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MARKSCHEME

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MATHEMATICS

Higher Level

Paper 2

22 pages

SECTION A

1. $\boldsymbol{n}_1 = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ and $\boldsymbol{n}_2 = \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix}$ ***(A1)(A1)***

use of $\cos\theta = \dfrac{\boldsymbol{n}_1 \bullet \boldsymbol{n}_2}{|\boldsymbol{n}_1||\boldsymbol{n}_2|}$ ***(M1)***

$\cos\theta = \dfrac{7}{\sqrt{21}\sqrt{19}} \left(= \dfrac{7}{\sqrt{399}} \right)$ ***(A1)(A1)***

Note: Award ***A1*** for a correct numerator and ***A1*** for a correct denominator.

$\theta = 69^{\circ}$ ***A1***

Note: Award ***A1*** for 111°.

Total [6 marks]

2. (a) $\mathrm{P}(X > x) = 0.99 \ (= \mathrm{P}(X < x) = 0.01)$ ***(M1)***

$\Rightarrow x = 54.6\,(\mathrm{cm})$ ***A1***

[2 marks]

(b) $\mathrm{P}(60.15 \le X \le 60.25)$ ***(M1)(A1)***

$= 0.0166$ ***A1***

[3 marks]

Total [5 marks]

3. use of $\mu = \frac{\sum_{i=1}^{k} f_i x_i}{n}$ to obtain $\frac{2 + x + y + 10 + 17}{5} = 8$ ***(M1)***

$x + y = 11$ ***A1***

EITHER

use of $\sigma^2 = \frac{\sum_{i=1}^{k} f_i (x_i - \mu)^2}{n}$ to obtain $\frac{(-6)^2 + (x-8)^2 + (y-8)^2 + 2^2 + 9^2}{5} = 27.6$ ***(M1)***

$(x-8)^2 + (y-8)^2 = 17$ ***A1***

OR

use of $\sigma^2 = \frac{\sum_{i=1}^{k} f_i x_i^2}{n} - \mu^2$ to obtain $\frac{2^2 + x^2 + y^2 + 10^2 + 17^2}{5} - 8^2 = 27.6$ ***(M1)***

$x^2 + y^2 = 65$ ***A1***

THEN

attempting to solve the two equations ***(M1)***

$x = 4$ and $y = 7$ (only as $x < y$) ***A1*** ***N4***

Note: Award ***A0*** for $x = 7$ and $y = 4$.

Note: Award ***(M1)A1(M0)A0(M1)A1*** for $x + y = 11 \Rightarrow x = 4$ and $y = 7$.

Total [6 marks]

4. **METHOD 1**

attempt to set up (diagram, vectors) ***(M1)***

correct distances $x = 15t,\ y = 20t$ ***(A1) (A1)***

the distance between the two cyclists at time t is $s = \sqrt{(15t)^2 + (20t)^2} = 25t$ (km) ***A1***

$\frac{\mathrm{d}s}{\mathrm{d}t} = 25\ (\mathrm{km\,h^{-1}})$ ***A1***

hence the rate is independent of time ***AG***

METHOD 2

attempting to differentiate $x^2 + y^2 = s^2$ implicitly ***(M1)***

$2x\frac{\mathrm{d}x}{\mathrm{d}t} + 2y\frac{\mathrm{d}y}{\mathrm{d}t} = 2s\frac{\mathrm{d}s}{\mathrm{d}t}$ ***(A1)***

the distance between the two cyclists at time t is $\sqrt{(15t)^2 + (20t)^2} = 25t$ (km) ***(A1)***

$2(15t)(15) + 2(20t)(20) = 2(25t)\frac{\mathrm{d}s}{\mathrm{d}t}$ ***M1***

Note: Award ***M1*** for substitution of correct values into their equation involving $\frac{\mathrm{d}s}{\mathrm{d}t}$.

$\frac{\mathrm{d}s}{\mathrm{d}t} = 25\ (\mathrm{km\,h^{-1}})$ ***A1***

hence the rate is independent of time ***AG***

METHOD 3

$s = \sqrt{x^2 + y^2}$ ***(A1)***

$\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{x\frac{\mathrm{d}x}{\mathrm{d}t} + y\frac{\mathrm{d}y}{\mathrm{d}t}}{\sqrt{x^2 + y^2}}$ ***(M1)(A1)***

Note: Award ***M1*** for attempting to differentiate the expression for s.

$\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{(15t)(15) + (20t)(20)}{\sqrt{(15t)^2 + (20t)^2}}$ ***M1***

Note: Award ***M1*** for substitution of correct values into their $\frac{\mathrm{d}s}{\mathrm{d}t}$.

$\frac{\mathrm{d}s}{\mathrm{d}t} = 25\ (\mathrm{km\ h^{-1}})$ ***A1***

hence the rate is independent of time ***AG***

Total [5 marks]

5. (a) attempting to find a normal to π *eg* $\begin{pmatrix}3\\2\\-2\end{pmatrix}\times\begin{pmatrix}8\\11\\6\end{pmatrix}$ ***(M1)***

$\begin{pmatrix}3\\2\\-2\end{pmatrix}\times\begin{pmatrix}8\\11\\6\end{pmatrix}=17\begin{pmatrix}2\\-2\\1\end{pmatrix}$ ***(A1)***

$\boldsymbol{r}\bullet\begin{pmatrix}2\\-2\\1\end{pmatrix}=\begin{pmatrix}1\\5\\12\end{pmatrix}\bullet\begin{pmatrix}2\\-2\\1\end{pmatrix}$ ***M1***

$2x-2y+z=4$ (or equivalent) ***A1***

[4 marks]

(b) $l_3:\boldsymbol{r}=\begin{pmatrix}4\\0\\8\end{pmatrix}+t\begin{pmatrix}2\\-2\\1\end{pmatrix}, t\in\mathbb{R}$ ***(A1)***

attempting to solve $\begin{pmatrix}4+2t\\-2t\\8+t\end{pmatrix}\bullet\begin{pmatrix}2\\-2\\1\end{pmatrix}=4$ for t *ie* $9t+16=4$ for t ***M1***

$t=-\frac{4}{3}$ ***A1***

$\left(\frac{4}{3},\frac{8}{3},\frac{20}{3}\right)$ ***A1***

[4 marks]

Total [8 marks]

6. using $p(a) = -7$ to obtain $3a^3 + a^2 + 5a + 7 = 0$ ***M1A1***

$(a+1)(3a^2 - 2a + 7) = 0$ ***(M1)(A1)***

> **Note:** Award ***M1*** for a cubic graph with correct shape and ***A1*** for clearly showing that the above cubic crosses the horizontal axis at $(-1, 0)$ only.

$a = -1$ ***A1***

EITHER

showing that $3a^2 - 2a + 7 = 0$ has no real (two complex) solutions for a ***R1***

OR

showing that $3a^3 + a^2 + 5a + 7 = 0$ has one real (and two complex) solutions for a ***R1***

> **Note:** Award ***R1*** for solutions that make specific reference to an appropriate graph.

Total [6 marks]

7. (a) using $r = \frac{u_2}{u_1} = \frac{u_3}{u_2}$ to form $\frac{a+2d}{a+6d} = \frac{a}{a+2d}$ ***(M1)***

$a(a+6d) = (a+2d)^2$ ***A1***

$2d(2d - a) = 0$ (or equivalent) ***A1***

since $d \neq 0 \Rightarrow d = \frac{a}{2}$ ***AG***

[3 marks]

(b) substituting $d = \frac{a}{2}$ into $a + 6d = 3$ and solving for a and d ***(M1)***

$a = \frac{3}{4}$ and $d = \frac{3}{8}$ ***(A1)***

$r = \frac{1}{2}$ ***A1***

$$\frac{n}{2}\left(2 \times \frac{3}{4} + (n-1)\frac{3}{8}\right) - \frac{3\left(1 - \left(\frac{1}{2}\right)^n\right)}{1 - \frac{1}{2}} \geq 200$$ ***(A1)***

attempting to solve for n ***(M1)***

$n \geq 31.68\ldots$

so the least value of n is 32 ***A1***

[6 marks]

Total [9 marks]

8. (a) $3-\frac{t}{2}=0 \Rightarrow t=6\,(\text{s})$ ***(M1)A1***

[2 marks]

> **Note:** Award ***A0*** if either $t=-0.236$ or $t=4.24$ or both are stated with $t=6$.

(b) let d be the distance travelled before coming to rest

$$d=\int_0^4 5-(t-2)^2\,\mathrm{d}t+\int_4^6 3-\frac{t}{2}\,\mathrm{d}t$$ ***(M1)(A1)***

> **Note:** Award ***M1*** for two correct integrals even if the integration limits are incorrect. The second integral can be specified as the area of a triangle.

$d=\frac{47}{3}\,(=15.7)\,(\text{m})$ ***(A1)***

attempting to solve $\int_6^T\left(\frac{t}{2}-3\right)\mathrm{d}t=\frac{47}{3}$ (or equivalent) for T ***M1***

$T=13.9\,(\text{s})$ ***A1***

[5 marks]

Total [7 marks]

9. (a) each triangle has area $\frac{1}{8}x^2\sin\frac{2\pi}{n}$ (use of $\frac{1}{2}ab\sin C$) ***(M1)***

there are n triangles so $A = \frac{1}{8}nx^2\sin\frac{2\pi}{n}$ ***A1***

$$C = \frac{4\left(\frac{1}{8}nx^2\sin\frac{2\pi}{n}\right)}{\pi x^2}$$ ***A1***

so $C = \frac{n}{2\pi}\sin\frac{2\pi}{n}$ ***AG***

[3 marks]

(b) attempting to find the least value of n such that $\frac{n}{2\pi}\sin\frac{2\pi}{n} > 0.99$ ***(M1)***

$n = 26$ ***A1***

attempting to find the least value of n such that $\frac{n\sin\frac{2\pi}{n}}{\pi\left(1+\cos\frac{\pi}{n}\right)} > 0.99$ ***(M1)***

$n = 21$ (and so a regular polygon with 21 sides) ***A1***

> **Note:** Award ***(M0)A0(M1)A1*** if $\frac{n}{2\pi}\sin\frac{2\pi}{n} > 0.99$ is not considered and $\frac{n\sin\frac{2\pi}{n}}{\pi\left(1+\cos\frac{\pi}{n}\right)} > 0.99$ is correctly considered.
>
> Award ***(M1)A1(M0)A0*** for $n = 26$.

[4 marks]

(c) **EITHER**

for even and odd values of n, the value of C seems to increase towards the limiting value of the circle $(C = 1)$ *ie* as n increases, the polygonal regions get closer and closer to the enclosing circular region ***R1***

OR

the differences between the odd and even values of n illustrate that this measure of compactness is not a good one. ***R1***

[1 mark]

Total [8 marks]

SECTION B

10. (a) use of $A=\frac{1}{2}qr\sin\theta$ to obtain $A=\frac{1}{2}(x+2)(5-x)^2\sin 30^\circ$ ***M1***

$=\frac{1}{4}(x+2)(25-10x+x^2)$ ***A1***

$A=\frac{1}{4}(x^3-8x^2+5x+50)$ ***AG***

[2 marks]

(b) (i) $\frac{\mathrm{d}A}{\mathrm{d}x}=\frac{1}{4}\left(3x^2-16x+5\right)=\frac{1}{4}(3x-1)(x-5)$ ***A1***

(ii) **METHOD 1**

EITHER

$\frac{\mathrm{d}A}{\mathrm{d}x}=\frac{1}{4}\left(3\left(\frac{1}{3}\right)^2-16\left(\frac{1}{3}\right)+5\right)=0$ ***M1A1***

OR

$\frac{\mathrm{d}A}{\mathrm{d}x}=\frac{1}{4}\left(3\left(\frac{1}{3}\right)-1\right)\left(\left(\frac{1}{3}\right)-5\right)=0$ ***M1A1***

THEN

so $\frac{\mathrm{d}A}{\mathrm{d}x}=0$ when $x=\frac{1}{3}$ ***AG***

METHOD 2

solving $\frac{\mathrm{d}A}{\mathrm{d}x}=0$ for x ***M1***

$-2<x<5\Rightarrow x=\frac{1}{3}$ ***A1***

so $\frac{\mathrm{d}A}{\mathrm{d}x}=0$ when $x=\frac{1}{3}$ ***AG***

METHOD 3

a correct graph of $\frac{\mathrm{d}A}{\mathrm{d}x}$ versus x ***M1***

the graph clearly showing that $\frac{\mathrm{d}A}{\mathrm{d}x}=0$ when $x=\frac{1}{3}$ ***A1***

so $\frac{\mathrm{d}A}{\mathrm{d}x}=0$ when $x=\frac{1}{3}$ ***AG***

[3 marks]

continued…

Question 10 continued

(c) (i) $\frac{\mathrm{d}^2 A}{\mathrm{d}x^2} = \frac{1}{2}(3x - 8)$ ***A1***

for $x = \frac{1}{3}$, $\frac{\mathrm{d}^2 A}{\mathrm{d}x^2} = -3.5 (< 0)$ ***R1***

so $x = \frac{1}{3}$ gives the maximum area of triangle PQR ***AG***

(ii) $A_{\max} = \frac{343}{27} (= 12.7) (\mathrm{cm}^2)$ ***A1***

(iii) $\mathrm{PQ} = \frac{7}{3}$ (cm) and $\mathrm{PR} = \left(\frac{14}{3}\right)^2$ (cm) ***(A1)***

$\mathrm{QR}^2 = \left(\frac{7}{3}\right)^2 + \left(\frac{14}{3}\right)^4 - 2\left(\frac{7}{3}\right)\left(\frac{14}{3}\right)^2 \cos 30^\circ$ ***(M1)(A1)***

$= 391.702\ldots$

$\mathrm{QR} = 19.8 (\mathrm{cm})$ ***A1***

[7 marks]

Total [12 marks]

11. (a) (i) $\mathrm{P}(X=0)=0.549\left(=\mathrm{e}^{-0.6}\right)$ ***A1***

(ii) $\mathrm{P}(X\geq 3)=1-\mathrm{P}(X\leq 2)$ ***(M1)***

$\mathrm{P}(X\geq 3)=0.0231$ ***A1***

[3 marks]

(b) **EITHER**

using $Y \sim \mathrm{Po}(3)$ ***(M1)***

OR

using $(0.549)^5$ ***(M1)***

THEN

$\mathrm{P}(Y=0)=0.0498\left(=\mathrm{e}^{-3}\right)$ ***A1***

[2 marks]

continued…

Question 11 continued

(c) $\mathrm{P}(X=0)$ (most likely number of complaints received is zero) ***A1***

EITHER

calculating $\mathrm{P}(X=0)=0.549$ and $\mathrm{P}(X=1)=0.329$ ***M1A1***

OR

sketching an appropriate (discrete) graph of $\mathrm{P}(X=x)$ against x ***M1A1***

OR

finding $\mathrm{P}(X=0)=e^{-0.6}$ and stating that $\mathrm{P}(X=0)>0.5$ ***M1A1***

OR

using $\mathrm{P}(X=x)=\mathrm{P}(X=x-1)\times\frac{\mu}{x}$ where $\mu<1$ ***M1A1***

[3 marks]

(d) $\mathrm{P}(X=0)=0.8(\Rightarrow e^{-\lambda}=0.8)$ ***(A1)***

$\lambda=0.223\left(=\ln\frac{5}{4},=-\ln\frac{4}{5}\right)$ ***A1***

[2 marks]

Total [10 marks]

12. (a) $\text{P}(\text{Ava wins on her first turn}) = \frac{1}{3}$ ***A1***

[1 mark]

(b) $\text{P}(\text{Barry wins on his first turn}) = \left(\frac{2}{3}\right)^2$ ***(M1)***

$= \frac{4}{9} (= 0.444)$ ***A1***

[2 marks]

(c) $\text{P}(\text{Ava wins in one of her first three turns})$

$= \frac{1}{3} + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\frac{1}{3} + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\frac{1}{3}$ ***M1A1A1***

> **Note:** Award ***M1*** for adding probabilities, award ***A1*** for a correct second term and award ***A1*** for a correct third term.
> Accept a correctly labelled tree diagram, awarding marks as above.

$= \frac{103}{243} (= 0.424)$ ***A1***

[4 marks]

(d) $\text{P}(\text{Ava eventually wins}) = \frac{1}{3} + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\frac{1}{3} + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\frac{1}{3} + \ldots$ ***(A1)***

using $S_\infty = \frac{a}{1-r}$ with $a = \frac{1}{3}$ and $r = \frac{2}{9}$ ***(M1)(A1)***

> **Note:** Award ***(M1)*** for using $S_\infty = \frac{a}{1-r}$ and award ***(A1)*** for $a = \frac{1}{3}$ and $r = \frac{2}{9}$.

$= \frac{3}{7} (= 0.429)$ ***A1***

[4 marks]

Total [11 marks]

13. (a) attempting to use $V=\pi\int_a^b x^2\mathrm{d}y$ ***(M1)***

attempting to express x^2 in terms of y *ie* $x^2=4(y+16)$ ***(M1)***

for $y=h$, $V=4\pi\int_0^h y+16\,\mathrm{d}y$ ***A1***

$V=4\pi\left(\frac{h^2}{2}+16h\right)$ ***AG***

[3 marks]

(b) (i) **METHOD 1**

$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}$ ***(M1)***

$\frac{\mathrm{d}V}{\mathrm{d}h}=4\pi(h+16)$ ***(A1)***

$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{1}{4\pi(h+16)}\times\frac{-250\sqrt{h}}{\pi(h+16)}$ ***M1A1***

Note: Award ***M1*** for substitution into $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}$.

$\frac{\mathrm{d}h}{\mathrm{d}t}=-\frac{250\sqrt{h}}{4\pi^2(h+16)^2}$ ***AG***

METHOD 2

$\frac{\mathrm{d}V}{\mathrm{d}t}=4\pi(h+16)\frac{\mathrm{d}h}{\mathrm{d}t}$ (implicit differentiation) ***(M1)***

$\frac{-250\sqrt{h}}{\pi(h+16)}=4\pi(h+16)\frac{\mathrm{d}h}{\mathrm{d}t}$ (or equivalent) ***A1***

$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{1}{4\pi(h+16)}\times\frac{-250\sqrt{h}}{\pi(h+16)}$ ***M1A1***

$\frac{\mathrm{d}h}{\mathrm{d}t}=-\frac{250\sqrt{h}}{4\pi^2(h+16)^2}$ ***AG***

(ii) $\frac{\mathrm{d}t}{\mathrm{d}h}=-\frac{4\pi^2(h+16)^2}{250\sqrt{h}}$ ***A1***

$t=\int-\frac{4\pi^2(h+16)^2}{250\sqrt{h}}\,\mathrm{d}h$ ***(M1)***

$t=\int-\frac{4\pi^2(h^2+32h+256)}{250\sqrt{h}}\,\mathrm{d}h$ ***A1***

$t=\frac{-4\pi^2}{250}\int\left(h^{\frac{3}{2}}+32h^{\frac{1}{2}}+256h^{-\frac{1}{2}}\right)\mathrm{d}h$ ***AG***

continued…

Question 13 continued

(iii) **METHOD 1**

$t = \dfrac{-4\pi^2}{250}\displaystyle\int_{48}^{0}\left(h^{\frac{3}{2}} + 32h^{\frac{1}{2}} + 256h^{-\frac{1}{2}}\right)\mathrm{d}h$ ***(M1)***

$t = 2688.756\ldots\,(\mathrm{s})$ ***(A1)***

45 minutes (correct to the nearest minute) ***A1***

METHOD 2

$t = \dfrac{-4\pi^2}{250}\left(\dfrac{2}{5}h^{\frac{5}{2}} + \dfrac{64}{3}h^{\frac{3}{2}} + 512h^{\frac{1}{2}}\right) + c$

when $t = 0, h = 48 \Rightarrow c = 2688.756...\left(c = \dfrac{4\pi^2}{250}\left(\dfrac{2}{5}\times 48^{\frac{5}{2}} + \dfrac{64}{3}\times 48^{\frac{3}{2}} + 512\times 48^{\frac{1}{2}}\right)\right)$ ***(M1)***

when $h = 0$, $t = 2688.756...\left(t = \dfrac{4\pi^2}{250}\left(\dfrac{2}{5}\times 48^{\frac{5}{2}} + \dfrac{64}{3}\times 48^{\frac{3}{2}} + 512\times 48^{\frac{1}{2}}\right)\right)(\mathrm{s})$ ***(A1)***

45 minutes (correct to the nearest minute) ***A1***

[10 marks]

(c) **EITHER**

the depth stabilises when $\dfrac{\mathrm{d}V}{\mathrm{d}t} = 0$ *ie* $8.5 - \dfrac{250\sqrt{h}}{\pi(h+16)} = 0$ ***R1***

attempting to solve $8.5 - \dfrac{250\sqrt{h}}{\pi(h+16)} = 0$ for h ***(M1)***

OR

the depth stabilises when $\dfrac{\mathrm{d}h}{\mathrm{d}t} = 0$ *ie* $\dfrac{1}{4\pi(h+16)}\left(8.5 - \dfrac{250\sqrt{h}}{\pi(h+16)}\right) = 0$ ***R1***

attempting to solve $\dfrac{1}{4\pi(h+16)}\left(8.5 - \dfrac{250\sqrt{h}}{\pi(h+16)}\right) = 0$ for h ***(M1)***

THEN

$h = 5.06\,(\mathrm{cm})$ ***A1***

[3 marks]

Total [16 marks]

14. (a) **METHOD 1**

squaring both equations *M1*

$9\sin^2 B + 24\sin B\cos C + 16\cos^2 C = 36$ *(A1)*

$9\cos^2 B + 24\cos B\sin C + 16\sin^2 C = 1$ *(A1)*

adding the equations and using $\cos^2\theta + \sin^2\theta = 1$ to obtain

$9 + 24\sin(B+C) + 16 = 37$ *M1*

$24(\sin B\cos C + \cos B\sin C) = 12$ *A1*

$24\sin(B+C) = 12$ *(A1)*

$\sin(B+C) = \frac{1}{2}$ *AG*

METHOD 2

substituting for $\sin B$ and $\cos B$ to obtain

$\sin(B+C) = \left(\frac{6-4\cos C}{3}\right)\cos C + \left(\frac{1-4\sin C}{3}\right)\sin C$ *M1*

$= \frac{6\cos C + \sin C - 4}{3}$ (or equivalent) *A1*

substituting for $\sin C$ and $\cos C$ to obtain

$\sin(B+C) = \sin B\left(\frac{6-3\sin B}{4}\right) + \cos B\left(\frac{1-3\cos B}{4}\right)$ *M1*

$= \frac{\cos B + 6\sin B - 3}{4}$ (or equivalent) *A1*

Adding the two equations for $\sin(B+C)$:

$2\sin(B+C) = \frac{(18\sin B + 24\cos C) + (4\sin C + 3\cos B) - 25}{12}$ *A1*

$\sin(B+C) = \frac{36+1-25}{24}$ *(A1)*

$\sin(B+C) = \frac{1}{2}$ *AG*

METHOD 3

substituting for $\sin B$ and $\sin C$ to obtain

$\sin(B+C) = \left(\frac{6-4\cos C}{3}\right)\cos C + \cos B\left(\frac{1-3\cos B}{4}\right)$ *M1*

substituting for $\cos B$ and $\cos C$ to obtain

$\sin(B+C) = \sin B\left(\frac{6-3\sin B}{4}\right) + \left(\frac{1-4\sin C}{3}\right)\sin C$ *M1*

Adding the two equations for $\sin(B+C)$:

$2\sin(B+C) = \frac{6\cos C + \sin C - 4}{3} + \frac{6\sin B + \cos B - 3}{4}$ (or equivalent) *A1A1*

$2\sin\left(B+C\right)=\frac{(18\sin B+24\cos C)+(4\sin C+3\cos B)-25}{12}$ ***A1***

$\sin\left(B+C\right)=\frac{36+1-25}{24}$ ***(A1)***

$\sin\left(B+C\right)=\frac{1}{2}$ ***AG***

[6 marks]

(b) $\sin A=\sin\left(180^{\circ}-(B+C)\right)$ so $\sin A=\sin(B+C)$ ***R1***

$\sin(B+C)=\frac{1}{2}\Rightarrow \sin A=\frac{1}{2}$ ***A1***

$\Rightarrow A=30^{\circ}$ or $A=150^{\circ}$ ***A1***

if $A=150^{\circ}$, then $B<30^{\circ}$ ***R1***

for example, $3\sin B+4\cos C<\frac{3}{2}+4<6$, *ie* a contradiction ***R1***

only one possible value ($A=30^{\circ}$) ***AG***

[5 marks]

Total [11 marks]