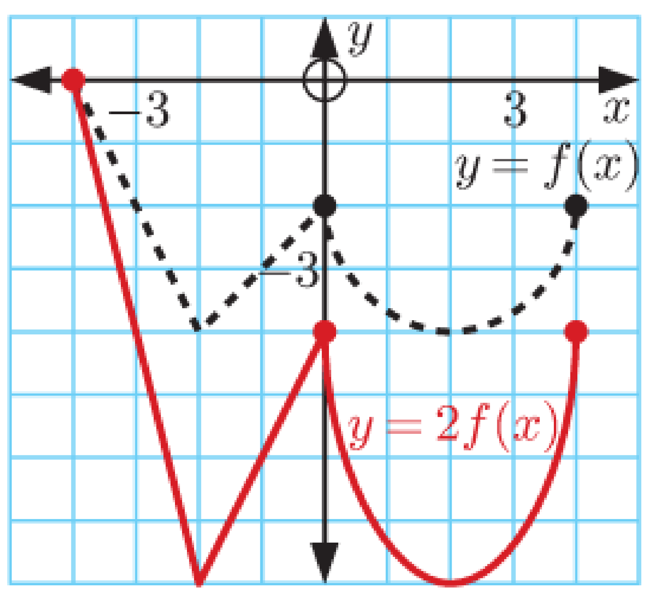
b



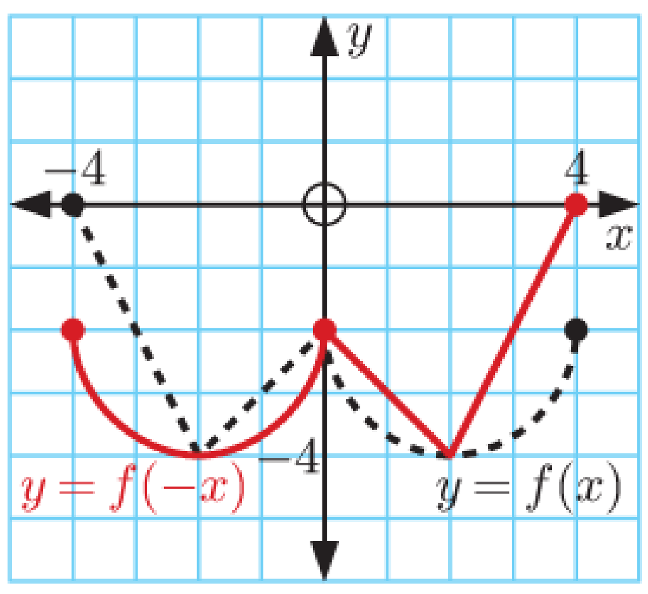
c



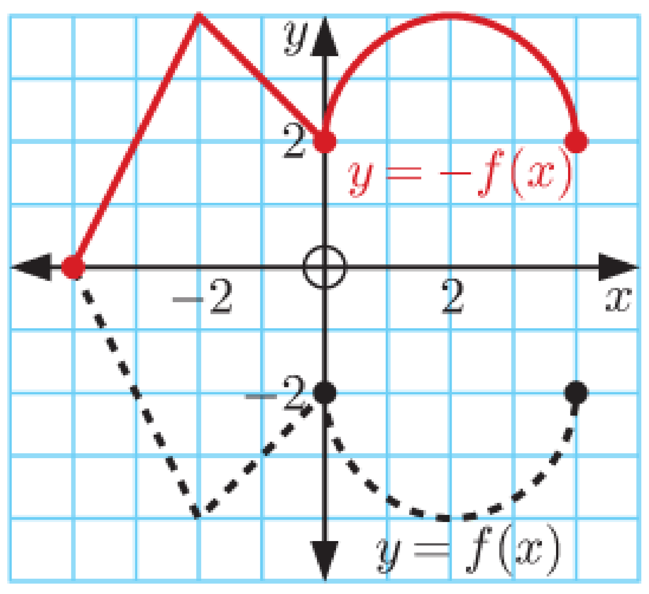
d



e



f

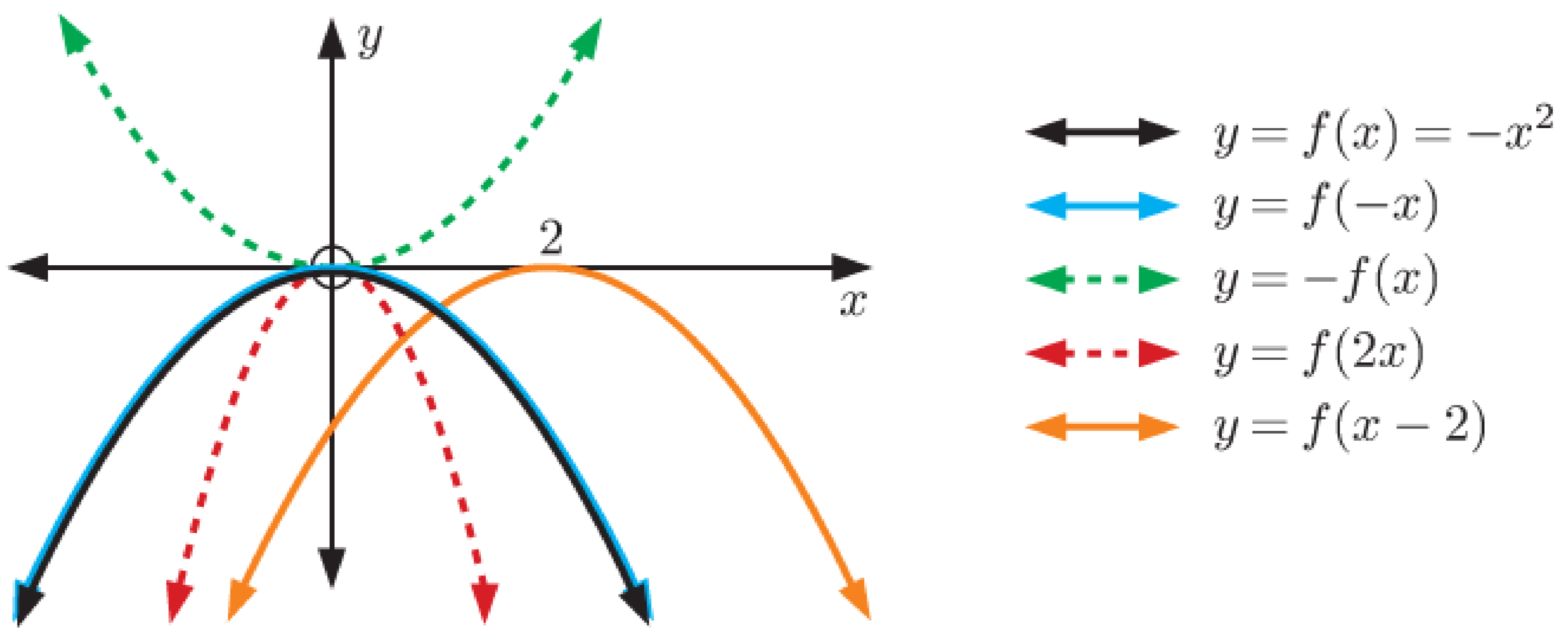


2 **a** $g(x) = 3x - x^2$ **b** $g(x) = 16 - x$

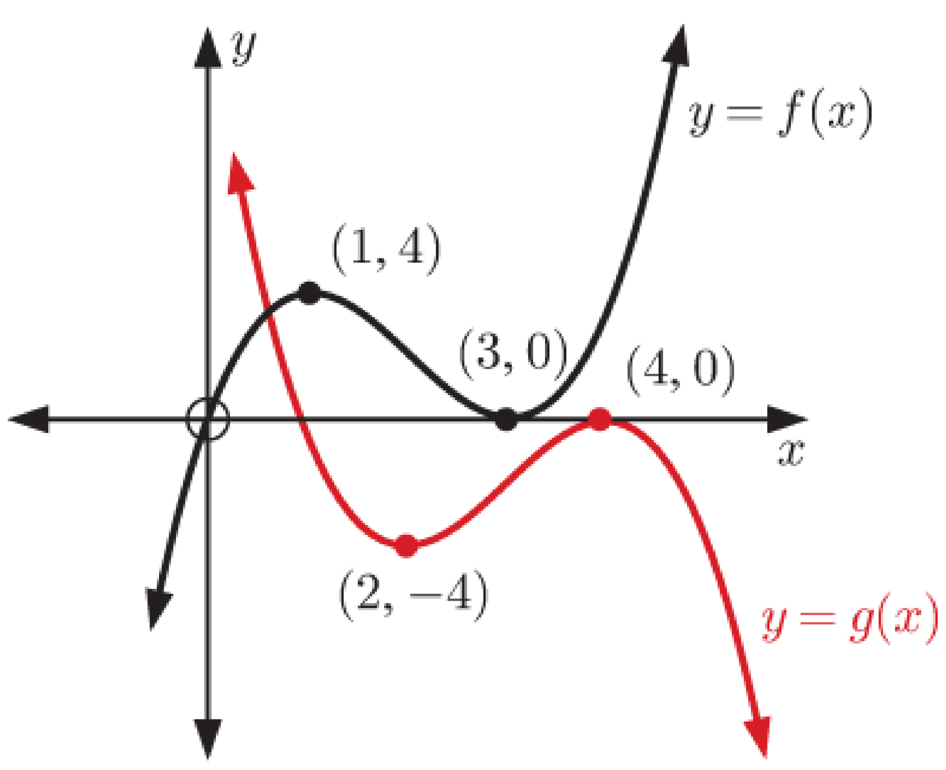
c $g(x) = \frac{1}{12}x + 2$

3 $g(x) = -x^2 - 6x - 7$

4



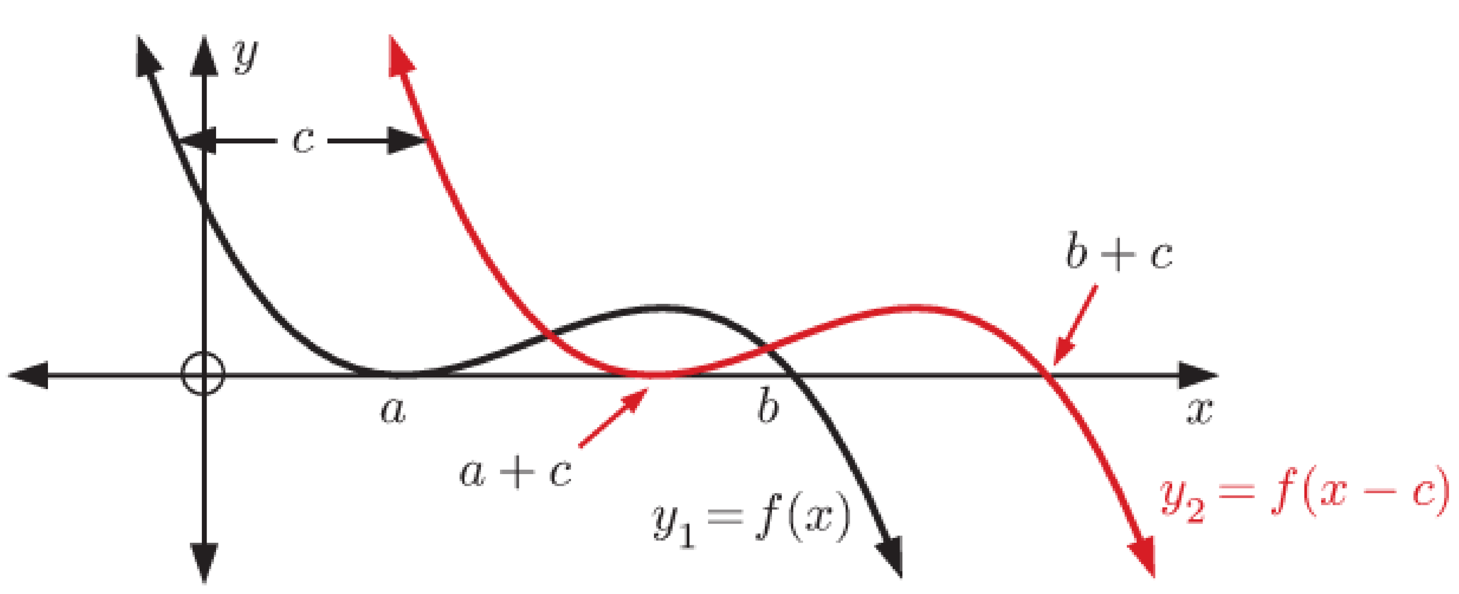
5 **a**



b $(2, -4)$ and $(4, 0)$

6 $y = -2x^2 + 5x - 3$

7



8 A reflection in the x-axis, then a translation through $\begin{pmatrix} \frac{5}{2} \\ -\frac{7}{2} \end{pmatrix}$.

9 $(1, 6)$

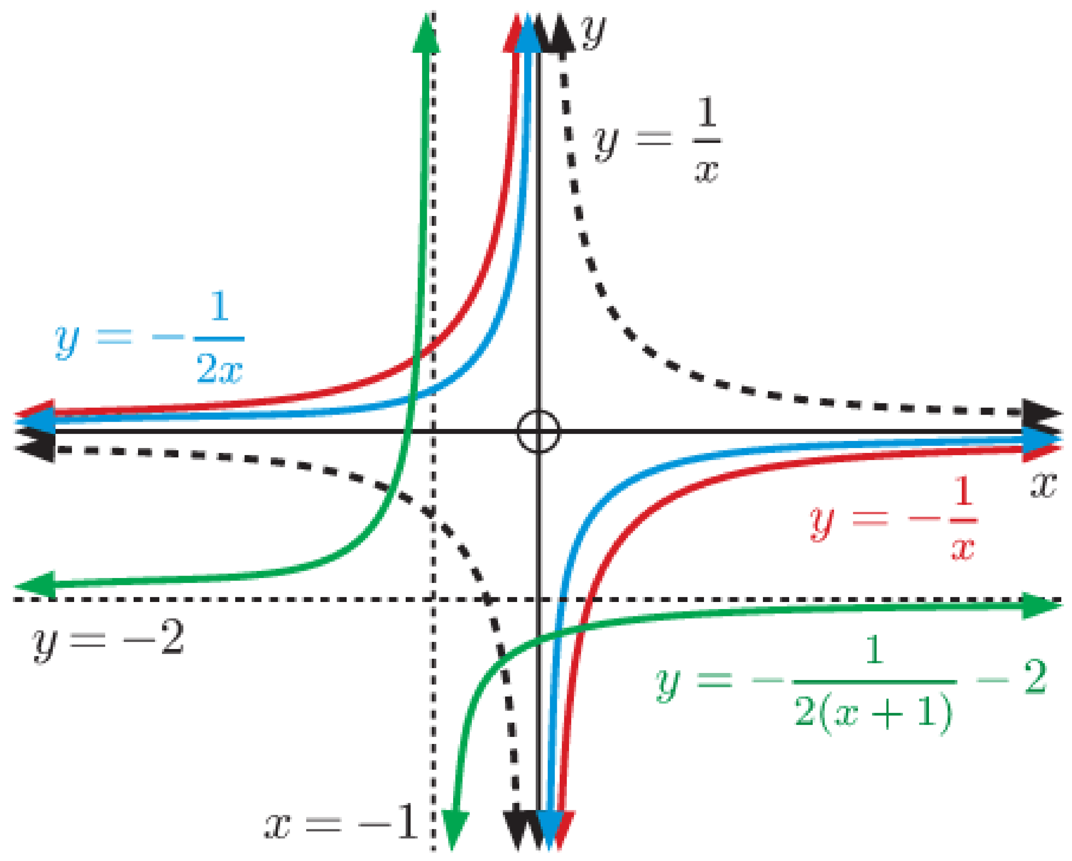
10 **a** A vertical stretch with scale factor 2, then a translation through $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

b A reflection in the x-axis, a horizontal stretch with scale factor $\frac{3}{2}$, then a translation through $\begin{pmatrix} 0 \\ -6 \end{pmatrix}$.

c A vertical stretch with scale factor $\frac{1}{3}$, a reflection in the y-axis, then a translation through $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

11 $b = 8, \quad c = -20$

12 **a**

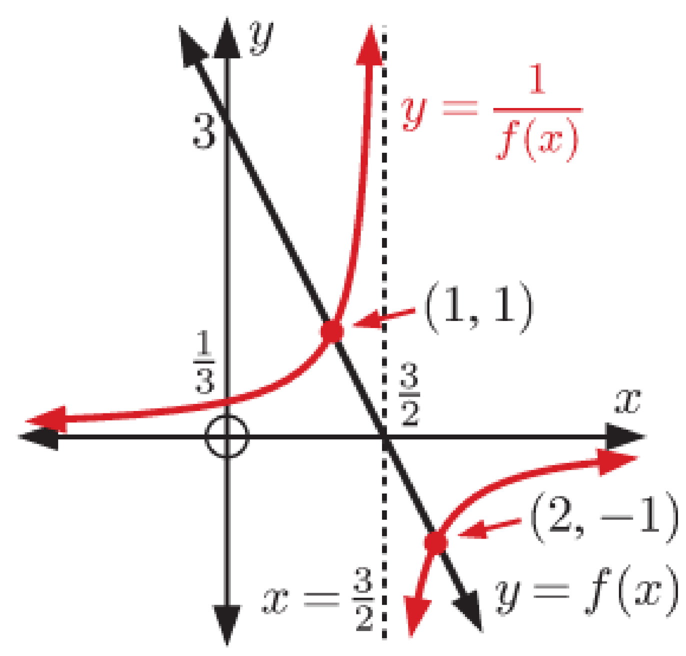


b A reflection in the x-axis, a vertical stretch with scale factor $\frac{1}{2}$, then a translation through $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$.

c $y = \dfrac{-4x - 5}{2x + 2}$

Domain is $\{x \mid x \neq -1\}$, Range is $\{y \mid y \neq -2\}$

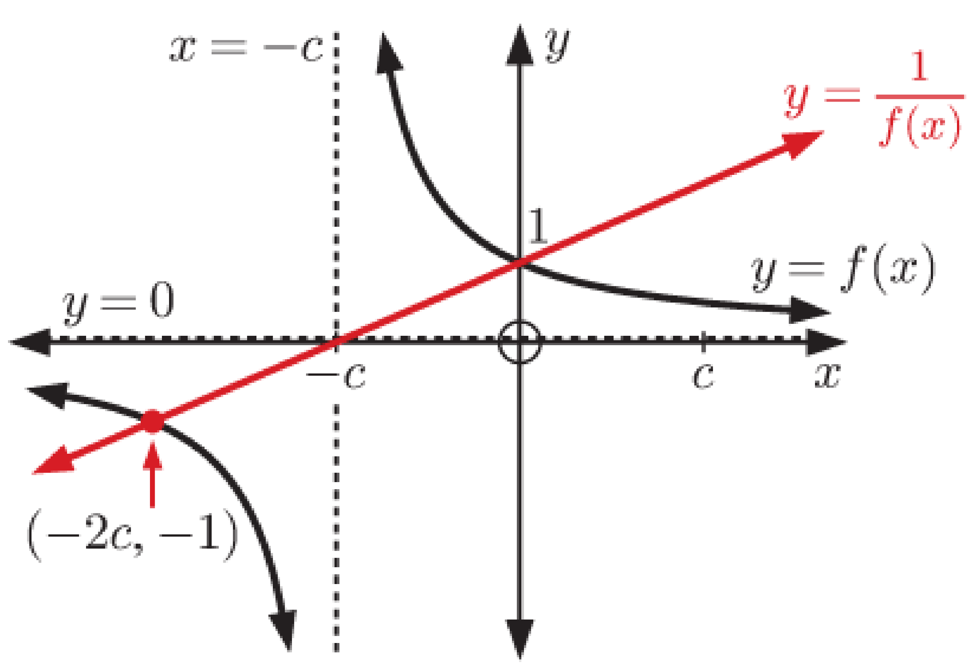
13 **a, d**



b $(1, 1)$ and $(2, -1)$

c y-intercept $\frac{1}{3}$ and vertical asymptote $x = \frac{3}{2}$

14

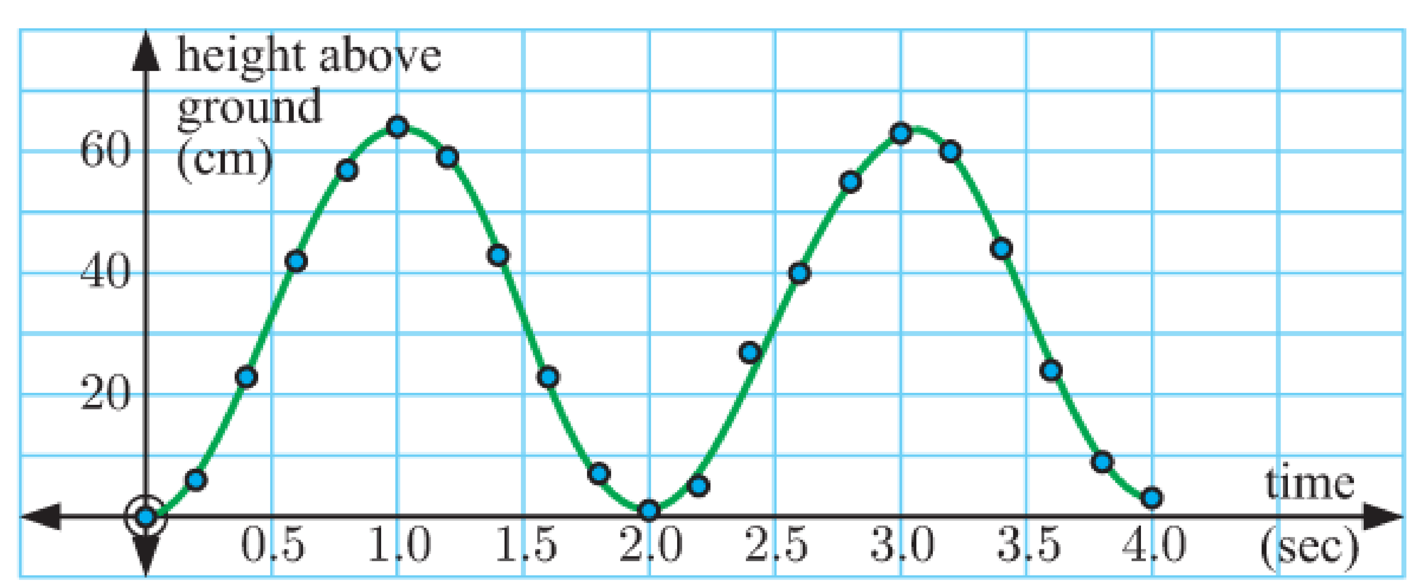


EXERCISE 17A

1 **a** periodic **b** periodic **c** periodic **d** not periodic

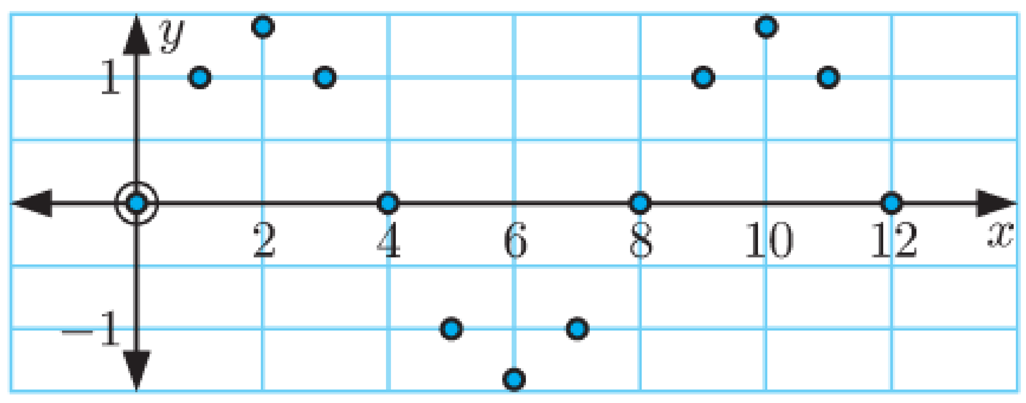
e periodic **f** periodic **g** not periodic **h** not periodic

2 **a**



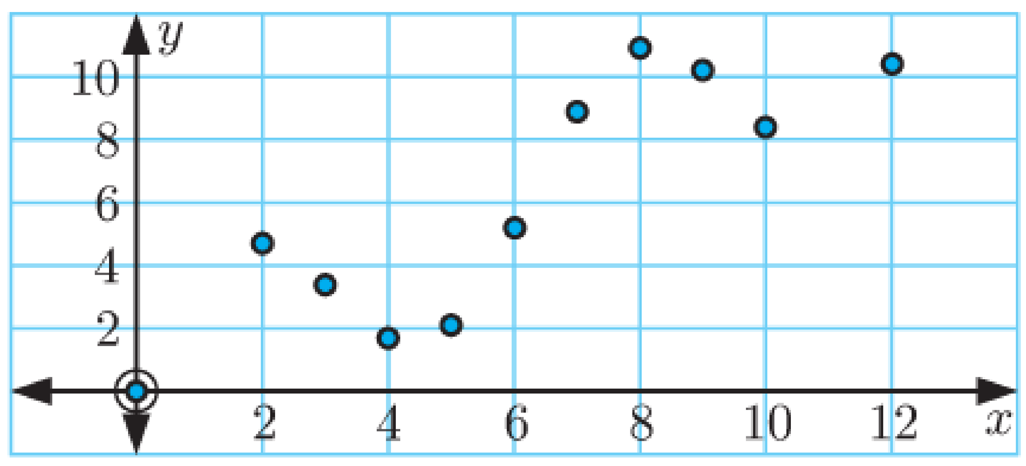
b A curve can be fitted to the data.
c The data is periodic.
 i $y = 32$ (approximately) **ii** ≈ 64 cm
 iii ≈ 2 seconds **iv** ≈ 32 cm

3 **a**



Data exhibits periodic behaviour.

b



Not enough information to say data is periodic.

EXERCISE 17B

1 **a** 0
b **i** $\theta = 0, \pi, 2\pi, 3\pi, 4\pi$ **ii** $\theta = \frac{3\pi}{2}, \frac{7\pi}{2}$
 iii $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ **iv** $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}$
c **i** $0 < \theta < \pi, \quad 2\pi < \theta < 3\pi$ **d** $\{y \mid -1 \leqslant y \leqslant 1\}$
 ii $\pi < \theta < 2\pi, \quad 3\pi < \theta < 4\pi$

2 **a** 1
b **i** $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ **ii** $\theta = 0, 2\pi, 4\pi$
 iii $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$ **iv** $\theta = \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}$
c **i** $0 \leqslant \theta < \frac{\pi}{2}, \quad \frac{3\pi}{2} < \theta < \frac{5\pi}{2}, \quad \frac{7\pi}{2} < \theta \leqslant 4\pi$
 ii $\frac{\pi}{2} < \theta < \frac{3\pi}{2}, \quad \frac{5\pi}{2} < \theta < \frac{7\pi}{2}$
d $\{y \mid -1 \leqslant y \leqslant 1\}$

EXERCISE 17C

1 **a** vertical translation 1 unit downwards
b horizontal translation $\frac{\pi}{4}$ units to the right
c vertical stretch, scale factor 2
d horizontal stretch, scale factor $\frac{1}{4}$
e horizontal stretch, scale factor 4
f translation $\frac{\pi}{3}$ units right and 2 units upwards

2 **a** vertical stretch, scale factor $\frac{1}{2}$ **b** reflection in the x-axis
c translation $\frac{\pi}{6}$ units left and 2 units downwards

3 **a** $\frac{2\pi}{5}$ **b** $\frac{10\pi}{3}$ **c** 2 **d** $\frac{2\pi}{3}$ **e** 6π **f** 100

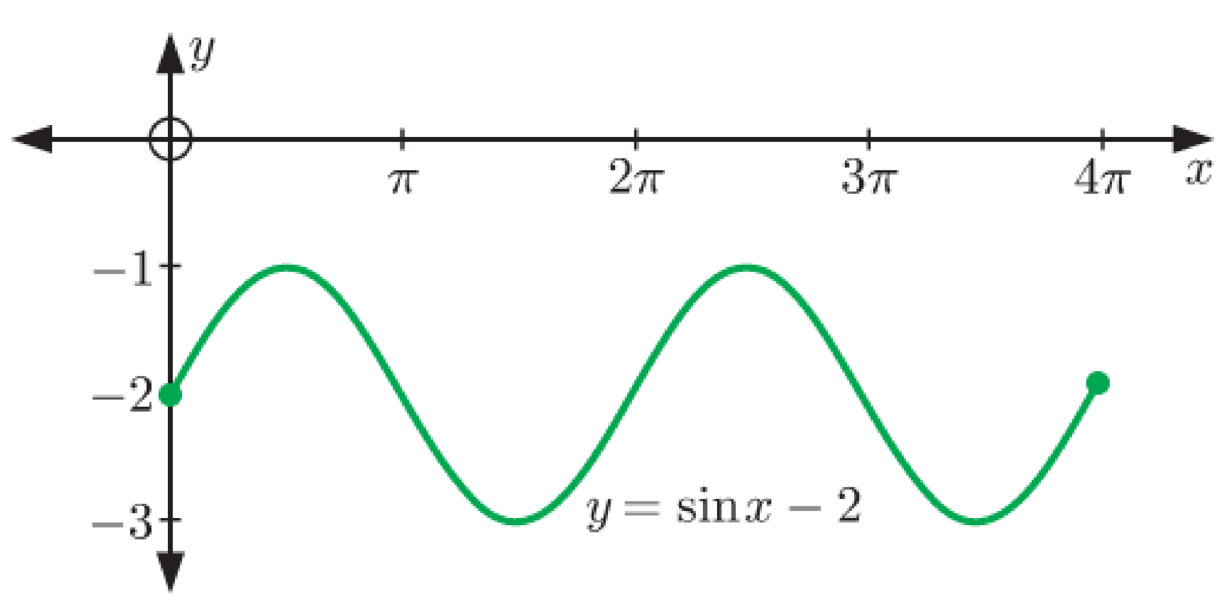
4 **a** $b = \frac{2}{5}$ **b** $b = 3$ **c** $b = \frac{1}{6}$ **d** $b = \frac{\pi}{2}$ **e** $b = \frac{\pi}{50}$

5 **a** maximum 4, minimum -4 **b** maximum 8, minimum 2
c maximum -2, minimum -6

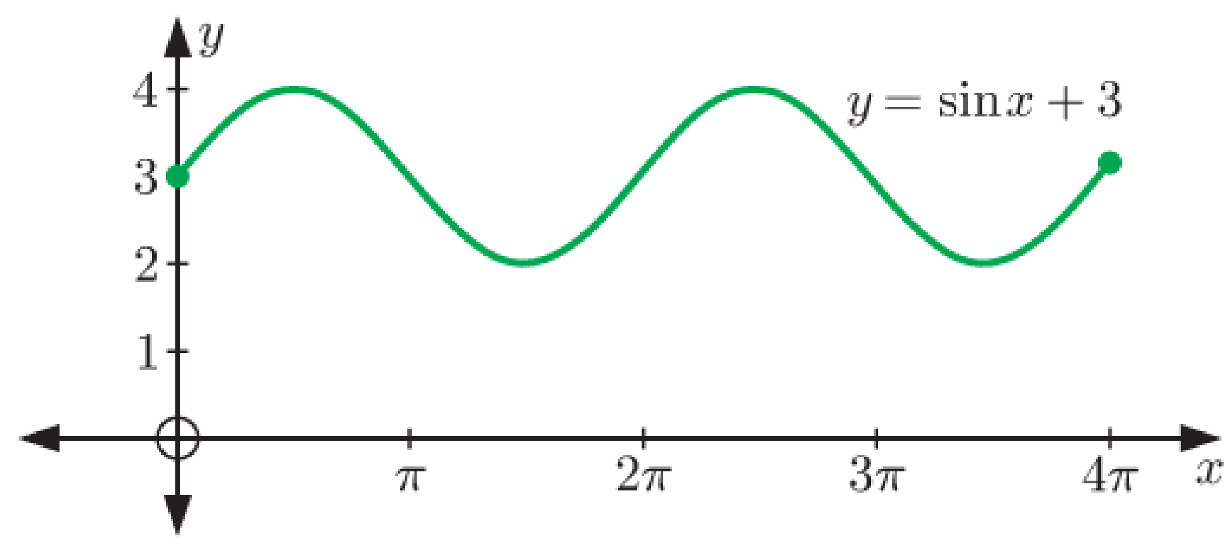
6 **a** 4 **b** $\frac{2\pi}{3}$ **c** $\{y \mid -2 \leqslant y \leqslant 6\}$

7 $|a| =$ amplitude, $b = \dfrac{2\pi}{\text{period}}$, $c =$ horizontal translation,
$d =$ vertical translation

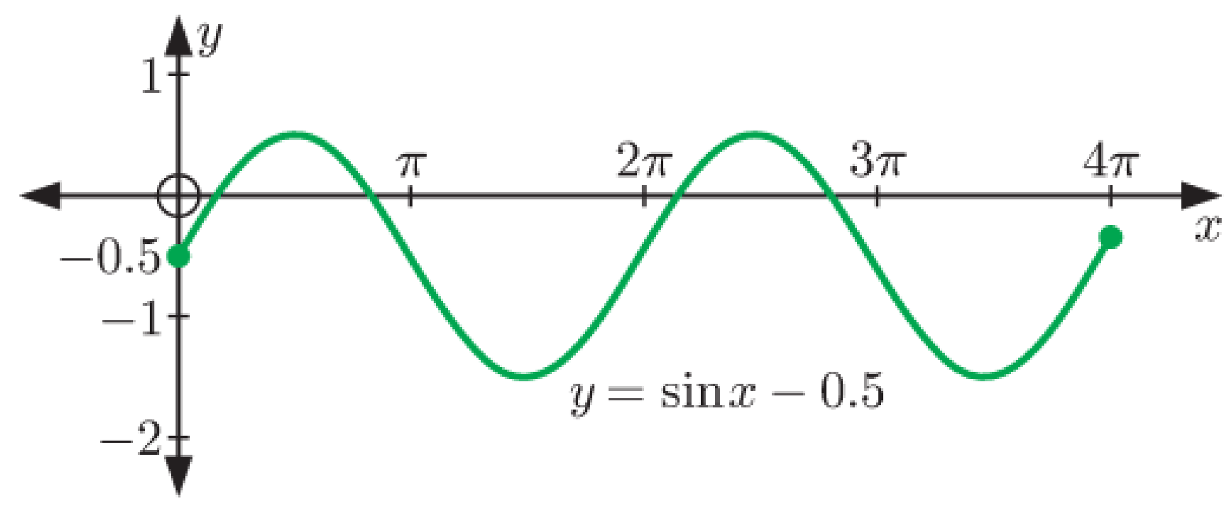
8 **a**



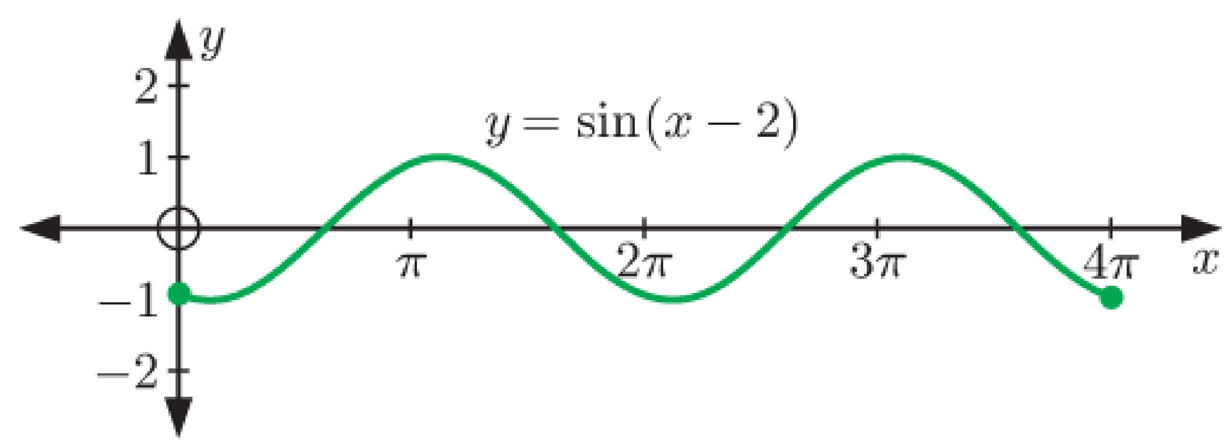
b



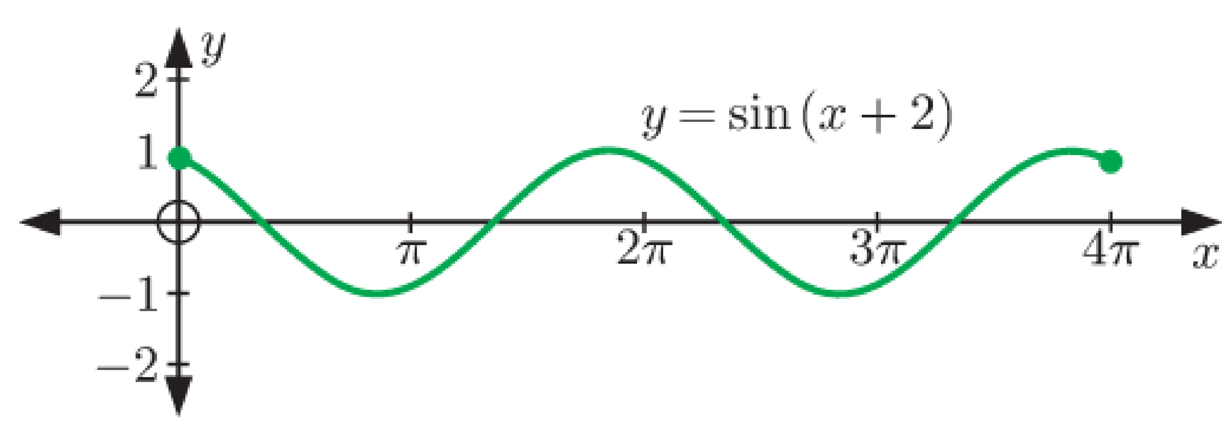
c



d



e



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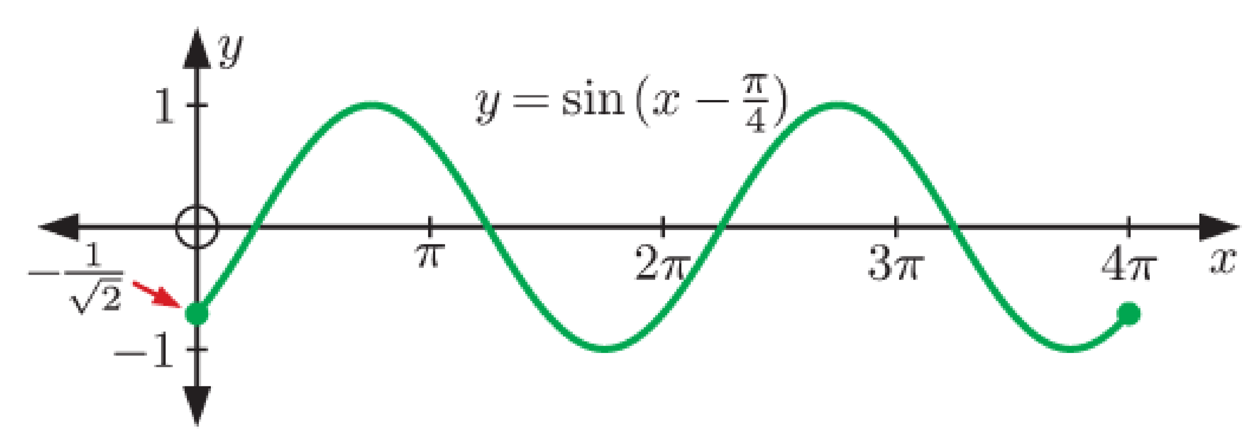
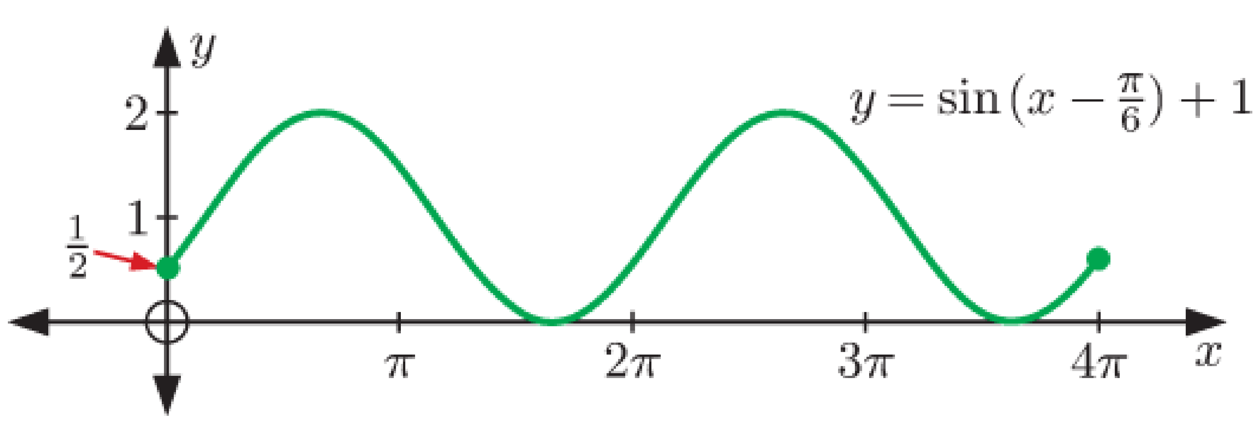


g



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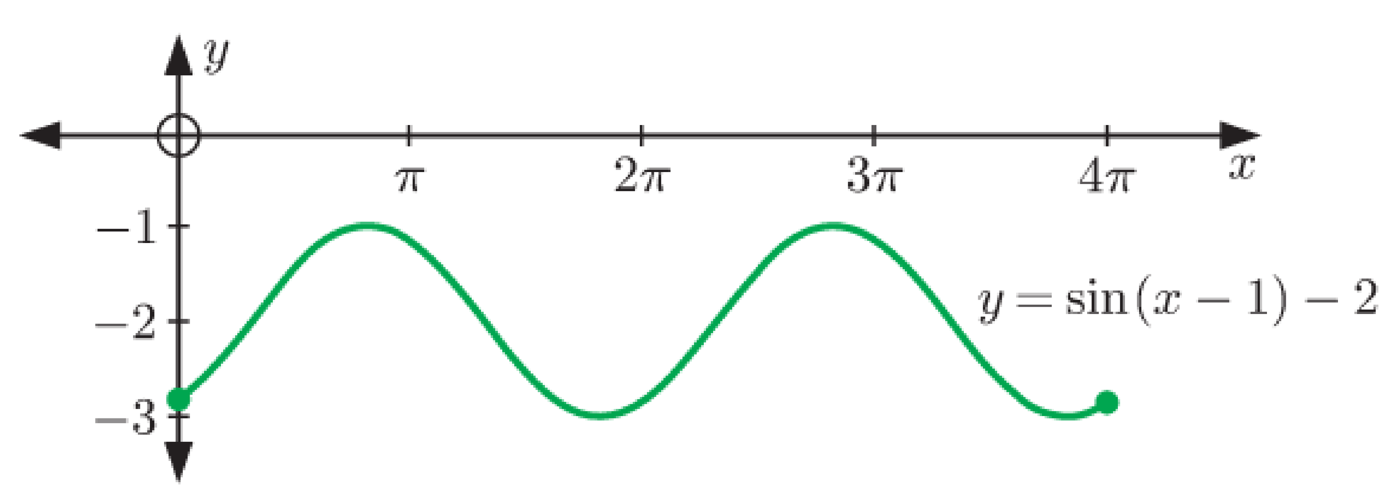
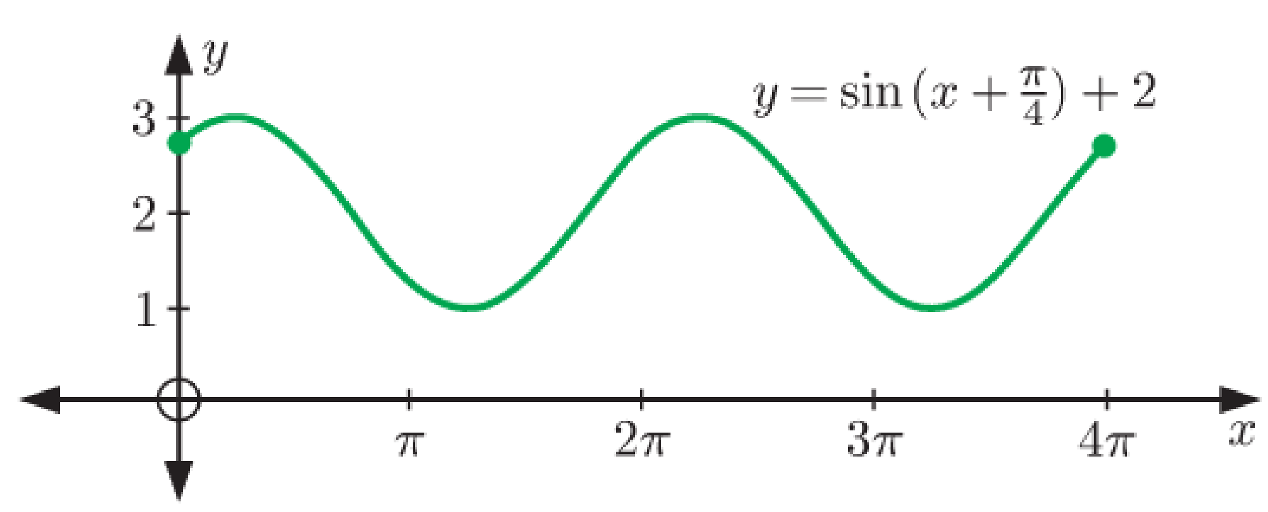


i



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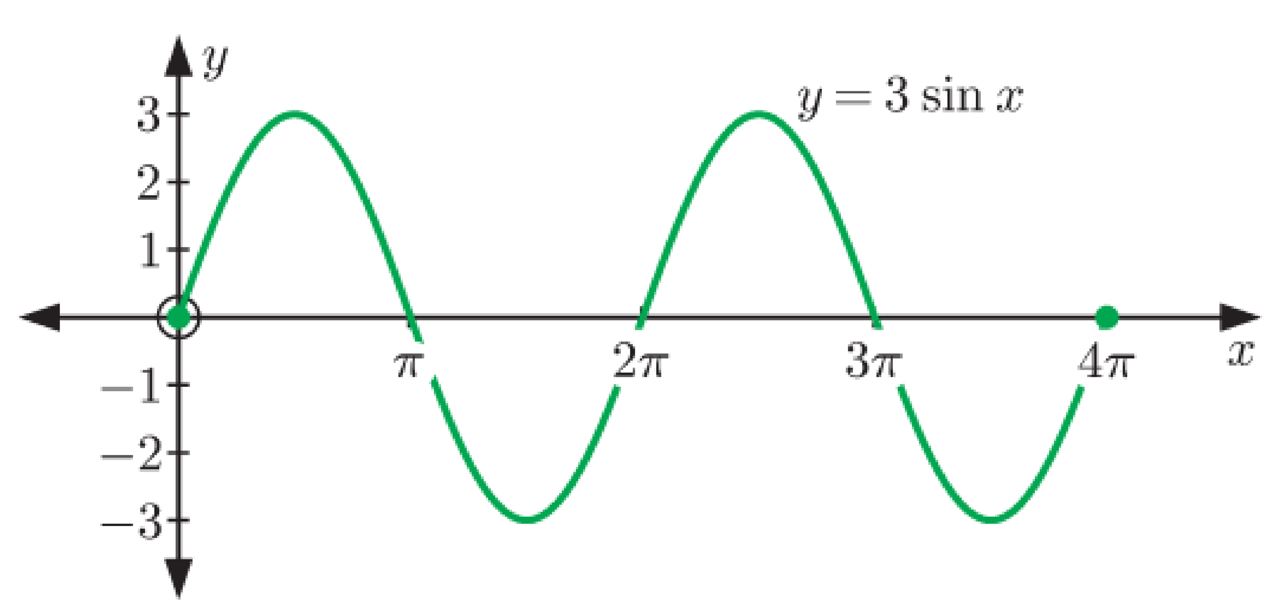
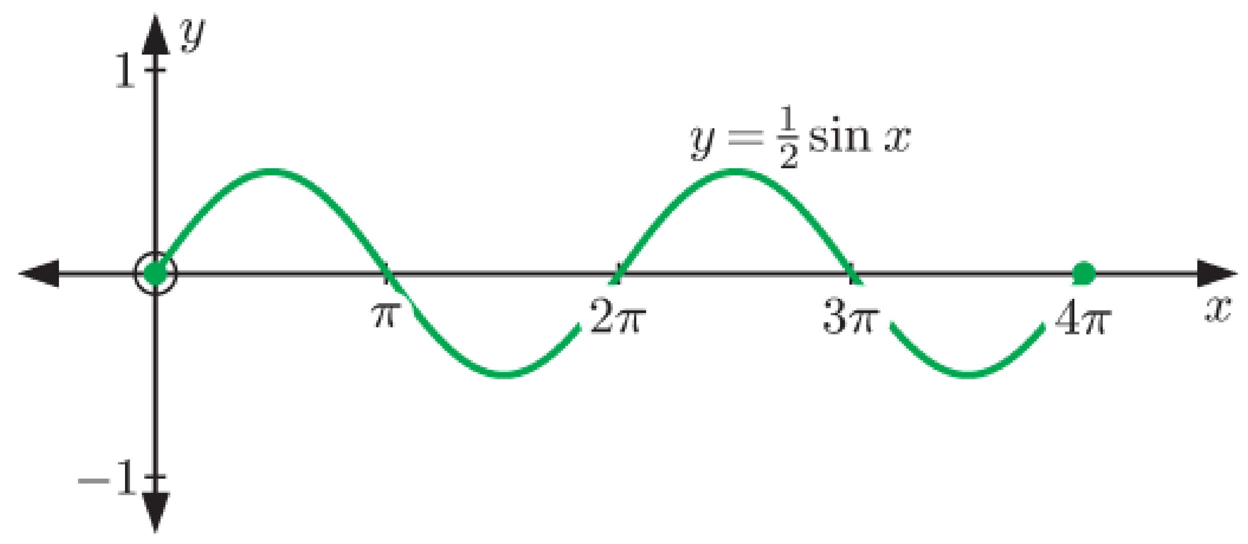
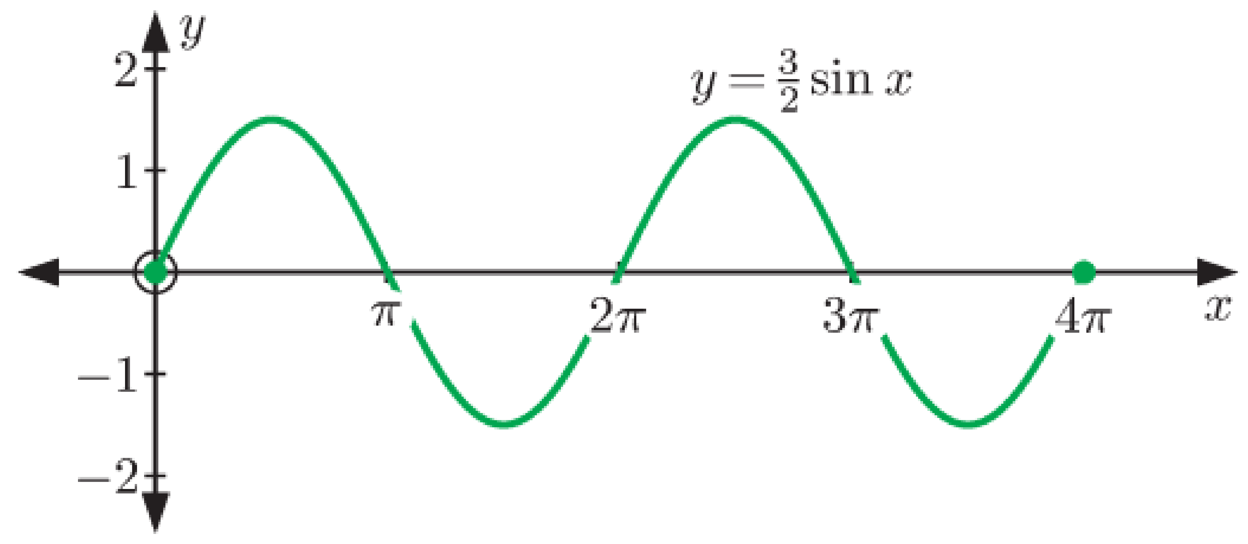


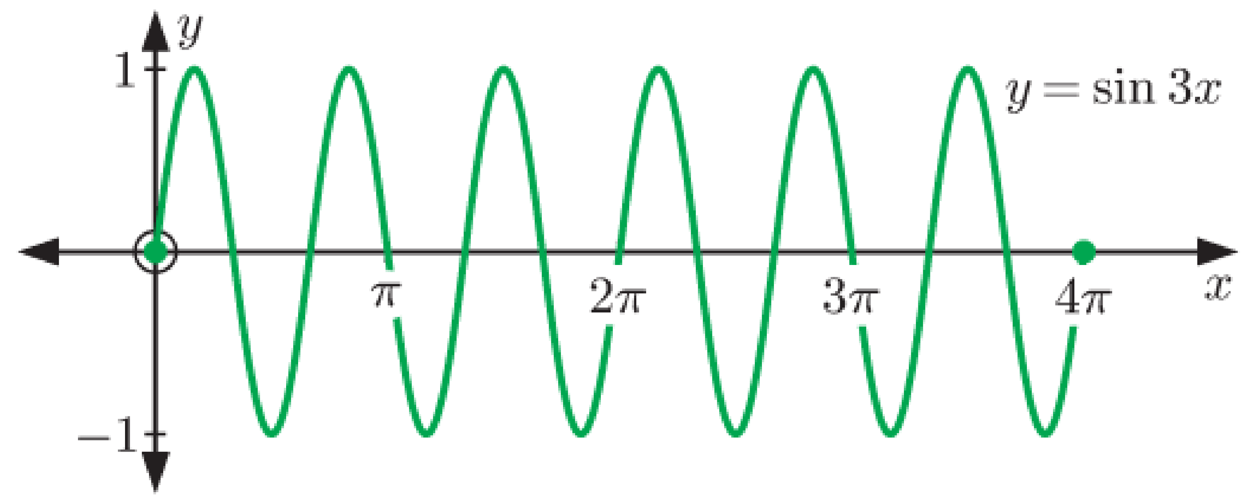
k



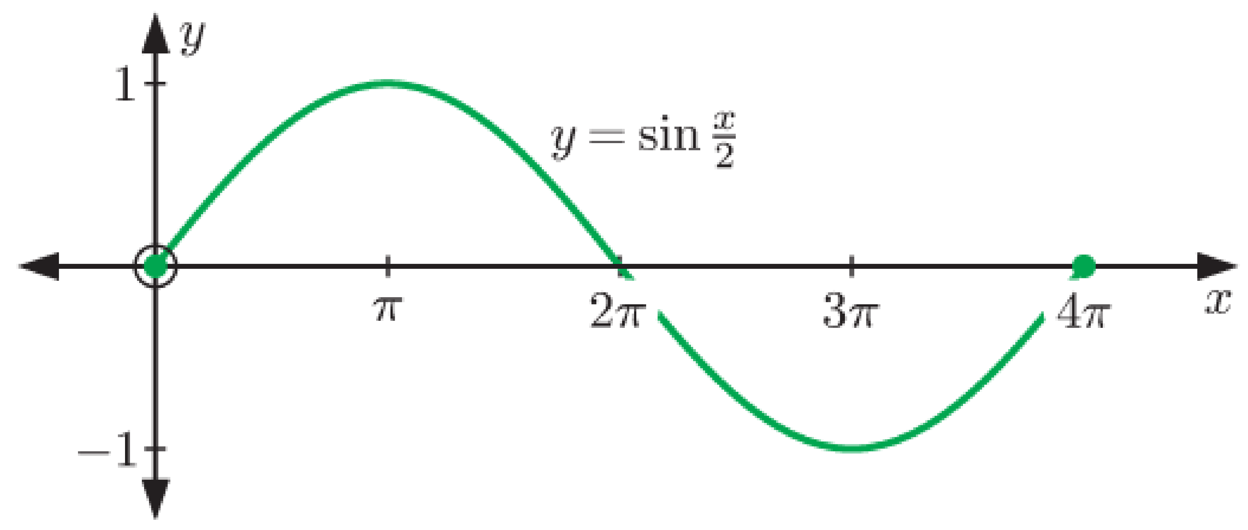
l



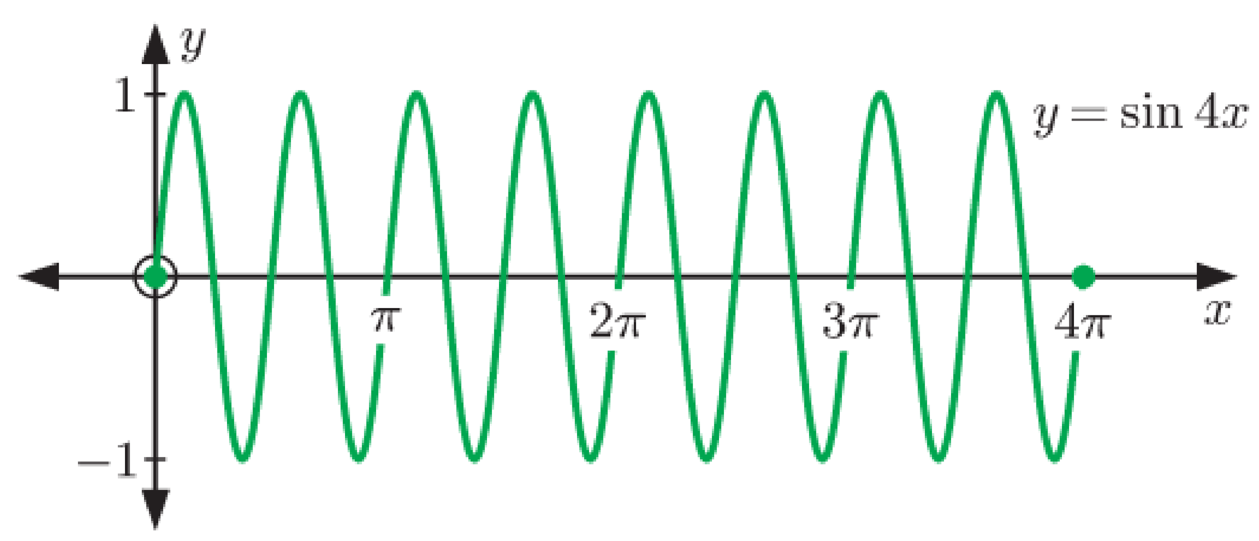
m



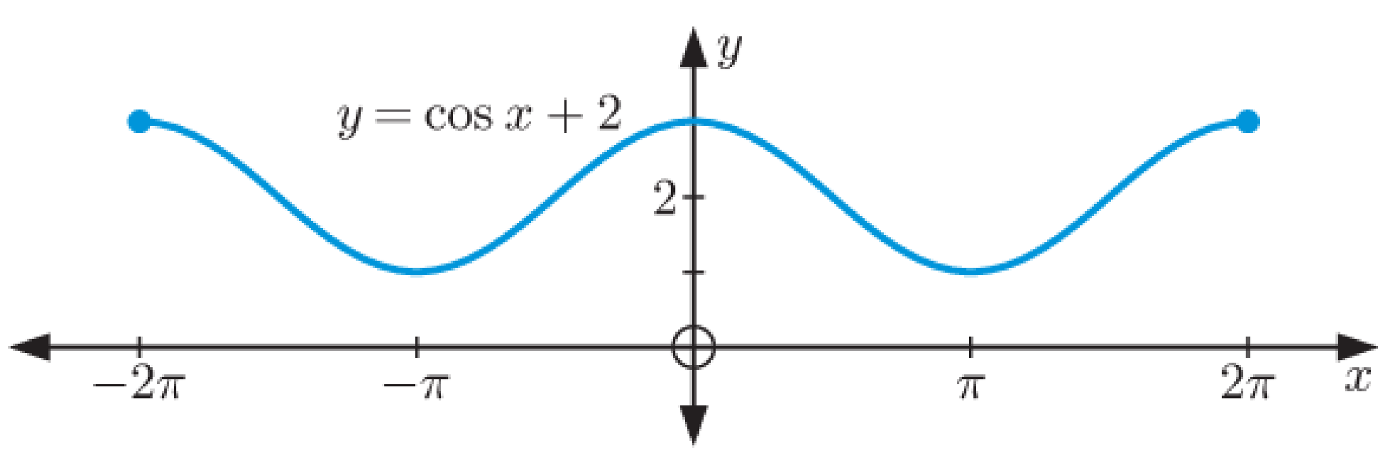
n



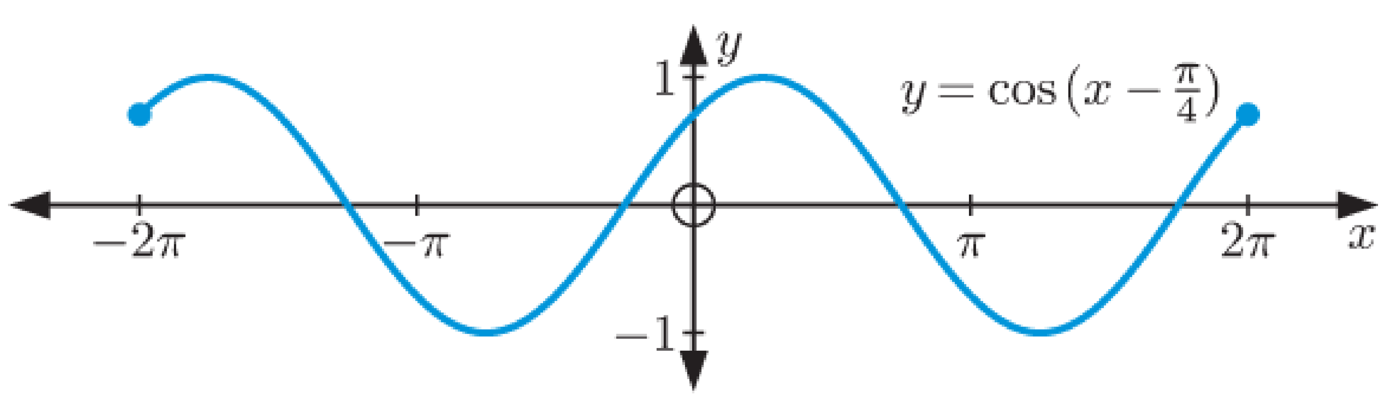
o



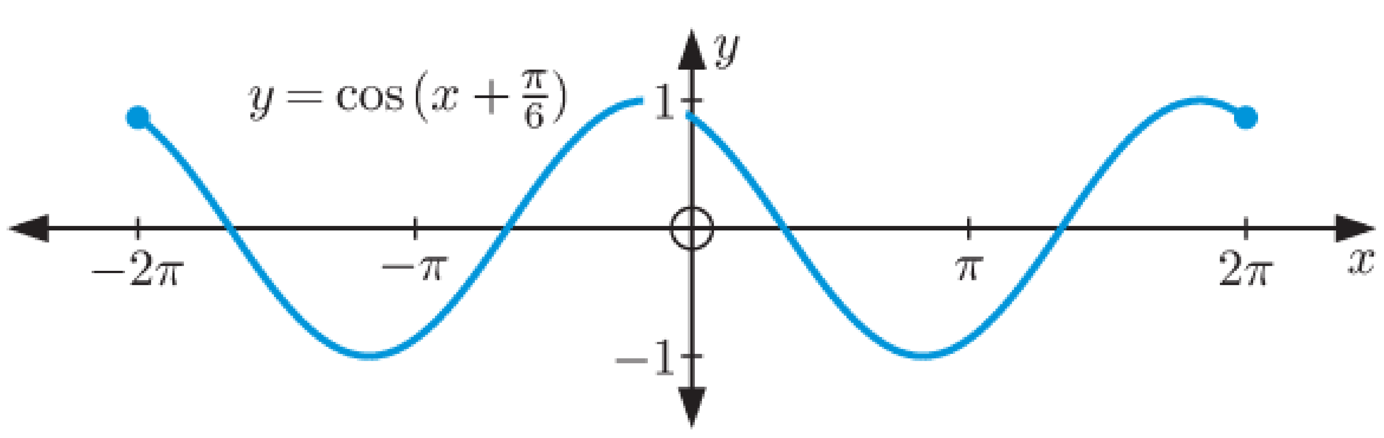
9 **a**



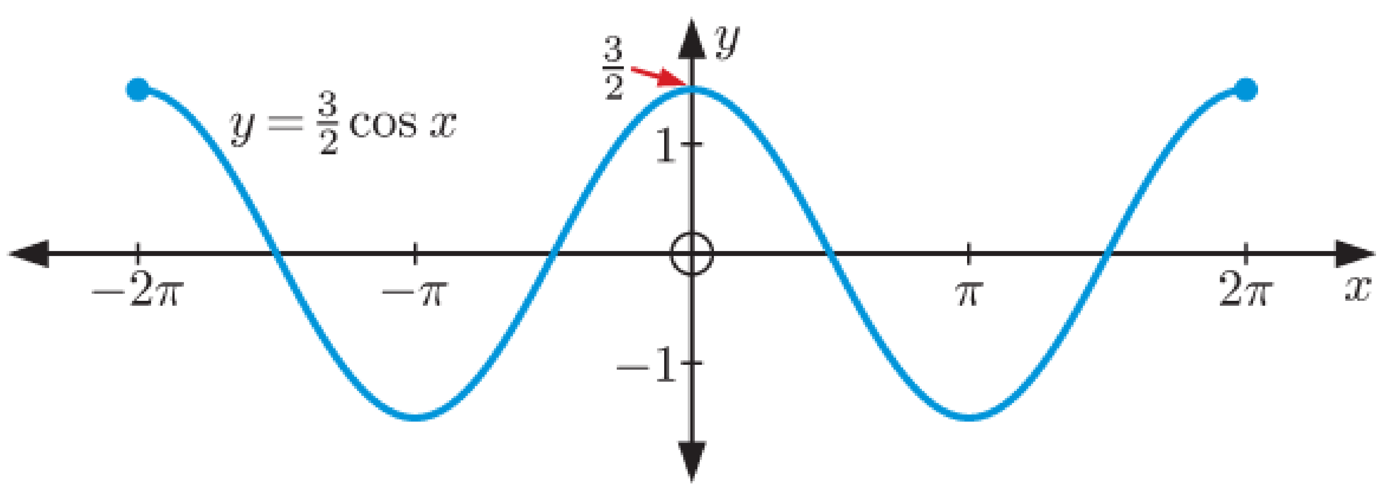
b



c



d



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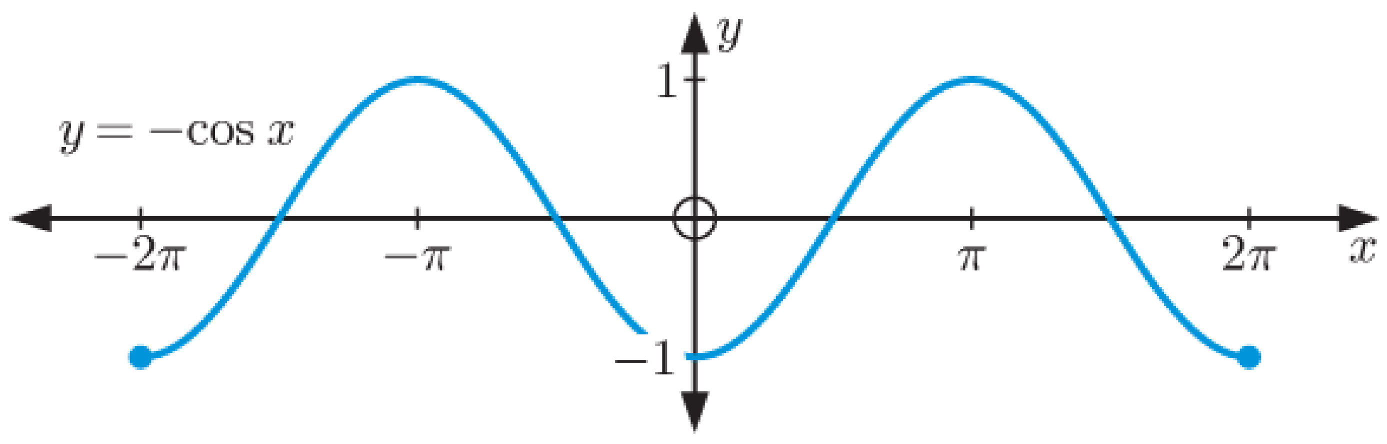
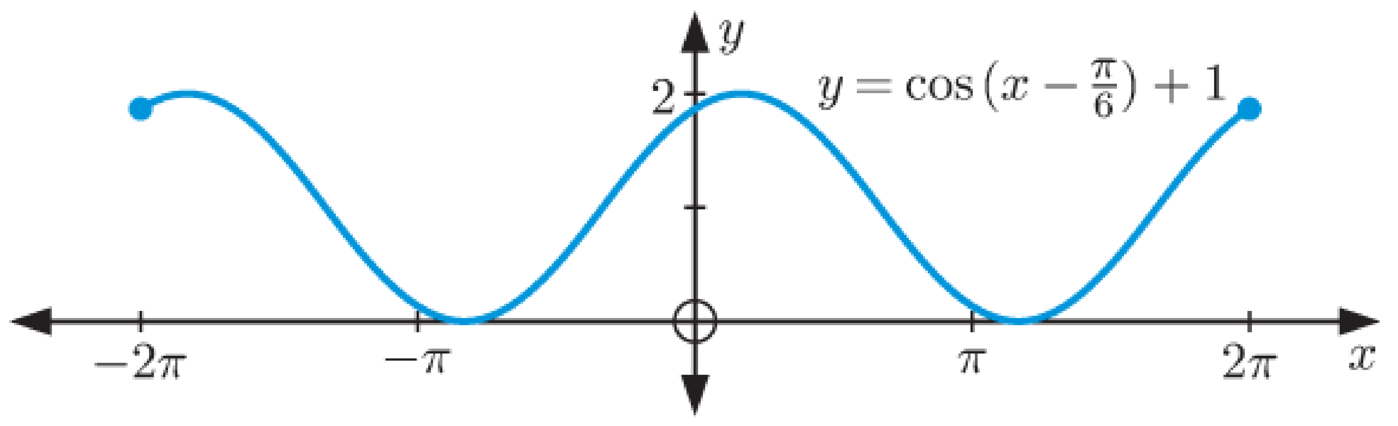
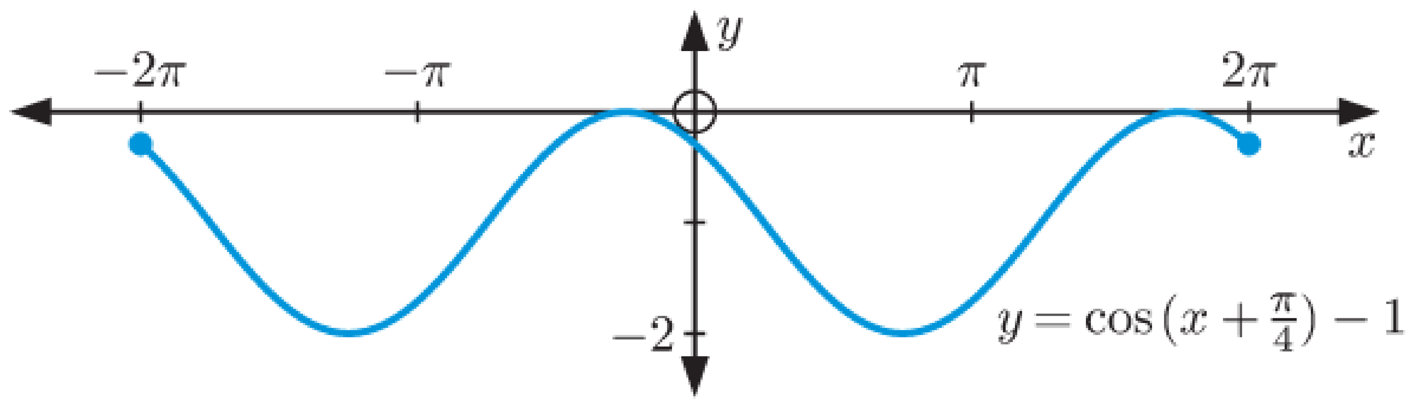


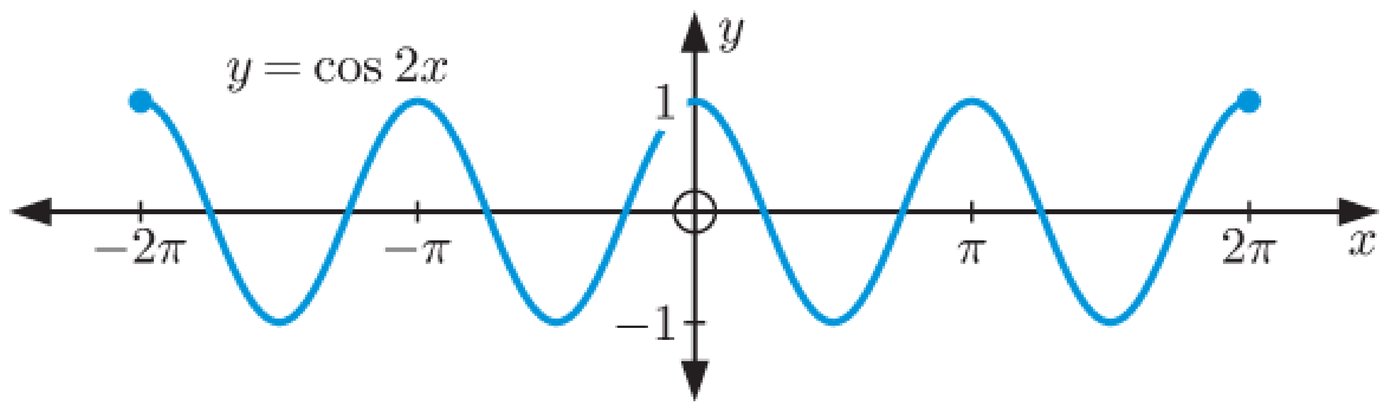
f



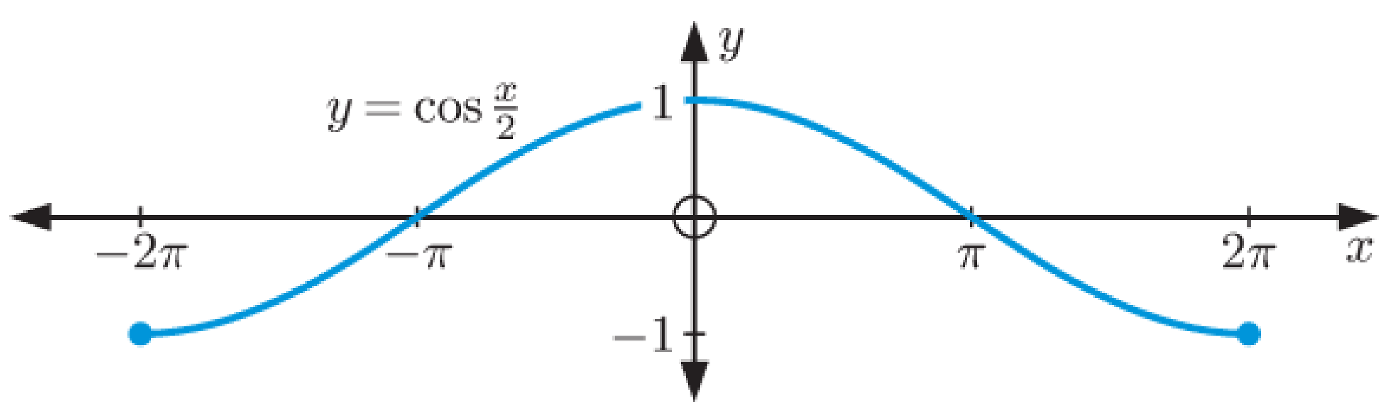
g



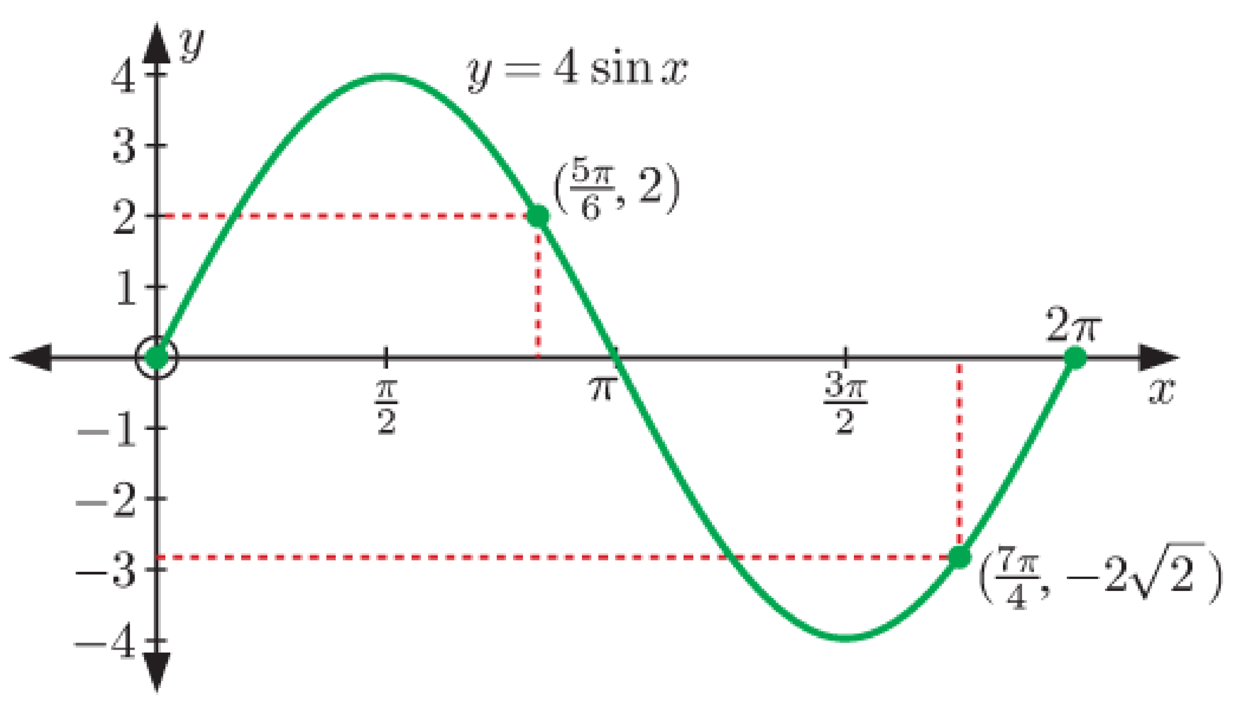
h



i



10 **a**



b **i** $y = 2$ **ii** $y = -2\sqrt{2} \approx -2.83$

11 **a** $d > 3$ **b** $d < -3$ **c** $-3 < d < 3$

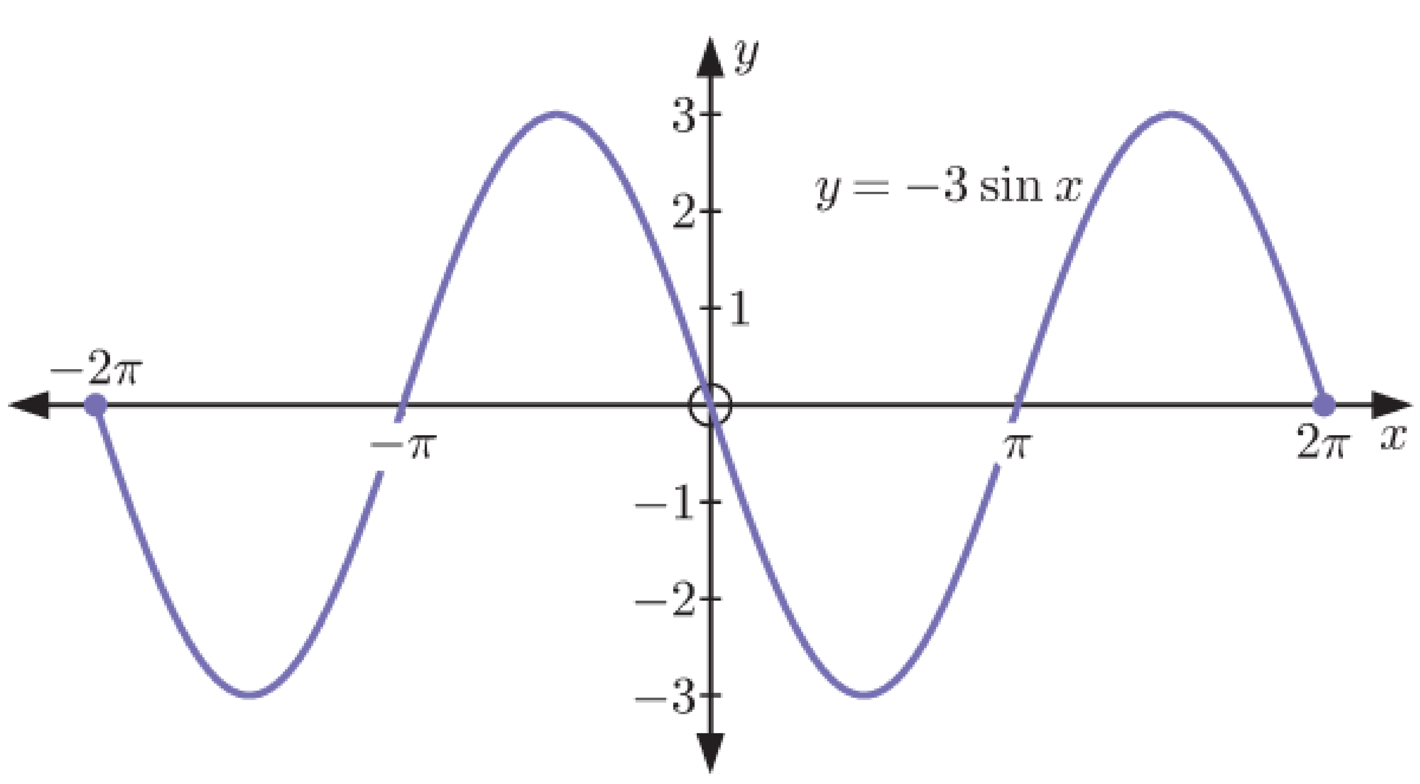
12 **a** A horizontal stretch with scale factor $\frac{1}{3}$, then a vertical stretch with scale factor 2.

b A vertical stretch with scale factor 2, then a reflection in the x-axis.

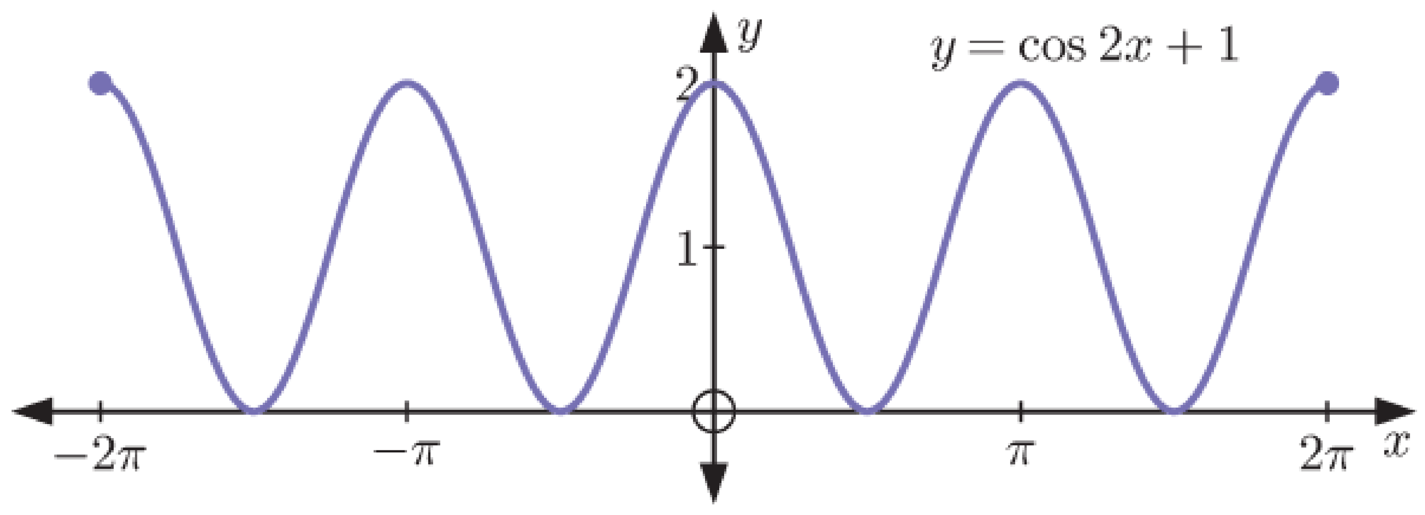
c A vertical stretch with scale factor 3, then a translation 5 units downwards.

d A horizontal stretch with scale factor $\frac{1}{2}$, then a translation $\frac{\pi}{6}$ units left.

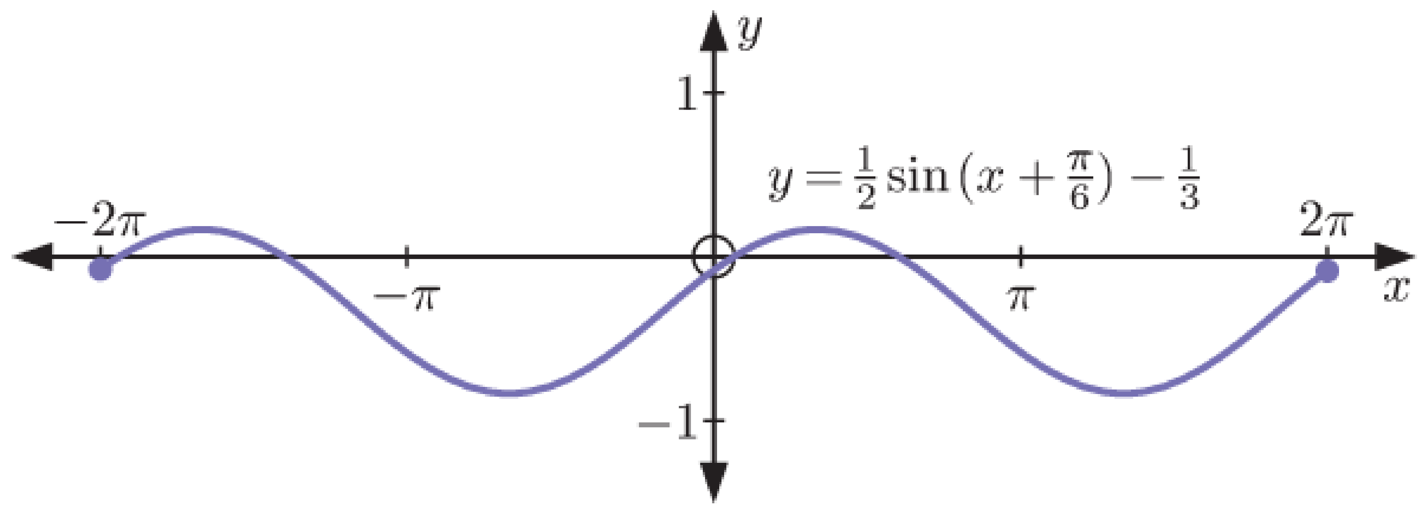
13 **a**



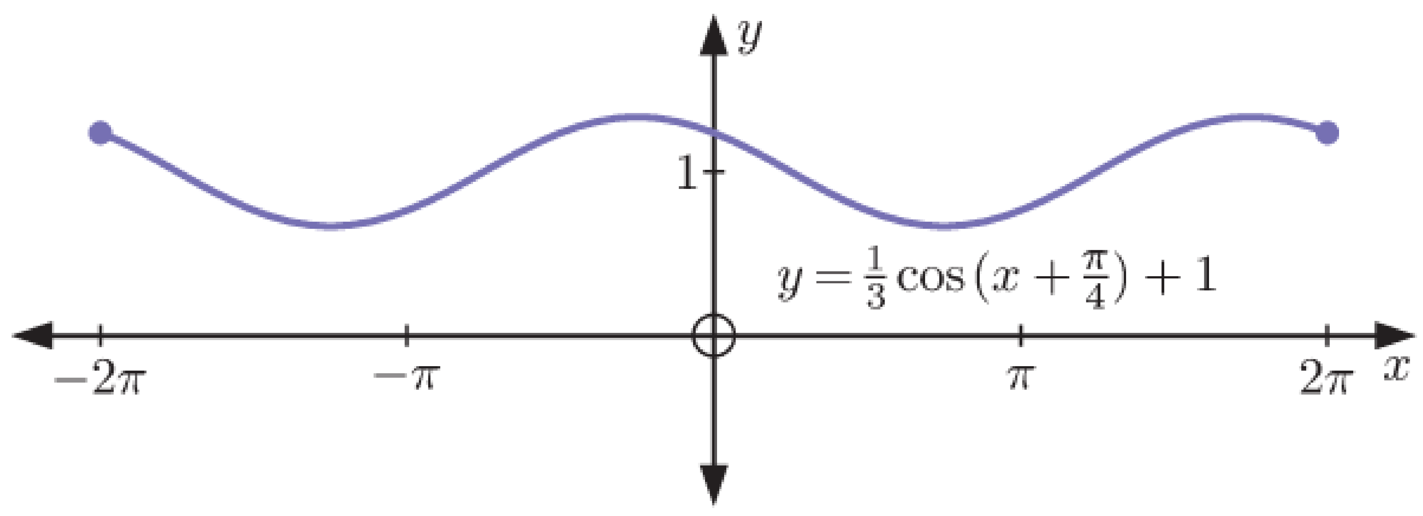
b



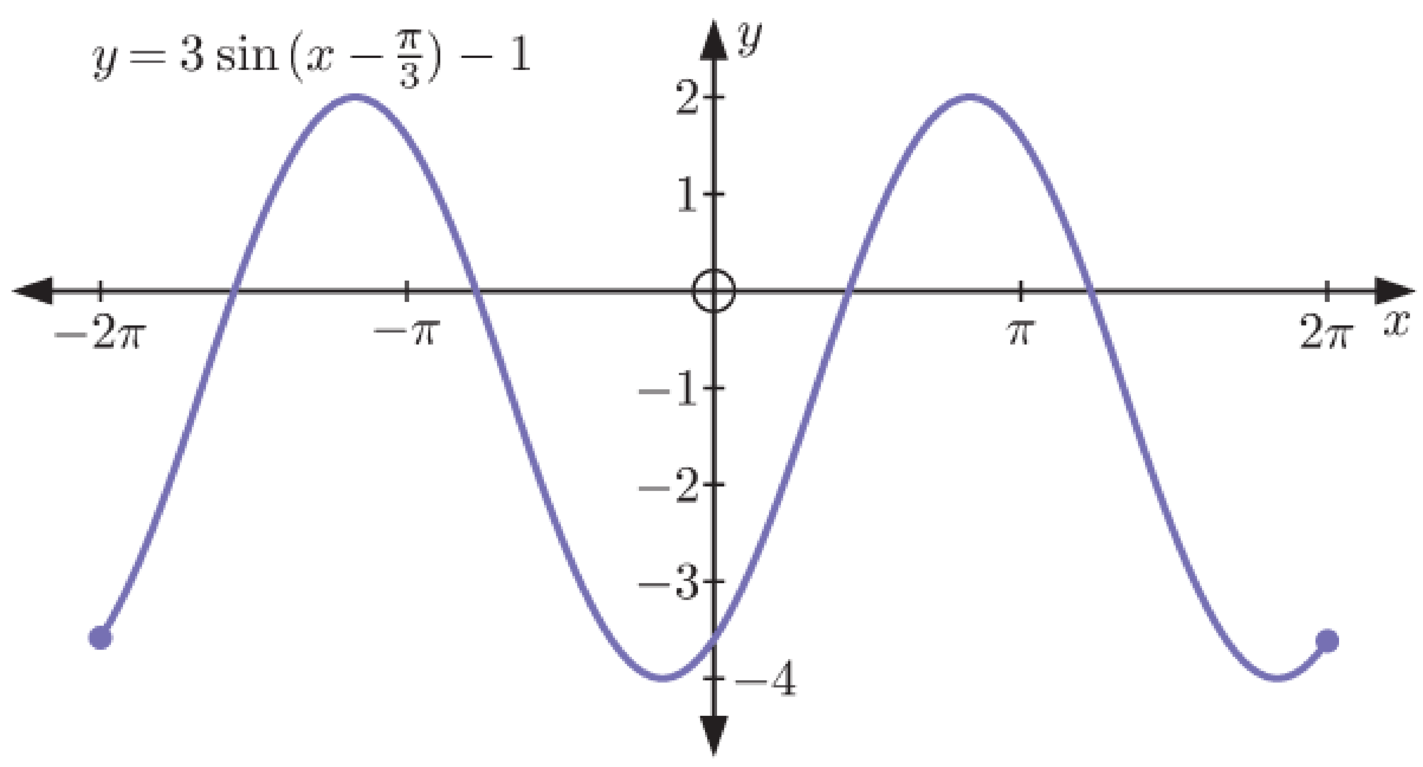
c



d



e



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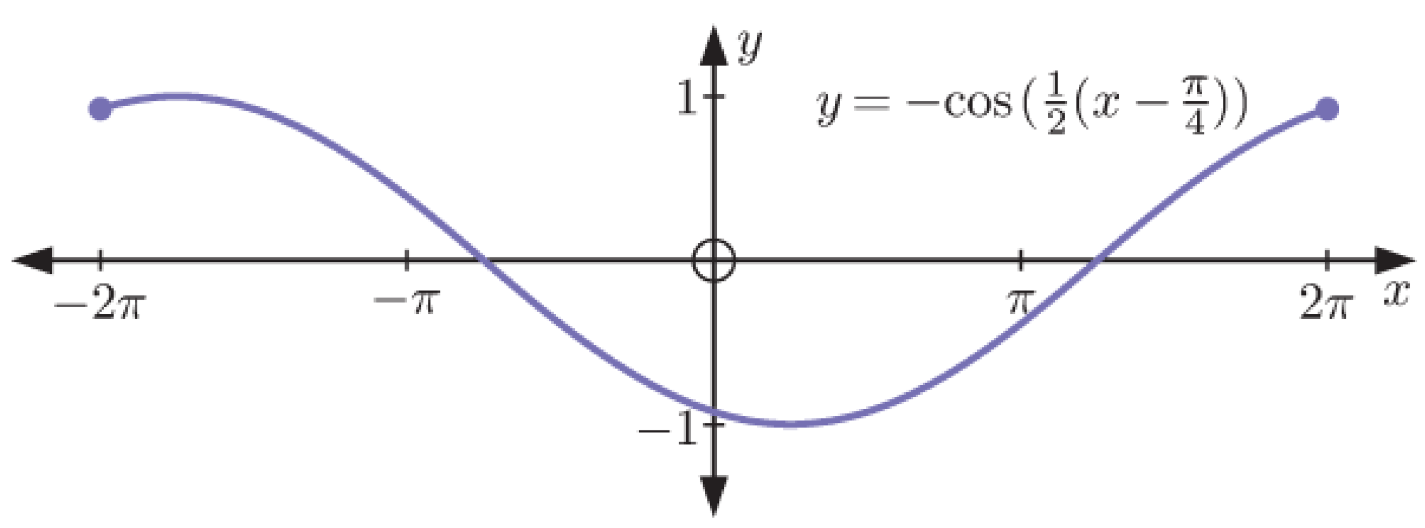


14 **a** b, c, d (provided the function has x-intercepts)
b d **c** d

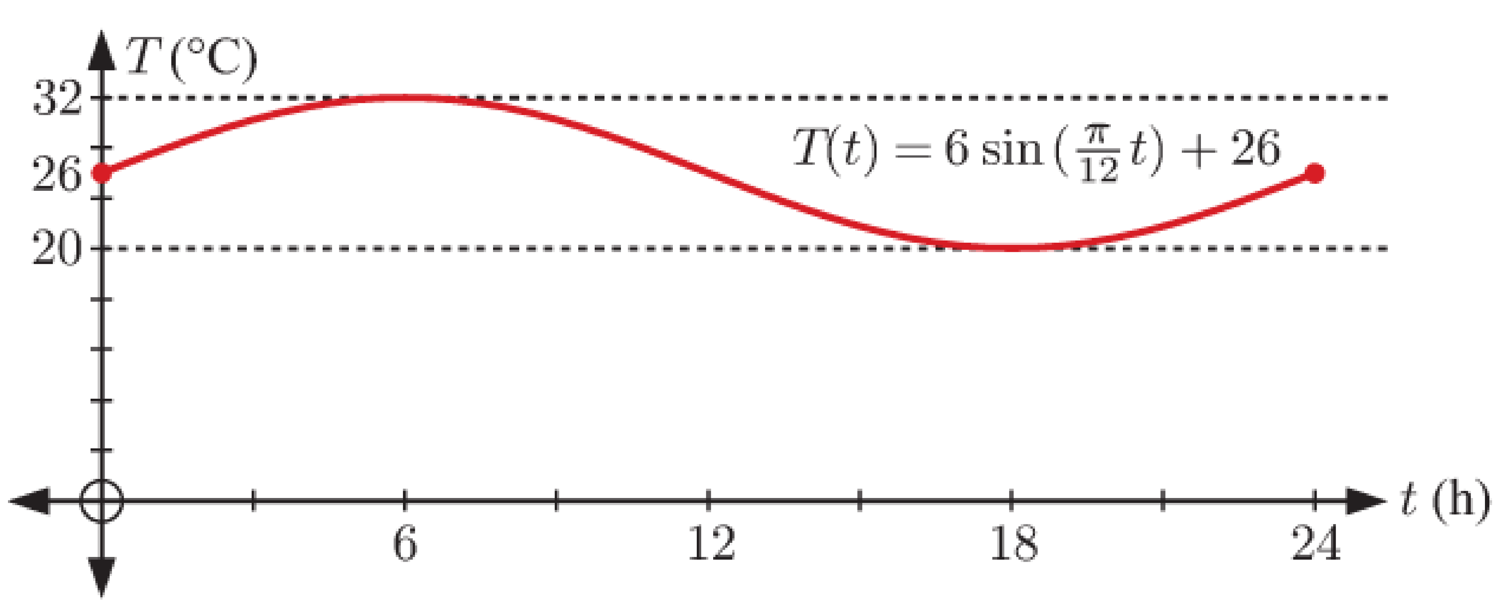
15 **a** $a = 4,\ d = 1$ **b** $a = -2,\ d = 3$ **c** $a = \frac{1}{3},\ d = \frac{4}{3}$

16 **a** $y = \sin x - 2$ **b** $y = \sin 3x$ **c** $y = \sin\left(x + \frac{\pi}{2}\right)$
d $y = 2\sin x + 1$ **e** $y = 4\sin\frac{x}{2} - 1$ **f** $y = 6\sin\frac{2\pi x}{5}$

17 **a** $y = 2\cos 2x$ **b** $y = \cos\frac{x}{2} + 2$ **c** $y = -5\cos\frac{\pi x}{3}$

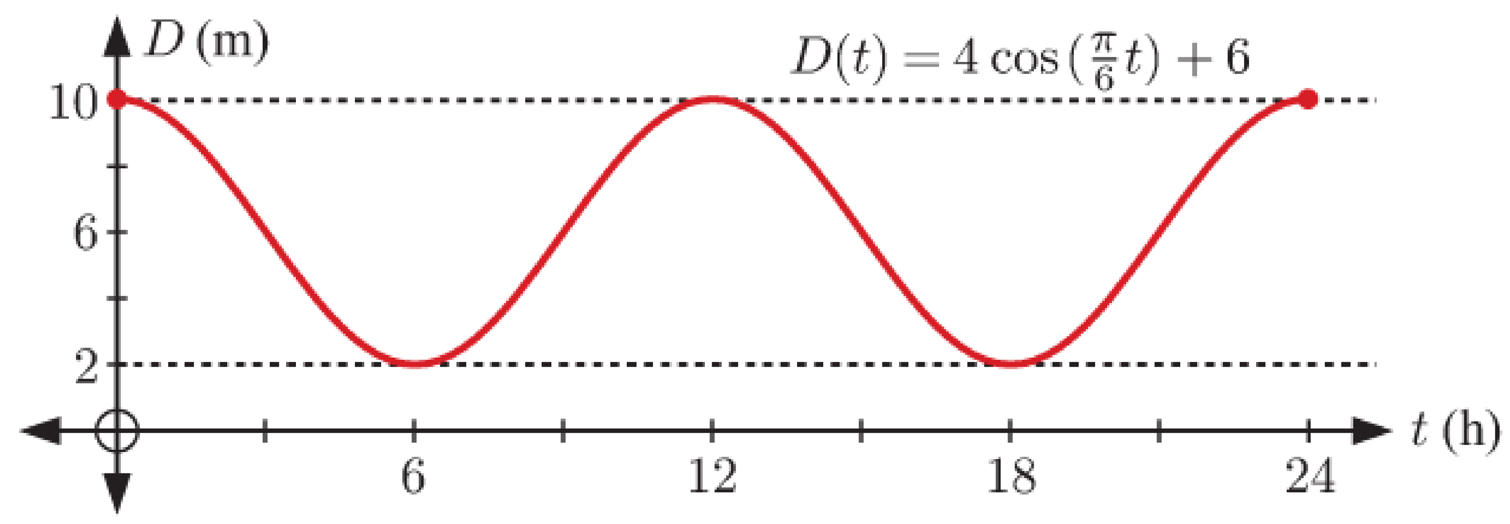
EXERCISE 17D

1 **a**



b **i** 26°C **ii** 29°C **c** 32°C, at 6 pm

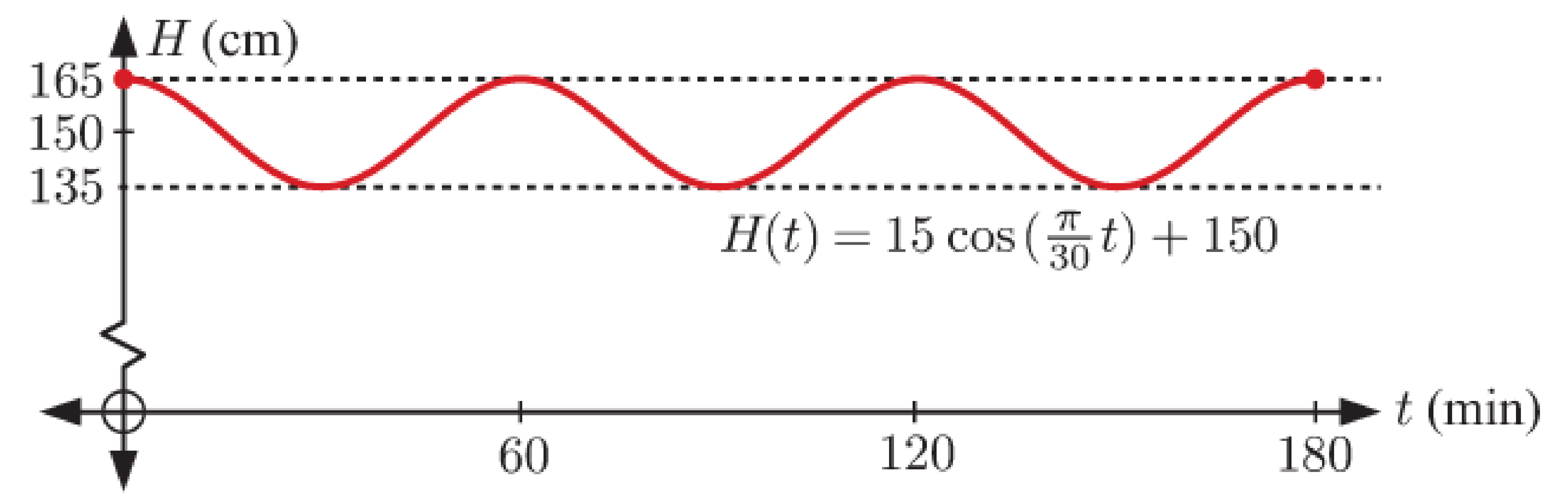
2 **a**



b highest = 10 m, at midnight, midday, and midnight the next day
lowest = 2 m, at 6 am and 6 pm

c no (water height is 4 m)

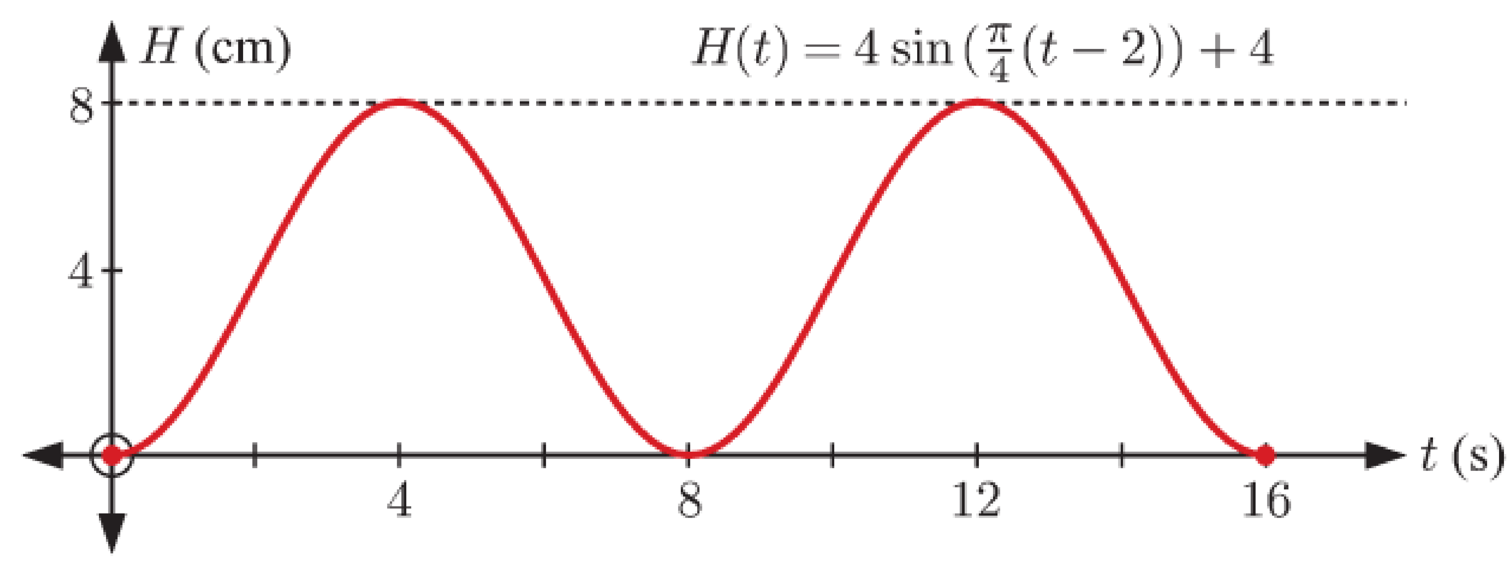
3 **a**



b 15 cm

c **i** ≈ 160.0 cm **ii** ≈ 138.9 cm **iii** ≈ 158.8 cm
iv ≈ 138.9 cm

4 **a**



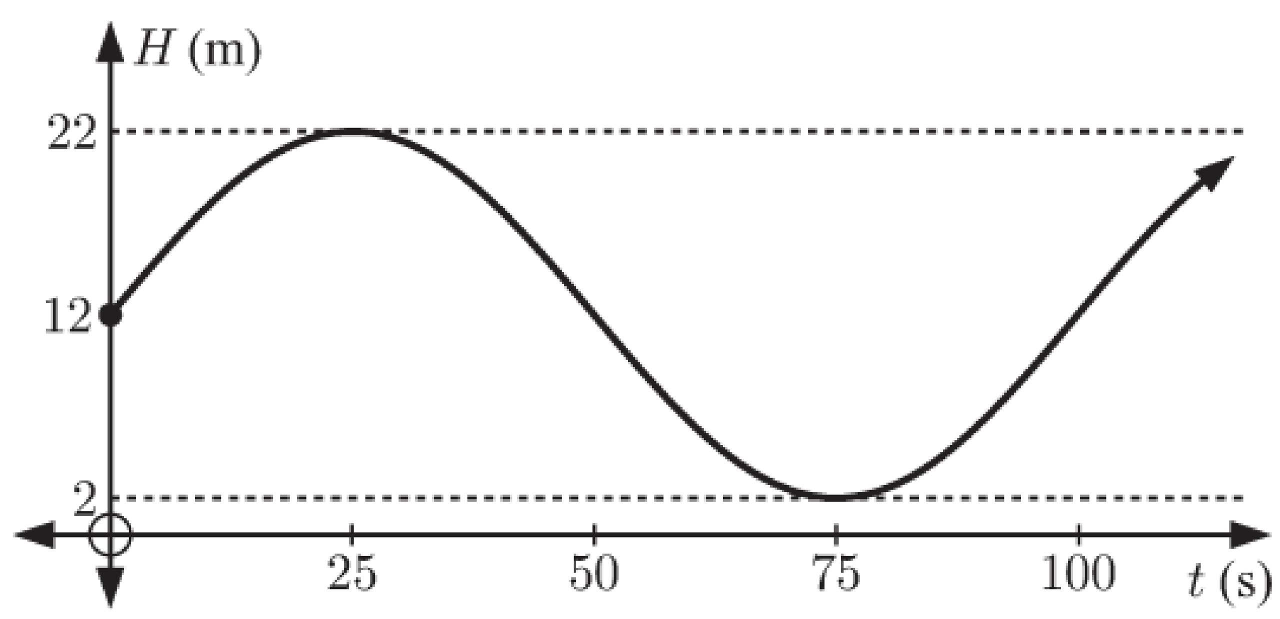
b 4 cm

c no (ball diameter is 4.28 cm, gate height is ≈ 3.07 cm)

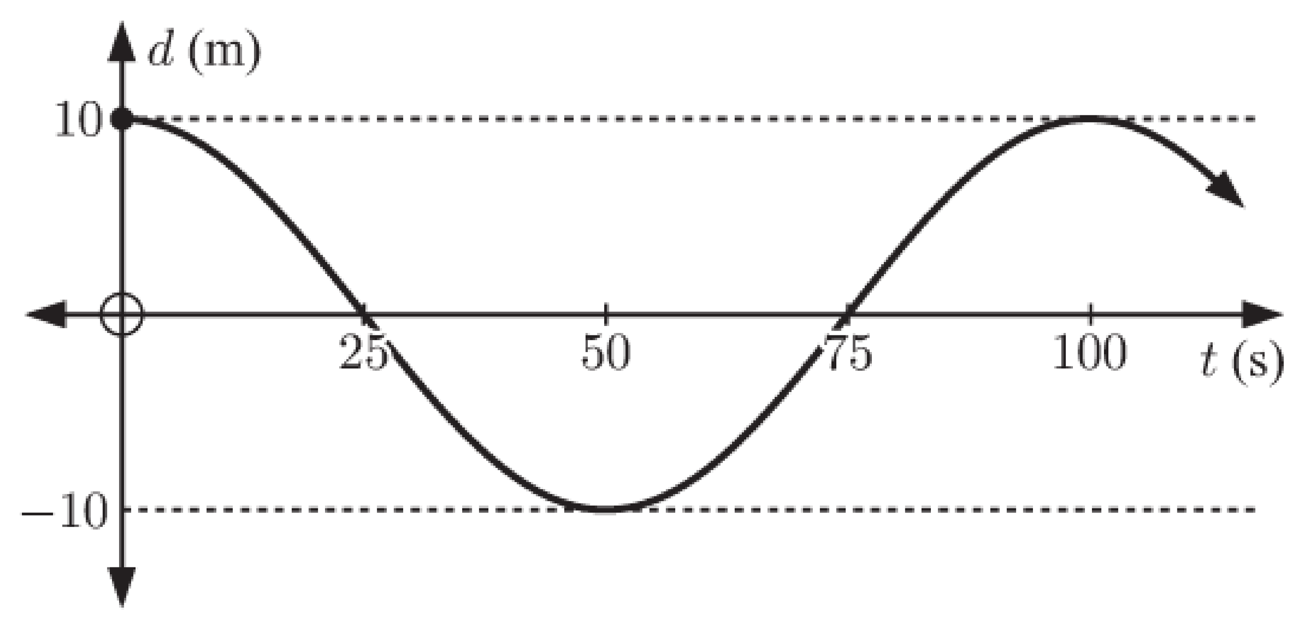
5 $T(t) = 5.2\sin\left(\frac{\pi}{12}(t - 8)\right) + 10.6$ °C

6 $H(t) = 0.6\cos\left(\frac{5\pi}{31}(t - 1.5)\right) + 0.76$ m

7 **a**



b



c Both graphs are periodic with an amplitude of 10 m and a period of 100 s. The graphs differ by a horizontal translation of 25 s and the principal axis is also translated by 12 m.

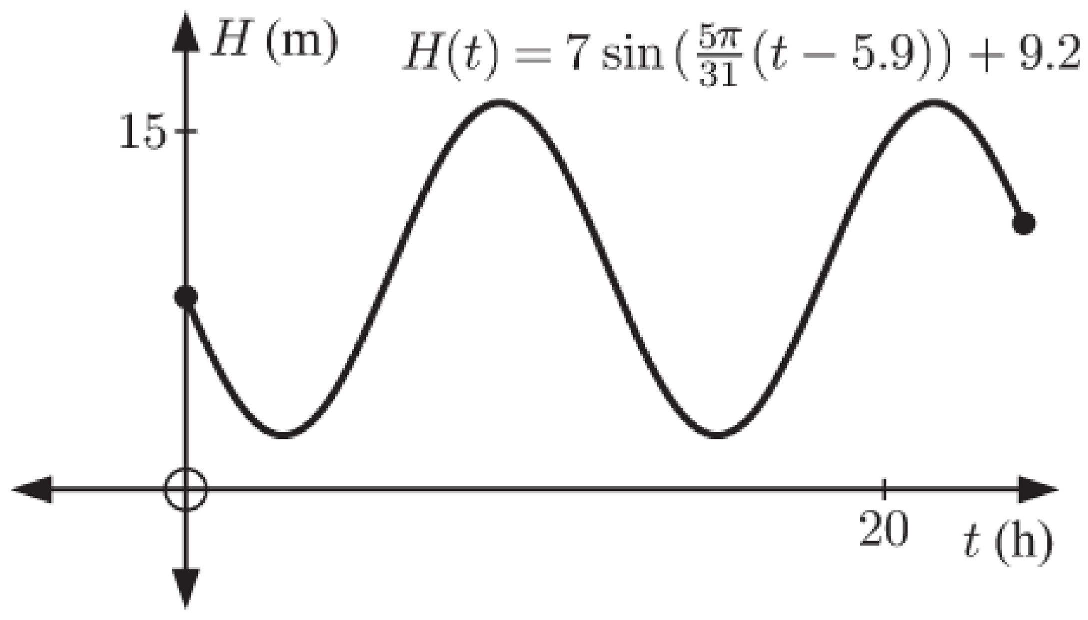
d **i** $H(t) = 10\sin\left(\frac{\pi}{50}t\right) + 12$ m

ii $d(t) = 10\sin\left(\frac{\pi}{50}(t + 25)\right)$ m

Note: The function of horizontal displacement of the light will be different depending on how the coordinate system is defined.

8 **a** $H(t) = 7\sin\left(\frac{5\pi}{31}(t-5.9)\right) + 9.2$ m

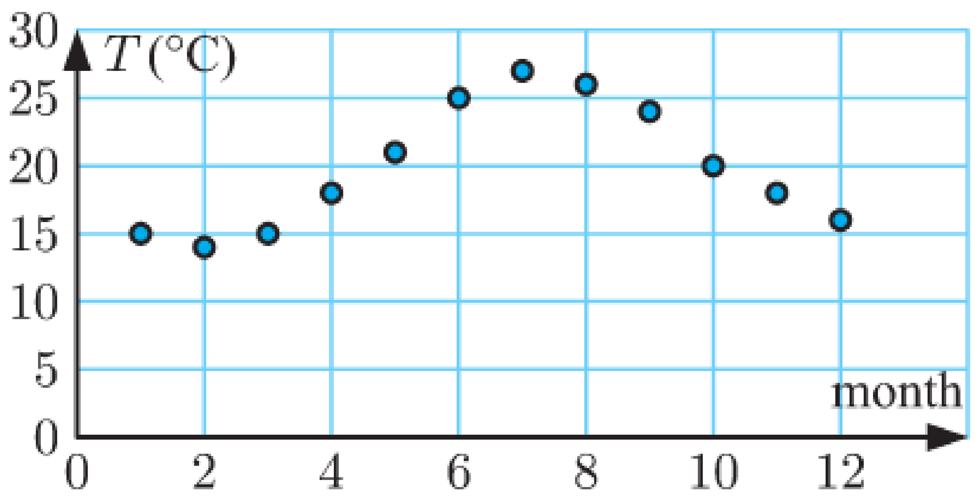
b



9 **a** $H(t) = 6\cos\left(\frac{\pi}{6}t\right)$ **b** $d(t) = 12\sin 2\pi t$

EXERCISE 17E

1 **a**



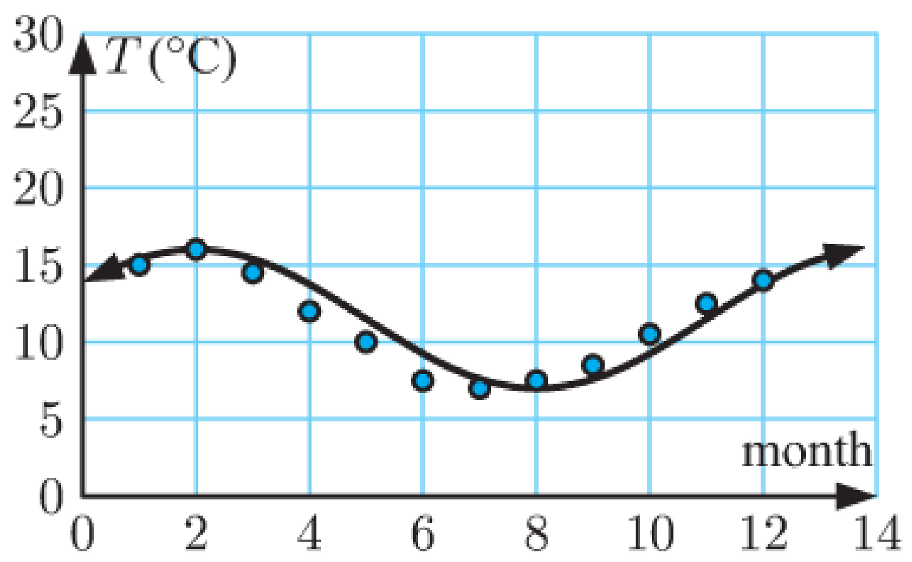
b The data appears to be periodic.

c **i** $b \approx \frac{\pi}{6}$ **ii** $a \approx 6.5$ **iii** $d \approx 20.5$ **iv** $c \approx 4.5$

d Using technology, $T \approx 6.15\sin(0.575t - 2.69) + 20.4$. Our model was a reasonable fit.

2 **a** $T \approx 4.5\cos\left(\frac{\pi}{6}(t-2)\right) + 11.5$

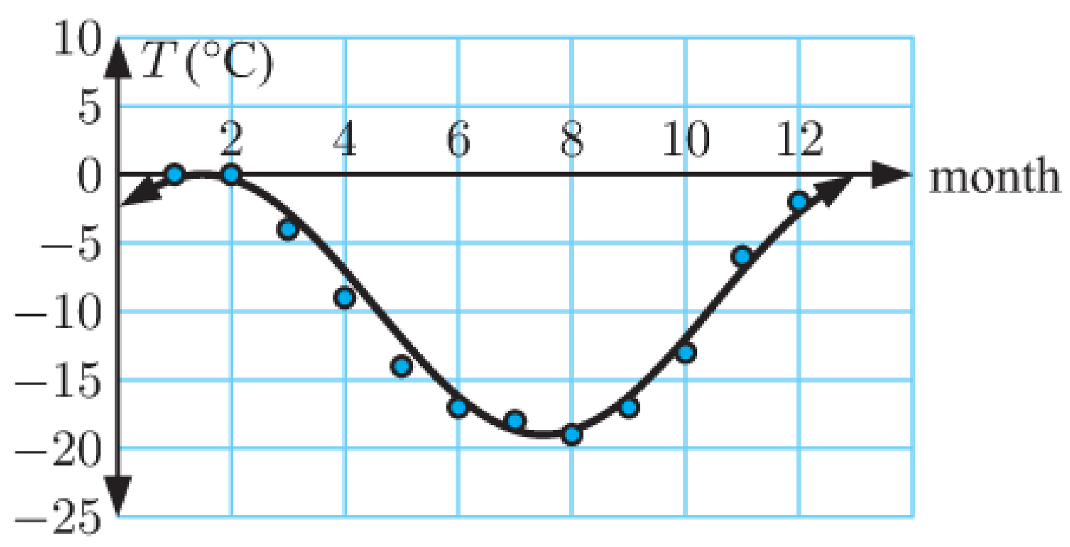
b



c Using technology, $T \approx 4.29\cos(0.533t - 0.805) + 11.2$.

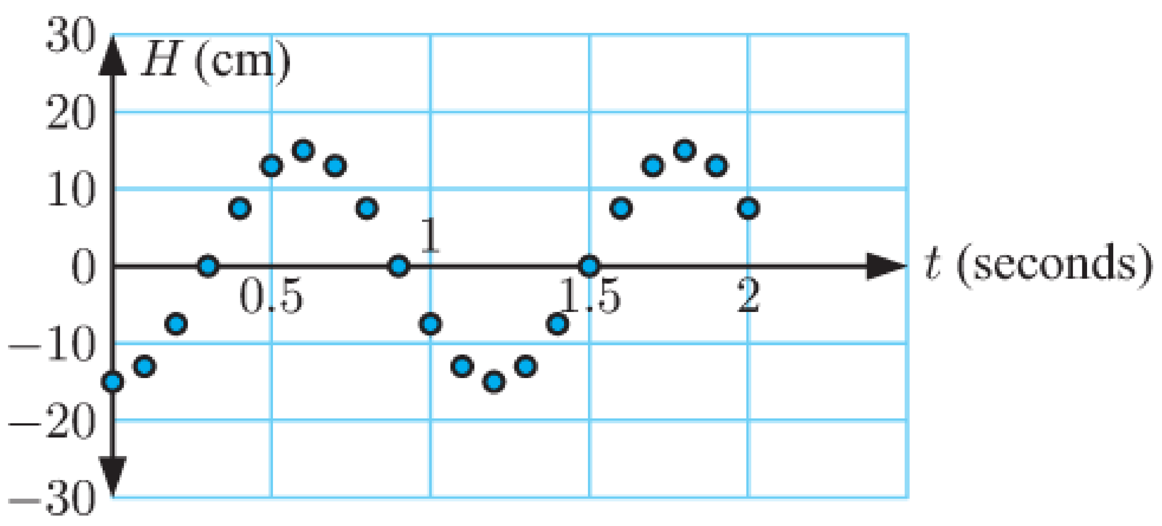
3 **a** $T \approx 9.5\sin\left(\frac{\pi}{6}(t-10.5)\right) - 9.5$

b



c The model is a reasonable fit, but not perfect.

4 **a**



b $H \approx 15.0\sin(5.24t - 1.57) + 0.000\,170$ **c** ≈ 14.5 cm

d The spring will not oscillate indefinitely at the same rate.

EXERCISE 17F

1 **a** A horizontal translation $\frac{\pi}{2}$ units to the right.

b A vertical stretch with scale factor 4.

c A horizontal stretch with scale factor $\frac{2}{\pi}$.

d A horizontal stretch with scale factor $\frac{1}{2}$, then a translation 1 unit downwards.

e A vertical stretch with scale factor $\frac{1}{2}$, then a reflection in the x-axis.

f A translation 2 units upwards.

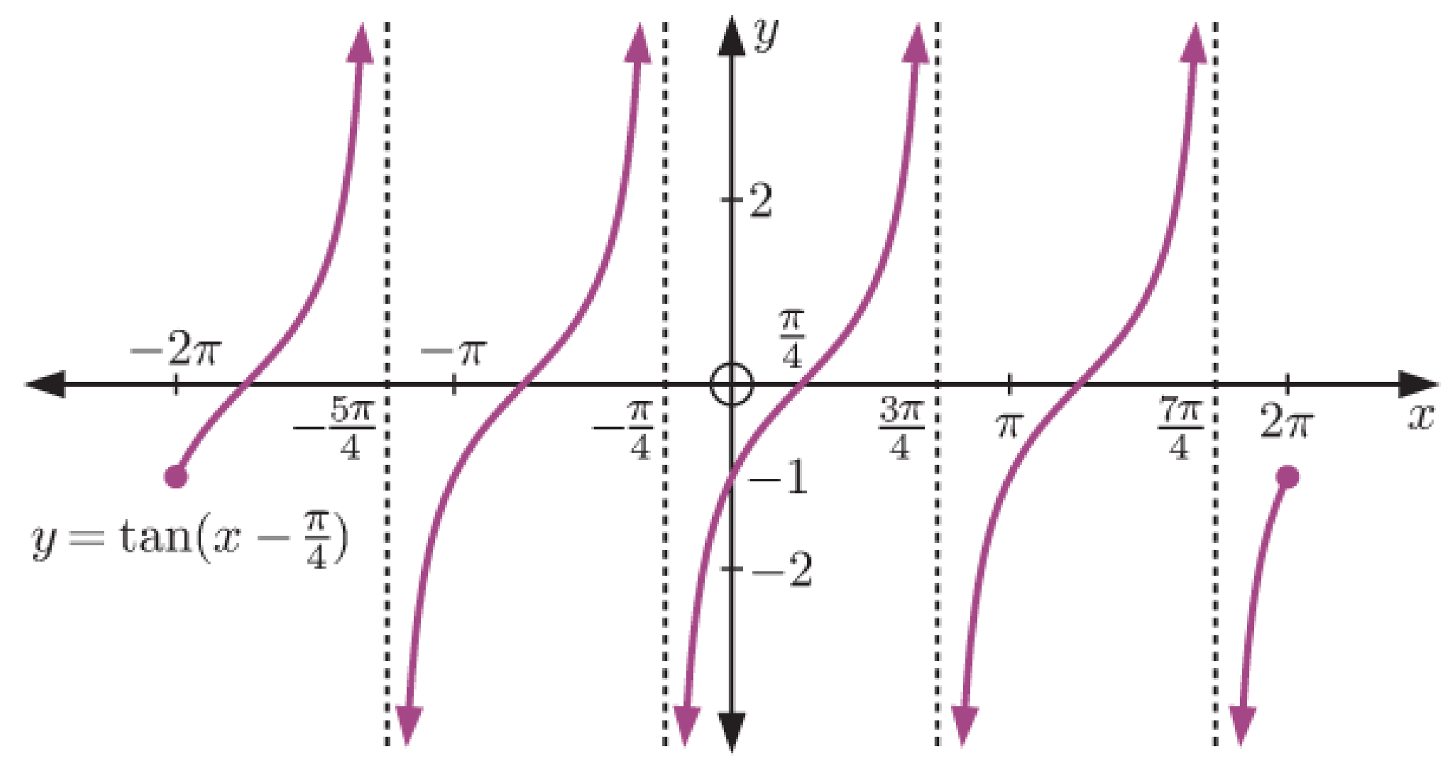
2 **a** $\frac{\pi}{3}$ **b** 4π **c** 1 **d** 2 **e** $\frac{3\pi}{2}$ **f** $\frac{\pi}{n}$

3 **a** **i** $\frac{k\pi}{2}$, $k \in \mathbb{Z}$ **ii** $x = \frac{\pi}{4} + \frac{k\pi}{2}$, $k \in \mathbb{Z}$

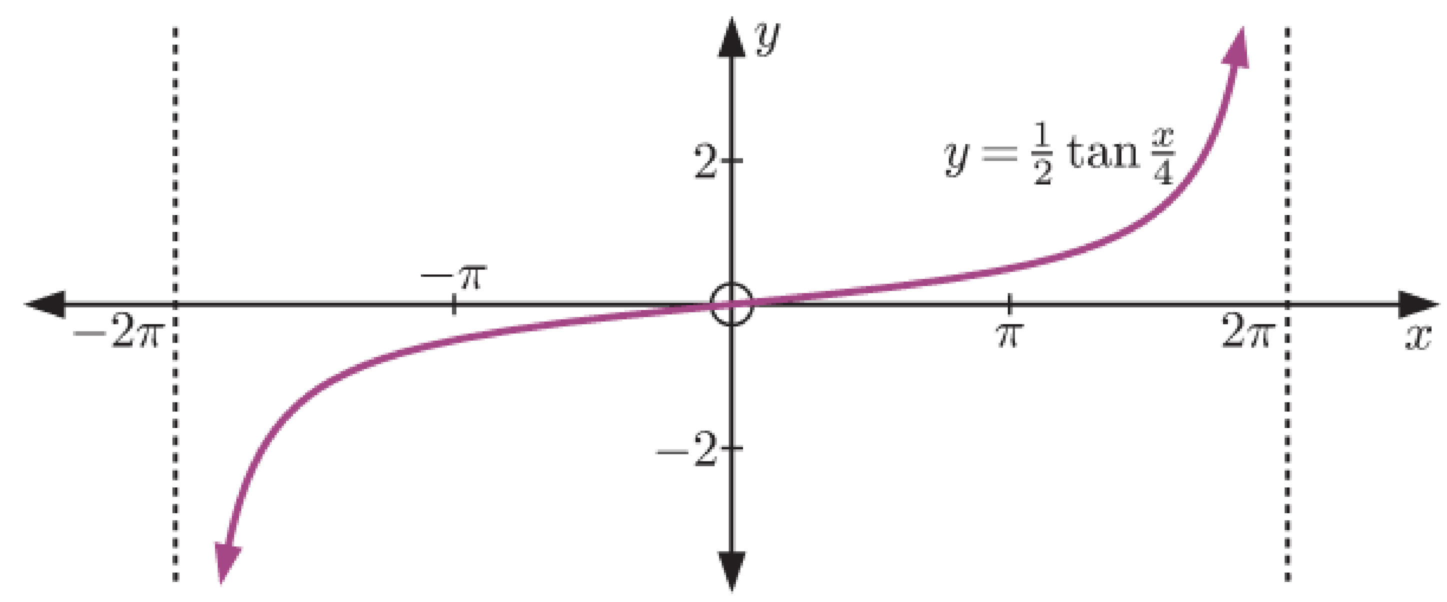
b **i** $\frac{2\pi}{3} + k\pi$, $k \in \mathbb{Z}$ **ii** $x = \frac{\pi}{6} + k\pi$, $k \in \mathbb{Z}$

c **i** $\frac{\pi}{6} + 2k\pi$, $k \in \mathbb{Z}$ **ii** $x = \frac{7\pi}{6} + 2k\pi$, $k \in \mathbb{Z}$

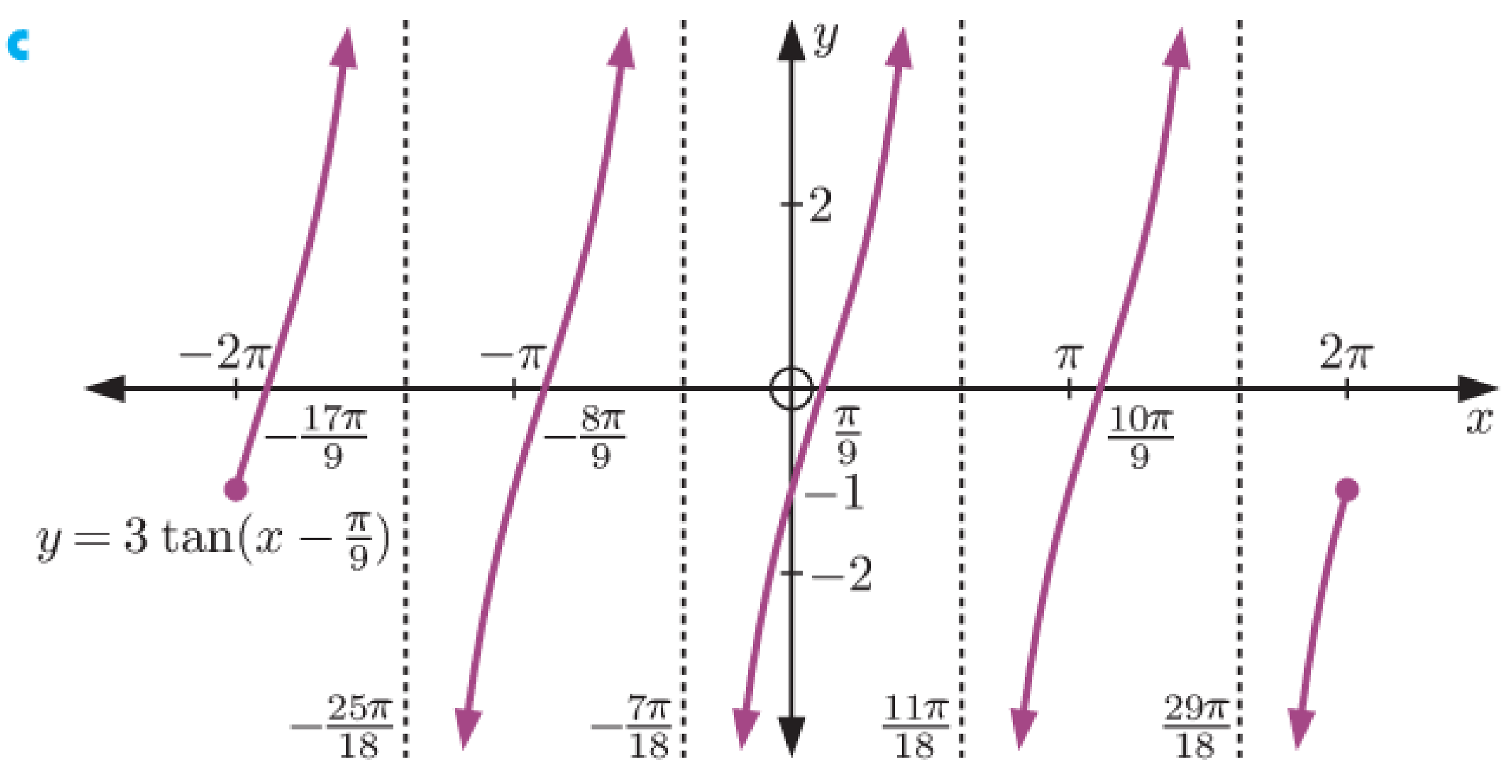
4 **a**



b



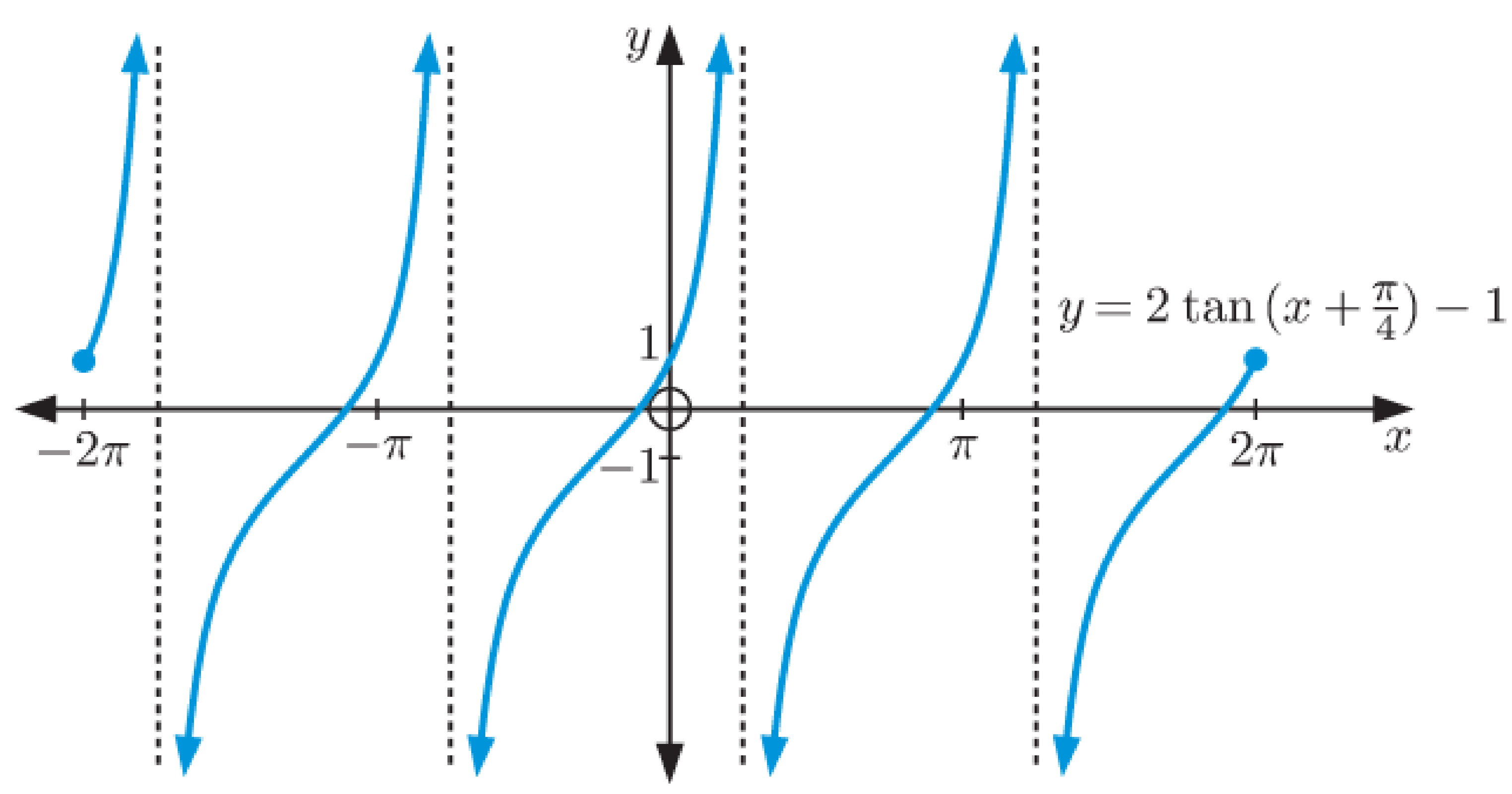
c



5 $p = \frac{1}{2}$, $q = 1$ **6** $a = \frac{3}{2}$, $b = -\frac{2\pi}{15} + \frac{2k\pi}{3}$, $k \in \mathbb{Z}$

7 **a** A vertical stretch with scale factor 2, then a translation $\frac{\pi}{4}$ units left and 1 unit downwards.

b



8 **Hint:** The function is undefined when $b(x - c) = \frac{\pi}{2} + k\pi$, $k \in \mathbb{Z}$.

9 **a** **i** $(f \circ g)(x) = \tan\left(2x - \frac{\pi}{2}\right)$

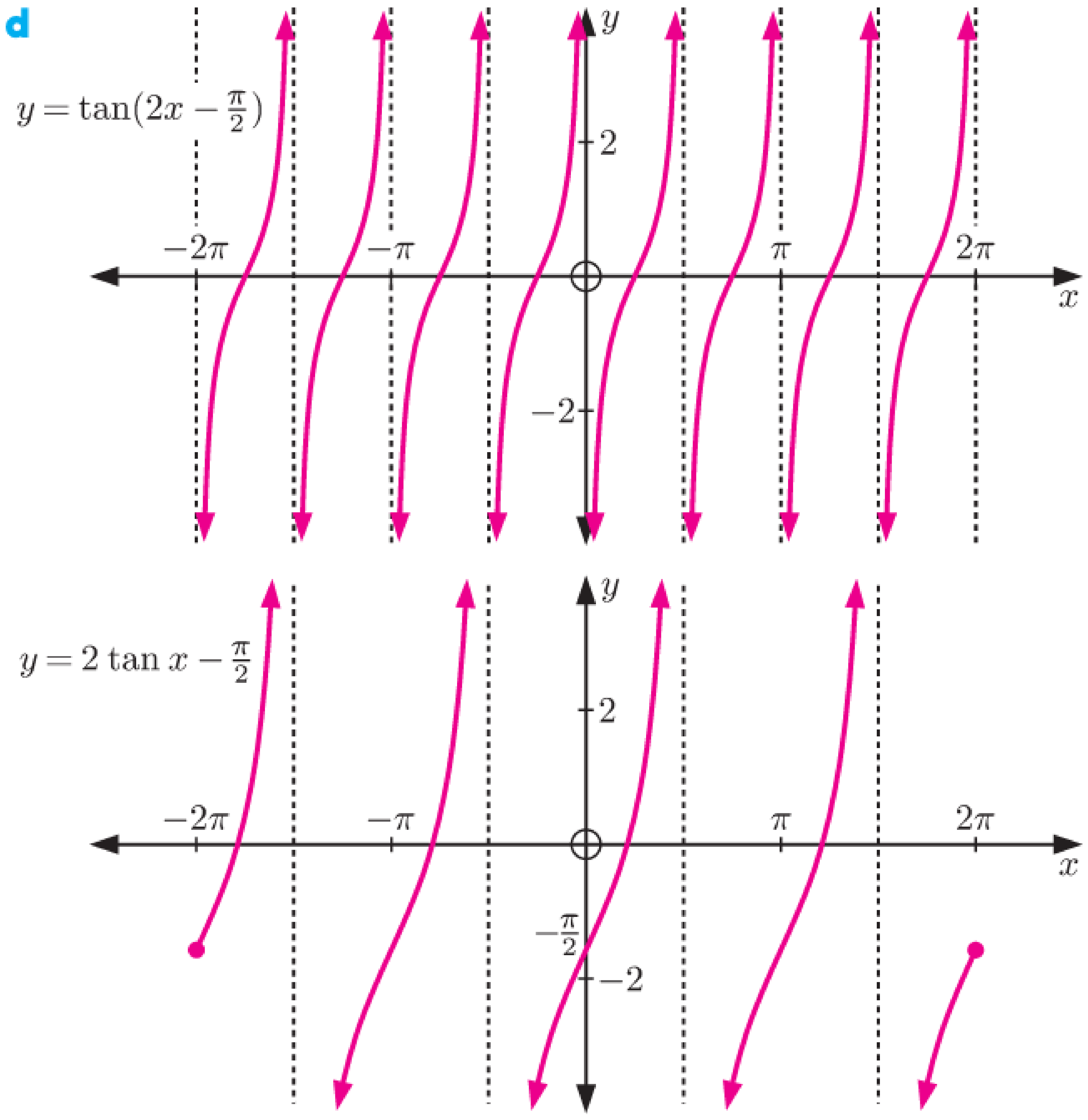
ii $(g \circ f)(x) = 2\tan x - \frac{\pi}{2}$

b **i** $\frac{1}{\sqrt{3}}$ **ii** $-\frac{\pi}{2}$

c **i** period $\frac{\pi}{2}$, vertical asymptotes $x = \dfrac{k\pi}{2}$, $k \in \mathbb{Z}$

ii period π, vertical asymptotes $x = \frac{\pi}{2} + k\pi$, $k \in \mathbb{Z}$

d



EXERCISE 17G.1

1 **a** $x \approx 0.3, 2.8, 6.6, 9.1, 12.9$ **b** $x \approx 5.9, 9.8, 12.2$
c $x \approx 0.3, 2.8$ **d** $x \approx 3.8, 5.6$

2 **a** $x \approx 1.2, 5.1, 7.4$ **b** $x \approx 4.4, 8.2, 10.7$
c $x \approx 5.2$ **d** $x \approx 2.5, 3.8$

3 **a** $x \approx 0.4, 1.2, 3.5, 4.3, 6.7, 7.5, 9.8, 10.6, 13.0, 13.7$
b $x \approx 1.7, 3.0, 4.9, 6.1, 8.0, 9.3, 11.1, 12.4, 14.3, 15.6$
c $x \approx 3.2, 4.6$ **d** $x \approx 1.6, 3.1, 4.8, 6.2$

4 **a** $x \approx 1.1, 4.2, 7.4$ **b** $x \approx 2.2, 5.3$
c $x \approx 1.3, 4.4$ **d** $x \approx 2.0$

EXERCISE 17G.2

1 **a** $x \approx 0.446, 2.70, 6.73, 8.98$
b $x \approx 2.52, 3.76, 8.80, 10.0$
c $x \approx 0.588, 3.73, 6.87, 10.0$

2 **a** $x \approx -0.644, 0.644$ **b** $x \approx -4.56, -1.42, 1.72, 4.87$
c $x \approx -2.76, -0.384, 3.53$

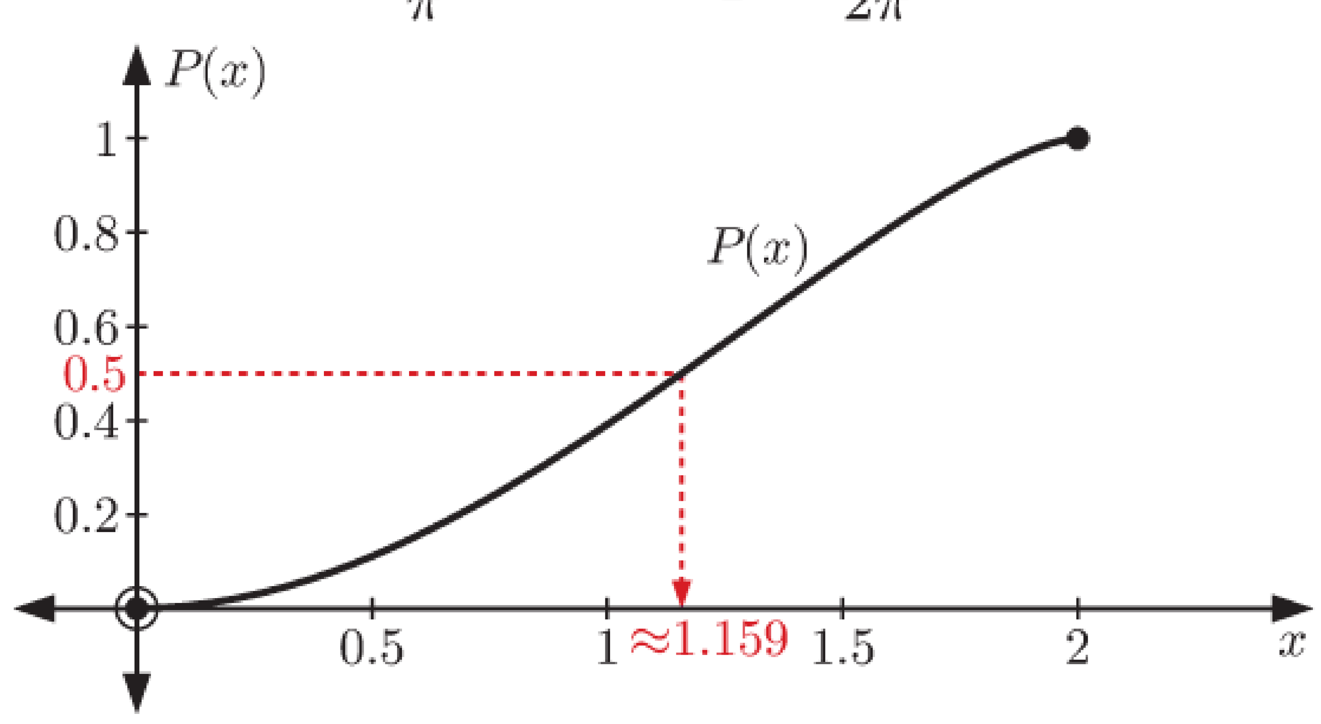
3 **a** $x \approx 1.08, 4.35$ **b** $x \approx 0.666, 2.48$

4 $x \approx -0.951, 0.234, 5.98$

5 **a** **Note:** The function $P(x)$ can be written in many different ways.

$$P(x) = 1 + \frac{x^2 - 2}{\pi} \arccos \frac{x}{2} - \frac{x}{2\pi}\sqrt{4 - x^2}$$

b



c $x \approx 1.159$

EXERCISE 17G.3

1 **a** $x = \frac{\pi}{3}$ or $\frac{5\pi}{3}$ **b** $x = \frac{5\pi}{4}$ or $\frac{7\pi}{4}$ **c** $x = \frac{\pi}{6}$ or $\frac{7\pi}{6}$
d $x = \frac{3\pi}{2}$ **e** $x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ **f** $x = 0, \pi$, or 2π

2 **a** $x = \frac{\pi}{3}$ or $\frac{2\pi}{3}$ **b** $x = \pi$ **c** $x = \frac{\pi}{4}$ or $\frac{5\pi}{4}$

3 **a** $x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}$, or $\frac{10\pi}{3}$ **b** $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}$, or $\frac{11\pi}{4}$
c $x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}$, or $\frac{13\pi}{4}$

4 **a** $x = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{4\pi}{3}$, or $\frac{5\pi}{3}$
b $x = -\frac{5\pi}{4}, -\frac{3\pi}{4}, \frac{3\pi}{4}$, or $\frac{5\pi}{4}$
c $x = -\frac{5\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}$, or $\frac{7\pi}{4}$

5 **a** $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$ **b** $x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$
c $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$, or $\frac{5\pi}{3}$

6 **a** $0 \leqslant 2x \leqslant 4\pi$ **b** $0 \leqslant \dfrac{x}{4} \leqslant \frac{\pi}{2}$
c $\frac{\pi}{2} \leqslant x + \frac{\pi}{2} \leqslant \frac{5\pi}{2}$ **d** $-\frac{\pi}{6} \leqslant x - \frac{\pi}{6} \leqslant \frac{11\pi}{6}$
e $-\frac{\pi}{2} \leqslant 2\left(x - \frac{\pi}{4}\right) \leqslant \frac{7\pi}{2}$ **f** $-2\pi \leqslant -x \leqslant 0$

7 **a** $-3\pi \leqslant 3x \leqslant 3\pi$ **b** $-\frac{\pi}{4} \leqslant \dfrac{x}{4} \leqslant \frac{\pi}{4}$
c $-\frac{3\pi}{2} \leqslant x - \frac{\pi}{2} \leqslant \frac{\pi}{2}$ **d** $-\frac{3\pi}{2} \leqslant 2x + \frac{\pi}{2} \leqslant \frac{5\pi}{2}$
e $-2\pi \leqslant -2x \leqslant 2\pi$ **f** $0 \leqslant \pi - x \leqslant 2\pi$

8 **a** $x = \frac{\pi}{3}, \frac{5\pi}{3}$, or $\frac{7\pi}{3}$
b $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}$, or $\frac{17\pi}{6}$
c $x = 0, \frac{4\pi}{3}$, or 2π

9 **a** $x = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}$, or $\frac{23\pi}{12}$
b $x = \frac{\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}, \frac{23\pi}{18}, \frac{25\pi}{18}$, or $\frac{35\pi}{18}$
c $x = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}$, or $\frac{5\pi}{3}$ **d** $x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$
e $x = \frac{4\pi}{3}$ **f** $x = \frac{3\pi}{4}$

10 **a** $x = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{14\pi}{9}$, $\frac{16\pi}{9}$, or $\frac{17\pi}{9}$
b $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$, or $\frac{7\pi}{4}$ **c** $x = \frac{\pi}{3}$ or $\frac{5\pi}{3}$

11 **a** $x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$ **b** $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{17\pi}{12}$, or $\frac{7\pi}{4}$
c $x = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}$, or $\frac{5\pi}{3}$

12 **a** $x = -\frac{5\pi}{3}, -\pi, \frac{\pi}{3}$, or π **b** $x = 0, \frac{3\pi}{2}$, or 2π
c $x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$, or π **d** $x = 0, \frac{\pi}{6}, \pi, \frac{7\pi}{6}$, or 2π

13 **a** $b = \frac{1}{3}$, $d = 2$ **b** $a = -2$, $b = \frac{1}{2}$
c $b = 4$, $d = -1$ **d** $b = \frac{1}{2}$, $d = -4$

14 $x = \frac{\pi}{3}$ or $\frac{4\pi}{3}$
a $x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$
b $x = \frac{\pi}{12}, \frac{\pi}{3}, \frac{7\pi}{12}, \frac{5\pi}{6}, \frac{13\pi}{12}, \frac{4\pi}{3}, \frac{19\pi}{12}$, or $\frac{11\pi}{6}$
c $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$, or $\frac{5\pi}{3}$

15 **a** $x = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi$, or 2π **b** $x = \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}$, or $\frac{5\pi}{3}$
c $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$, or 2π **d** $x = \frac{7\pi}{6}, \frac{3\pi}{2}$, or $\frac{11\pi}{6}$

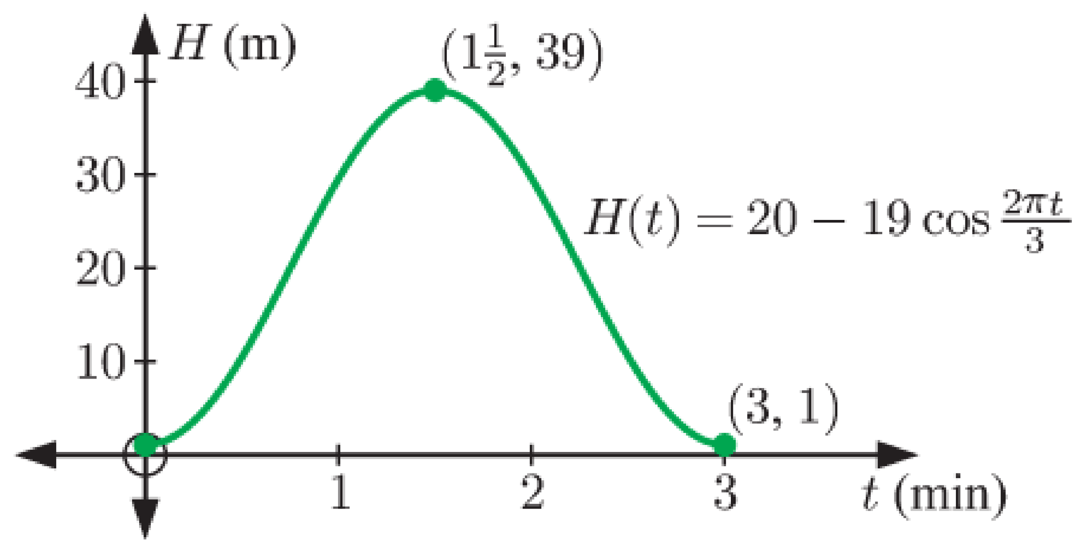
EXERCISE 17H

1 **a** **i** 7500 grasshoppers **ii** $\approx 10\,300$ grasshoppers
b 10 500 grasshoppers, when $t = 4$ weeks
c **i** at $t = 1\frac{1}{3}$ weeks and $6\frac{2}{3}$ weeks
ii at $t = 9\frac{1}{3}$ weeks

d $2.51 \leqslant t \leqslant 5.49$

2 **a** 1 m above ground **b** at $t = 1\frac{1}{2}$ min **c** 3 min

d



e $0.570 \leqslant t \leqslant 2.43$ min

3 **a** 400 water buffalo

b **i** 577 water buffalo **ii** 400 water buffalo

c 650, which is the maximum population.

d 150, after 3 years **e** $t \approx 0.262$ years

4 **a** **i** true **ii** true **b** 116.8 cents L^{-1}

c on the 5th, 11th, 19th, and 25th days

d 98.6 cents L^{-1} on the 1st and 15th days

5 **a** $H(t) = 3\cos\left(\frac{\pi}{2}t\right) + 4$ **b** $t \approx 1.46$ s

REVIEW SET 17A

1 **a** not periodic **b** periodic

2 **a** minimum = 0, maximum = 2

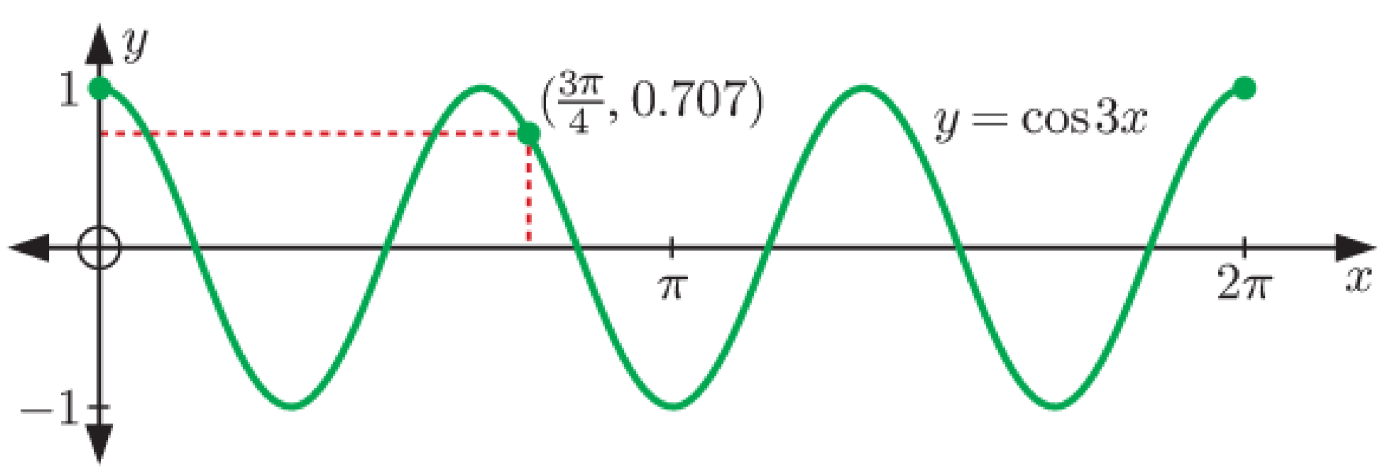
b minimum = −2, maximum = 2

3 **a** 10π **b** $\frac{\pi}{2}$ **c** 4π **d** $\frac{\pi}{3}$

4

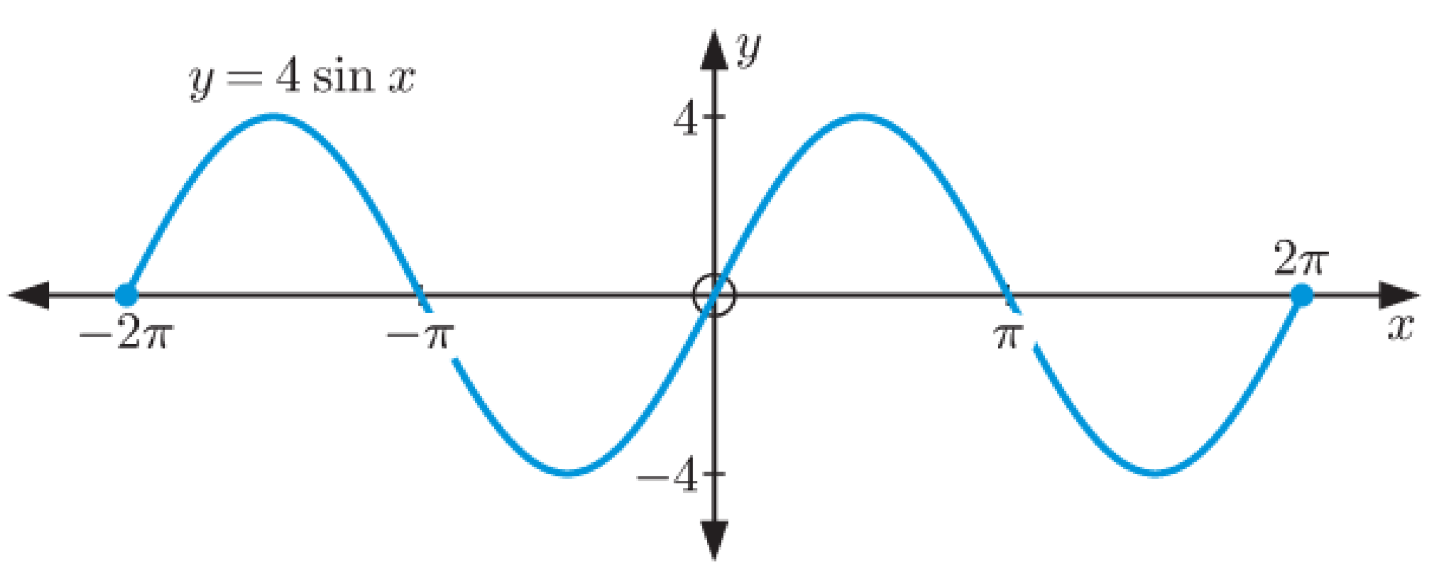
Function	*Period*	*Amplitude*	*Range*
$y = -3\sin\frac{x}{4} + 1$	8π	3	$-2 \leqslant y \leqslant 4$
$y = 3\cos\pi x$	2	3	$-3 \leqslant y \leqslant 3$

5 **a**



b $y = \frac{1}{\sqrt{2}} \approx 0.707$

6 **a**



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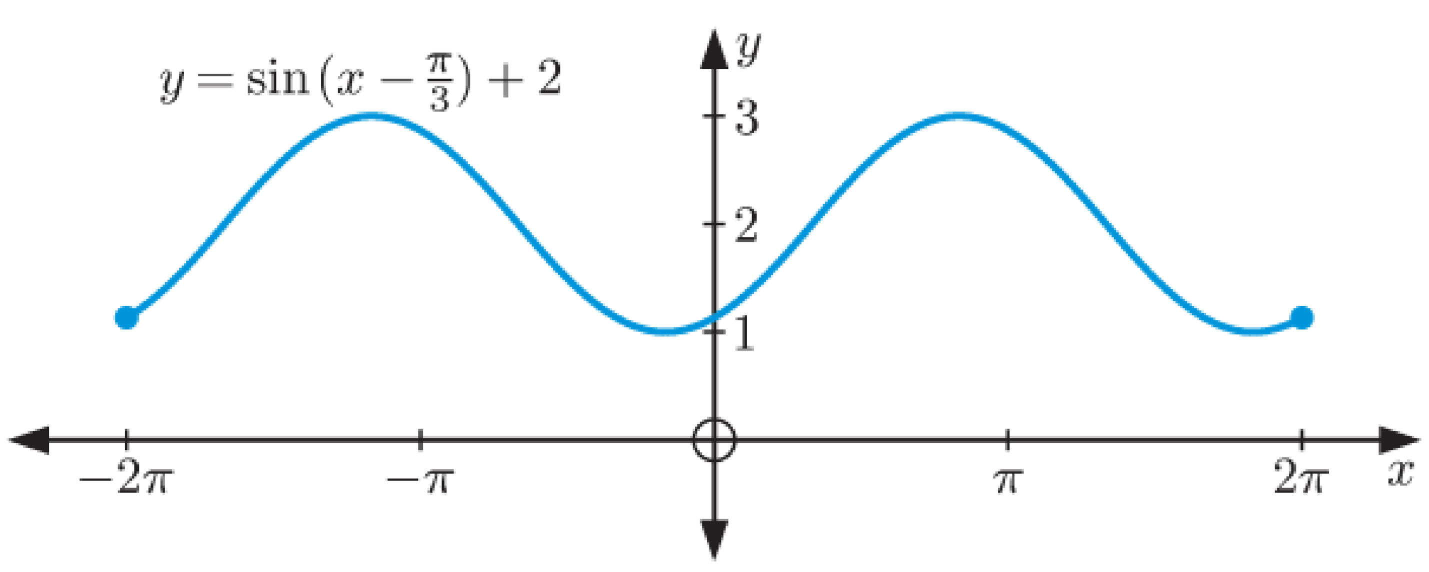
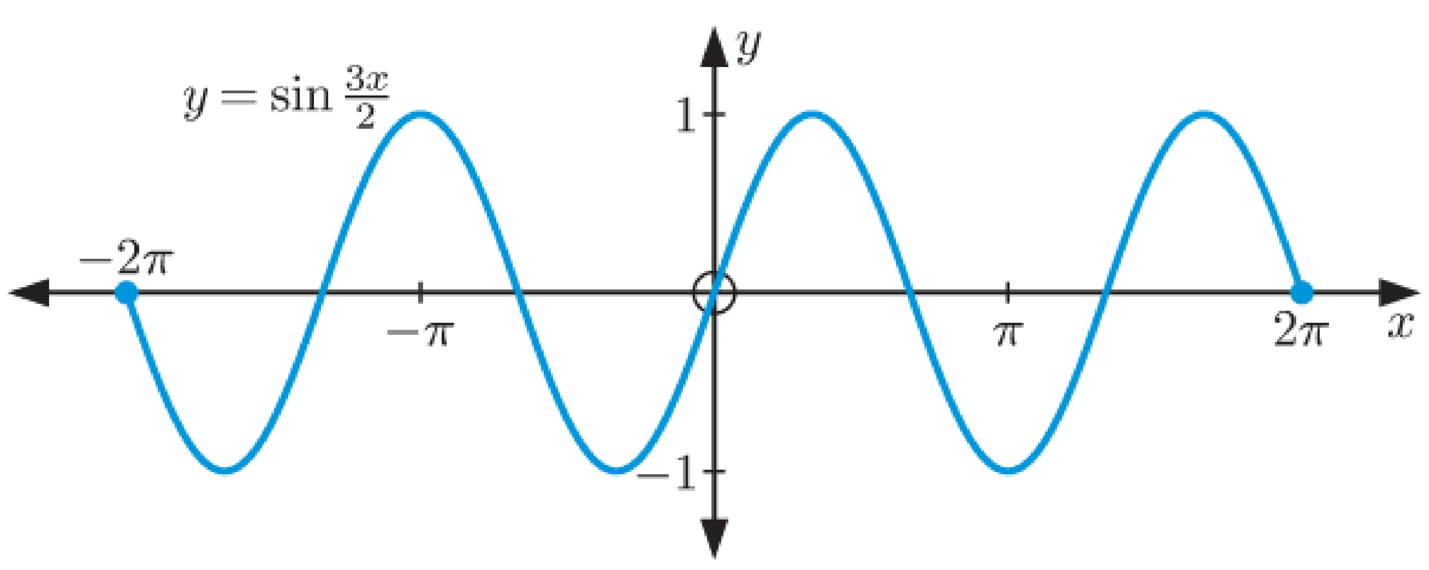
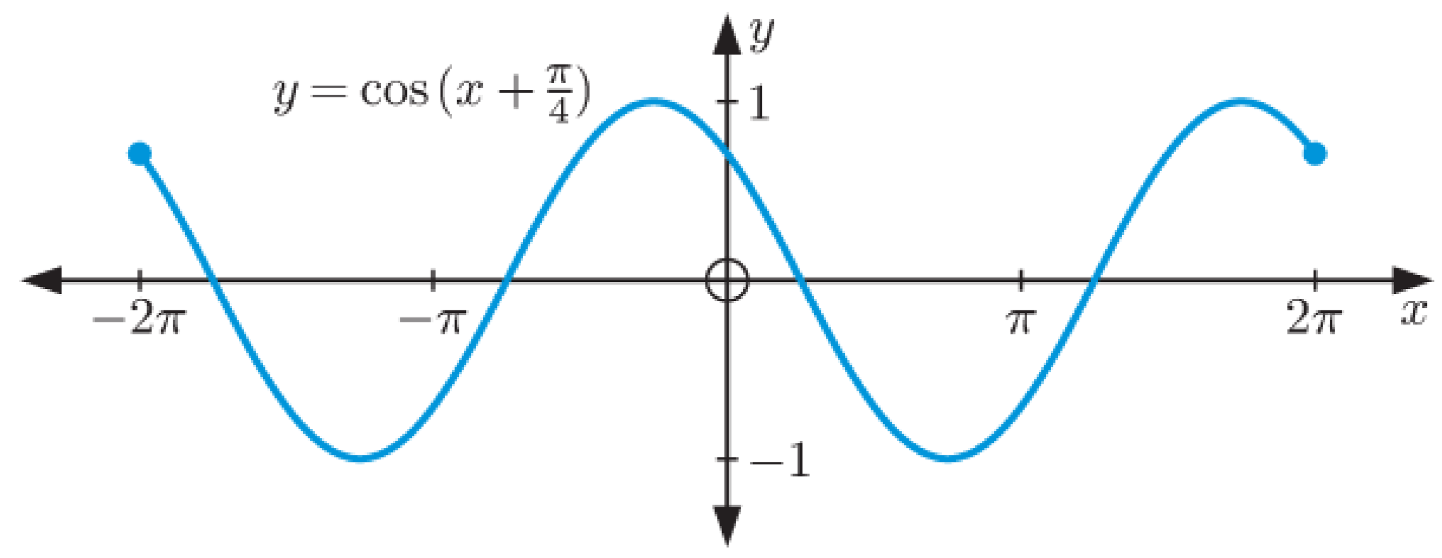


c



d



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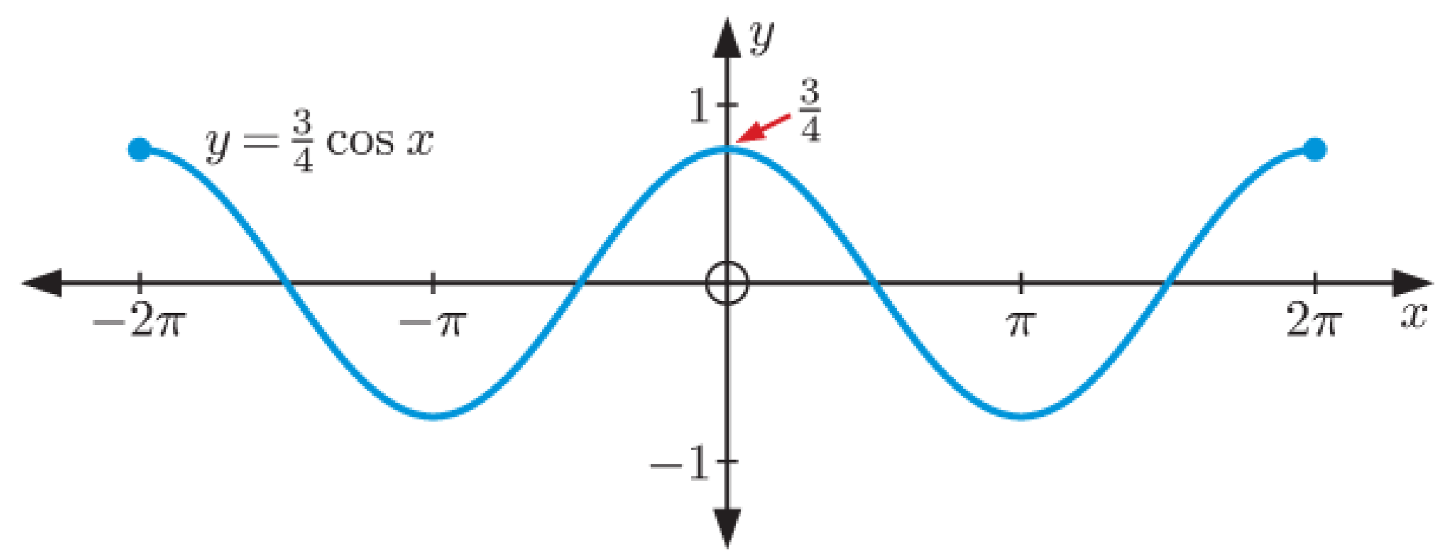

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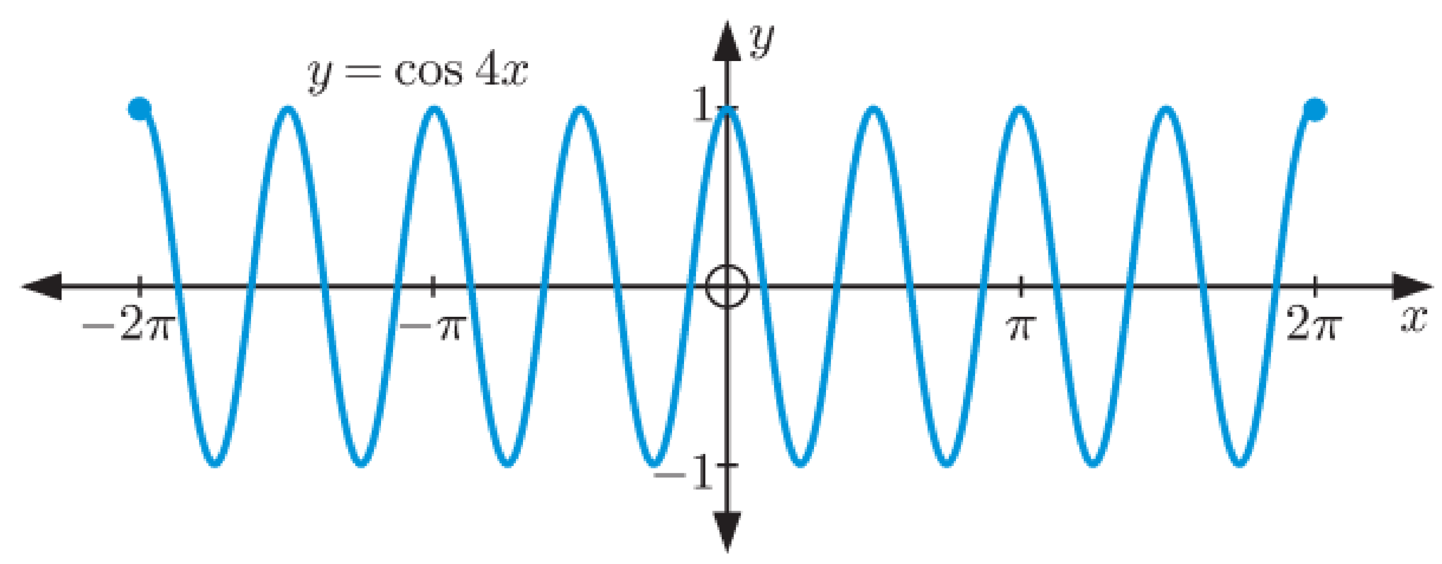


7 **a** A vertical stretch with scale factor 3, then a horizontal stretch with scale factor $\frac{1}{2}$.

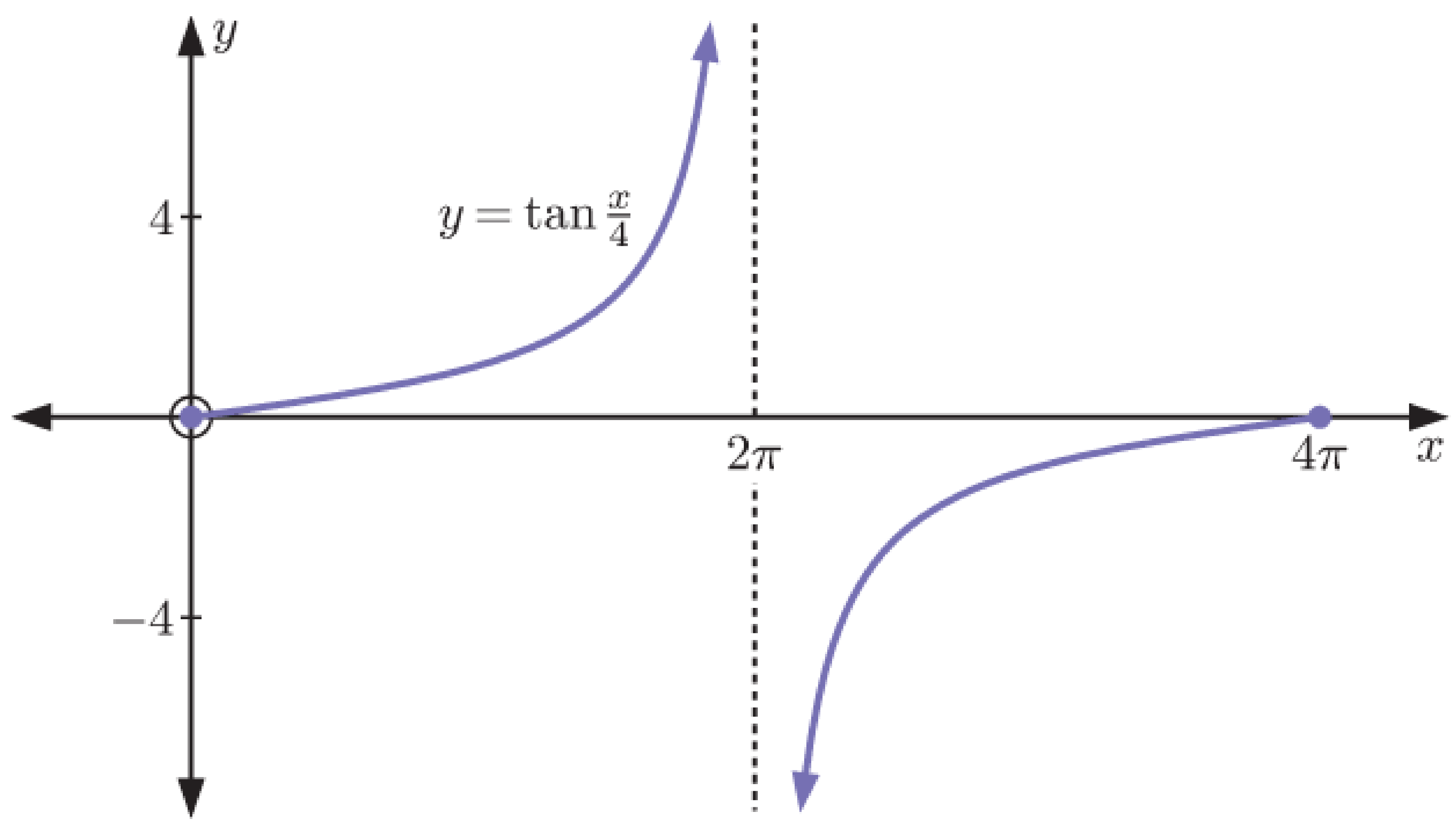
b A translation $\frac{\pi}{3}$ units right and 1 unit downwards.

c A reflection in the x-axis, then a horizontal stretch with scale factor $\frac{1}{2}$.

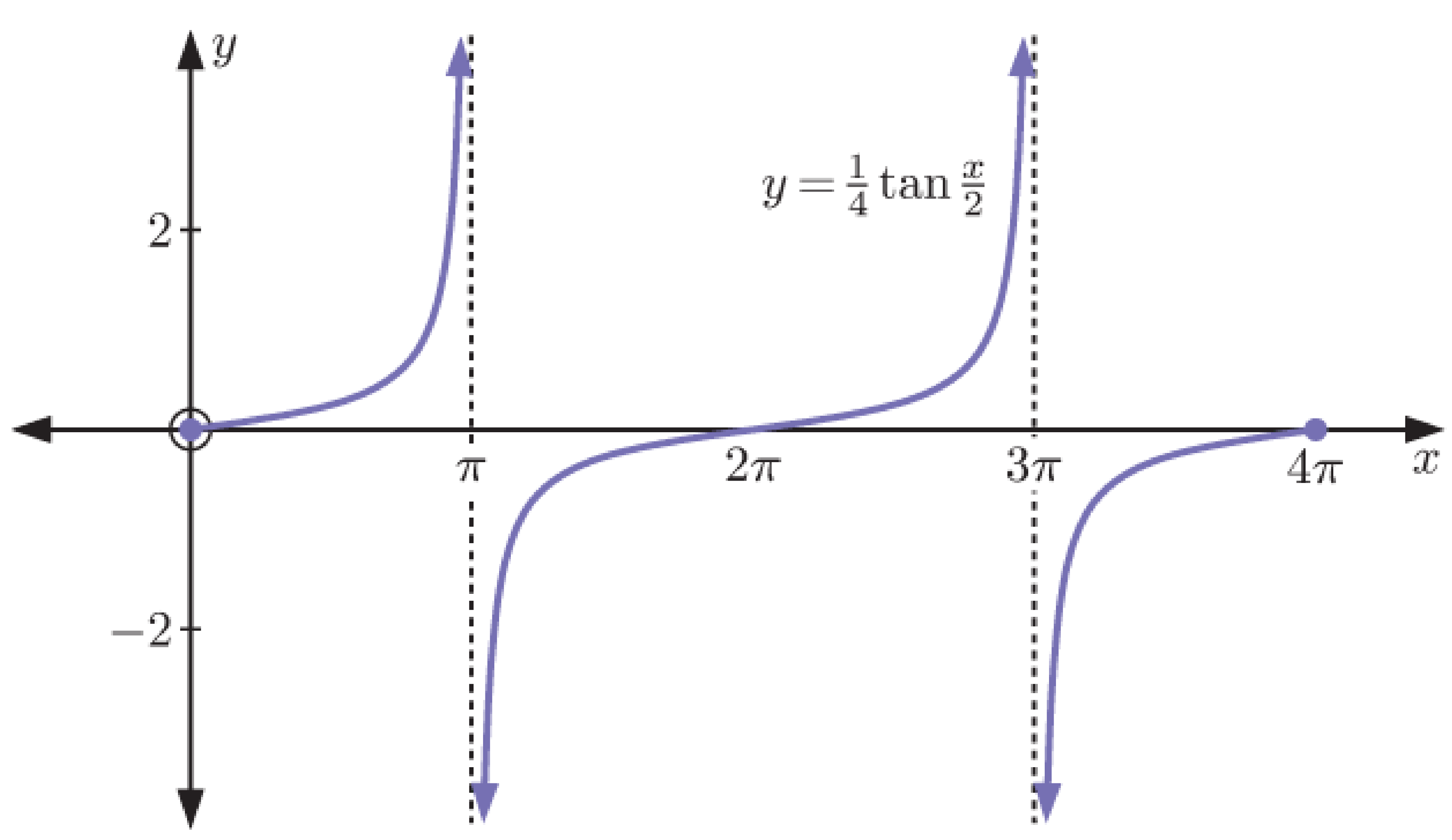
d A vertical stretch with scale factor 2, then a horizontal stretch with scale factor 2, then a translation $\frac{\pi}{2}$ units right and $\frac{1}{2}$ unit upwards.

8 **a** $y = -4\cos 2x$ **b** $y = \cos\frac{\pi x}{4} + 2$

9 **a**

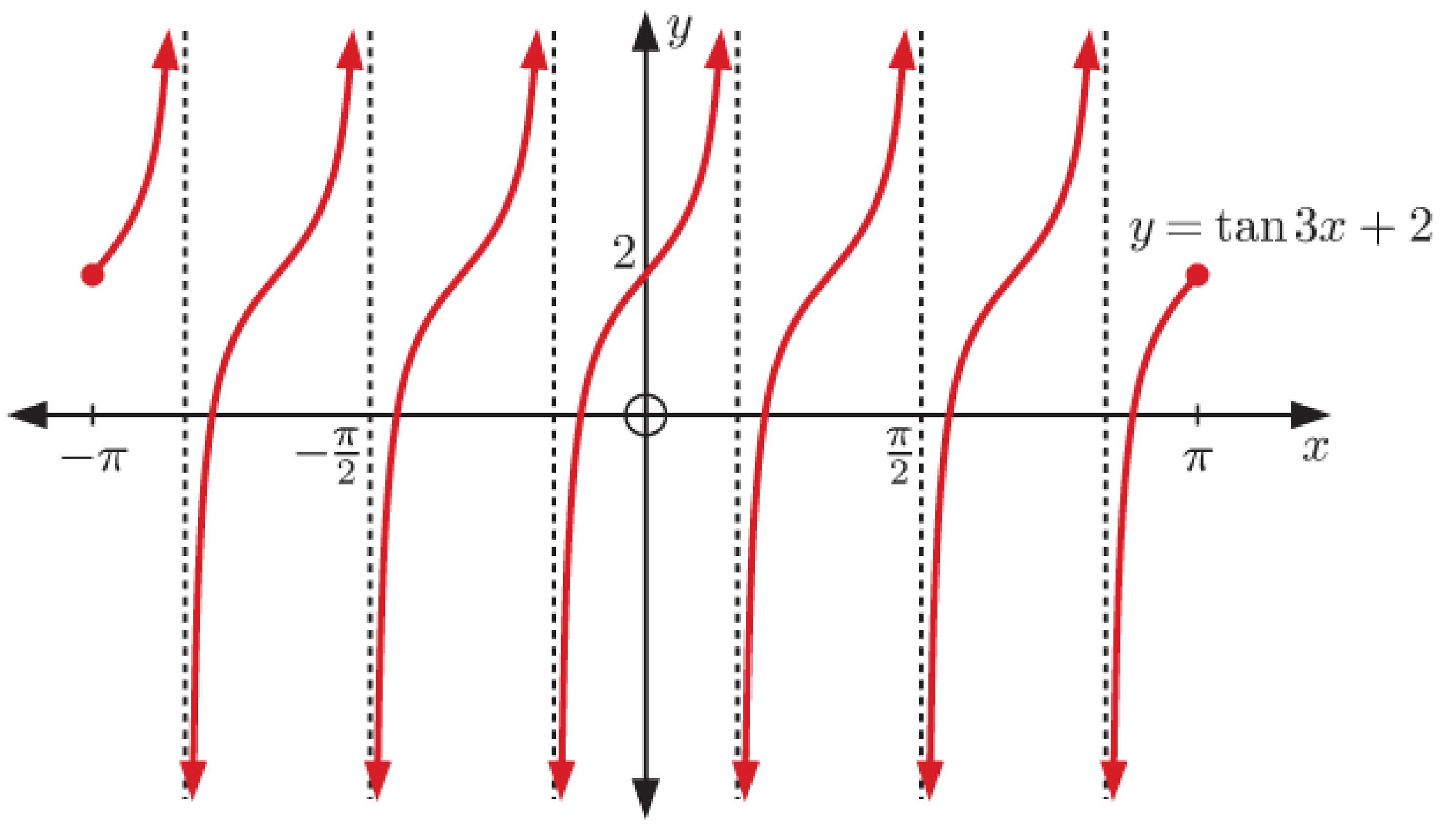


b



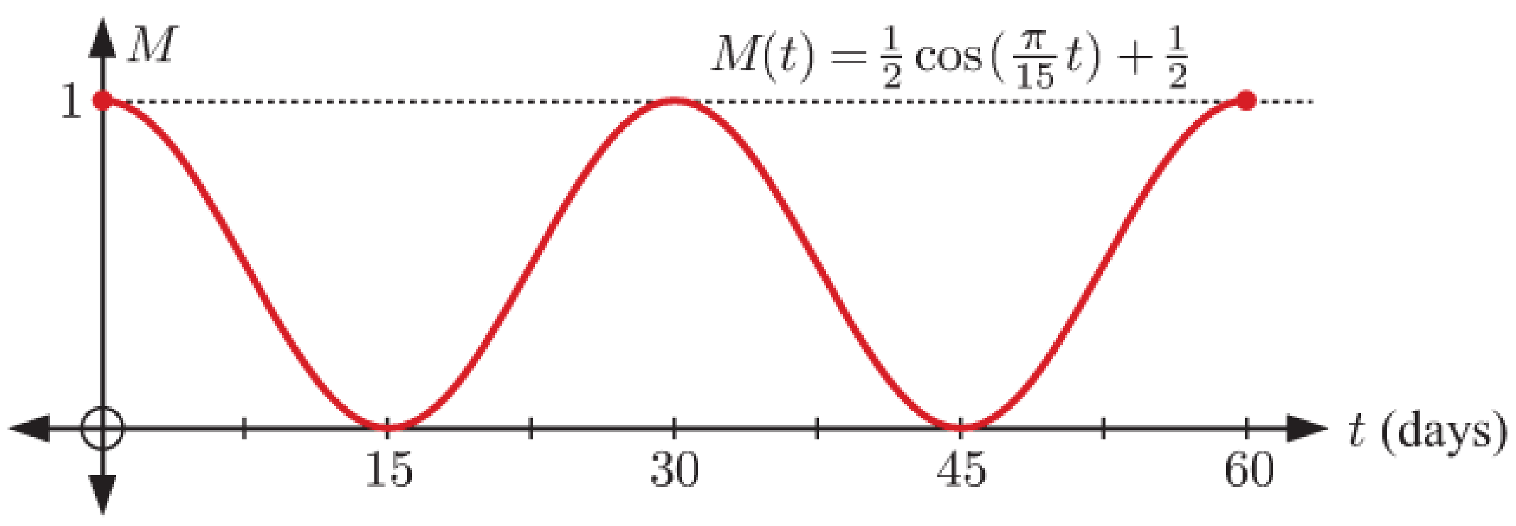
10 **a** A horizontal stretch with scale factor $\frac{1}{3}$, then a vertical translation 2 units upwards.

b $\frac{\pi}{3}$

c


11 a $a = 7,\ b = \frac{\pi}{8},\ c = 1,\ d = 10$

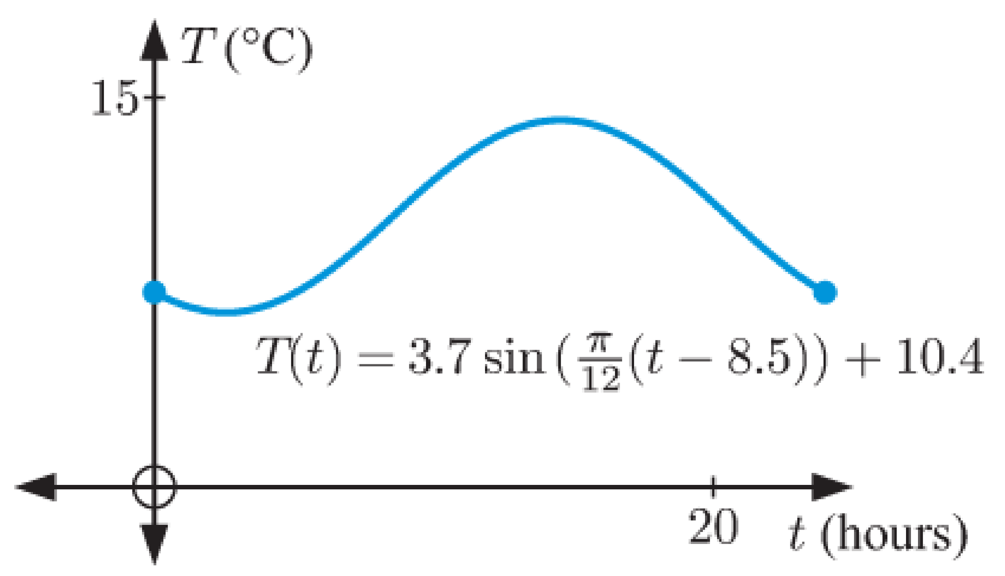
b $g(x) = 14\sin\left(\frac{\pi}{8}(x-3)\right) + 14$

12 a


b i 0.75 ii 0.25 iii ≈ 0.835 iv ≈ 0.165

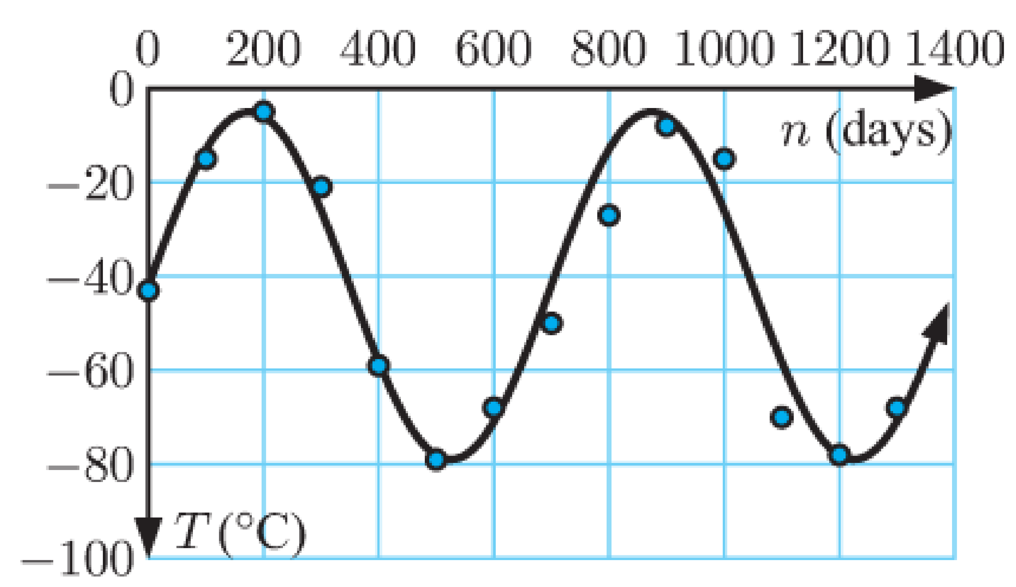
c once every 30 days d January 16, February 15

13 a $T(t) = 3.7\sin\left(\frac{\pi}{12}(t-8.5)\right) + 10.4$ °C

b


14 a maximum: -5°C, minimum: -79°C

b ≈ 700 Mars days c $T \approx 37\sin(0.008\,98n) - 42$

d


e Using technology,
$T \approx 36.5\sin(0.009\,01x - 0.0903) - 43.2$.
Our model fits the data well.

15 a $x \approx 2.0, 4.3, 8.3, 10.6$ b $x \approx 0.5, 5.8, 6.7, 12.1$

16 a $x \approx 0.392, 2.75, 6.68$ b $x \approx 5.42$

17 a $x \approx 1.12, 5.17, 7.40$ b $x \approx 0.184, 4.62$

18 a $x = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$ b $x = \frac{\pi}{4}$ or $\frac{7\pi}{4}$

c $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$, or $\frac{5\pi}{3}$

19 a $x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}$, or $\frac{15\pi}{8}$

b $x = \frac{3\pi}{2}$ c $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$

20 a $x = 0, \frac{3\pi}{2}, 2\pi, \frac{7\pi}{2}$, or 4π b $x = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}$, or $\frac{5\pi}{3}$

21 a 5000 beetles b smallest 3000, largest 7000

c $0.5 < t < 2.5$ and $6.5 < t \leqslant 8$

REVIEW SET 17B

1 a The function repeats itself over and over in a horizontal direction, in intervals of length 8 units.

b i 8 ii 5 iii -1

2 a A translation $\frac{\pi}{3}$ units right and 1 unit upwards.

b A horizontal stretch with scale factor $\frac{1}{3}$.

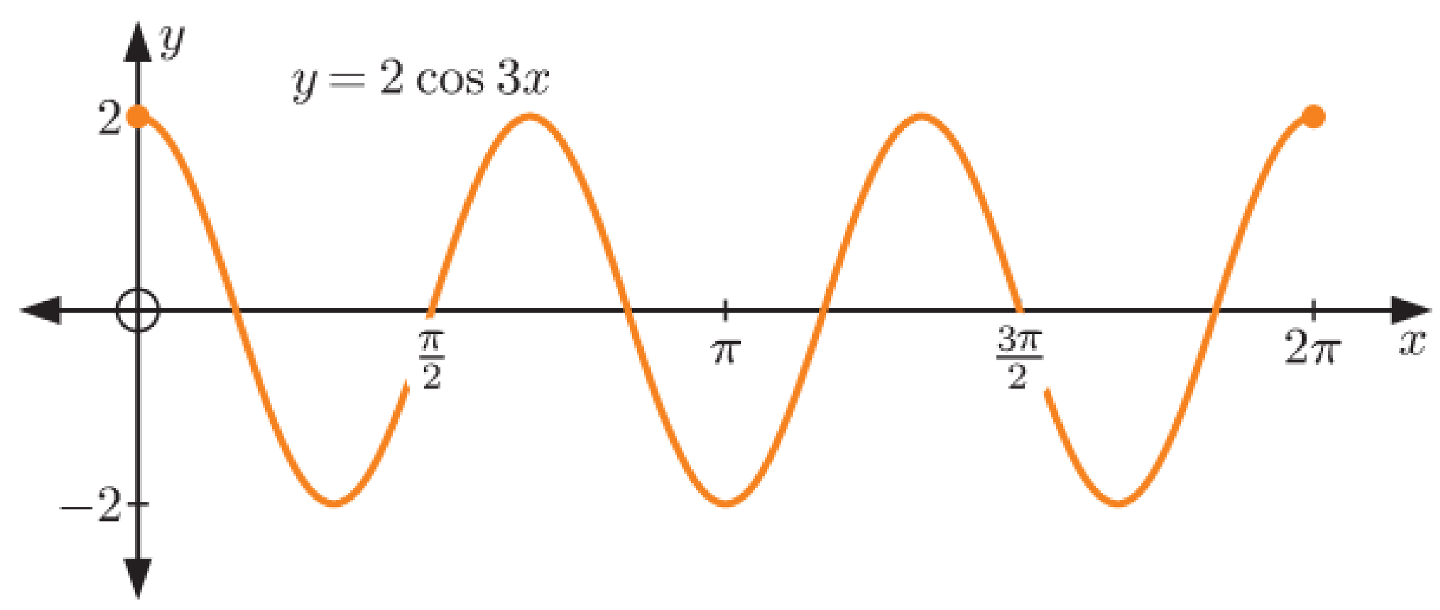
3 a 6π b $\frac{\pi}{4}$

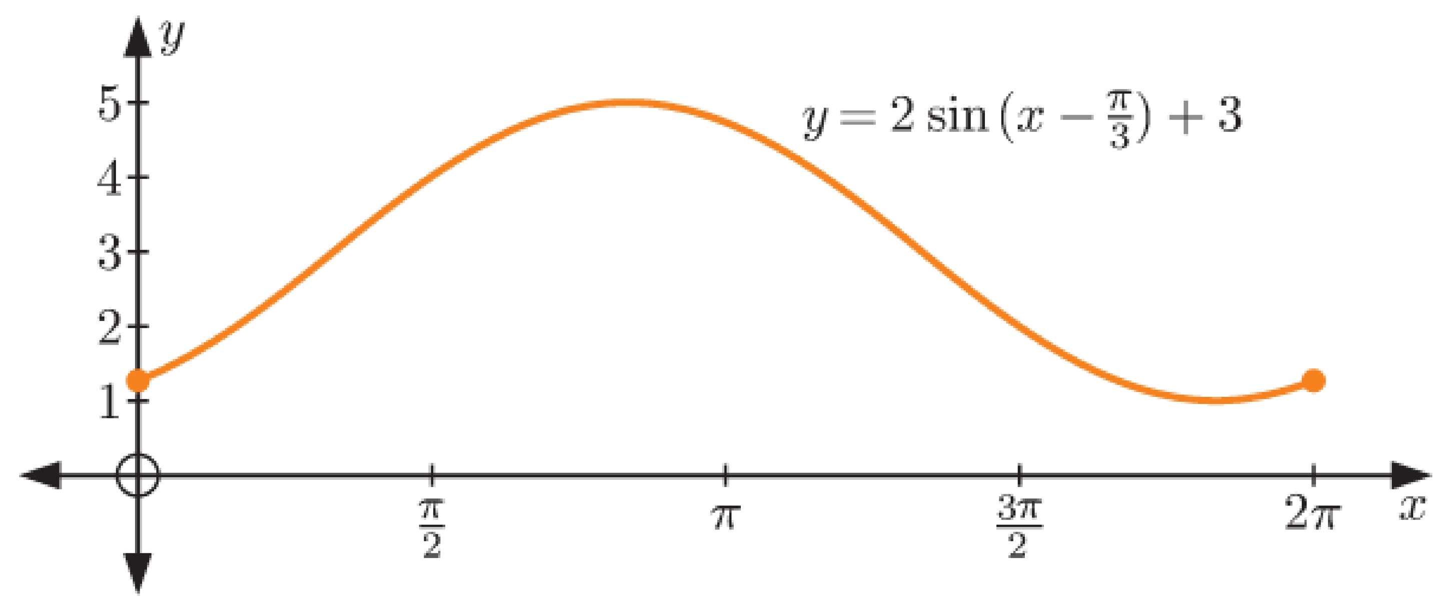
4 a $b = \frac{1}{3}$ b $b = 24$ c $b = \frac{2\pi}{9}$

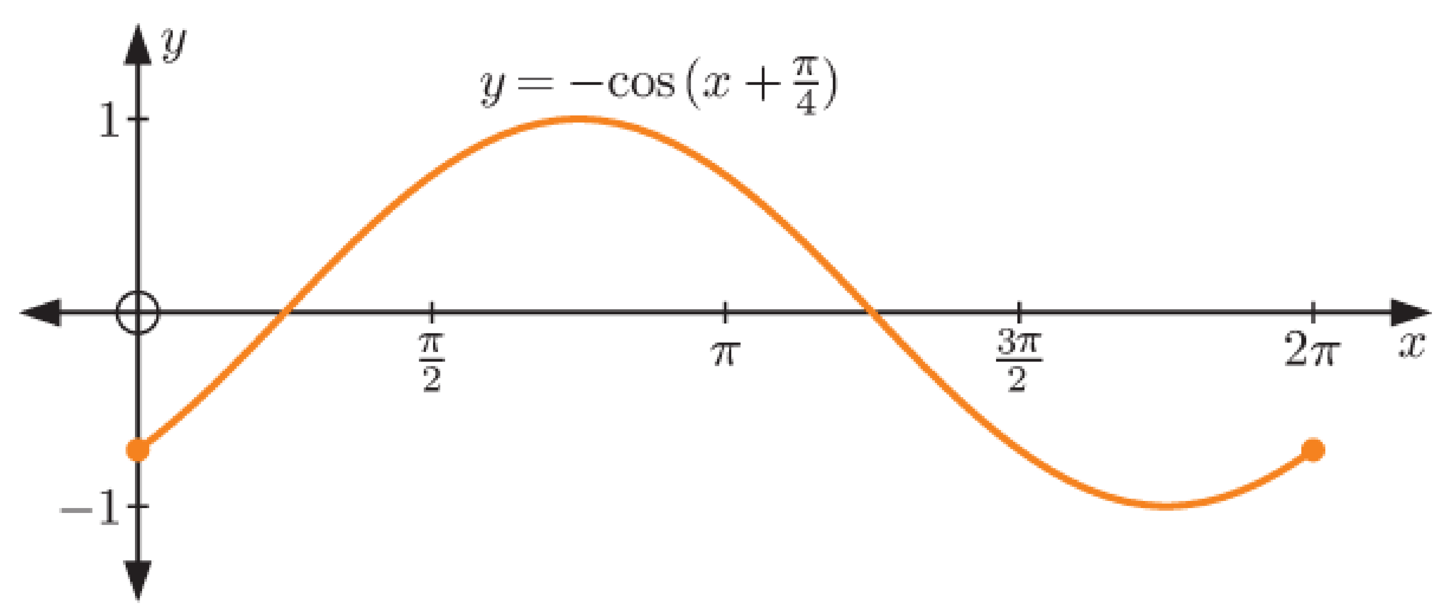
5 a minimum $= -8$, maximum $= 2$

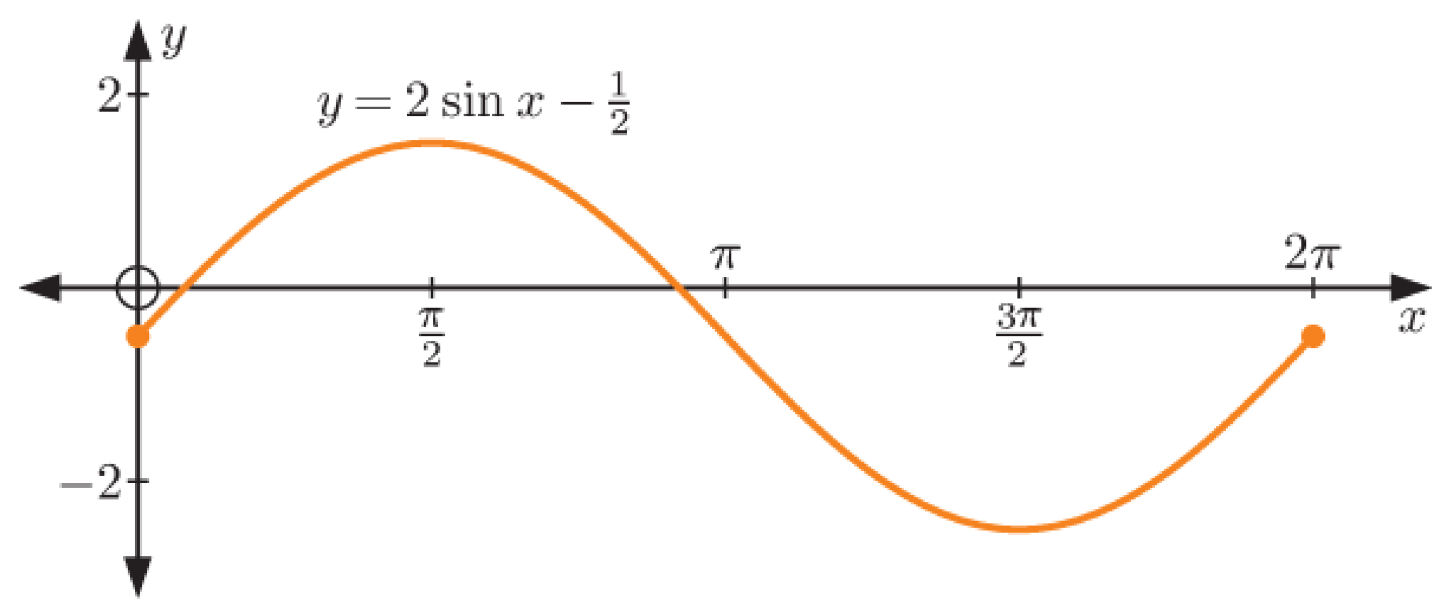
b minimum $= \frac{2}{3}$, maximum $= 1\frac{1}{3}$

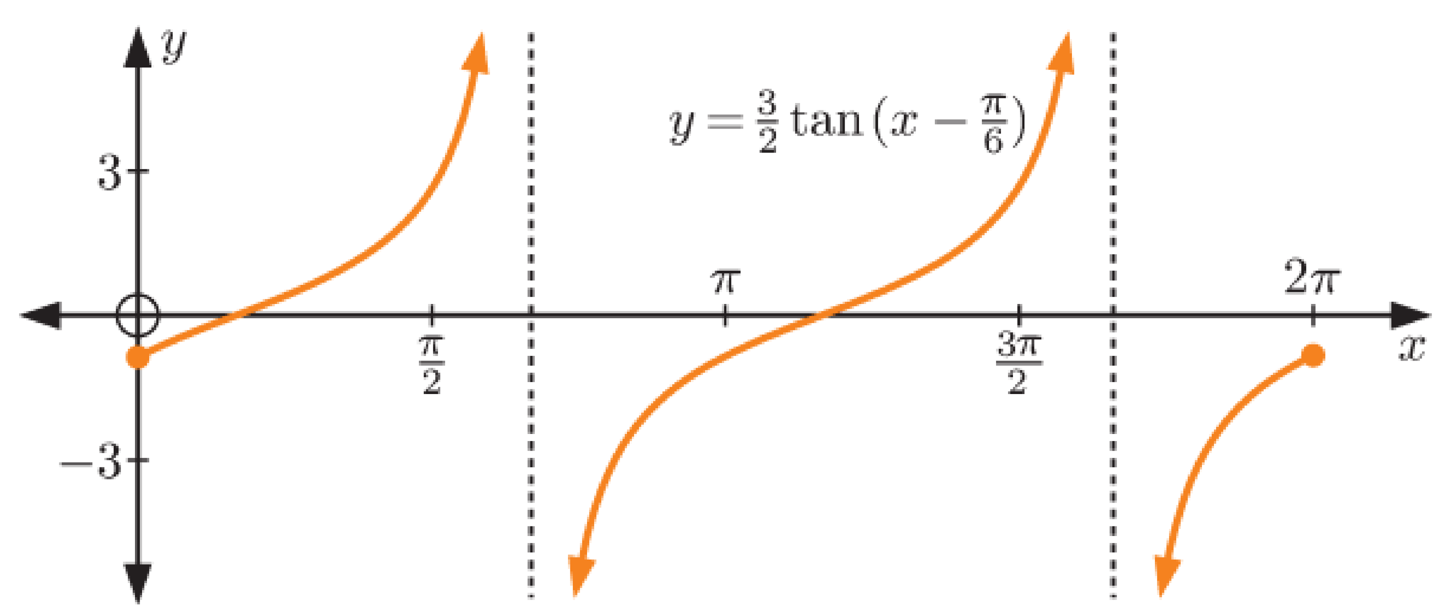
6 a $y = 5$ b $y = -4$

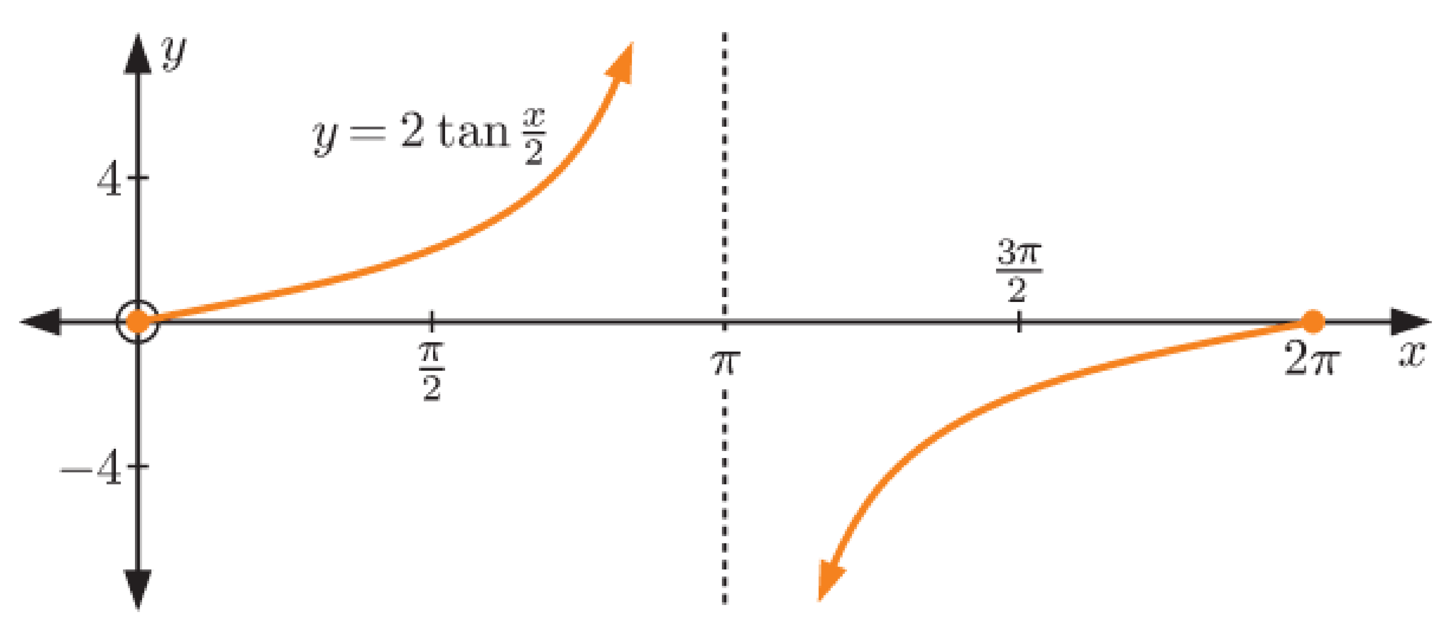
7 a


b


c


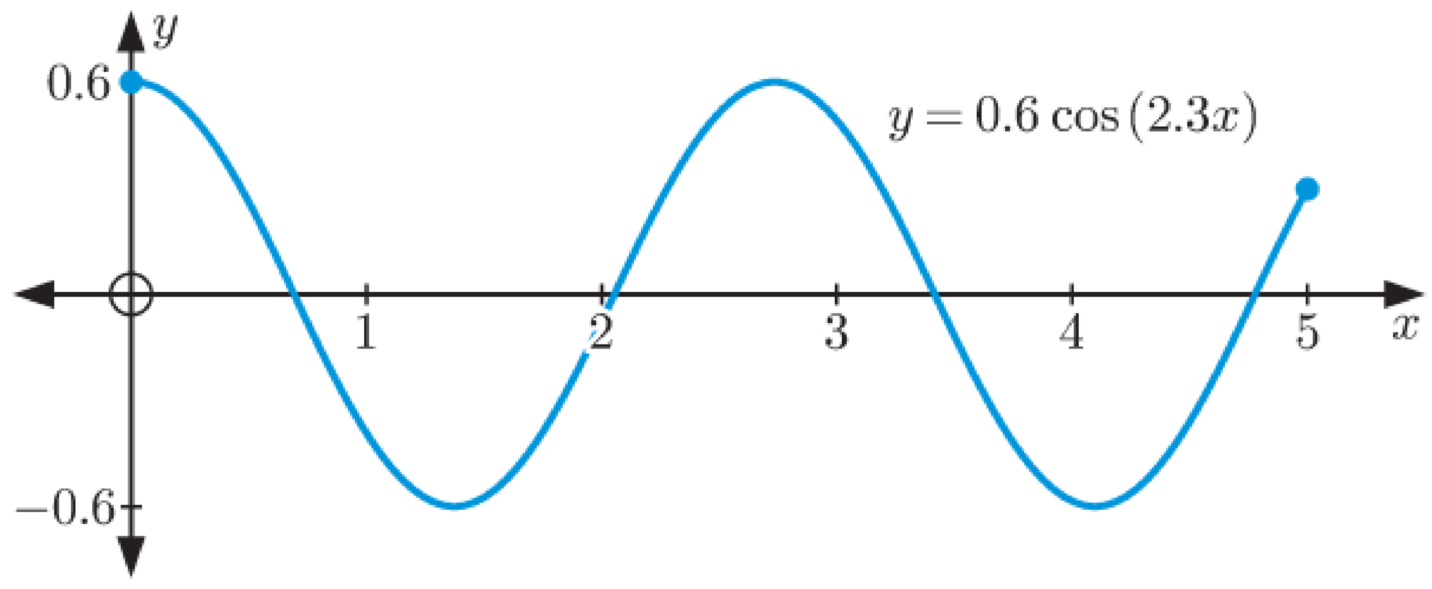
d


e


f


8 **a** $y = 4\sin x + 6$ **b** $y = 4\cos\left(x - \frac{\pi}{2}\right) + 6$

9

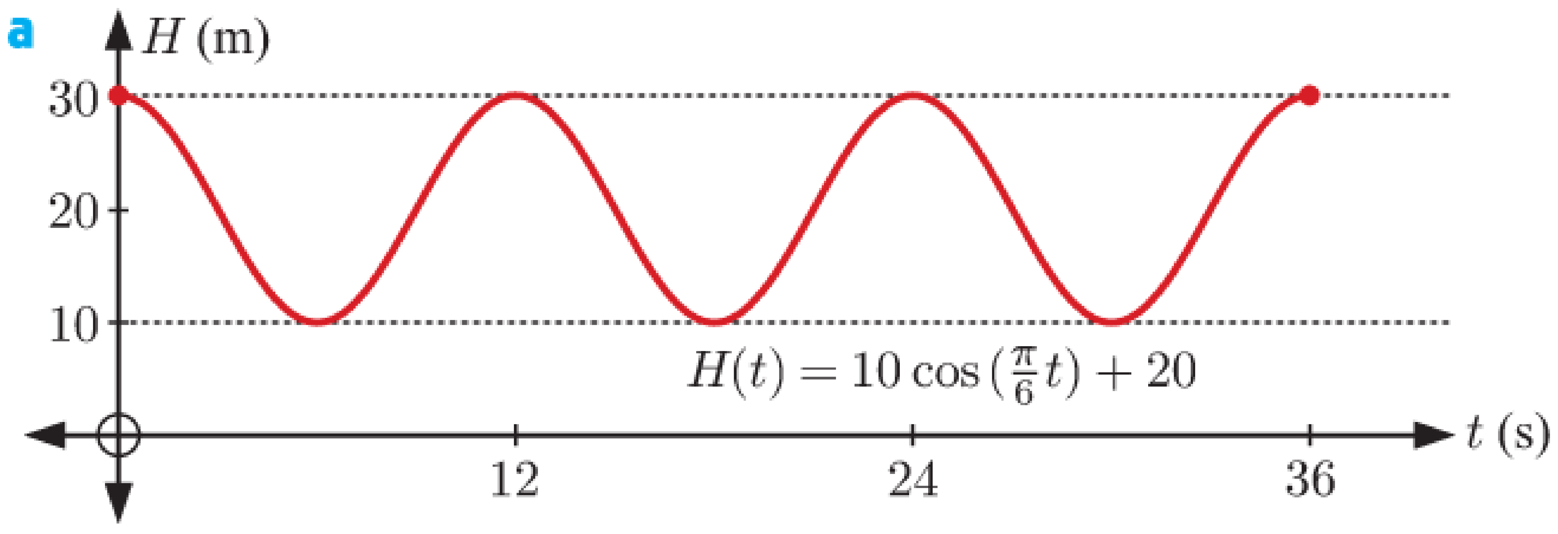


10 $a = \frac{3}{2}$, $b = -\frac{1}{2}$

11 **a** A reflection in the x-axis, then a horizontal stretch with scale factor $\frac{1}{2}$.

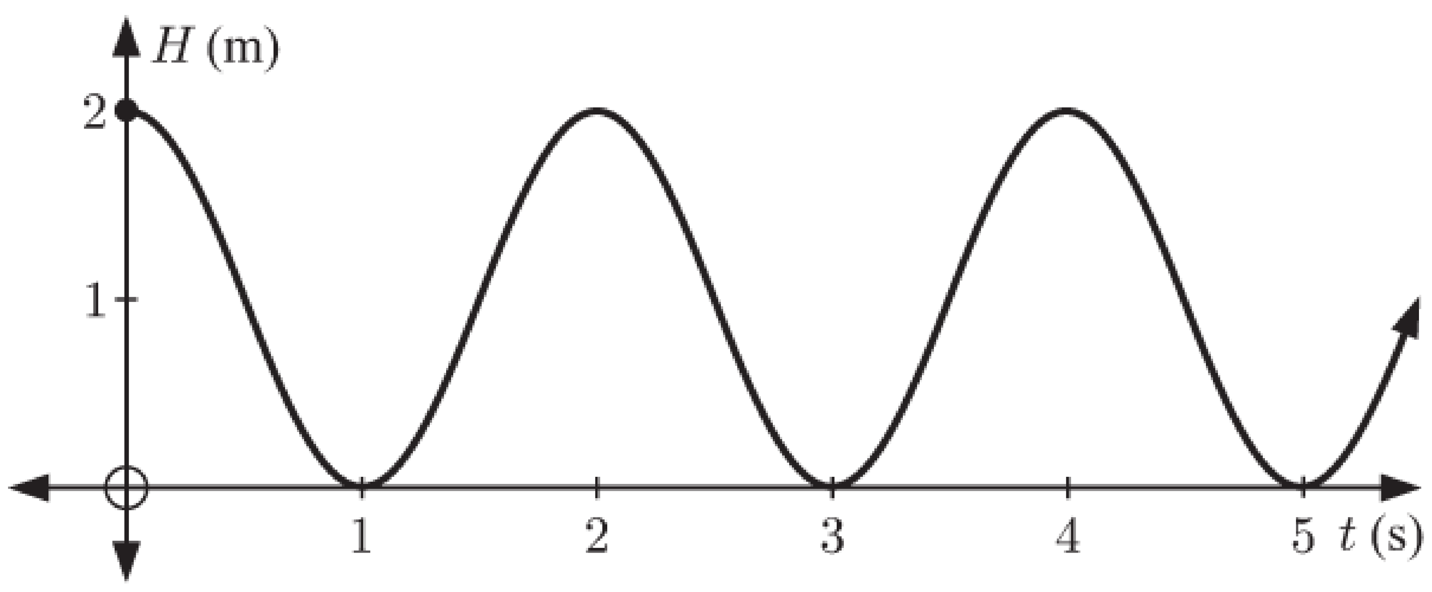
b A vertical stretch with scale factor 2, then a horizontal stretch with scale factor 2, then a translation $\frac{\pi}{2}$ units right and $\frac{1}{2}$ unit upwards.

12 **a**



b 20 m **c** 10 m **d** 12 seconds

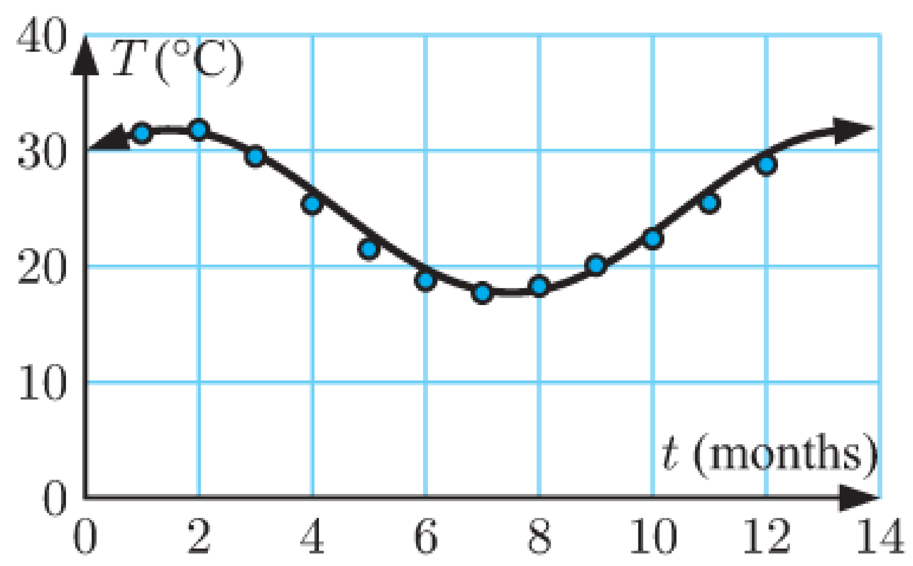
13 **a**



b $H(t) = \sin(\pi(t - 1.5)) + 1$

14 **a** $a \approx 7.05$, $b \approx \frac{\pi}{6}$, $c \approx 10.5$, $d \approx 24.75$

b



c Using technology, $T \approx 7.20\sin(0.488t - 1.08) + 24.7$.
The model fits reasonably well but not perfectly.

15 **a** $x \approx -6.1, -3.4$ **b** $x \approx 0.8$

16 **a** $x \approx 1.27, 5.02$ **b** $x \approx 1.09, 2.05$

17 **a** $x \approx 1.33, 4.47, 7.61$ **b** $x \approx 5.30$

c $x \approx 2.83, 5.97, 9.11$

18 **a** $x = \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{16\pi}{9}$, or $\frac{17\pi}{9}$ **b** $x = \frac{5\pi}{3}$

c $x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}$, or $\frac{19\pi}{12}$

19 **a** $x = 0, \pi$, or 2π **b** $x = \frac{\pi}{3}$ or $\frac{5\pi}{3}$

20 **a** $x = -\frac{\pi}{2}$ or $\frac{\pi}{2}$ **b** $x = -\frac{2\pi}{3}, -\frac{\pi}{6}, \frac{\pi}{3}$, or $\frac{5\pi}{6}$

c $x = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}$, or $\frac{2\pi}{3}$

21 **a** $\frac{4\pi}{9}, \frac{5\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}$ **b** $\frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}$

22 **a** 28 milligrams per m^3 **b** 8:00 am Monday