Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2015

Mathematics

Higher level

Paper 1

27 pages

Section A

1. (a) $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = 0.25 + 0.6 - 0.7$ ***M1***

$= 0.15$ ***A1***

[2 marks]

(b) **EITHER**

$\mathrm{P}(A)\mathrm{P}(B)\left(= 0.25 \times 0.6\right) = 0.15$ ***A1***

$= \mathrm{P}(A \cap B)$ so independent ***R1***

OR

$P\left(A|B\right) = \dfrac{P\left(A \cap B\right)}{P\left(B\right)} = \dfrac{0.15}{0.6} = 0.25$ ***A1***

$= P\left(A\right)$ so independent ***R1***

Note:	Allow follow through for incorrect answer to (a) that will result in events being dependent in (b).

[2 marks]

Total [4 marks]

2. $(3-x)^4 = 1.3^4 + 4.3^3(-x) + 6.3^2(-x)^2 + 4.3(-x)^3 + 1(-x)^4$ or equivalent (***M1***)(***A1***)

$= 81 - 108x + 54x^2 - 12x^3 + x^4$ ***A1A1***

Note:	***A1*** for ascending powers, ***A1*** for correct coefficients including signs.

[4 marks]

3. $\tan x + \tan 2x = 0$

$\tan x + \dfrac{2\tan x}{1 - \tan^2 x} = 0$ ***M1***

$\tan x - \tan^3 x + 2\tan x = 0$ ***A1***

$\tan x\left(3 - \tan^2 x\right) = 0$ (***M1***)

$\tan x = 0 \Rightarrow x = 0,\ x = 180^\circ$ ***A1***

Note: If $x = 360^\circ$ seen anywhere award ***A0***

$\tan x = \sqrt{3} \Rightarrow x = 60^\circ,\ 240^\circ$ ***A1***

$\tan x = -\sqrt{3} \Rightarrow x = 120^\circ,\ 300^\circ$ ***A1***

[6 marks]

4. (a) attempt to differentiate $f(x) = x^3 - 3x^2 + 4$ ***M1***

$f'(x) = 3x^2 - 6x$ ***A1***

$= 3x(x-2)$

(Critical values occur at) $x = 0$, $x = 2$ ***(A1)***

so f decreasing on $x \in]0, 2[$ (or $0 < x < 2$) ***A1***

[4 marks]

(b) $f''(x) = 6x - 6$ ***(A1)***

setting $f''(x) = 0$ ***M1***

$\Rightarrow x = 1$

coordinate is $(1, 2)$ ***A1***

[3 marks]

Total [7 marks]

5. any attempt at integration by parts ***M1***

$u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x}$ ***(A1)***

$\frac{dv}{dx} = x^3 \Rightarrow v = \frac{x^4}{4}$ ***(A1)***

$= \left[\frac{x^4}{4}\ln x\right]_1^2 - \int_1^2 \frac{x^3}{4}dx$ ***A1***

Note: Condone absence of limits at this stage.

$= \left[\frac{x^4}{4}\ln x\right]_1^2 - \left[\frac{x^4}{16}\right]_1^2$ ***A1***

Note: Condone absence of limits at this stage.

$= 4\ln 2 - \left(1 - \frac{1}{16}\right)$ ***A1***

$= 4\ln 2 - \frac{15}{16}$ ***AG***

[6 marks]

6. (a) any attempt to use sine rule ***M1***

$$\frac{\text{AB}}{\sin\frac{\pi}{3}}=\frac{\sqrt{3}}{\sin\left(\frac{2\pi}{3}-\theta\right)}$$ ***A1***

$$=\frac{\sqrt{3}}{\sin\frac{2\pi}{3}\cos\theta-\cos\frac{2\pi}{3}\sin\theta}$$ ***A1***

Note: Condone use of degrees.

$$=\frac{\sqrt{3}}{\frac{\sqrt{3}}{2}\cos\theta+\frac{1}{2}\sin\theta}$$ ***A1***

$$\frac{\text{AB}}{\frac{\sqrt{3}}{2}}=\frac{\sqrt{3}}{\frac{\sqrt{3}}{2}\cos\theta+\frac{1}{2}\sin\theta}$$

$$\therefore \text{AB}=\frac{3}{\sqrt{3}\cos\theta+\sin\theta}$$ ***AG***

[4 marks]

(b) **METHOD 1**

$$(\text{AB})'=\frac{-3\left(-\sqrt{3}\sin\theta+\cos\theta\right)}{\left(\sqrt{3}\cos\theta+\sin\theta\right)^2}$$ ***M1A1***

setting $(\text{AB})'=0$ ***M1***

$$\tan\theta=\frac{1}{\sqrt{3}}$$

$$\theta=\frac{\pi}{6}$$ ***A1***

continued…

Question 6 continued

METHOD 2

$$\text{AB} = \frac{\sqrt{3}\sin\frac{\pi}{3}}{\sin\left(\frac{2\pi}{3} - \theta\right)}$$

AB minimum when $\sin\left(\frac{2\pi}{3} - \theta\right)$ is maximum ***M1***

$\sin\left(\frac{2\pi}{3} - \theta\right) = 1$ ***(A1)***

$\frac{2\pi}{3} - \theta = \frac{\pi}{2}$ ***M1***

$\theta = \frac{\pi}{6}$ ***A1***

METHOD 3

shortest distance from B to AC is perpendicular to AC ***R1***

$\theta = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$ ***M1A2***

[4 marks]

Total [8 marks]

7. (a) **METHOD 1**

$z^3 = -\frac{27}{8} = \frac{27}{8}(\cos\pi + \mathrm{i}\sin\pi)$ ***M1(A1)***

$= \frac{27}{8}\left(\cos(\pi + 2n\pi) + \mathrm{i}\sin(\pi + 2n\pi)\right)$ (***A1)***

$z = \frac{3}{2}\left(\cos\left(\frac{\pi + 2n\pi}{3}\right) + \mathrm{i}\sin\left(\frac{\pi + 2n\pi}{3}\right)\right)$ ***M1***

$z_1 = \frac{3}{2}\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right)$,

$z_2 = \frac{3}{2}\left(\cos\pi + \mathrm{i}\sin\pi\right)$,

$z_3 = \frac{3}{2}\left(\cos\frac{5\pi}{3} + \mathrm{i}\sin\frac{5\pi}{3}\right)$. ***A2***

Note: Accept $-\frac{\pi}{3}$ as the argument for z_3.

Note: Award ***A1*** for 2 correct roots.

Note: Allow solutions expressed in Eulerian $\left(re^{\mathrm{i}\theta}\right)$ form.

Note: Allow use of degrees in mod-arg (r-cis) form only.

[6 marks]

continued…

Question 7 continued

METHOD 2

$8z^3 + 27 = 0$

$\Rightarrow z = -\frac{3}{2}$ so $(2z+3)$ is a factor

Attempt to use long division or factor theorem: ***M1***

$\Rightarrow 8z^3 + 27 \equiv (2z+3)(4z^2 - 6z + 9)$

$\Rightarrow 4z^2 - 6z + 9 = 0$ ***A1***

Attempt to solve quadratic: ***M1***

$z = \frac{3 \pm 3\sqrt{3}\,\mathrm{i}}{4}$ ***A1***

$z_1 = \frac{3}{2}\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right)$,

$z_2 = \frac{3}{2}(\cos\pi + \mathrm{i}\sin\pi)$,

$z_3 = \frac{3}{2}\left(\cos\frac{5\pi}{3} + \mathrm{i}\sin\frac{5\pi}{3}\right)$. ***A2***

Note: Accept $-\frac{\pi}{3}$ as the argument for z_3.

Note: Award ***A1*** for 2 correct roots.

Note: Allow solutions expressed in Eulerian $\left(re^{\mathrm{i}\theta}\right)$ form.

Note: Allow use of degrees in mod-arg (r-cis) form only.

[6 marks]

continued…

Question 7 continued

METHOD 3

$8z^3 + 27 = 0$

Substitute $z = x + \mathrm{i}\,y$ ***M1***

$8\left(x^3 + 3\mathrm{i}\,x^2 y - 3xy^2 - \mathrm{i}\,y^3\right) + 27 = 0$

$\Rightarrow 8x^3 - 24xy^2 + 27 = 0$ and $24x^2 y - 8y^3 = 0$ ***A1***

Attempt to solve simultaneously: ***M1***

$8y\left(3x^2 - y^2\right) = 0$

$y = 0, y = x\sqrt{3}, y = -x\sqrt{3}$

$\Rightarrow \left(x = -\frac{3}{2}, y = 0\right), x = \frac{3}{4}, y = \pm\frac{3\sqrt{3}}{4}$ ***A1***

$z_1 = \frac{3}{2}\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right),$

$z_2 = \frac{3}{2}\left(\cos\pi + \mathrm{i}\sin\pi\right),$

$z_3 = \frac{3}{2}\left(\cos\frac{5\pi}{3} + \mathrm{i}\sin\frac{5\pi}{3}\right).$ ***A2***

Note: Accept $-\frac{\pi}{3}$ as the argument for z_3.

Note: Award ***A1*** for 2 correct roots.

Note: Allow solutions expressed in Eulerian $\left(re^{\mathrm{i}\theta}\right)$ form.

Note: Allow use of degrees in mod-arg (r-cis) form only.

[6 marks]

continued…

Question 7 continued

(b) **EITHER**

Valid attempt to use area $= 3\left(\frac{1}{2}ab\sin C\right)$ ***M1***

$= 3 \times \frac{1}{2} \times \frac{3}{2} \times \frac{3}{2} \times \frac{\sqrt{3}}{2}$ ***A1A1***

Note: Award ***A1*** for correct sides, ***A1*** for correct $\sin C$.

OR

Valid attempt to use area $= \frac{1}{2}$ base $\times$ height ***M1***

area $= \frac{1}{2} \times \left(\frac{3}{4} + \frac{3}{2}\right) \times \frac{6\sqrt{3}}{4}$ ***A1A1***

Note: ***A1*** for correct height, ***A1*** for correct base.

THEN

$= \frac{27\sqrt{3}}{16}$ ***AG***

[3 marks]

Total [9 marks]

8. EITHER

$x = \arctan t$ ***(M1)***

$\frac{dx}{dt} = \frac{1}{1+t^2}$ ***A1***

OR

$t = \tan x$

$\frac{dt}{dx} = \sec^2 x$ ***(M1)***

$= 1 + \tan^2 x$ ***A1***

$= 1 + t^2$

THEN

$\sin x = \frac{t}{\sqrt{1+t^2}}$ ***(A1)***

Note: This ***A1*** is independent of the first two marks

$\int \frac{dx}{1+\sin^2 x} = \int \frac{\frac{dt}{1+t^2}}{1+\left(\frac{t}{\sqrt{1+t^2}}\right)^2}$ ***M1A1***

Note: Award ***M1*** for attempting to obtain integral in terms of t and dt

$= \int \frac{dt}{(1+t^2)+t^2} = \int \frac{dt}{1+2t^2}$ ***A1***

$= \frac{1}{2}\int \frac{dt}{\frac{1}{2}+t^2} = \frac{1}{2} \times \frac{1}{\frac{1}{\sqrt{2}}} \arctan\left(\frac{t}{\frac{1}{\sqrt{2}}}\right)$ ***A1***

$= \frac{\sqrt{2}}{2}\arctan\left(\sqrt{2}\tan x\right)(+c)$ ***A1***

[8 marks]

9. (a) $a > 0$ ***A1***

$a \neq 1$ ***A1***

[2 marks]

(b) **METHOD 1**

$\log_x y = \dfrac{\ln y}{\ln x}$ and $\log_y x = \dfrac{\ln x}{\ln y}$ ***M1A1***

Note: Use of any base is permissible here, not just "e".

$\left(\dfrac{\ln y}{\ln x}\right)^2 = 4$ ***A1***

$\ln y = \pm 2 \ln x$ ***A1***

$y = x^2$ or $\dfrac{1}{x^2}$ ***A1A1***

METHOD 2

$\log_y x = \dfrac{\log_x x}{\log_x y} = \dfrac{1}{\log_x y}$ ***M1A1***

$(\log_x y)^2 = 4$ ***A1***

$\log_x y = \pm 2$ ***A1***

$y = x^2$ or $y = \dfrac{1}{x^2}$ ***A1A1***

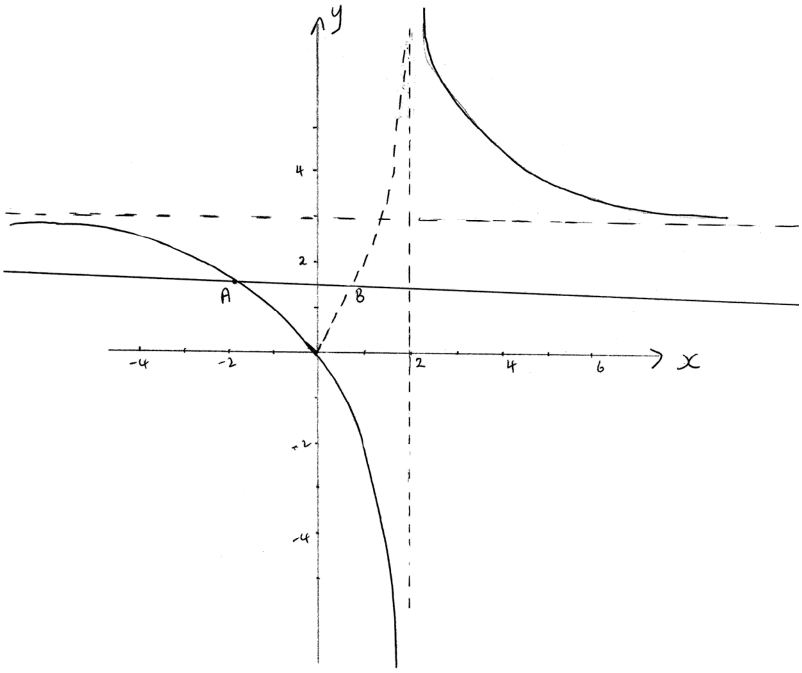
Note: The final two ***A*** marks are independent of the one coming before.

[6 marks]

Total [8 marks]

Section B

10. (a)



> **Note:** In the diagram, points marked A and B refer to part (d) and do not need to be seen in part (a).

shape of curve ***A1***

> **Note:** This mark can only be awarded if there appear to be both horizontal and vertical asymptotes.

intersection at $(0, 0)$ ***A1***
horizontal asymptote at $y = 3$ ***A1***
vertical asymptote at $x = 2$ ***A1***

[4 marks]

(b) $y = \dfrac{3x}{x-2}$

$xy - 2y = 3x$ ***M1A1***

$xy - 3x = 2y$

$x = \dfrac{2y}{y-3}$

$\left(f^{-1}(x)\right) = \dfrac{2x}{x-3}$ ***M1A1***

> **Note:** Final M1 is for interchanging of x and y, which may be seen at any stage.

[4 marks]

continued…

Question 10 continued

(c) **METHOD 1**

attempt to solve $\frac{2x}{x-3}=\frac{3x}{x-2}$ ***(M1)***

$2x(x-2)=3x(x-3)$

$x\left[2(x-2)-3(x-3)\right]=0$

$x(5-x)=0$

$x=0$ or $x=5$ ***A1A1***

METHOD 2

$x=\frac{3x}{x-2}$ or $x=\frac{2x}{x-3}$ ***(M1)***

$x=0$ or $x=5$ ***A1A1***

[3 marks]

(d) **METHOD 1**

at $\mathrm{A}:\frac{3x}{x-2}=\frac{3}{2}$ AND at $\mathrm{B}:\frac{3x}{x-2}=-\frac{3}{2}$ ***M1***

$6x=3x-6$

$x=-2$ ***A1***

$6x=6-3x$

$x=\frac{2}{3}$ ***A1***

solution is $-2<x<\frac{2}{3}$ ***A1***

[4 marks]

METHOD 2

$\left(\frac{3x}{x-2}\right)^2<\left(\frac{3}{2}\right)^2$ ***M1***

$9x^2<\frac{9}{4}\left(x-2\right)^2$

$3x^2+4x-4<0$

$\left(3x-2\right)\left(x+2\right)<0$

$x=-2$ ***(A1)***

$x=\frac{2}{3}$ ***(A1)***

solution is $-2<x<\frac{2}{3}$ ***A1***

[4 marks]

continued…

Question 10 continued

(e) $-2 < x < 2$ ***A1A1***

Note: ***A1*** for correct end points, ***A1*** for correct inequalities.

Note: If working is shown, then ***A*** marks may only be awarded following correct working.

[2 marks]

Total [17 marks]

11. (a) $g \circ f(x) = \dfrac{\tan x + 1}{\tan x - 1}$ ***A1***

$x \neq \dfrac{\pi}{4}$, $0 \leq x < \dfrac{\pi}{2}$ ***A1***

[2 marks]

(b) $\dfrac{\tan x + 1}{\tan x - 1} = \dfrac{\frac{\sin x}{\cos x} + 1}{\frac{\sin x}{\cos x} - 1}$ ***M1A1***

$= \dfrac{\sin x + \cos x}{\sin x - \cos x}$ ***AG***

[2 marks]

(c) **METHOD 1**

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{(\sin x - \cos x)(\cos x - \sin x) - (\sin x + \cos x)(\cos x + \sin x)}{(\sin x - \cos x)^2}$ ***M1(A1)***

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\left(2\sin x\cos x - \cos^2 x - \sin^2 x\right) - \left(2\sin x\cos x + \cos^2 x + \sin^2 x\right)}{\cos^2 x + \sin^2 x - 2\sin x\cos x}$

$= \dfrac{-2}{1 - \sin 2x}$

Substitute $\dfrac{\pi}{6}$ into any formula for $\dfrac{dy}{dx}$ ***M1***

$\dfrac{-2}{1 - \sin\frac{\pi}{3}}$

$= \dfrac{-2}{1 - \frac{\sqrt{3}}{2}}$ ***A1***

$= \dfrac{-4}{2 - \sqrt{3}}$

$= \dfrac{-4}{2 - \sqrt{3}}\left(\dfrac{2 + \sqrt{3}}{2 + \sqrt{3}}\right)$ ***M1***

$= \dfrac{-8 - 4\sqrt{3}}{1} = -8 - 4\sqrt{3}$ ***A1***

continued…

Question 11 continued

METHOD 2

$$\frac{dy}{dx} = \frac{(\tan x - 1)\sec^2 x - (\tan x + 1)\sec^2 x}{(\tan x - 1)^2}$$ ***M1A1***

$$= \frac{-2\sec^2 x}{(\tan x - 1)^2}$$ ***A1***

$$= \frac{-2\sec^2 \frac{\pi}{6}}{\left(\tan\frac{\pi}{6} - 1\right)^2} = \frac{-2\left(\frac{4}{3}\right)}{\left(\frac{1}{\sqrt{3}} - 1\right)^2} = \frac{-8}{\left(1 - \sqrt{3}\right)^2}$$ ***M1***

> **Note:** Award ***M1*** for substitution of $\frac{\pi}{6}$.

$$\frac{-8}{\left(1 - \sqrt{3}\right)^2} = \frac{-8}{\left(4 - 2\sqrt{3}\right)}\frac{\left(4 + 2\sqrt{3}\right)}{\left(4 + 2\sqrt{3}\right)} = -8 - 4\sqrt{3}$$ ***M1A1***

[6 marks]

continued…

Question 11 continued

(d) Area $= \left| \int_0^{\frac{\pi}{6}} \frac{\sin x + \cos x}{\sin x - \cos x} \mathrm{d}x \right|$ ***M1***

$= \left| \left[\ln\left|\sin x - \cos x\right| \right]_0^{\frac{\pi}{6}} \right|$ ***A1***

Note: Condone absence of limits and absence of modulus signs at this stage.

$= \left| \ln\left|\sin\frac{\pi}{6} - \cos\frac{\pi}{6}\right| - \ln\left|\sin 0 - \cos 0\right| \right|$ ***M1***

$= \left| \ln\left|\frac{1}{2} - \frac{\sqrt{3}}{2}\right| - 0 \right|$

$= \left| \ln\left(\frac{\sqrt{3} - 1}{2}\right) \right|$ ***A1***

$= -\ln\left(\frac{\sqrt{3} - 1}{2}\right) = \ln\left(\frac{2}{\sqrt{3} - 1}\right)$ ***A1***

$= \ln\left(\frac{2}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}\right)$ ***M1***

$= \ln\left(\sqrt{3} + 1\right)$ ***AG***

[6 marks]

Total [16 marks]

12. (a) (i)–(iii) given the three roots α, β, γ, we have

$x^3 + px^2 + qx + c = (x-\alpha)(x-\beta)(x-\gamma)$ ***M1***

$= \left(x^2 - (\alpha+\beta)x + \alpha\beta\right)(x-\gamma)$ ***A1***

$= x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)x - \alpha\beta\gamma$ ***A1***

comparing coefficients:

$p = -(\alpha+\beta+\gamma)$ ***AG***

$q = (\alpha\beta+\beta\gamma+\gamma\alpha)$ ***AG***

$c = -\alpha\beta\gamma$ ***AG***

[3 marks]

(b) **METHOD 1**

i) Given $-\alpha-\beta-\gamma = -6$

And $\alpha\beta+\beta\gamma+\gamma\alpha = 18$

Let the three roots be α, β, γ.

So $\beta-\alpha = \gamma-\beta$ ***M1***

or $2\beta = \alpha+\gamma$

Attempt to solve simultaneous equations: ***M1***

$\beta + 2\beta = 6$ ***A1***

$\beta = 2$ ***AG***

ii) $\alpha+\gamma = 4$

$2\alpha + 2\gamma + \alpha\gamma = 18$

$\Rightarrow \gamma^2 - 4\gamma + 10 = 0$

$\Rightarrow \gamma = \dfrac{4 \pm \mathrm{i}\sqrt{24}}{2}$ ***(A1)***

Therefore $c = -\alpha\beta\gamma = -\left(\dfrac{4+\mathrm{i}\sqrt{24}}{2}\right)\left(\dfrac{4-\mathrm{i}\sqrt{24}}{2}\right)2 = -20$ ***A1***

[5 marks]

continued…

Question 12 continued

METHOD 2

(i) let the three roots be α, $\alpha - d$, $\alpha + d$ ***M1***
adding roots ***M1***
to give $3\alpha = 6$ ***A1***
$\alpha = 2$ ***AG***

(ii) α is a root, so $2^3 - 6 \times 2^2 + 18 \times 2 + c = 0$ ***M1***

$8 - 24 + 36 + c = 0$
$c = -20$ ***A1***

[5 marks]

METHOD 3

(i) let the three roots be α, $\alpha - d$, $\alpha + d$ ***M1***
adding roots ***M1***
to give $3\alpha = 6$ ***A1***
$\alpha = 2$ ***AG***

(ii) $q = 18 = 2(2 - d) + (2 - d)(2 + d) + 2(2 + d)$ ***M1***

$d^2 = -6 \Rightarrow d = \sqrt{6}\,\mathrm{i}$
$\Rightarrow c = -20$ ***A1***

[5 marks]

continued…

Question 12 continued

(c) **METHOD 1**

Given $-\alpha-\beta-\gamma=-6$
And $\alpha\beta+\beta\gamma+\gamma\alpha=18$

Let the three roots be α,β,γ.

So $\frac{\beta}{\alpha}=\frac{\gamma}{\beta}$ ***M1***

or $\beta^2=\alpha\gamma$

Attempt to solve simultaneous equations: ***M1***

$\alpha\beta+\gamma\beta+\beta^2=18$
$\beta(\alpha+\beta+\gamma)=18$
$6\beta=18$
$\beta=3$ ***A1***

$\alpha+\gamma=3$, $\alpha=\frac{9}{\gamma}$
$\Rightarrow\gamma^2-3\gamma+9=0$
$\Rightarrow\gamma=\frac{3\pm\mathrm{i}\sqrt{27}}{2}$ ***(A1)(A1)***

Therefore $c=-\alpha\beta\gamma=-\left(\frac{3+\mathrm{i}\sqrt{27}}{2}\right)\left(\frac{3-\mathrm{i}\sqrt{27}}{2}\right)3=-27$ ***A1***

[6 marks]

METHOD 2

let the three roots be a, ar, ar^2 ***M1***
attempt at substitution of a, ar, ar^2 and *p* and *q* into equations from (a) ***M1***
$6=a+ar+ar^2\left(=a\left(1+r+r^2\right)\right)$ ***A1***
$18=a^2r+a^2r^3+a^2r^2\left(=a^2r\left(1+r+r^2\right)\right)$ ***A1***

therefore $3=ar$ ***A1***
therefore $c=-a^3r^3=-3^3=-27$ ***A1***

[6 marks]

Total [14 marks]

13. (a) $\frac{1}{\sqrt{n}+\sqrt{n+1}} = \frac{1}{\sqrt{n}+\sqrt{n+1}} \times \frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n+1}-\sqrt{n}}$ ***M1***

$= \frac{\sqrt{n+1}-\sqrt{n}}{(n+1)-n}$ ***A1***

$= \sqrt{n+1}-\sqrt{n}$ ***AG***

[2 marks]

(b) $\sqrt{2}-1 = \frac{1}{\sqrt{2}+\sqrt{1}}$ ***A2***

$< \frac{1}{\sqrt{2}}$ ***AG***

[2 marks]

(c) consider the case $n = 2$: required to prove that $1+\frac{1}{\sqrt{2}} > \sqrt{2}$ ***M1***

from part (b) $\frac{1}{\sqrt{2}} > \sqrt{2}-1$

hence $1+\frac{1}{\sqrt{2}} > \sqrt{2}$ is true for $n = 2$ ***A1***

now assume true for $n = k$: $\sum_{r=1}^{r=k} \frac{1}{\sqrt{r}} > \sqrt{k}$ ***M1***

$\frac{1}{\sqrt{1}}+\ldots+\frac{1}{\sqrt{k}} > \sqrt{k}$

attempt to prove true for $n = k+1$: $\frac{1}{\sqrt{1}}+\ldots+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}} > \sqrt{k+1}$ (***M1***)

from assumption, we have that $\frac{1}{\sqrt{1}}+\ldots+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}} > \sqrt{k}+\frac{1}{\sqrt{k+1}}$ ***M1***

so attempt to show that $\sqrt{k}+\frac{1}{\sqrt{k+1}} > \sqrt{k+1}$ (***M1***)

continued…

Question 13 continued

EITHER

$\frac{1}{\sqrt{k+1}} > \sqrt{k+1} - \sqrt{k}$ ***A1***

$\frac{1}{\sqrt{k+1}} > \frac{1}{\sqrt{k} + \sqrt{k+1}}$, (from part a), which is true ***A1***

OR

$\sqrt{k} + \frac{1}{\sqrt{k+1}} = \frac{\sqrt{k+1}\sqrt{k}+1}{\sqrt{k+1}}$. ***A1***

$> \frac{\sqrt{k}\sqrt{k}+1}{\sqrt{k+1}} = \sqrt{k+1}$ ***A1***

THEN

so true for $n = 2$ and $n = k$ true $\Rightarrow n = k+1$ true. Hence true for all $n \geq 2$ ***R1***

Note: Award ***R1*** only if all previous ***M*** marks have been awarded.

[9 marks]

Total [13 marks]