

Name:

1. (12 points) Let X be a discrete random variable with a probability mass function:

$$P(X = x) = kx(10 - x^2), \quad x = 1, 2, 3$$

where k is a constant.

(a) Find the value of k . [3]

(b) State the mode of X . [1]

(c) Calculate: [6]

(i) $E(X)$,

(ii) $Var(X)$.

(d) A new discrete random variable Y is given by $Y = 3 - 2X$. Calculate: [2]

(i) $E(Y)$,

(ii) $\sigma(Y)$.

2. (9 points)

The function f is defined by:

$$f(x) = \begin{cases} ax - 1, & x \leq \frac{\pi}{4} \\ \sqrt{2} \sin x, & x > \frac{\pi}{4} \end{cases}$$

$f(x)$ is continuous.

(a) Find the value of a . [3]

(b) Show that $f(x)$ is not differentiable at $\frac{\pi}{4}$. [3]

(c) The graph of g is obtained by translating the graph of f by a vector $\begin{pmatrix} -\frac{\pi}{4} \\ 1 \end{pmatrix}$.

Find $g(x)$. [3]

3. (8 points)

Given that $\sin x - \cos x = \frac{1}{3}$, find the possible values of $\cos 6x$.

4. (11 points)

(a) Write $\frac{1}{36}(x + 5)(2x + 4)$ in the form $ax^2 + bx + c$. [1]

(b) Tomasz rolls a die with 1 red and 5 blue faces once and Maria rolls a die with 2 red and 4 blue faces once. They each get a point if their die shows a red face. Find the probability that: [3]

(i) they both get a point,

(ii) only one of them gets a point,

(iii) no one gets a point.

(c) They now play a different game. Tomasz rolls a die with 4 red and 2 blue faces once. Maria rolls a die with 3 red and 3 blue faces twice. They get a point for each red faces that comes up. Let X denote the total number of points they score. By expanding

$$\frac{1}{216}(Ax + B)(Cx + D)^2$$

for appropriate values of A, B, C and D , find the probability distribution for X . [5]

(d) Calculate $E(X)$. [2]