Name:

Mathematics Analysis & Approaches
Higher level
Paper 2

December 22, 2020 (morning)

2 hours

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the space provided.
- Section B: answer all questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the Mathematics Analysis & Approaches formula booklet is required for this paper.
- The maximum mark for this examination paper is [**110 marks**].

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working

Section A

Answer **all** questions in the space provided.

1. [Maximum mark: 4]

Triangle ABC has $\angle ACB = 42^\circ$, $BC = 1.74\ cm$ and area $1.19\ cm^2$.

(a) Find AC [2]

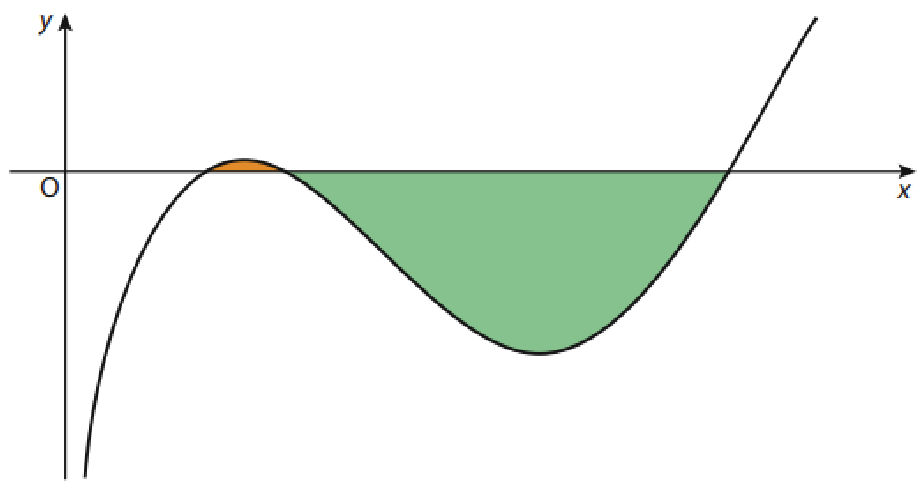
(b) Find AB [2]

2. [Maximum mark: 5]

A bag contains 2 red balls, 3 blue balls and 4 green balls. A ball is chosen at random from the bag and is not replaced. A second ball is chosen. Find the probability of choosing one green ball and one blue ball in any order.

3. [Maximum mark: 5]

The diagram below shows the graph of $y = \cos x \ln x$ for $0 \leqslant x \leqslant 2\pi$.



Find the ratio of the shaded area above the x-axis to the shaded area below the x-axis.

4. [Maximum mark: 4]

A geometric sequence has a first term of 2 and a common ratio of 1.05. Find the least value of n so that the n-th term is greater than 500.

5. [Maximum mark: 5]

Two ships, A and B, are observed from an origin O. Relative to O, their position vectors at time t hours after midday are given by:

$$r_A = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + t\begin{pmatrix} 5 \\ 8 \end{pmatrix}$$
$$r_B = \begin{pmatrix} 7 \\ -3 \end{pmatrix} + t\begin{pmatrix} 0 \\ 12 \end{pmatrix}$$

where distances are measured in kilometres.
Find the minimum distance between the two ships.

6. [Maximum mark: 5]

Find the values of a and b, where $a, b \in \mathbb{R}$, given that:

$$(a + bi)(2 - i) = 5 - i$$

7. [Maximum mark: 6]

There are 10 seats in a row in a waiting room. There are 6 people in the room.

(a) In how many different way can they be seated? [2]

(b) In this group of six people, there are 3 sisters who must sit next to each other. In how many different ways can the group be seated? [4]

8. [Maximum mark: 7]

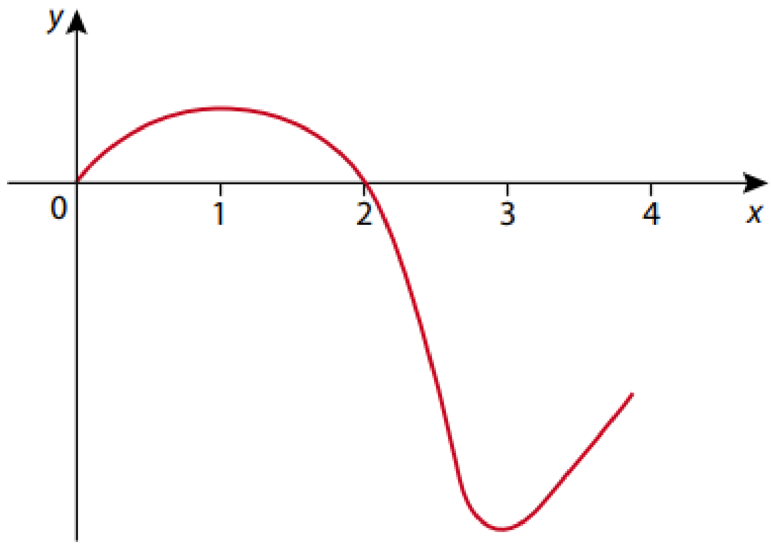
Find the equation of the tangent line to the curve

$$\ln(xy) = 2x$$

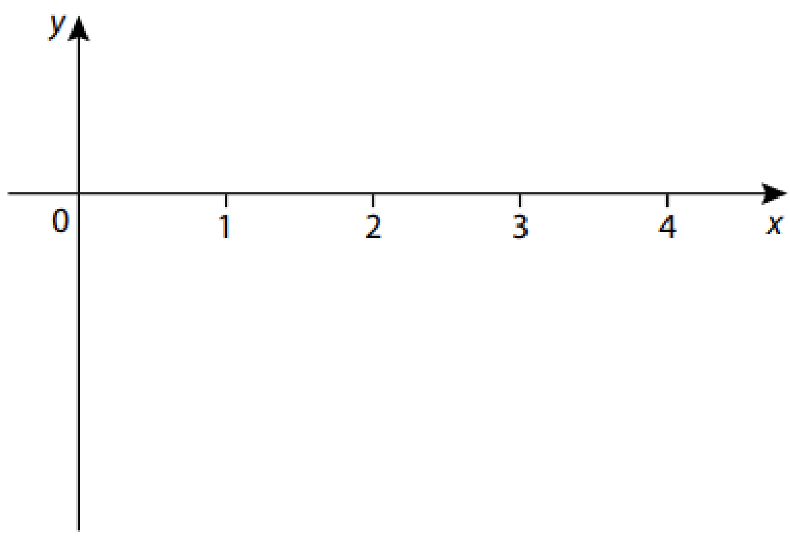
when $x = 1$.

9. [Maximum mark: 7]

The diagram below shows the graph of $y = f(x)$ for $0 \leqslant x \leqslant 4$.



Sketch the graph of $y = \int_0^x f(t)dt$ on the set of axes below, marking clearly any maxima or minima and points of inflexion.



10. [Maximum mark: 8]

Consider the following system of equations:

$$\begin{cases} x + 2y - 3z = k \\ 3x + y + 2z = 4 \\ 5x + 7z = 5 \end{cases}$$

(a) Find the value of k for which the system has more than one solution. [4]

(b) For the value of k found in part (a) find the vector equation of the line of intersection of the three planes. [4]

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

11. [Maximum mark: 14]

(a) There is a group of 10 teachers in teacher's lounge. This group includes Tomasz. Find the number of ways 4 teachers can be chosen (the order of choice is irrelevant) if: [3]

(i) there are no restrictions.

(ii) Tomasz must be among the chosen four.

(iii) Tomasz cannot be among the chosen four.

(b) Prove using the formula for the binomial coefficient that [4]

$$\binom{n}{r} + \binom{n}{r+1} \equiv \binom{n+1}{r+1}$$

(c) Prove by induction that [7]

$$\frac{d^n}{dx^n}(f(x) \times g(x)) \equiv \sum_{i=0}^{n} \binom{n}{i} \frac{d^i}{dx^i}(f(x)) \times \frac{d^{n-i}}{dx^{n-i}}(g(x))$$

12. [Maximum mark: 21]

Particles A and B move in a straight line, starting at the origin, such that their velocities in metres per second for $0 \leqslant t \leqslant 10$ are given by:

$$v_A(t) = -\frac{1}{2}t^2 + 3t + \frac{3}{2}$$

$$v_B(t) = e^{0.2t}$$

respectively.

(a) Write down initial velocities of both particles. [2]

(b) Find the time t when particle A changes direction. [2]

(c) Find the total distance travelled by particle A. [2]

Let s_A and s_B be displacements from the origin of particles A and B respectively.

(d) Find and expression, in terms of t, for s_A and s_B [3]

(e) Sketch the graphs of $s_A(t)$ and $s_B(t)$ and hence: [7]

(i) find the value of t at which $s_A = s_B$,

(ii) decide if particle A comes back to the origin and justify your answer,

(iii) state the largest distance from each particle to the origin.

Now consider only the part of the journey for which the two particles were moving in the same direction.

(e) Find the largest distance between the particles. [5]

13. [Maximum mark: 19]

Consider the function $f(x) = \dfrac{3\cos x}{2 - \sin x}$ with $0 \leqslant x \leqslant 2\pi$.

(a) Show that $f'(x) = \dfrac{3 - 6\sin x}{(2 - \sin x)^2}$. [3]

(b) Hence find the exact values of x-coordinates of stationary points of $y = f(x)$. [2]

(c) Show that $f''(x) = \dfrac{6\cos x(\sin x + 1)}{(\sin x - 2)^3}$ [4]

(d) Hence show that points where the graph of $f(x)$ intercepts the x-axis coincide with points of inflexion of $y = f(x)$. [3]

(e) Sketch the graph of $y = f(x)$. Clearly indicate axes intercepts, stationary points and points of inflexion. [4]

(f) State the range of value of $f(x)$. [1]

(h) Show that $g(x) = f(x - \frac{\pi}{2})$ is an odd function. [2]