Mathematics
Higher level
Paper 1

Monday 13 May 2019 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

2219 – 7203

13 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

Let $\boldsymbol{a} = \begin{pmatrix} 2 \\ k \\ -1 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} -3 \\ k+2 \\ k \end{pmatrix}$, $k \in \mathbb{R}$.

Given that $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular, find the possible values of k.

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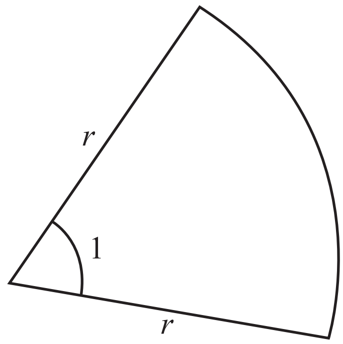
2. [Maximum mark: 4]

Determine the first three terms of $(1 - 2x)^{11}$ in ascending powers of x, giving each term in its simplest form.

3. [Maximum mark: 4]

A sector of a circle with radius r cm, where $r > 0$, is shown on the following diagram. The sector has an angle of 1 radian at the centre.



Let the area of the sector be $A\,\text{cm}^2$ and the perimeter be $P\,\text{cm}$. Given that $A = P$, find the value of r.

4. [Maximum mark: 7]

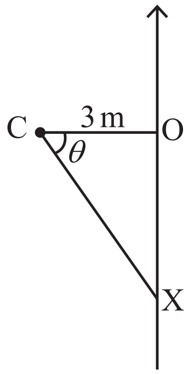
The lengths of two of the sides in a triangle are $4\,\text{cm}$ and $5\,\text{cm}$. Let θ be the angle between the two given sides. The triangle has an area of $\frac{5\sqrt{15}}{2}\,\text{cm}^2$.

(a) Show that $\sin\theta = \frac{\sqrt{15}}{4}$. [1]

(b) Find the two possible values for the length of the third side. [6]

5. [Maximum mark: 6]

A camera at point C is $3\,\mathrm{m}$ from the edge of a straight section of road as shown in the following diagram. The camera detects a car travelling along the road at $t = 0$. It then rotates, always pointing at the car, until the car passes O, the point on the edge of the road closest to the camera.



A car travels along the road at a speed of $24\,\mathrm{m\,s^{-1}}$. Let the position of the car be X and let $\mathrm{O\hat{C}X} = \theta$.

Find $\dfrac{\mathrm{d}\theta}{\mathrm{d}t}$, the rate of rotation of the camera, in radians per second, at the instant the car passes the point O.

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6. [Maximum mark: 7]

Let X be a random variable which follows a normal distribution with mean μ. Given that $\mathrm{P}(X < \mu - 5) = 0.2$, find

(a) $\mathrm{P}(X > \mu + 5)$; [2]

(b) $\mathrm{P}(X < \mu + 5 \mid X > \mu - 5)$. [5]

Please **do not** write on this page.

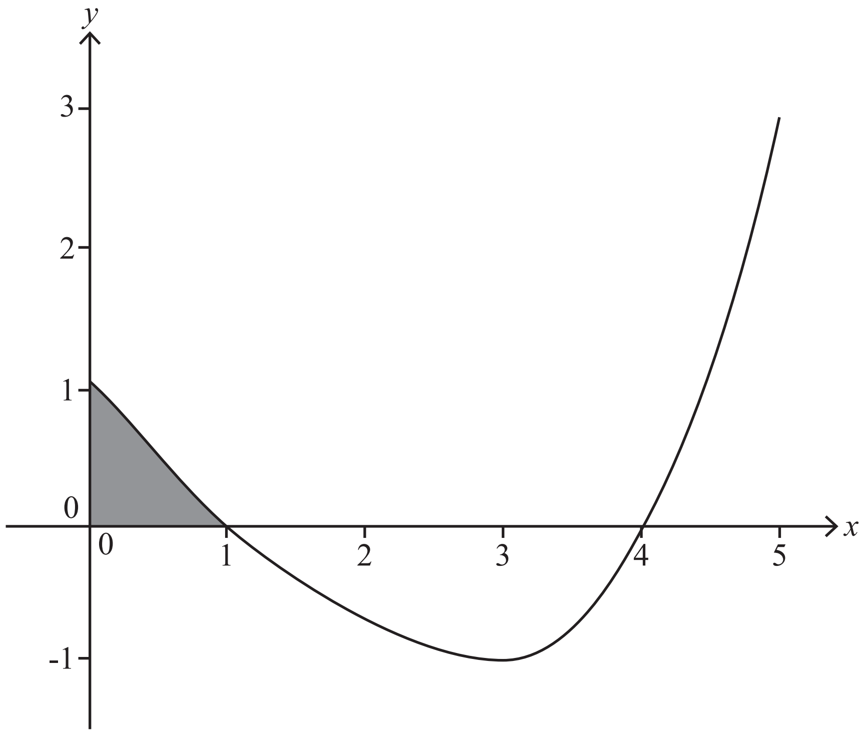
Answers written on this page
will not be marked.

7. [Maximum mark: 9]

Find the coordinates of the points on the curve $y^3 + 3xy^2 - x^3 = 27$ at which $\frac{dy}{dx} = 0$.

8. [Maximum mark: 9]

The graph of $y = f'(x)$, $0 \le x \le 5$ is shown in the following diagram. The curve intercepts the x-axis at $(1, 0)$ and $(4, 0)$ and has a local minimum at $(3, -1)$.



(a) Write down the x-coordinate of the point of inflexion on the graph of $y = f(x)$. [1]

The shaded area enclosed by the curve $y = f'(x)$, the x-axis and the y-axis is 0.5.
Given that $f(0) = 3$,

(b) find the value of $f(1)$. [3]

The area enclosed by the curve $y = f'(x)$ and the x-axis between $x = 1$ and $x = 4$ is 2.5.

(c) Find the value of $f(4)$. [2]

(d) Sketch the curve $y = f(x)$, $0 \le x \le 5$ indicating clearly the coordinates of the maximum and minimum points and any intercepts with the coordinate axes. [3]

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(Question 8 continued)

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

(a) Show that $(\sin x + \cos x)^2 = 1 + \sin 2x$. [2]

(b) Show that $\sec 2x + \tan 2x = \dfrac{\cos x + \sin x}{\cos x - \sin x}$. [4]

(c) Hence or otherwise find $\int_0^{\frac{\pi}{6}} (\sec 2x + \tan 2x)\,\mathrm{d}x$ in the form $\ln\left(a + \sqrt{b}\right)$ where $a, b \in \mathbb{Z}$. [9]

10. [Maximum mark: 15]

The function $p(x)$ is defined by $p(x) = x^3 - 3x^2 + 8x - 24$ where $x \in \mathbb{R}$.

(a) Find the remainder when $p(x)$ is divided by

(i) $(x - 2)$

(ii) $(x - 3)$. [3]

(b) Prove that $p(x)$ has only one real zero. [4]

(c) Write down the transformation that will transform the graph of $y = p(x)$ onto the graph of $y = 8x^3 - 12x^2 + 16x - 24$. [2]

The random variable X follows a Poisson distribution with a mean of λ and $6\mathrm{P}(X = 3) = 3\mathrm{P}(X = 2) - 2\mathrm{P}(X = 1) + 3\mathrm{P}(X = 0)$.

(d) Find the value of λ. [6]

Do **not** write solutions on this page.

11. [Maximum mark: 20]

Two distinct lines, l_1 and l_2, intersect at a point P. In addition to P, four distinct points are marked out on l_1 and three distinct points on l_2. A mathematician decides to join some of these eight points to form polygons.

(a) (i) Find how many sets of four points can be selected which can form the vertices of a quadrilateral.

(ii) Find how many sets of three points can be selected which can form the vertices of a triangle. [6]

The line l_1 has vector equation $\boldsymbol{r}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \lambda \in \mathbb{R}$ and the line l_2 has vector equation

$$\boldsymbol{r}_2 = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 6 \\ 2 \end{pmatrix}, \mu \in \mathbb{R}.$$

The point P has coordinates $(4, 6, 4)$.

(b) Verify that P is the point of intersection of the two lines. [3]

The point A has coordinates $(3, 4, 3)$ and lies on l_1.

(c) Write down the value of λ corresponding to the point A. [1]

The point B has coordinates $(-1, 0, 2)$ and lies on l_2.

(d) Write down $\overrightarrow{\mathrm{PA}}$ and $\overrightarrow{\mathrm{PB}}$. [2]

Let C be the point on l_1 with coordinates $(1, 0, 1)$ and D be the point on l_2 with parameter $\mu = -2$.

(e) Find the area of the quadrilateral CDBA. [8]

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