M10/5/MATHL/HP2/ENG/TZ1/XX/M+

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MARKSCHEME

May 2010

MATHEMATICS

Higher Level

Paper 2

Samples to team leaders	June 8 2010
Everything (marks, scripts etc) to IB Cardiff	June 15 2010

13 pages

SECTION A

1. $a = 3$ ***A1***

$c = 2$ ***A1***

period $= \frac{2\pi}{b} = 3$ ***(M1)***

$b = \frac{2\pi}{3}$ $(= 2.09)$ ***A1***

[4 marks]

2. **EITHER**

using row reduction (or attempting to eliminate a variable) ***M1***

$$\left(\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ 3 & 1 & 2 & -2 \\ -1 & 2 & a & b \end{array}\right) \begin{array}{l} \\ \to 2R2 - 3R1 \\ \to 2R3 + R1 \end{array}$$

$$\left(\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ 0 & 5 & -5 & -10 \\ 0 & 3 & 2a+3 & 2b+2 \end{array}\right) \to R2/5$$ ***A1***

Note: For an algebraic solution award ***A1*** for **two** correct equations in two variables.

$$\left(\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ 0 & 1 & -1 & -2 \\ 0 & 3 & 2a+3 & 2b+2 \end{array}\right) \to R3 - 3R2$$

$$\left(\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 2a+6 & 2b+8 \end{array}\right)$$

Note: Accept alternative correct row reductions.

recognition of the need for 4 zeroes ***M1***

so for multiple solutions $a = -3$ and $b = -4$ ***A1A1***

[5 marks]

OR

$$\begin{vmatrix} 2 & -1 & 3 \\ 3 & 1 & 2 \\ -1 & 2 & a \end{vmatrix} = 0$$ ***M1***

$\Rightarrow 2(a-4) + (3a+2) + 3(6+1) = 0$

$\Rightarrow 5a + 15 = 0$

$\Rightarrow a = -3$ ***A1***

$$\begin{vmatrix} 2 & -1 & 2 \\ 3 & 1 & -2 \\ -1 & 2 & b \end{vmatrix} = 0$$ ***M1***

$\Rightarrow 2(b+4) + (3b-2) + 2(6+1) = 0$ ***A1***

$\Rightarrow 5b + 20 = 0$

$\Rightarrow b = -4$ ***A1***

[5 marks]

3. $AC = AB = 10$ (cm) ***A1***
triangle OBC is equilateral ***(M1)***
$BC = 6$ (cm) ***A1***

EITHER

$B\hat{A}C = 2\arcsin\frac{3}{10}$ ***M1A1***

$B\hat{A}C = 34.9^\circ$ (accept 0.609 radians) ***A1***

OR

$\cos B\hat{A}C = \frac{10^2 + 10^2 - 6^2}{2 \times 10 \times 10} = \frac{164}{200}$ ***M1A1***

$B\hat{A}C = 34.9^\circ$ (accept 0.609 radians) ***A1***

Note: Other valid methods may be seen.

[6 marks]

4. (a) $z^3 = 2\sqrt{2}\,e^{\frac{3\pi i}{4}}$ ***(M1)(A1)***

$z_1 = \sqrt{2}\,e^{\frac{\pi i}{4}}$ ***A1***

adding or subtracting $\frac{2\pi i}{3}$ ***M1***

$z_2 = \sqrt{2}\,e^{\frac{\pi i}{4}+\frac{2\pi i}{3}} = \sqrt{2}\,e^{\frac{11\pi i}{12}}$ ***A1***

$z_3 = \sqrt{2}\,e^{\frac{\pi i}{4}-\frac{2\pi i}{3}} = \sqrt{2}\,e^{-\frac{5\pi i}{12}}$ ***A1***

Notes: Accept equivalent solutions *e.g.* $z_3 = \sqrt{2}\,e^{\frac{19\pi i}{12}}$

Award marks as appropriate for solving $(a + bi)^3 = -2 + 2i$.

Accept answers in degrees.

(b) $\sqrt{2}e^{\frac{\pi i}{4}}\left(= \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)\right)$ ***A1***

$= 1 + i$ ***AG***

Note: Accept geometrical reasoning.

[7 marks]

5. (a) $(\boldsymbol{A}+\boldsymbol{B})^2 = \boldsymbol{A}^2 + \boldsymbol{AB} + \boldsymbol{BA} + \boldsymbol{B}^2$ ***A2***

(b) $(\boldsymbol{A} - k\boldsymbol{I})^3 = \boldsymbol{A}^3 - 3k\boldsymbol{A}^2 + 3k^2\boldsymbol{A} - k^3\boldsymbol{I}$ ***A2***

(c) $\boldsymbol{CA} = \boldsymbol{B} \Rightarrow \boldsymbol{C} = \boldsymbol{BA}^{-1}$ ***A2***

Note: Award ***A1*** in parts (a) to (c) if error is correctly identified, but not corrected.

[6 marks]

6. $(100+101+102+...+999)-(102+105+...+999)$ ***(M1)***

$=\frac{900}{2}(100+999)-\frac{300}{2}(102+999)$ ***M1A1A1***

$=329\,400$ ***A1*** ***N5***

Note: A variety of other acceptable methods may be seen including for example $\frac{300}{2}(201+1995)$ or $\frac{600}{2}(100+998)$.

[5 marks]

7. (a) There are $3!$ ways of arranging the Mathematics books, $5!$ ways of arranging the English books and $4!$ ways of arranging the Science books. ***(A1)***

Then we have 4 types of books which can be arranged in $4!$ ways. ***(A1)***

$3!\times 5!\times 4\times 4!=414\,720$ ***(M1)A1***

(b) There are $3!$ ways of arranging the subject books, and for each of these there are 2 ways of putting the dictionary next to the Mathematics books. ***(M1)(A1)***

$3!\times 5!\times 4!\times 3!\times 2=207\,360$ ***A1***

[7 marks]

8. weight of glass $=X$

$X\sim \text{N}(160,\sigma^2)$

$\text{P}(X<160+14)=\text{P}(X<174)=0.75$ ***(M1)(A1)***

Note: $\text{P}(X<160-14)=\text{P}(X<146)=0.25$ can also be used.

$\text{P}\left(Z<\frac{14}{\sigma}\right)=0.75$ ***(M1)***

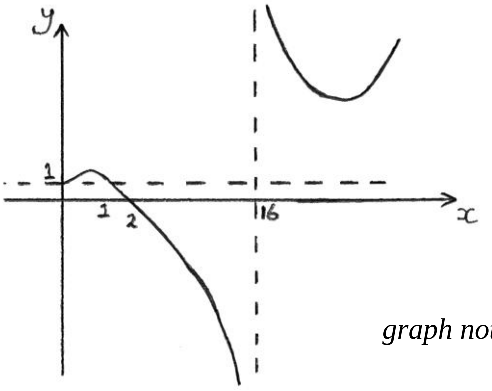
$\frac{14}{\sigma}=0.6745...$ ***(M1)(A1)***

$\sigma=20.8$ ***A1***

[6 marks]

9. (a) $x \geq 0$ and $x \neq 16$ ***A1A1***

(b)



graph not to scale

finding crossing points ***(M1)***

e.g. $4 - x^2 = 4 - \sqrt{x}$

$x = 0$ or $x = 1$ ***(A1)***

$0 \leq x \leq 1$ or $x > 16$ ***A1A1***

Note: Award ***M1A1A1A0*** for solving the inequality only for the case $x < 16$.

[6 marks]

10. $\left|\frac{3}{2}x - 3\right| = 0$ when $x = 2$ ***(A1)***

the equation of the parabola is $y = p(x-2)^2 - 3$ ***(M1)***

through $(0, 3) \Rightarrow 3 = 4p - 3 \Rightarrow p = \frac{3}{2}$ ***(M1)***

the equation of the parabola is $y = \frac{3}{2}(x-2)^2 - 3 \quad \left(= \frac{3}{2}x^2 - 6x + 3\right)$ ***A1***

area $= 2\int_0^2 \left(3 - \frac{3}{2}x\right) - \left(\frac{3}{2}x^2 - 6x + 3\right) \mathrm{d}x$ ***M1M1A1***

Note: Award ***M1*** for recognizing symmetry to obtain $2\int_0^2$, ***M1*** for the difference, ***A1*** for getting all parts correct.

$= \int_0^2 (-3x^2 + 9x)\,\mathrm{d}x$ ***A1***

[8 marks]

SECTION B

11. (a) **EITHER**

normal to plane given by

$$\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 2 & 3 & 2 \\ 6 & -3 & 2 \end{vmatrix}$$ ***M1A1***

$= 12\boldsymbol{i} + 8\boldsymbol{j} - 24\boldsymbol{k}$ ***A1***

equation of π is $3x + 2y - 6z = d$ ***(M1)***

as goes through $(-2, 3, -2)$ so $d = 12$ ***M1A1***

$\pi : 3x + 2y - 6z = 12$ ***AG***

OR

$x = -2 + 2\lambda + 6\mu$

$y = 3 + 3\lambda - 3\mu$

$z = -2 + 2\lambda + 2\mu$

eliminating μ

$x + 2y = 4 + 8\lambda$

$2y + 3z = 12\lambda$ ***M1A1A1***

eliminating λ

$3(x + 2y) - 2(2y + 3z) = 12$ ***M1A1A1***

$\pi : 3x + 2y - 6z = 12$ ***AG***

[6 marks]

(b) therefore $\text{A}(4, 0, 0)$, $\text{B}(0, 6, 0)$ and $\text{C}(0, 0, -2)$ ***A1A1A1***

Note: Award ***A1A1A0*** if position vectors given instead of coordinates.

[3 marks]

(c) area of base OAB $= \frac{1}{2} \times 4 \times 6 = 12$ ***M1***

$V = \frac{1}{3} \times 12 \times 2 = 8$ ***M1A1***

[3 marks]

(d) $\begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 3 = 7 \times 1 \times \cos\phi$ ***M1A1***

$\phi = \arccos\frac{3}{7}$

so $\theta = 90 - \arccos\frac{3}{7} = 25.4^\circ$ (accept 0.443 radians) ***M1A1***

[4 marks]

continued …

Question 11 continued

(e) $d = 4\sin\theta = \frac{12}{7}$ $(= 1.71)$ ***(M1)A1***

[2 marks]

(f) $8 = \frac{1}{3} \times \frac{12}{7} \times \text{area} \Rightarrow \text{area} = 14$ ***M1A1***

Note:	If answer to part (f) is found in an earlier part, award ***M1A1***, regardless of the fact that it has not come from their answers to part (c) and part (e).

[2 marks]

Total [20 marks]

12. (a) number of patients in 30 minute period $= X$

$X \sim \text{Po}(3)$ ***(A1)***

$\text{P}(X = 0) = 0.0498$ ***(M1)A1***

[3 marks]

(b) number of patients in working period $= Y$

$Y \sim \text{Po}(12)$ ***(A1)***

$\text{P}(X < 10) = \text{P}(X \le 9) = 0.242$ ***(M1)A1***

[3 marks]

(c) number of working period with less than 10 patients $= W$

$W \sim \text{B}(6, 0.2424...)$ ***(M1)(A1)***

$\text{P}(W \le 3) = 0.966$ ***(M1)A1***

[4 marks]

Note:	Accept exact answers in parts (a) to (c).

(d) number of patients in t minute interval $= X$

$X \sim \text{Po}(T)$

$\text{P}(X \ge 2) = 0.95$

$\text{P}(X = 0) + \text{P}(X = 1) = 0.05$ ***(M1)(A1)***

$\text{e}^{-T}(1 + T) = 0.05$ ***(M1)***

$T = 4.74$ ***(A1)***

$t = 47.4$ minutes ***A1***

[5 marks]

Total [15 marks]

13. (a) $O\hat{A}B = \pi - \theta$ (allied) *A1*

recognizing OAB as an isosceles triangle *M1*

so $A\hat{B}O = \pi - \theta$ *A1*

$B\hat{O}C = \pi - \theta$ (alternate) *AG*

Note: This can be done in many ways, including a clear diagram.

[3 marks]

(b) area of trapezium is $T = \text{area}_{\Delta BOC} + \text{area}_{\Delta AOB}$ *(M1)*

$$= \frac{1}{2}r^2 \sin(\pi - \theta) + \frac{1}{2}r^2 \sin(2\theta - \pi)$$ *M1A1*

$$= \frac{1}{2}r^2 \sin\theta - \frac{1}{2}r^2 \sin 2\theta$$ *AG*

[3 marks]

(c) (i) $\frac{dT}{d\theta} = \frac{1}{2}r^2 \cos\theta - r^2 \cos 2\theta$ *M1A1*

for maximum area $\frac{1}{2}r^2 \cos\theta - r^2 \cos 2\theta = 0$ *M1*

$\cos\theta = 2\cos 2\theta$ *AG*

(ii) $\theta_{max} = 2.205...$ *(A1)*

$\frac{1}{2}\sin\theta_{max} - \frac{1}{2}\sin\ 2\theta_{max}\ = 0.880$ *A1*

[5 marks]

Total [11 marks]

14. (a) $\frac{dv}{dt} = -\frac{v^2}{200} - 32 \left(= \frac{-v^2 - 6400}{200} \right)$ ***(M1)***

$\int_0^T dt = \int_{40}^{V} -\frac{200}{v^2 + 80^2} dv$ ***M1A1A1***

$T = 200 \int_V^{40} \frac{1}{v^2 + 80^2} dv$ ***AG***

[4 marks]

(b) (i) $a = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt}$ ***R1***

$= v \frac{dv}{ds}$ ***AG***

(ii) $v \frac{dv}{ds} = \frac{-v^2 - 80^2}{200}$ ***(M1)***

$\int_0^S ds = \int_{40}^{V} -\frac{200v}{v^2 + 80^2} dv$ ***M1A1A1***

$\int_0^S ds = \int_V^{40} \frac{200v}{v^2 + 80^2} dv$ ***M1***

$S = 200 \int_V^{40} \frac{v}{v^2 + 80^2} dv$ ***A1***

[7 marks]

(c) letting $V = 0$ ***(M1)***

distance $= 200 \int_0^{40} \frac{v}{v^2 + 80^2} dv = 22.3$ metres ***A1***

time $= 200 \int_0^{40} \frac{1}{v^2 + 80^2} dv = 1.16$ seconds ***A1***

[3 marks]

Total [14 marks]