M14/5/MATHL/HP2/ENG/TZ1/XX/M

International Baccalaureate®
Baccalauréat International
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MARKSCHEME

May 2014

MATHEMATICS

Higher Level

Paper 2

20 pages

SECTION A

1. **METHOD 1**

substituting
$-5+12\text{i}+a(2+3\text{i})+b=0$ *(A1)*
equating real or imaginary parts *(M1)*
$12+3a=0 \Rightarrow a=-4$ *A1*
$-5+2a+b=0 \Rightarrow b=13$ *A1*

METHOD 2

other root is $2-3\text{i}$ *(A1)*
considering either the sum or product of roots or multiplying factors *(M1)*
$4=-a$ (sum of roots) so $a=-4$ *A1*
$13=b$ (product of roots) *A1*

[4 marks]

2. $X \sim \text{N}\left(100, \sigma^2\right)$
$\text{P}(X<124)=0.68$ *(M1)(A1)*
$\frac{24}{\sigma}=0.4676....$ *(M1)*
$\sigma=51.315\ldots$ *(A1)*
$\text{variance}=2630$ *A1*

[5 marks]

Notes: Accept use of $\text{P}(X<124.5)=0.68$ leading to variance $=2744$.

3. the number of ways of allocating presents to the first child is $\binom{7}{3}\left(\text{or } \binom{7}{2}\right)$ *(A1)*

multiplying by $\binom{4}{2}\left(\text{or } \binom{5}{3} \text{ or } \binom{5}{2}\right)$ *(M1)(A1)*

Note: Award ***M1*** for multiplication of combinations.

$\binom{7}{3}\binom{4}{2}=210$ *A1*

[4 marks]

4. (a) $\begin{bmatrix} x+2y-z=2 \\ 2x+y+z=1 \\ -x+4y+az=4 \end{bmatrix}$

$\rightarrow \begin{bmatrix} x+2y-z=2 \\ -3y+3z=-3 \\ 6y+(a-1)z=6 \end{bmatrix}$ ***M1A1***

$\rightarrow \begin{bmatrix} x+2y-z=2 \\ -3y+3z=-3 \\ (a+5)z=0 \end{bmatrix}$ ***A1***

(or equivalent)
if not a unique solution then $a=-5$ ***A1***

Note:	The first ***M1*** is for attempting to eliminate a variable, the first ***A1*** for obtaining two expression in just two variables (plus *a*), and the second ***A1*** for obtaining an expression in just *a* and one other variable

[4 marks]

(b) if $a=-5$ there are an infinite number of solutions as last equation always true ***R1***
and if $a \neq -5$ there is a unique solution ***R1***
hence always a solution ***AG***

[2 marks]

Total [6 marks]

5. (a) $\frac{\pi}{2}(1.57)$, $\frac{3\pi}{2}(4.71)$ ***A1A1***

hence the coordinates are $\left(\frac{\pi}{2}, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, \frac{3\pi}{2}\right)$ ***A1***

[3 marks]

(b) (i) $\pi\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left(x^2 - (x + 2\cos x)^2\right) \mathrm{d}x$ ***A1A1A1***

> **Note:** Award ***A1*** for $x^2 - (x + 2\cos x)^2$, ***A1*** for correct limits and ***A1*** for π.

(ii) $6\pi^2 (= 59.2)$ ***A2***

> **Notes:** Do not award **ft** from (b)(i).

[5 marks]

Total [8 marks]

6. (a) **METHOD 1**

sketch showing where the lines cross or zeros of $y = x(x+2)^6 - x$ ***(M1)***

$x = 0$ ***(A1)***

$x = -1$ and $x = -3$ ***(A1)***

the solution is $-3 < x < -1$ or $x > 0$ ***A1A1***

Note: Do not award either final ***A1*** mark if strict inequalities are not given.

METHOD 2

separating into two cases $x > 0$ and $x < 0$ ***(M1)***

if $x > 0$ then $(x+2)^6 > 1 \Rightarrow$ always true ***(M1)***

if $x < 0$ then $(x+2)^6 < 1 \Rightarrow -3 < x < -1$ ***(M1)***

so the solution is $-3 < x < -1$ or $x > 0$ ***A1A1***

Note: Do not award either final ***A1*** mark if strict inequalities are not given.

METHOD 3

$f(x) = x^7 + 12x^6 + 60x^5 + 160x^4 + 240x^3 + 192x^2 + 64x$ ***(A1)***

solutions to $x^7 + 12x^6 + 60x^5 + 160x^4 + 240x^3 + 192x^2 + 63x = 0$ are ***(M1)***

$x = 0$, $x = -1$ and $x = -3$ ***(A1)***

so the solution is $-3 < x < -1$ or $x > 0$ ***A1A1***

Note: Do not award either final ***A1*** mark if strict inequalities are not given.

METHOD 4

$f(x) = x$ when $x(x+2)^6 = x$

either $x = 0$ or $(x+2)^6 = 1$ ***(A1)***

if $(x+2)^6 = 1$ then $x + 2 = \pm 1$ so $x = -1$ or $x = -3$ ***(M1)(A1)***

the solution is $-3 < x < -1$ or $x > 0$ ***A1A1***

Note: Do not award either final ***A1*** mark if strict inequalities are not given.

[5 marks]

continued ...

Question 6 continued

(b) **METHOD 1** (by substitution)

substituting $u = x + 2$ ***(M1)***
$\mathrm{d}u = \mathrm{d}x$
$\int (u-2)u^6 \mathrm{d}u$ ***M1A1***
$= \frac{1}{8}u^8 - \frac{2}{7}u^7 \ (+c)$ ***(A1)***
$= \frac{1}{8}(x+2)^8 - \frac{2}{7}(x+2)^7 \ (+c)$ ***A1***

METHOD 2 (by parts)

$u = x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 1, \ \frac{\mathrm{d}v}{\mathrm{d}x} = (x+2)^6 \Rightarrow v = \frac{1}{7}(x+2)^7$ ***(M1)(A1)***
$\int x(x+2)^6 \, \mathrm{d}x = \frac{1}{7}x(x+2)^7 - \frac{1}{7}\int (x+2)^7 \, \mathrm{d}x$ ***M1***
$= \frac{1}{7}x(x+2)^7 - \frac{1}{56}(x+2)^8 \ (+c)$ ***A1A1***

METHOD 3 (by expansion)

$\int f(x)\,\mathrm{d}x = \int \left(x^7 + 12x^6 + 60x^5 + 160x^4 + 240x^3 + 192x^2 + 64x\right)\mathrm{d}x$ ***M1A1***
$= \frac{1}{8}x^8 + \frac{12}{7}x^7 + 10x^6 + 32x^5 + 60x^4 + 64x^3 + 32x^2 \ (+c)$ are ***M1A2***

Note: Award ***M1A1*** if at least four terms are correct.

[5 marks]

Total [10 marks]

7. if $n = 0$

$7^3 + 2 = 345$ which is divisible by 5, hence true for $n = 0$ ***A1***

> **Note:** Award ***A0*** for using $n = 1$ but do not penalize further in question.

assume true for $n = k$ ***M1***

> **Note:** Only award the ***M1*** if truth is assumed.

so $7^{8k+3} + 2 = 5p,\ p \in$ • ***A1***

if $n = k + 1$

$7^{8(k+1)+3} + 2$ ***M1***

$= 7^8 7^{8k+3} + 2$ ***M1***

$= 7^8 (5p - 2) + 2$ ***A1***

$= 7^8 .5p - 2.7^8 + 2$

$= 7^8 .5p - 11529600$

$= 5\left(7^8 p - 2305920\right)$ ***A1***

hence if true for $n = k$, then also true for $n = k + 1$. Since true for $n = 0$, then true for all $n \in$ • ***R1***

[8 marks]

> **Note:** Only award the ***R1*** if the first two ***M1***s have been awarded.

8. (a) $\left(A\binom{6}{5}2^5B+3\binom{6}{4}2^4B^2\right)x^5$ ***M1A1A1***

$=\left(192AB+720B^2\right)x^5$ ***A1***

[4 marks]

(b) **METHOD 1**

$x=\frac{1}{6},\ A=\frac{3}{6}\left(=\frac{1}{2}\right),\ B=\frac{4}{6}\left(=\frac{2}{3}\right)$ ***A1A1A1***

probability is $\frac{4}{81}\ (=0.0494)$ ***A1***

METHOD 2

$\mathrm{P}(5\text{ eaten})=\mathrm{P}(\text{M eats }1)\ \mathrm{P}(\text{N eats }4)+\mathrm{P}(\text{M eats }0)\ \mathrm{P}(\text{N eats }5)$ ***(M1)***

$=\frac{1}{2}\binom{6}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2+\frac{1}{2}\binom{6}{5}\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)$ ***(A1)(A1)***

$=\frac{4}{81}\ (=0.0494)$ ***A1***

[4 marks]

Total [8 marks]

9. (a) mean for week is 40.88 ***(A1)***

$\mathrm{P}(S > 40) = 1 - \mathrm{P}(S \leq 40) = 0.513$ ***A1***

[2 marks]

(b) $\dfrac{\text{probability there were more than 10 on Monday AND more than 40 over the week}}{\text{probability there were more than 10 on Monday}}$

M1

possibilities for the numerator are:

there were more than 40 birds on the power line on Monday ***R1***
11 on Monday and more than 29 over the course of the next 6 days ***R1***
12 on Monday and more than 28 over the course of the next 6 days … until
40 on Monday and more than 0 over the course of the next 6 days ***R1***
hence if X is the number on the power line on Monday and Y, the number on the power line Tuesday – Sunday then the numerator is ***M1***

$\mathrm{P}(X > 40) + \mathrm{P}(X = 11) \times \mathrm{P}(Y > 29) + \mathrm{P}(X = 12) \times \mathrm{P}(Y > 28) + \ldots$

$+ \mathrm{P}(X = 40) \times \mathrm{P}(Y > 0)$

$= \mathrm{P}(X > 40) + \sum_{r=11}^{40} \mathrm{P}(X = r)\mathrm{P}(Y > 40 - r)$

hence solution is $\dfrac{\mathrm{P}(X > 40) + \sum_{r=11}^{40} \mathrm{P}(X = r)\mathrm{P}(Y > 40 - r)}{\mathrm{P}(X > 10)}$ ***AG***

[5 marks]

Total [7 marks]

SECTION B

10. (a) $x \to -\infty \Rightarrow y \to -\frac{1}{2}$ so $y = -\frac{1}{2}$ is an asymptote ***(M1)A1***

$e^x - 2 = 0 \Rightarrow x = \ln 2$ so $x = \ln 2 (= 0.693)$ is an asymptote ***(M1)A1***

[4 marks]

(b) (i) $f'(x) = \frac{2(e^x - 2)e^{2x} - (e^{2x} + 1)e^x}{(e^x - 2)^2}$ ***M1A1***

$= \frac{e^{3x} - 4e^{2x} - e^x}{(e^x - 2)^2}$

(ii) $f'(x) = 0$ when $e^{3x} - 4e^{2x} - e^x = 0$ ***M1***

$e^x(e^{2x} - 4e^x - 1) = 0$

$e^x = 0,\ e^x = -0.236,\ e^x = 4.24\ \left(\text{or } e^x = 2 \pm \sqrt{5}\right)$ ***A1A1***

> **Note:** Award ***A1*** for zero, ***A1*** for other two solutions.
> Accept any answers which show a zero, a negative and a positive.

as $e^x > 0$ exactly one solution ***R1***

> **Note:** Do not award marks for purely graphical solution.

(iii) $(1.44, 8.47)$ ***A1A1***

[8 marks]

(c) $f'(0) = -4$ ***(A1)***

so gradient of normal is $\frac{1}{4}$ ***(M1)***

$f(0) = -2$ ***(A1)***

so equation of L_1 is $y = \frac{1}{4}x - 2$ ***A1***

[4 marks]

continued ...

Question 10 continued

(d) $f'(x) = \frac{1}{4}$ ***M1***

so $x = 1.46$ ***(M1)A1***

$f(1.46) = 8.47$ ***(A1)***

equation of L_2 is $y - 8.47 = \frac{1}{4}(x - 1.46)$ ***A1***

(or $y = \frac{1}{4}x + 8.11$)

[5 marks]

Total [21 marks]

11. (a) $\int_2^3 (ax+b)\,\mathrm{d}x\,(=1)$ ***M1A1***

$\left[\frac{1}{2}ax^2+bx\right]_2^3 (=1)$ ***A1***

$\frac{5}{2}a+b=1$ ***M1***

$5a+2b=2$ ***AG***

[4 marks]

(b) (i) $\int_2^3 \left(ax^2+bx\right)\mathrm{d}x\,(=\mu)$ ***M1A1***

$\left[\frac{1}{3}ax^3+\frac{1}{2}bx^2\right]_2^3 (=\mu)$ ***A1***

$\frac{19}{3}a+\frac{5}{2}b=\mu$ ***A1***

eliminating b ***M1***

eg

$\frac{19}{3}a+\frac{5}{2}\left(1-\frac{5}{2}a\right)=\mu$ ***A1***

$\frac{1}{12}a+\frac{5}{2}=\mu$

$a=12\mu-30$ ***AG***

Note: Elimination of *b* could be at different stages.

(ii) $b=1-\frac{5}{2}(12\mu-30)$

$=76-30\mu$ ***A1***

Note: This solution may be seen in part (i).

[7 marks]

(c) (i) $\int_2^{2.3}(ax+b)\,\mathrm{d}x\,(=0.5)$ ***(M1)(A1)***

$\left[\frac{1}{2}ax^2+bx\right]_2^{2.3} (=0.5)$

$0.645a+0.3b\,(=0.5)$ ***(A1)***

$0.645(12\mu-30)+0.3(76-30\mu)=0.5$ ***M1***

$\mu=2.34\left(=\frac{295}{126}\right)$ ***A1***

continued …

Question 11 continued

(ii) $\mathrm{E}\left(X^2\right)=\int_2^3 x^2\,(ax+b)\,\mathrm{d}x$ ***(M1)***

$a=12\mu-30=-\frac{40}{21},\ b=76-30\mu=\frac{121}{21}$ ***(A1)***

$\mathrm{E}\left(X^2\right)=\int_2^3 x^2\left(-\frac{40}{21}x+\frac{121}{21}\right)\mathrm{d}x=5.539\ldots\left(=\frac{349}{63}\right)$ ***(A1)***

$\mathrm{Var}\,(X)=5.539\mathrm{K}\ -(2.341\mathrm{K}\ )^2=0.05813\ldots$ ***(M1)***

$\sigma=0.241$ ***A1***

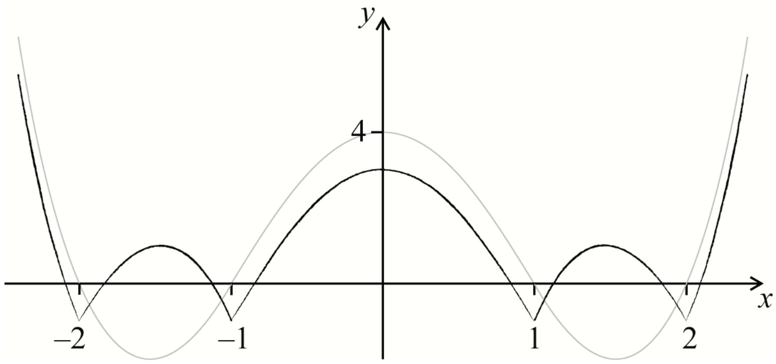
[10 marks]

Total [21 marks]

12. (a) (i) $f(0) = -1$ ***(M1)A1***

(ii) $(f \circ g)(0) = f(4) = 3$ ***A1***

(iii)

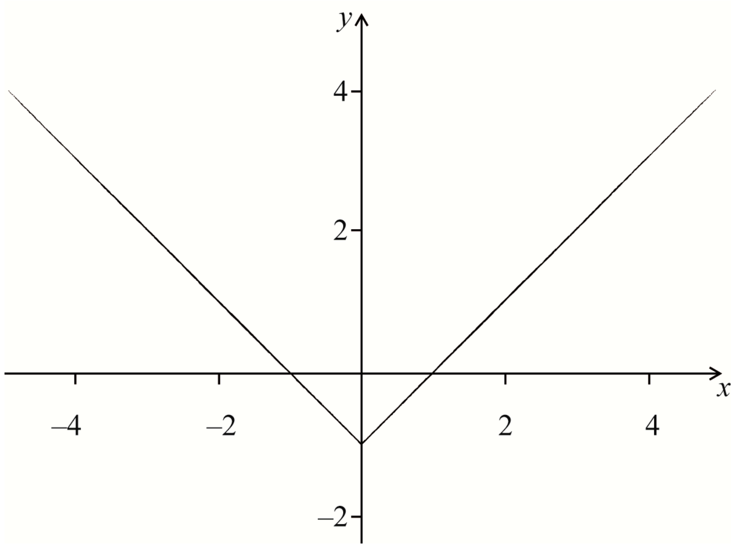


(M1)A1

Note:	Award ***M1*** for evidence that the lower part of the graph has been reflected and ***A1*** correct shape with y-intercept below 4.

[5 marks]

(b) (i)



(M1)A1

Note:	Award ***M1*** for any translation of $y = \|x\|$.

(ii) ± 1 ***A1***

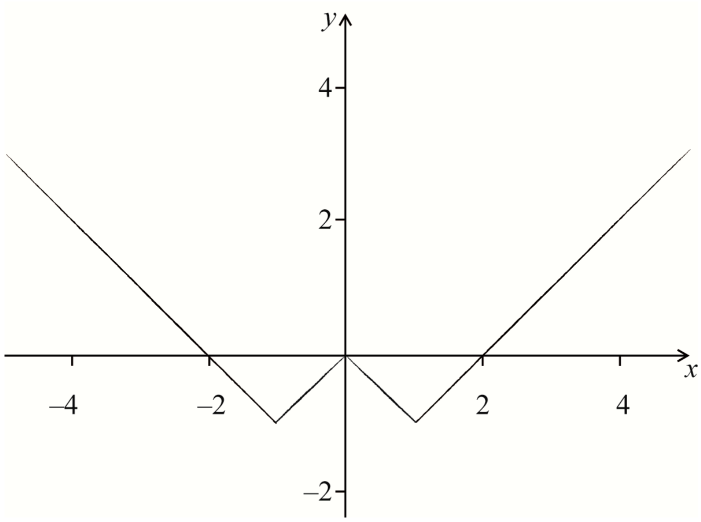
Note:	Do not award the ***A1*** if coordinates given, but do not penalise in the rest of the question

[3 marks]

continued …

Question 12 continued

(c) (i)



(M1)A1

> **Note:** Award ***M1*** for evidence that lower part of (b) has been reflected in the x-axis and translated.

(ii) $0, \pm 2$ ***A1***

[3 marks]

(d) (i) $\pm 1, \pm 3$ ***A1***

(ii) $0, \pm 2, \pm 4$ ***A1***

(iii) $0, \pm 2, \pm 4, \pm 6, \pm 8$ ***A1***

[3 marks]

(e) (i) $(1, 3), (2, 5), \ldots$ ***(M1)***

$N = 2n+1$ ***A1***

(ii) Using the formula of the sum of an arithmetic series ***(M1)***

EITHER

$4(1+2+3+\ldots+n) = \frac{4}{2}n(n+1)$

$= 2n(n+1)$ ***A1***

OR

$2(2+4+6+\ldots+2n) = \frac{2}{2}n(2n+2)$

$= 2n(n+1)$ ***A1***

[4 marks]

Total [18 marks]