Name:

Mathematics Analysis & Approaches
Higher level
Paper 1

December 21, 2020 (afternoon)

2 hours

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the space provided.
- Section B: answer all questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the Mathematics Analysis & Approaches formula booklet is required for this paper.
- The maximum mark for this examination paper is [**110 marks**].

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working

Section A

Answer **all** questions in the space provided.

1. [Maximum mark: 5]

$x+2, x-2$ and $\dfrac{x}{6}$ are the first three terms of a geometric sequence.

Find the possible values of x and the sum to infinity of each sequence.

2. [Maximum mark: 5]

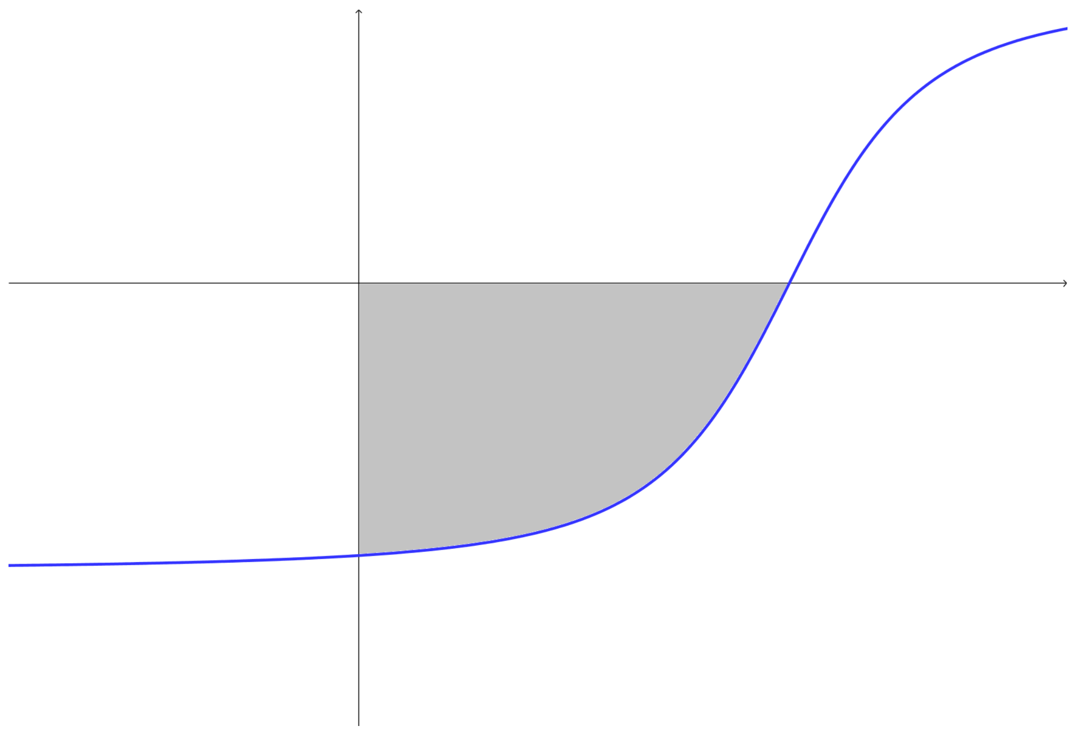
Let $f(x) = \dfrac{x+1}{x-3}$ and $g(x) = 2^x$.

(a) Find $f^{-1}(x)$. [3]

(b) Calculate $(f \circ g)(-1)$. [2]

3. [Maximum mark: 6]

Graph of the function $f(x) = \dfrac{2x-6}{\sqrt{x^2-6x+10}}$ is shown below. The region enclosed by the graph and the axes is shaded. Find the area of this region.



6. [Maximum mark: 4]

Two of the roots of the polynomial:

$$P(x) = 2x^3 + px^2 + qx + r$$

are $\frac{1}{2}$ and $2 + i\sqrt{3}$. Find the values of p, q and r.

5. [Maximum mark: 5]

Differentiate the function $f(x) = 2x^3 + x$ from first principles.

6. [Maximum mark: 8]

If $x^2 + y^2 = 9$, show that $\dfrac{d^2y}{dx^2} = -\dfrac{9}{y^3}$ (for $y \neq 0$).

7. [Maximum mark: 7]

(a) Prove that $\sin 3\theta = \sin\theta(3 - 4\sin^2\theta)$ [3]

(b) Hence, or otherwise, solve:

$$\sin 3\theta + 4\sin^2\theta = 0$$

for $0 \leqslant \theta \leqslant 2\pi$. [4]

8. [Maximum mark: 5]

(a) Find the first three terms of the binomial expansion of $(1+x)^{-\frac{1}{2}}$.

(b) Hence, or otherwise, find the first three terms of the expansion of $\dfrac{2+3x}{\sqrt{1+x}}$.

9. [Maximum mark: 10]

A box contains 3 red and 2 blue balls. Two balls are picked at random one after the other without replacement.

(a) Find the probability that both balls are red. [2]

(b) Find the probability that both balls are blue. [1]

(c) Write down the probability that the balls are of different colour. [1]

Tomasz plays the following game. He picks two balls from the box at random one after the other without replacement. If they are both red, the game ends and he wins. If they are both blue, the game ends and he loses. If they are of different colour, then they're returned to the box and the Tomasz plays again.

(d) Find the probability that Tomasz wins. [4]

(e) Find the probability that Tomasz wins, given that the first ball he picked was blue. [2]

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 19]

A function f is defined by

$$f(x) = \frac{x^2 + x - 2}{x^2 - 2x - 3}$$

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(a) State the largest possible domain of f. [1]

(b) Find $f'(x)$ and show that the derivative has a constant sign over its domain. [4]

(c) Write down the equations of the asymptotes of the graph of $y = f(x)$. [3]

(d) Sketch the graph of $y = f(x)$. [3]

(e) Find real numbers P, Q and R such that

$$f(x) \equiv P + \frac{Q}{x+1} + \frac{R}{x-3}$$

[4]

(f) Find the exact value of the area of the region enclosed by the axes and the graph of $y = f(x)$. [4]

11. [Maximum mark: 17]

Consider the plane Π with the equation $2x + 3y - z = 11$ and the line l with the equation:

$$2 - x = \frac{y-1}{2} = \frac{z-3}{5}$$

(a) Show that the point $A(1, 3, 8)$ lies on the line l, but does not lie on the plane Π. [3]

(b) Find the coordinates of point B, the intersection between line l and plane Π. [3]

(c) Find the equation of line k containing A and perpendicular to Π. [3]

(d) Find the point C symmetric to A about Π. [4]

(e) Find the Cartesian equation of the line m which is the symmetric image of l about the plane Π. [4]

12. [Maximum mark: 19]

(a) Use de Moivre's theorem to show that, if $z = \cos\theta + i\sin\theta$, then $z^n + \dfrac{1}{z^n} = 2\cos n\theta$. [3]

(b) By considering the binomial expansion of $\left(z + \dfrac{1}{z}\right)^4$, show that

$$\cos^4\theta \equiv \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$$

[5]

(c) Hence find the exact value of $\int_0^{\frac{\pi}{6}} \cos^4\theta d\theta$. [4]

(d) Show that $\cos^4\theta - \sin^4\theta \equiv \cos 2\theta$. [3]

(e) Using parts (c) and (d) find the exact value of $\int_0^{\frac{\pi}{6}} \sin^4\theta d\theta$. [4]