Name:

Mathematics Analysis & Approaches
Higher level
Paper 3

December 21, 2020 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the Mathematics Analysis & Approaches formula booklet is required for this paper.
- The maximum mark for this examination paper is [**55 marks**].

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working

1. [Maximum mark: 30]

This question asks you to investigate some properties of the graph of the function $f(x) = e^{-x}\sin x$.

Important: In this question zeroes and points are arranged in ascending order with respect to their x-coordinate.

Let $f(x) = e^{-x}\sin x$, with $x \geqslant 0$.

(a) Write down the first three zeroes of the graph of $y = f(x)$. [2]

Let x_n be the n-th zero of $f(x)$.

(b) The sequence $\{x_n\}$ with $n \in \mathbb{Z}^+$ is an arithmetic sequence. State the value of the common difference of this sequence. [1]

(c) Find $f'(x)$. [2]

Let P_n be the n-th stationary point of the graph of $y = f(x)$.

(d) Find the exact value of the x-coordinate of P_1. [2]

(e) Find an expression for the x-coordinate of P_n. [2]

(f) Show that $\sin(\theta + \pi) \equiv -\sin\theta$ [2]

(g) Show that y-coordinates of the sequence of points $\{P_n\}$ form a geometric sequence and find the exact value of the ratio of this sequence. [4]

(h) Sketch the graph of $y = f(x)$ for $0 \leqslant x \leqslant 2\pi$. [2]

Let $A_n = \int_{x_n}^{x_{n+1}} f(x)dx$.

(i) Find the exact value of A_1. [5]

(j) Show that the sequence $\{A_n\}$ with $n \in \mathbb{Z}^+$ is a geometric sequence with common ratio equal to $-e^{-\pi}$. [6]

(l) Hence or otherwise calculate $\int_0^{\infty} f(x)dx$ [2]

2. [Maximum mark: 25]

This question asks you to investigate a strategy in a duel among three opponents.

Alan, Phill and Sean play a game in which they each take turns shooting a ball at each other. Each player gets one shot per turn and each has unlimited number of balls available. If a player is hit, then he is immediately eliminated from the game. The last player remaining wins the game.

Alan hits the target 10% of the time, Phill hits the target 60% of the time and Sean hits the target 90% of the time.

In each turn Alan shoots first, then Phill, and Sean shoots last.

Assume that each player always shoots at the most skilful player still in the game.

(a) Find the probability that in the first turn: [2]

(i) Alan missed and Phill hit his target.

(ii) Alan hit his target and Phill missed.

(b) If one of the above happened, write down the players remaining after the first round, and find the probability that Alan wins the whole game. [6]

(c) Show that the probability that Alan wins the whole game if only Sean hit his target in the first turn is $\frac{10}{91}$. [4]

(d) Write down the probability that all three players missed in their first turn. [1]

Let p be the probability that Alan wins the game.

(e) Explain why

$$p = 0.58 \times \frac{5}{32} + 0.324 \times \frac{10}{91} + 0.036p$$

[3]

(f) Calculate p. [1]

Now suppose that Alan deliberately misses his first shot in the first turn. In the later turns the probability that he hits the target is still 10%.

(g) Show that in this case:

$$p = 0.6 \times \frac{5}{32} + 0.36 \times \frac{10}{91} + 0.04p$$

[6]

(h) Calculate p. [1]

(i) Comment on your answers to parts (f) and (h). [1]