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MARKSCHEME

May 2010

MATHEMATICS

Higher Level

Paper 2

Samples to team leaders	June 8 2010
Everything (marks, scripts etc) to IB Cardiff	June 15 2010

13 pages

SECTION A

1. (a) $18n-10$ (or equivalent) ***A1***

(b) $\sum_{1}^{n}(18r-10)$ (or equivalent) ***A1***

(c) by use of GDC or algebraic summation or sum of an AP ***(M1)***

$\sum_{1}^{15}(18r-10)=2010$ ***A1***

[4 marks]

2. (a) $p+q=0.44$ ***A1***

$2.5p+3.5q=1.25$ ***(M1)A1***

$p=0.29,\ q=0.15$ ***A1***

(b) use of $\text{Var}(X)=\text{E}(X^2)-\text{E}(X)^2$ ***(M1)***

$\text{Var}(X)=2.10$ ***A1***

[6 marks]

3. (a) required to solve $\text{P}\left(Z<\frac{21-15}{\sigma}\right)=0.8$ ***(M1)***

$\frac{6}{\sigma}=0.842...$ (or equivalent) ***(M1)***

$\Rightarrow \sigma=7.13$ (days) ***A1*** ***N1***

(b) $\text{P}(\text{survival after } 21 \text{ days})=0.337$ ***(M1)A1***

[5 marks]

4. (a) rewrite the equation as $(4x-1)\ln 2=(x+5)\ln 8+(1-2x)\log_2 16$ ***(M1)***

$(4x-1)\ln 2=(3x+15)\ln 2+4-8x$ ***(M1)(A1)***

$x=\frac{4+16\ln 2}{8+\ln 2}$ ***A1***

(b) $x=a^2$ ***(M1)***

$a=1.318$ ***A1***

Note:	Treat 1.32 as an ***AP***. Award ***A0*** for $\pm$.

[6 marks]

5. use of cosine rule: $\text{BC} = \sqrt{(8^2 + 7^2 - 2\times 7\times 8\cos 70)} = 8.6426...$ ***(M1)A1***

> **Note:** Accept an expression for BC^2.

$\text{BD} = 5.7617\ldots \quad (\text{CD} = 2.88085\ldots)$ ***A1***

use of sine rule: $\hat{B} = \arcsin\left(\dfrac{7\sin 70}{\text{BC}}\right) = 49.561...^\circ \quad (\hat{C} = 60.4387...^\circ)$ ***(M1)A1***

use of cosine rule: $\text{AD} = \sqrt{8^2 + \text{BD}^2 - 2\times \text{BD}\times 8\cos B} = 6.12\ (\text{cm})$ ***A1***

> **Note:** Scale drawing method not acceptable.

[6 marks]

6. (a) required to solve $e^{-\lambda} + \lambda e^{-\lambda} = 0.123$ ***M1A1***
solving to obtain $\lambda = 3.63$ ***A2*** ***N2***

> **Note:** Award ***A2*** if an additional negative solution is seen but ***A0*** if only a negative solution is seen.

(b) $\text{P}(0 < X < 9)$
$= \text{P}(X \le 8) - \text{P}(X = 0)$ (or equivalent) ***(M1)***
$= 0.961$ ***A1***

[6 marks]

7. (a) use GDC or manual method to find a, b and c ***(M1)***
obtain $a = 2,\ b = -1,\ c = 3$ (in any identifiable form) ***A1***

(b) use GDC or manual method to solve second set of equations ***(M1)***

obtain $x = \dfrac{4-11t}{2};\ y = \dfrac{-7t}{2};\ z = t$ (or equivalent) ***(A1)***

$\boldsymbol{r} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} -5.5 \\ -3.5 \\ 1 \end{pmatrix}$ (accept equivalent vector forms) ***M1A1***

> **Note:** Final ***A1*** requires $\boldsymbol{r} =$ or equivalent.

[6 marks]

8. (a) the expression is

$\dfrac{n!}{(n-3)!\,3!} - \dfrac{(2n)!}{(2n-2)!\,2!}$ ***(A1)***

$\dfrac{n(n-1)(n-2)}{6} - \dfrac{2n(2n-1)}{2}$ ***M1A1***

$= \dfrac{n(n^2 - 15n + 8)}{6} \quad \left(= \dfrac{n^3 - 15n^2 + 8n}{6}\right)$ ***A1***

(b) the inequality is

$\dfrac{n^3 - 15n^2 + 8n}{6} > 32n$

attempt to solve cubic inequality or equation ***(M1)***

$n^3 - 15n^2 - 184n > 0 \quad n(n-23)(n+8) > 0$

$n > 23 \quad (n \ge 24)$ ***A1***

[6 marks]

9. (a) using de Moivre's theorem

$z^n + \frac{1}{z^n} = \cos n\theta + \mathrm{i}\sin n\theta + \cos n\theta - \mathrm{i}\sin n\theta \quad (= 2\cos n\theta)$, imaginary part of which is 0

M1A1

so $\mathrm{Im}\left(z^n + \frac{1}{z^n}\right) = 0$ ***AG***

(b) $\frac{z-1}{z+1} = \frac{\cos\theta + \mathrm{i}\sin\theta - 1}{\cos\theta + \mathrm{i}\sin\theta + 1}$

$= \frac{(\cos\theta - 1 + \mathrm{i}\sin\theta)(\cos\theta + 1 - \mathrm{i}\sin\theta)}{(\cos\theta + 1 + \mathrm{i}\sin\theta)(\cos\theta + 1 - \mathrm{i}\sin\theta)}$ ***M1A1***

Note: Award ***M1*** for an attempt to multiply numerator and denominator by the complex conjugate of their denominator.

$\Rightarrow \mathrm{Re}\left(\frac{z-1}{z+1}\right) = \frac{(\cos\theta - 1)(\cos\theta + 1) + \sin^2\theta}{\text{real denominator}}$ ***M1A1***

Note: Award ***M1*** for multiplying out the numerator.

$= \frac{\cos^2\theta + \sin^2\theta - 1}{\text{real denominator}}$ ***A1***

$= 0$ ***AG***

[7 marks]

10. (a) the distance of the spot from P is $x = 500\tan\theta$ ***A1***

the speed of the spot is

$\frac{\mathrm{d}x}{\mathrm{d}t} = 500\sec^2\theta\frac{\mathrm{d}\theta}{\mathrm{d}t} \quad (= 4000\pi\sec^2\theta)$ ***M1A1***

when $x = 2000$, $\sec^2\theta = 17$ $(\theta = 1.32581...)\left(\frac{\mathrm{d}\theta}{\mathrm{d}t} = 8\pi\right)$

$\Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t} = 500 \times 17 \times 8\pi$ ***M1A1***

speed is $214\,000$ (metres per minute) ***AG***

Note: If their displayed answer does not round to 214 000, they lose the final ***A1***.

(b) $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 8000\pi\sec^2\theta\tan\theta\frac{\mathrm{d}\theta}{\mathrm{d}t}$ **or** $500 \times 2\sec^2\theta\tan\theta\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2$ ***M1A1***

$\left(\text{since } \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = 0\right)$

$= 43\,000\,000 \quad (= 4.30 \times 10^7)$ (metres per minute^2) ***A1***

[8 marks]

SECTION B

11. (a) solving to obtain one root: $1, -2$ or -5 ***A1***
obtain other roots ***A1***

[2 marks]

(b) $D = x \in [-5, -2] \cup [1, \infty)$ (or equivalent) ***M1A1***

Note: ***M1*** is for 1 finite and 1 infinite interval.

[2 marks]

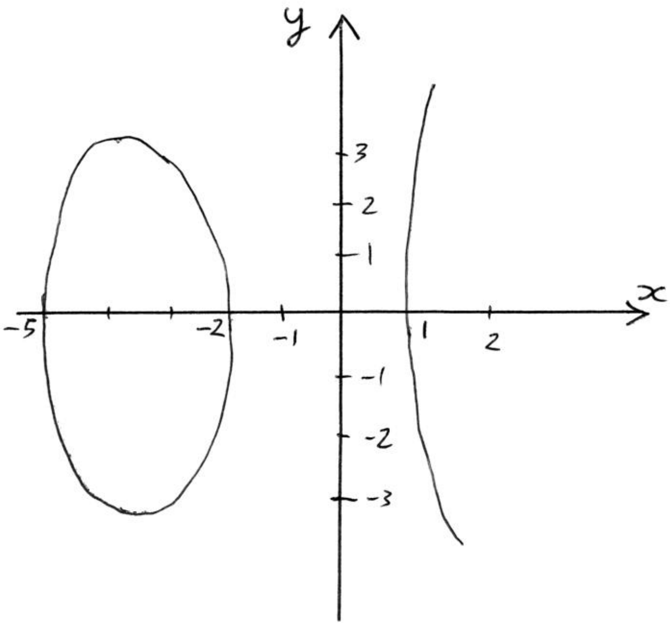
(c) coordinates of local maximum -3.73 $-2-\sqrt{3}$, 3.22 $\sqrt{6\sqrt{3}}$ ***A1A1***

[2 marks]

(d) use GDC to obtain one root: $1.41, -3.18$ or -4.23 ***A1***
obtain other roots ***A1***

[2 marks]

(e)



A1A1A1

Note: Award ***A1*** for shape, ***A1*** for max and for min clearly in correct places, ***A1*** for all intercepts. Award ***A1A0A0*** if only the complete top half is shown.

[3 marks]

(f) required area is twice that of $y = f(x)$ between -5 and -2 ***M1A1***
answer 14.9 ***A1*** ***N3***

Note: Award ***M1A0A0*** for $\int_{-5}^{-2} f(x)\,dx = 7.47...$ or ***N1*** for 7.47.

[3 marks]

Total [14 marks]

12. (a) (i) the median height is 1.18 ***A1***

(ii) the interquartile range is UQ – LQ

$= 1.22 - 1.13 = 0.09$ (accept answers that round to 0.09) ***A1A1***

Note: Award ***A1*** for the quartiles, ***A1*** for final answer.

[3 marks]

(b) (i)

$1.00 < h \le 1.05$	$1.05 < h \le 1.10$	$1.10 < h \le 1.15$	$1.15 < h \le 1.20$	$1.20 < h \le 1.25$	$1.25 < h \le 1.30$
5	9	13	24	19	10

A1A1

Note: Award ***A1*** for entries within ± 1 of the above values and ***A1*** for a total of 80.

(ii) unbiased estimate of the population mean

$$\left(\frac{5\times 1.025 + 9\times 1.075 + 13\times 1.125 + 24\times 1.175 + 19\times 1.225 + 10\times 1.275}{80}\right) = 1.17$$ ***A1***

unbiased estimate of the population variance

use of $s^2_{n-1} = \left(\frac{n}{n-1}\right) s^2_n$ or GDC ***(M1)***

obtain 0.00470 ***A1***

[5 marks]

(c) (i) $\mathrm{P}(h \le 1.15 \text{ m}) = \frac{27}{80}$ (0.3375 or 0.338) $\left(\text{allow } \frac{26}{80} \text{ (0.325)}\right)$ ***A1***

(ii) use of the conditional probability formula $\mathrm{P}(A \mid B) = \mathrm{P}(A \cap B) / \mathrm{P}(B)$ ***(M1)***

obtain $\frac{18}{80} \div \frac{27}{80}$ ***(A1)(A1)***

$= \frac{2}{3}$ (0.667) $\left(\text{allow } \frac{18}{26} \text{ (0.692)}\right)$ ***A1***

[5 marks]

Total [13 marks]

13. (a) the area of the first sector is $\frac{1}{2}2^2\theta$ ***(A1)***

the sequence of areas is $2\theta, 2k\theta, 2k^2\theta\ldots$ ***(A1)***

the sum of these areas is $2\theta(1+k+k^2+\ldots)$ ***(M1)***

$=\frac{2\theta}{1-k}=4\pi$ ***M1A1***

hence $\theta=2\pi(1-k)$ ***AG***

Note:	Accept solutions where candidates deal with angles instead of area.

[5 marks]

(b) the perimeter of the first sector is $4+2\theta$ ***(A1)***

the perimeter of the third sector is $4+2k^2\theta$ ***(A1)***

the given condition is $4+2k^2\theta=2+\theta$ ***M1***

which simplifies to $2=\theta(1-2k^2)$ ***A1***

eliminating θ, obtain cubic in k: $\pi(1-k)(1-2k^2)-1=0$ ***A1***

or equivalent

solve for $k=0.456$ and then $\theta=3.42$ ***A1A1***

[7 marks]

Total [12 marks]

14. (a) $g \circ f(x) = \frac{1}{1+e^x}$ ***A1***

$1 < 1 + e^x < \infty$ ***(M1)***

range $g \circ f$ is $]0, 1[$ ***A1*** ***N3***

[3 marks]

(b)

Note: Interchange of variables and rearranging can be done in either order.

attempt at solving $y = \frac{1}{1+e^x}$ ***M1***

rearranging

$e^x = \frac{1-y}{y}$ ***M1***

$(g \circ f)^{-1}(x) = \ln\left(\frac{1-x}{x}\right)$ ***A1***

Note: The ***A1*** is for RHS.

domain is $]0, 1[$ ***A1***

Note: Final ***A1*** is independent of the ***M*** marks.

[4 marks]

(c) (i) $y = f \circ g \circ h = 1 + e^{\cos x}$ ***M1A1***

$\frac{dy}{dx} = -\sin x e^{\cos x}$ ***M1A1***

$= (1-y)\sin x$ ***AG***

Note: Second ***M1A1*** could also be obtained by solving the differential equation.

(ii) **EITHER**

rearranging

$y \sin x = \sin x - \frac{dy}{dx}$ ***A1***

$\int y \sin x \, dx = \int \sin x \, dx - \int \frac{dy}{dx} dx$ ***M1***

$= -\cos x - y(+c)$ ***A1***

$= -\cos x - e^{\cos x}(+d)$ ***A1***

OR

$\int y \sin x \, dx = \int (1 + e^{\cos x}) \sin x \, dx$ ***A1***

$= \int \sin x \, dx + \int \sin x \times e^{\cos x} \, dx$

Note: Either the first or second line gains the ***A1***.

$= -\cos x - e^{\cos x}(+d)$ ***A1M1A1***

continued …

Question 14 continued

(iii) use of definition of y and the differential equation or GDC to identify first minimum at $x = \pi$ (3.14...) ***(M1)A1***

EITHER

the required integral is

$\pi \int_{y_{\min}}^{y_{\max}} x^2 \mathrm{d}y$ ***M1A1***

Note: $y_{\max} = 1 + \mathrm{e}$ and $y_{\min} = 1 + \mathrm{e}^{-1}$ but these do not need to be specified.

$= \pi \int_{\pi}^{0} -x^2 \sin x \, \mathrm{e}^{\cos x} \mathrm{d}x = \pi \times 4.32... = 13.6$ ***(M1)A1***

OR

the required integral is

$\pi \int_{1+\mathrm{e}^{-1}}^{1+\mathrm{e}} x^2 \, \mathrm{d}y$ ***M1A1***

$= \pi \int_{1+\mathrm{e}^{-1}}^{1+\mathrm{e}} \left(\arccos\left(\ln(y-1)\right)\right)^2 \mathrm{d}y = \pi \times 4.32... = 13.6$ ***M1A1***

Note: $1 + \mathrm{e} = 3.7182...$ and $1 + \mathrm{e}^{-1} = 1.3678...$

[14 marks]

Total [21 marks]