SECTION A

1. using the factor theorem or long division ***M1***
$-A+B-1+6=0 \Rightarrow A-B=5$ ***A1***
$8A+4B+2+6=0 \Rightarrow 2A+B=-2$ ***A1***
$3A=3 \Rightarrow A=1$ ***A1***
$B=-4$ ***A1*** ***N3***

> **Note:** Award ***M1A0A0A1A1*** for using $(x-3)$ as the third factor, without justification that the leading coefficient is 1.

[5 marks]

2. $g(x)=0$ or 3 ***(M1)(A1)***
$x=-1$ or 4 or 1 or 2 ***A1A1***

> **Notes:** Award ***A1A1*** for all four correct values,
> ***A1A0*** for two or three correct values,
> ***A0A0*** for less than two correct values.
>
> Award ***M1*** and corresponding ***A*** marks for correct attempt to find expressions for f and g.

[4 marks]

3. (a) for using normal vectors ***(M1)***

$$\begin{pmatrix}1\\2\\-1\end{pmatrix}\bullet\begin{pmatrix}1\\0\\1\end{pmatrix}=1-1=0$$ ***M1A1***

hence the two planes are perpendicular ***AG***

(b) **METHOD 1**

EITHER

$$\begin{vmatrix}\boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k}\\1 & 2 & -1\\1 & 0 & 1\end{vmatrix}=2\boldsymbol{i}-2\boldsymbol{j}-2\boldsymbol{k}$$ ***M1A1***

OR

if $\begin{pmatrix}a\\b\\c\end{pmatrix}$ is normal to π_3, then

$a+2b-c=0$ and $a+c=0$ ***M1***
a solution is $a=1$, $b=-1$, $c=-1$ ***A1***

THEN

π_3 has equation $x-y-z=d$ ***(M1)***
as it goes through the origin, $d=0$ so π_3 has equation $x-y-z=0$ ***A1***

> **Note:** The final ***(M1)A1*** are independent of previous working.

continued ...

Question 3 continued

METHOD 2

$$\boldsymbol{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$ ***A1(A1)A1A1***

[7 marks]

4. $2^{2x-2} = 2^x + 8$ ***(M1)***

$\frac{1}{4}2^{2x} = 2^x + 8$ ***(A1)***

$2^{2x} - 4 \times 2^x - 32 = 0$ ***A1***

$(2^x - 8)(2^x + 4) = 0$ ***(M1)***

$2^x = 8 \Rightarrow x = 3$ ***A1***

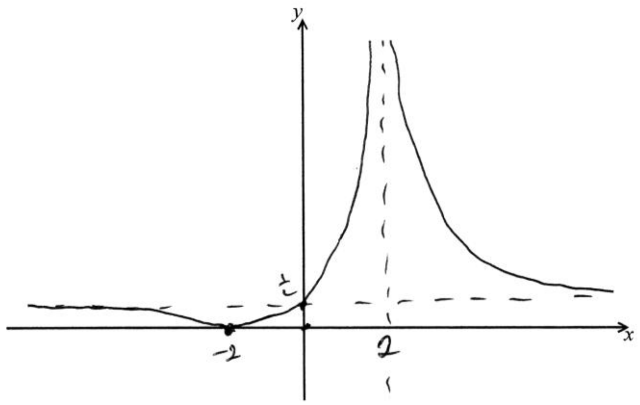
Notes:	Do not award final ***A1*** if more than 1 solution is given.

[5 marks]

5. (a) an attempt to use either asymptotes or intercepts ***(M1)***

$a = -2,\ b = 1,\ c = \frac{1}{2}$ ***A1A1A1***

(b)



A4

Note:	Award ***A1*** for both asymptotes, ***A1*** for both intercepts, ***A1***, ***A1*** for the shape of each branch, ignoring shape at $(x = -2)$.

[8 marks]

6. $(\boldsymbol{a}+\boldsymbol{b})\bullet(\boldsymbol{a}-\boldsymbol{b}) = \boldsymbol{a}\bullet\boldsymbol{a}+\boldsymbol{b}\bullet\boldsymbol{a}-\boldsymbol{a}\bullet\boldsymbol{b}-\boldsymbol{b}\bullet\boldsymbol{b}$ ***M1***

$= \boldsymbol{a}\bullet\boldsymbol{a}-\boldsymbol{b}\bullet\boldsymbol{b}$ ***A1***

$= |\boldsymbol{a}|^2 - |\boldsymbol{b}|^2 = 0$ since $|\boldsymbol{a}| = |\boldsymbol{b}|$ ***A1***

the **diagonals** are perpendicular ***R1***

> **Note:** Accept geometric proof, awarding ***M1*** for recognizing OACB is a rhombus, ***R1*** for a clear indication that $(\boldsymbol{a}+\boldsymbol{b})$ and $(\boldsymbol{a}-\boldsymbol{b})$ are the diagonals, ***A1*** for stating that diagonals cross at right angles and ***A1*** for "hence dot product is zero".
>
> Accept solutions using components in 2 or 3 dimensions.

[4 marks]

7. $\text{P(six in first throw)} = \frac{1}{6}$ ***(A1)***

$\text{P(six in third throw)} = \frac{25}{36}\times\frac{1}{6}$ ***(M1)(A1)***

$\text{P(six in fifth throw)} = \left(\frac{25}{36}\right)^2\times\frac{1}{6}$

$\text{P(A obtains first six)} = \frac{1}{6}+\frac{25}{36}\times\frac{1}{6}+\left(\frac{25}{36}\right)^2\times\frac{1}{6}+\ldots$ ***(M1)***

recognizing that the common ratio is $\frac{25}{36}$ ***(A1)***

$\text{P(A obtains first six)} = \frac{\frac{1}{6}}{1-\frac{25}{36}}$ (by summing the infinite GP) ***M1***

$= \frac{6}{11}$ ***A1***

[7 marks]

8. $\sqrt{x}\,\mathrm{e}^{x} = \mathrm{e}\sqrt{x} \Rightarrow x = 0 \text{ or } 1$ ***(A1)***

attempt to find $\int y^2\,\mathrm{d}x$ ***M1***

$V_1 = \pi\int_0^1 \mathrm{e}^2 x\,\mathrm{d}x$

$= \pi\left[\frac{1}{2}\mathrm{e}^2x^2\right]_0^1$

$= \frac{\pi\mathrm{e}^2}{2}$ ***A1***

$V_2 = \pi\int_0^1 x\mathrm{e}^{2x}\,\mathrm{d}x$

$= \pi\left(\left[\frac{1}{2}x\mathrm{e}^{2x}\right]_0^1 - \int_0^1 \frac{1}{2}\mathrm{e}^{2x}\,\mathrm{d}x\right)$ ***M1A1***

Note: Award ***M1*** for attempt to integrate by parts.

$= \frac{\pi\mathrm{e}^2}{2} - \pi\left[\frac{1}{4}\mathrm{e}^{2x}\right]_0^1$

finding difference of volumes ***M1***

volume $= V_1 - V_2$

$= \pi\left[\frac{1}{4}\mathrm{e}^{2x}\right]_0^1$

$= \frac{1}{4}\pi(\mathrm{e}^2 - 1)$ ***A1***

[7 marks]

9. (a) $u = \frac{1}{x} \Rightarrow \mathrm{d}u = -\frac{1}{x^2}\mathrm{d}x$ ***M1***

$\Rightarrow \mathrm{d}x = -\frac{\mathrm{d}u}{u^2}$ ***A1***

$\int_1^{\alpha} \frac{1}{1+x^2}\,\mathrm{d}x = -\int_1^{\frac{1}{\alpha}} \frac{1}{1+\left(\frac{1}{u}\right)^2}\frac{\mathrm{d}u}{u^2}$ ***A1M1A1***

Note: Award ***A1*** for correct integrand and ***M1A1*** for correct limits.

$= \int_{\frac{1}{\alpha}}^{1} \frac{1}{1+u^2}\,\mathrm{d}u$ (upon interchanging the two limits) ***AG***

(b) $\left[\arctan x\right]_1^{\alpha} = \left[\arctan u\right]_{\frac{1}{\alpha}}^{1}$ ***A1***

$\arctan\alpha - \frac{\pi}{4} = \frac{\pi}{4} - \arctan\frac{1}{\alpha}$ ***A1***

$\arctan\alpha + \arctan\frac{1}{\alpha} = \frac{\pi}{2}$ ***AG***

[7 marks]

10. **EITHER**

let $y_i = x_i - 12$

$\bar{x} = 10 \Rightarrow \bar{y} = -2$ ***M1A1***

$\sigma_x = \sigma_y = 3$ ***A1***

$\dfrac{\sum_{i=1}^{10} y_i^2}{10} - \bar{y}^2 = 9$ ***M1A1***

$\sum_{i=1}^{10} y_i^2 = 10(9+4) = 130$ ***A1***

OR

$\sum_{i=1}^{10}(x_i - 12)^2 = \sum_{i=1}^{10} x_i^2 - 24\sum_{i=1}^{10} x_i + 144\sum_{i=1}^{10} 1$ ***M1A1***

$\bar{x} = 10 \Rightarrow \sum_{i=1}^{10} x_i = 100$ ***A1***

$\sigma_x = 3,\ \dfrac{\sum_{i=1}^{10} x_i^2}{10} - \bar{x}^2 = 9$ ***(M1)***

$\Rightarrow \sum_{i=1}^{10} x_i^2 = 10(9+100)$ ***A1***

$\sum_{i=1}^{10}(x_i - 12)^2 = 1090 - 2400 + 1440 = 130$ ***A1***

[6 marks]

SECTION B

11. (a) $x^2+5x+4=0 \Rightarrow x=-1$ or $x=-4$ *(M1)*

so vertical asymptotes are $x=-1$ and $x=-4$ ***A1***

as $x \to \infty$ then $y \to 1$ so horizontal asymptote is $y=1$ ***(M1)A1***

[4 marks]

(b) $x^2-5x+4=0 \Rightarrow x=1$ or $x=4$ ***A1***

$x=0 \Rightarrow y=1$ ***A1***

so intercepts are $(1, 0)$, $(4, 0)$ and $(0, 1)$

[2 marks]

(c) (i) $f'(x)=\dfrac{(x^2+5x+4)(2x-5)-(x^2-5x+4)(2x+5)}{(x^2+5x+4)^2}$ ***M1A1A1***

$=\dfrac{10x^2-40}{(x^2+5x+4)^2} \quad \left(=\dfrac{10(x-2)(x+2)}{(x^2+5x+4)^2}\right)$ ***A1***

$f'(x)=0 \Rightarrow x=\pm 2$ ***M1***

so the points under consideration are $(-2, -9)$ and $\left(2, -\dfrac{1}{9}\right)$ ***A1A1***

looking at the sign either side of the points (or attempt to find $f''(x)$) ***M1***

e.g. if $x=-2^-$ then $(x-2)(x+2)>0$ and if $x=-2^+$ then $(x-2)(x+2)<0$,

therefore $(-2, -9)$ is a maximum ***A1***

(ii) *e.g.* if $x=2^-$ then $(x-2)(x+2)<0$ and if $x=2^+$ then $(x-2)(x+2)>0$,

therefore $\left(2, -\dfrac{1}{9}\right)$ is a minimum ***A1***

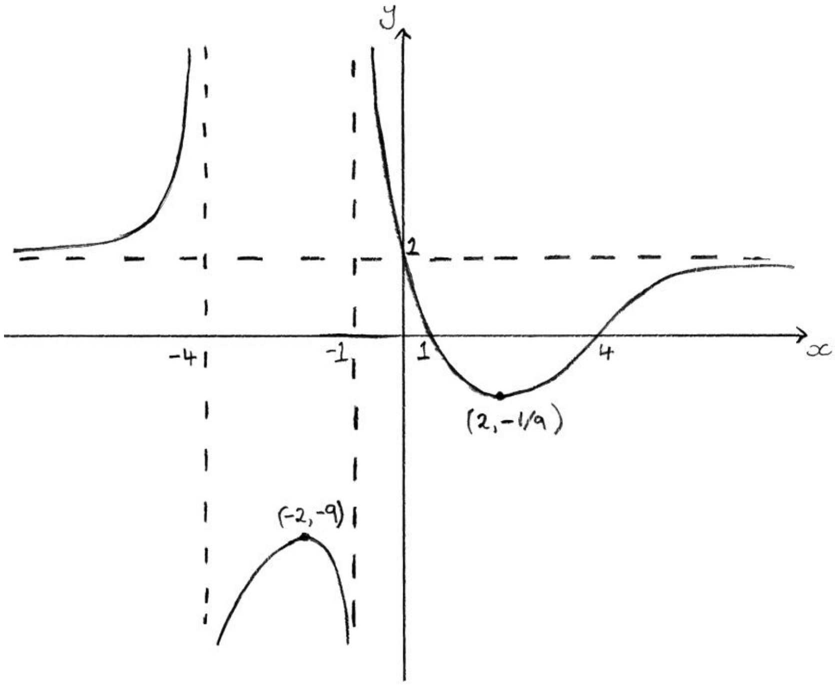
Note: Candidates may find the minimum first.

[10 marks]

continued …

Question 11 continued

(d)



A3

Note: Award ***A1*** for each branch consistent with and including the features found in previous parts.

[3 marks]

(e) one ***A1***

[1 mark]

Total [20 marks]

12. (a) $\int_0^1 a\mathrm{e}^{-ax}\,\mathrm{d}x = 1 - \frac{1}{\sqrt{2}}$ ***M1A1***

$\left[-\mathrm{e}^{-ax}\right]_0^1 = 1 - \frac{1}{\sqrt{2}}$ ***M1A1***

$-\mathrm{e}^{-a} + 1 = 1 - \frac{1}{\sqrt{2}}$ ***A1***

Note: Accept e^0 instead of 1.

$\mathrm{e}^{-a} = \frac{1}{\sqrt{2}}$

$\mathrm{e}^{a} = \sqrt{2}$

$a = \ln 2^{\frac{1}{2}}$ $\left(\text{accept } -a = \ln 2^{-\frac{1}{2}}\right)$ ***A1***

$a = \frac{1}{2}\ln 2$ ***AG***

[6 marks]

(b) $\int_0^M a\mathrm{e}^{-ax}\,\mathrm{d}x = \frac{1}{2}$ ***M1A1***

$\left[-\mathrm{e}^{-ax}\right]_0^M = \frac{1}{2}$ ***A1***

$-\mathrm{e}^{-Ma} + 1 = \frac{1}{2}$

$\mathrm{e}^{-Ma} = \frac{1}{2}$ ***A1***

$Ma = \ln 2$

$M = \frac{\ln 2}{a} = 2$ ***A1***

[5 marks]

continued …

Question 12 continued

(c) $\mathrm{P}(1 < X < 3) = \int_1^3 a\mathrm{e}^{-ax}\,\mathrm{d}x$ ***M1A1***

$= -\mathrm{e}^{-3a} + \mathrm{e}^{-a}$ ***A1***

$\mathrm{P}(X < 3 \mid X > 1) = \dfrac{\mathrm{P}(1 < X < 3)}{\mathrm{P}(X > 1)}$ ***M1A1***

$= \dfrac{-\mathrm{e}^{-3a} + \mathrm{e}^{-a}}{1 - \mathrm{P}(X < 1)}$ ***A1***

$= \dfrac{-\mathrm{e}^{-3a} + \mathrm{e}^{-a}}{\frac{1}{\sqrt{2}}}$ ***A1***

$= \sqrt{2}\,(-\mathrm{e}^{-3a} + \mathrm{e}^{-a})$

$= \sqrt{2}\left(-2^{-\frac{3}{2}} + 2^{-\frac{1}{2}}\right)$ ***A1***

$= \dfrac{1}{2}$ ***A1***

Note: Award full marks for $\mathrm{P}(X < 3 / X > 1) = \mathrm{P}(X < 2) = \dfrac{1}{2}$ or quoting properties of exponential distribution.

[9 marks]

Total [20 marks]

13. (a) $\sin\ (2n+1)x\ \cos x - \cos\ (2n+1)x\ \sin x = \sin\ (2n+1)x - x$ ***M1A1***

$= \sin 2nx$ ***AG***

[2 marks]

(b) if $n = 1$ ***M1***

$\text{LHS} = \cos x$

$\text{RHS} = \dfrac{\sin 2x}{2\sin x} = \dfrac{2\sin x \cos x}{2\sin x} = \cos x$ ***M1***

so $\text{LHS} = \text{RHS}$ and the statement is true for $n = 1$ ***R1***

assume true for $n = k$ ***M1***

> **Note:** Only award ***M1*** if the word **true** appears.
> Do **not** award ***M1*** for ‘let $n = k$ ’ only.
> Subsequent marks are independent of this ***M1***.

so $\cos x + \cos 3x + \cos 5x + ... + \cos(2k-1)x = \dfrac{\sin 2kx}{2\sin x}$

if $n = k+1$ then

$\cos x + \cos 3x + \cos 5x + ... + \cos(2k-1)x + \cos(2k+1)x$ ***M1***

$= \dfrac{\sin 2kx}{2\sin x} + \cos(2k+1)x$ ***A1***

$= \dfrac{\sin 2kx + 2\cos\ (2k+1)x\ \sin x}{2\sin x}$ ***M1***

$= \dfrac{\sin\ (2k+1)x\ \cos x - \cos\ (2k+1)x\ \sin x + 2\cos\ (2k+1)x\ \sin x}{2\sin x}$ ***M1***

$= \dfrac{\sin\ (2k+1)x\ \cos x + \cos\ (2k+1)x\ \sin x}{2\sin x}$ ***A1***

$= \dfrac{\sin\ (2k+2)x}{2\sin x}$ ***M1***

$= \dfrac{\sin\ 2(k+1)x}{2\sin x}$ ***A1***

so if true for $n = k$, then also true for $n = k+1$

as true for $n = 1$ then true for all $n \in \mathbb{Z}^+$ ***R1***

> **Note:** Final ***R1*** is independent of previous work.

[12 marks]

continued ...

Question 13 continued

(c) $\frac{\sin 4x}{2\sin x} = \frac{1}{2}$ ***M1A1***

$\sin 4x = \sin x$

$4x = x \Rightarrow x = 0$ but this is impossible

$4x = \pi - x \Rightarrow x = \frac{\pi}{5}$ ***A1***

$4x = 2\pi + x \Rightarrow x = \frac{2\pi}{3}$ ***A1***

$4x = 3\pi - x \Rightarrow x = \frac{3\pi}{5}$ ***A1***

for not including any answers outside the domain ***R1***

Note:	Award the first ***M1A1*** for correctly obtaining $8\cos^3 x - 4\cos x - 1 = 0$ or equivalent and subsequent marks as appropriate including the answers $\arccos\left(-\frac{1}{2}, \frac{1 \pm \sqrt{5}}{4}\right)$.

[6 marks]

Total [20 marks]