Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2015

Mathematics

Higher level

Paper 2

Section A

1. (a) $A = \frac{1}{2} \times 5 \times 12 \times \sin 100^\circ$ ***(M1)***

$= 29.5 \left(\text{cm}^2\right)$ ***A1***

[2 marks]

(b) $\text{AC}^2 = 5^2 + 12^2 - 2 \times 5 \times 12 \times \cos 100^\circ$ ***(M1)***

therefore $\text{AC} = 13.8 \text{ (cm)}$ ***A1***

[2 marks]

Total [4 marks]

2. (a) $\binom{11}{4} = \frac{11 \times 10 \times 9 \times 8}{4 \times 3 \times 2 \times 1} = 330$ ***(M1)A1***

[2 marks]

(b) $\binom{5}{2} \times \binom{6}{2} = \frac{5 \times 4}{2 \times 1} \times \frac{6 \times 5}{2 \times 1}$ ***M1***

$= 150$ ***A1***

[2 marks]

(c) **METHOD1**

number of ways all men $= \binom{5}{4} = 5$

$330 - 5 = 325$ ***M1A1***

Note: Allow ***FT*** from answer obtained in part (a).

[2 marks]

continued…

Question 2 continued.

METHOD 2

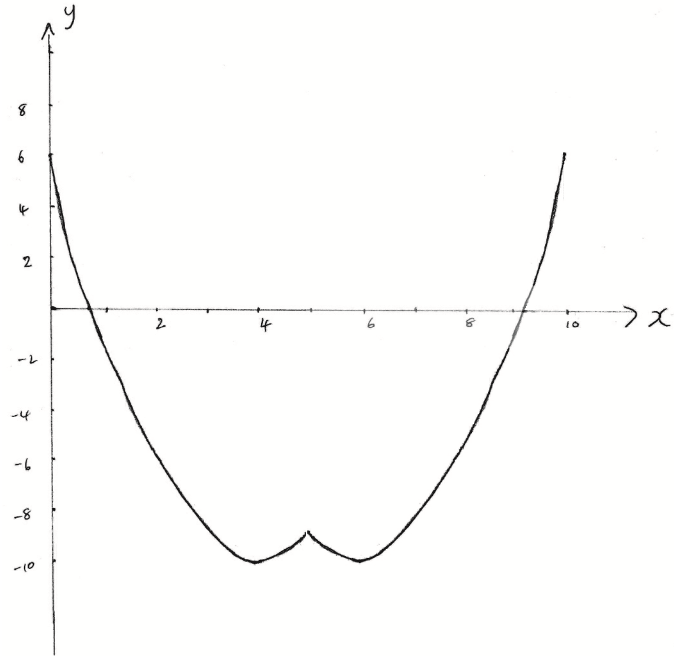
$\binom{6}{1}\binom{5}{3}+\binom{6}{2}\binom{5}{2}+\binom{6}{3}\binom{5}{1}+\binom{6}{4}$ ***M1***

=325 ***A1***

[2 marks]

Total [6 marks]

3. (a)



general shape including 2 minimums, cusp ***A1A1***
correct domain and symmetrical about the middle $(x=5)$ ***A1***

[3 marks]

(b) $x=9.16$ or $x=0.838$ ***A1A1***

[2 marks]

Total [5 marks]

4. (a) (i) $X \sim Po(5)$

$\mathrm{P}(X \geq 8) = 0.133$ ***(M1)A1***

(ii) $7 \times 0.133\ldots$ ***M1***

≈ 0.934 days ***A1***

> **Note:** Accept "1 day".

[4 marks]

(b) $7 \times 5 = 35 \ \left(Y \sim Po(35)\right)$ ***(A1)***

$\mathrm{P}(Y \leq 29) = 0.177$ ***(M1)A1***

[3 marks]

Total [7 marks]

5. (a) $\boldsymbol{u} \times \boldsymbol{v} = \begin{pmatrix} 2(0) + 2b \\ -2a - 1(0) \\ b - 2a \end{pmatrix} = \begin{pmatrix} 2b \\ -2a \\ b - 2a \end{pmatrix}$ ***(M1)(A1)***

$\begin{pmatrix} 2b \\ -2a \\ b - 2a \end{pmatrix} = \begin{pmatrix} 4 \\ b \\ c \end{pmatrix}$ ***(M1)***

$\Rightarrow a = -1,\ b = 2,\ c = 4$ ***A2***

> **Note:** Award ***A1*** for two correct.

[5 marks]

(b) $\boldsymbol{n} = \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix}$ ***(A1)***

$\Rightarrow 4x + 2y + 4z = 0 \ (2x + y + 2z = 0)$ ***A1***

[2 marks]

Total [7 marks]

6. (a) **EITHER**

$y = \ln(x - a) + b = \ln(5x + 10)$ ***(M1)***

$y = \ln(x - a) + \ln c = \ln(5x + 10)$

$y = \ln\left(c(x - a)\right) = \ln(5x + 10)$ ***(M1)***

OR

$y = \ln(5x + 10) = \ln\left(5(x + 2)\right)$ ***(M1)***

$y = \ln(5) + \ln(x + 2)$ ***(M1)***

THEN

$a = -2$, $b = \ln 5$ ***A1A1***

Note: Accept graphical approaches.

Note: Accept $a = 2,\ b = 1.61$

[4marks]

(b) $V = \pi \int_{e}^{2e} \left[\ln(5x + 10)\right]^2 \mathrm{d}x$ ***(M1)***

$= 99.2$ ***A1***

[2marks]

Total [6 marks]

7. (a)

$$2x + y + 6z = 0$$
$$4x + 3y + 14z = 4$$
$$2x - 2y + (\alpha - 2)z = \beta - 12$$

attempt at row reduction ***M1***

eg $R_2 - 2R_1$ and $R_3 - R_1$

$$2x + y + 6z = 0$$
$$y + 2z = 4$$
$$-3y + (\alpha - 8)z = \beta - 12$$ ***A1***

eg $R_3 + 3R_2$

$$2x + y + 6z = 0$$
$$y + 2z = 4$$ ***A1***
$$(\alpha - 2)z = \beta$$

(i) no solutions if $\alpha = 2$, $\beta \neq 0$ ***A1***

(ii) one solution if $\alpha \neq 2$ ***A1***

(iii) infinite solutions if $\alpha = 2$, $\beta = 0$ ***A1***

Note: Accept alternative methods e.g. determinant of a matrix

Note: Award ***A1A1A0*** if all three consistent with their reduced form, ***A1A0A0*** if two or one answer consistent with their reduced form.

[6 marks]

(b) $y + 2z = 4 \Rightarrow y = 4 - 2z$ ***A1***

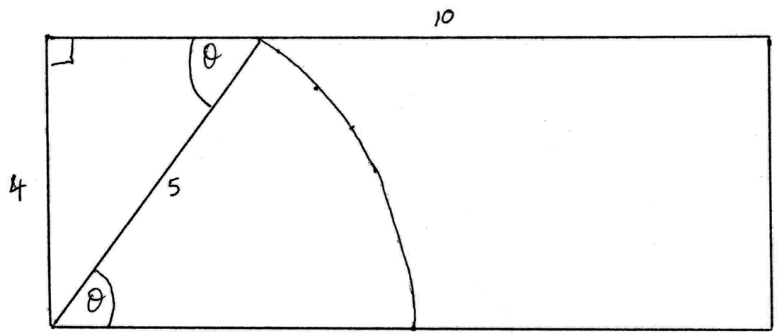
$2x = -y - 6z = 2z - 4 - 6z = -4z - 4 \Rightarrow x = -2z - 2$ ***A1***

therefore Cartesian equation is $\frac{x+2}{-2} = \frac{y-4}{-2} = \frac{z}{1}$ or equivalent ***A1***

[3 marks]

Total [9 marks]

8. (a)



EITHER

area of triangle $=\frac{1}{2}\times 3\times 4\ (=6)$ ***A1***

area of sector $=\frac{1}{2}\arcsin\left(\frac{4}{5}\right)\times 5^2\ (=11.5911\ldots)$ ***A1***

OR

$\int_0^4 \sqrt{25-x^2}\,\mathrm{d}x$ ***M1A1***

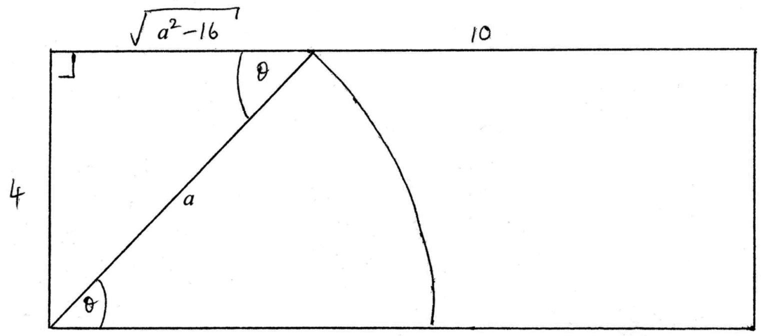
THEN

total area $=17.5911\ldots\text{m}^2$ ***(A1)***

percentage $=\frac{17.5911\ldots}{40}\times 100=44\%$ ***A1***

[4 marks]

(b) **METHOD 1**



area of triangle $=\frac{1}{2}\times 4\times\sqrt{a^2-16}$ ***A1***

$\theta=\arcsin\left(\frac{4}{a}\right)$ ***(A1)***

area of sector $=\frac{1}{2}r^2\theta=\frac{1}{2}a^2\arcsin\left(\frac{4}{a}\right)$ ***A1***

therefore total area $=2\sqrt{a^2-16}+\frac{1}{2}a^2\arcsin\left(\frac{4}{a}\right)=20$ ***A1***

rearrange to give: $a^2\arcsin\left(\frac{4}{a}\right)+4\sqrt{a^2-16}=40$ ***AG***

continued…

Question 8 continued

METHOD 2

$$\int_0^4 \sqrt{a^2 - x^2}\,dx = 20$$ ***M1***

use substitution $x = a\sin\theta,\ \frac{\mathrm{d}x}{\mathrm{d}\theta} = a\cos\theta$

$$\int_0^{\arcsin\left(\frac{4}{a}\right)} a^2\cos^2\theta\,\mathrm{d}\theta = 20$$

$$\frac{a^2}{2}\int_0^{\arcsin\left(\frac{4}{a}\right)} (\cos 2\theta + 1)\,\mathrm{d}\theta = 20$$ ***M1***

$$a^2\left[\left(\frac{\sin 2\theta}{2} + \theta\right)\right]_0^{\arcsin\left(\frac{4}{a}\right)} = 40$$ ***A1***

$$a^2\left[(\sin\theta\cos\theta + \theta)\right]_0^{\arcsin\left(\frac{4}{a}\right)} = 40$$

$$a^2\arcsin\left(\frac{4}{a}\right) + a^2\left(\frac{4}{a}\right)\sqrt{\left(1 - \left(\frac{4}{a}\right)^2\right)} = 40$$ ***A1***

$$a^2\arcsin\left(\frac{4}{a}\right) + 4\sqrt{a^2 - 16} = 40$$ ***AG***

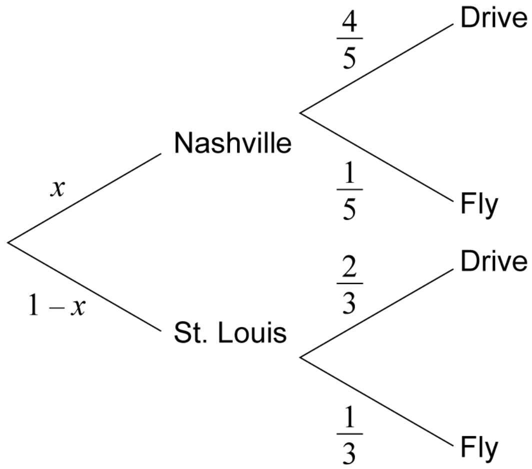
[4 marks]

(c) solving using $\text{GDC} \Rightarrow a = 5.53\,\text{cm}$ ***A2***

[2 marks]

Total [10 marks]

9.



(a) attempt to set up the problem using a tree diagram and/or an equation, with the unknown x ***M1***

$\frac{4}{5}x+\frac{2}{3}(1-x)=\frac{13}{18}$ ***A1***

$\frac{4x}{5}-\frac{2x}{3}=\frac{13}{18}-\frac{2}{3}$

$\frac{2x}{15}=\frac{1}{18}$

$x=\frac{5}{12}$ ***A1***

[3 marks]

(b) attempt to set up the problem using conditional probability ***M1***

EITHER

$\dfrac{\frac{5}{12}\times\frac{1}{5}}{1-\frac{13}{18}}$ ***A1***

OR

$\dfrac{\frac{5}{12}\times\frac{1}{5}}{\frac{1}{12}+\frac{7}{36}}$ ***A1***

THEN

$=\frac{3}{10}$ ***A1***

[3 marks]

Total [6 marks]

Section B

10. (a) (i) $P(110 < X < 130) = = 0.49969... = 0.500 = 50.0\%$ ***(M1)A1***

Note: Accept 50

Note: Award ***M1A0*** for 0.50 (0.500)

(ii) $P(X > 130) = (1 - 0.707\ldots) = 0.293\ldots$ ***M1***

expected number of turnips $= 29.3$ ***A1***

Note: Accept 29.

(iii) no of turnips weighing more than 130 is $Y \sim B(100,\ 0.293)$ ***M1***

$P(Y \geq 30) = 0.478$ ***A1***

[6 marks]

(b) (i) $X \sim N\left(144,\ \sigma^2\right)$

$P(X \leq 130) = \frac{1}{15} = 0.0667$ ***(M1)***

$P\left(Z \leq \frac{130 - 144}{\sigma}\right) = 0.0667$

$\frac{14}{\sigma} = 1.501$ ***(A1)***

$\sigma = 9.33\,g$ ***A1***

(ii) $P(X > 150 \mid X > 130) = \frac{P(X > 150)}{P(X > 130)}$ ***M1***

$= \frac{0.26008\ldots}{1 - 0.06667} = 0.279$ ***A1***

expected number of turnips $= 55.7$ ***A1***

[6 marks]

Total [12 marks]

11. (a) attempt at implicit differentiation ***M1***

$$2x - 5x\frac{\mathrm{d}y}{\mathrm{d}x} - 5y + 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$$ ***A1A1***

> **Note:** ***A1*** for differentiation of $x^2 - 5xy$, ***A1*** for differentiation of y^2 and 7.

$$2x - 5y + \frac{\mathrm{d}y}{\mathrm{d}x}(2y - 5x) = 0$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5y - 2x}{2y - 5x}$$ ***AG***

[3 marks]

(b) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5\times1 - 2\times6}{2\times1 - 5\times6} = \frac{1}{4}$ ***A1***

gradient of normal $= -4$ ***A1***

equation of normal $y = -4x + c$ ***M1***

substitution of $(6, 1)$

$y = -4x + 25$ ***A1***

> **Note:** Accept $y - 1 = -4(x - 6)$

[4 marks]

(c) setting $\frac{5y - 2x}{2y - 5x} = 1$ ***M1***

$y = -x$ ***A1***

substituting into original equation ***M1***

$x^2 + 5x^2 + x^2 = 7$ ***(A1)***

$7x^2 = 7$

$x = \pm 1$ ***A1***

points $(1, -1)$ and $(-1, 1)$ ***(A1)***

distance $= \sqrt{8}\left(= 2\sqrt{2}\right)$ ***(M1)A1***

[8 marks]

Total [15 marks]

12. (a) **METHOD 1**

$s = \int (9t - 3t^2)\,dt = \frac{9}{2}t^2 - t^3 (+c)$ ***(M1)***

$t = 0,\ s = 3 \Rightarrow c = 3$ ***(A1)***

$t = 4 \Rightarrow s = 11$ ***A1***

[3 marks]

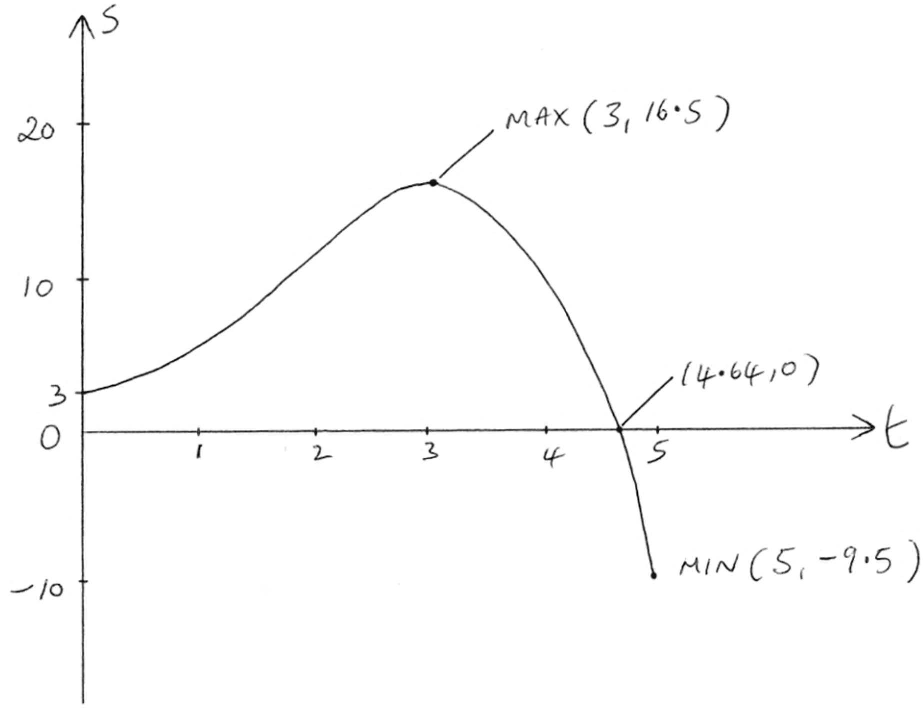
METHOD 2

$s = 3 + \int_0^4 (9t - 3t^2)\,dt$ ***(M1)(A1)***

$s = 11$ ***A1***

[3 marks]

(b) $s = 3 + \frac{9}{2}t^2 - t^3$ ***(A1)***



correct shape over correct domain ***A1***

maximum at $(3, 16.5)$ ***A1***

t intercept at 4.64, *s* intercept at 3 ***A1***

minimum at $(5, -9.5)$ ***A1***

[5 marks]

continued…

Question 12 continued

(c) $-9.5 = a + b\cos 2\pi$

$16.5 = a + b\cos 3\pi$ ***(M1)***

Note: Only award ***M1*** if two simultaneous equations are formed over the correct domain.

$a = \frac{7}{2}$ ***A1***

$b = -13$ ***A1***

[3 marks]

(d) at t_1:

$3 + \frac{9}{2}t^2 - t^3 = 3$ ***(M1)***

$t^2\left(\frac{9}{2} - t\right) = 0$

$t_1 = \frac{9}{2}$ ***A1***

solving $\frac{7}{2} - 13\cos\frac{2\pi t}{5} = 3$ ***(M1)***

$\text{GDC} \Rightarrow t_2 = 6.22$ ***A1***

Note: Accept graphical approaches.

[4 marks]

Total [15 marks]

13. (a) L_1 and L_2 are not parallel, since $\begin{pmatrix}-1\\1\\2\end{pmatrix} \neq k\begin{pmatrix}2\\1\\6\end{pmatrix}$ ***R1***

if they meet, then $1-\lambda = 1+2\mu$ and $2+\lambda = 2+\mu$ ***M1***

solving simultaneously $\Rightarrow \lambda = \mu = 0$ ***A1***

$2+2\lambda = 4+6\mu \Rightarrow 2 \neq 4$ contradiction, ***R1***

so lines are skew ***AG***

Note: Do not award the second ***R1*** if their values of parameters are incorrect.

[4 marks]

(b) $\begin{pmatrix}-1\\1\\2\end{pmatrix} \bullet \begin{pmatrix}2\\1\\6\end{pmatrix}(=11) = \sqrt{6}\sqrt{41}\cos\theta$ ***M1A1***

$\cos\theta = \frac{11}{\sqrt{246}}$ ***(A1)***

$\theta = 45.5^\circ$ (0.794 radians) ***A1***

[4 marks]

(c) (i) $\begin{pmatrix}-1\\1\\2\end{pmatrix} \times \begin{pmatrix}2\\1\\6\end{pmatrix} = \begin{pmatrix}6-2\\4+6\\-1-2\end{pmatrix}$ ***(M1)***

$= \begin{pmatrix}4\\10\\-3\end{pmatrix} = 4\boldsymbol{i} + 10\boldsymbol{j} - 3\boldsymbol{k}$ ***A1***

continued…

Question 13 continued

(ii) **METHOD 1**

let P be the intersection of L_1 and L_3
let Q be the intersection of L_2 and L_3

$\overrightarrow{OP} = \begin{pmatrix} 1-\lambda \\ 2+\lambda \\ 2+2\lambda \end{pmatrix}$ $\overrightarrow{OQ} = \begin{pmatrix} 1+2\mu \\ 2+\mu \\ 4+6\mu \end{pmatrix}$ ***M1***

therefore $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \begin{pmatrix} 2\mu+\lambda \\ \mu-\lambda \\ 2+6\mu-2\lambda \end{pmatrix}$ ***M1A1***

$\begin{pmatrix} 2\mu+\lambda \\ \mu-\lambda \\ 2+6\mu-2\lambda \end{pmatrix} = t\begin{pmatrix} 4 \\ 10 \\ -3 \end{pmatrix}$ ***M1***

$2\mu+\lambda-4t=0$
$\mu-\lambda-10t=0$
$6\mu-2\lambda+3t=-2$

solving simultaneously ***(M1)***

$\lambda = \frac{32}{125}(0.256)$, $\mu = -\frac{28}{125}(-0.224)$ ***A1***

Note: Award ***A1*** for either correct λ or μ.

EITHER

therefore $\overrightarrow{OP} = \begin{pmatrix} 1-\lambda \\ 2+\lambda \\ 2+2\lambda \end{pmatrix} = \begin{pmatrix} \frac{93}{125} \\ \frac{282}{125} \\ \frac{314}{125} \end{pmatrix} = \begin{pmatrix} 0.744 \\ 2.256 \\ 2.512 \end{pmatrix}$ ***A1***

therefore $L_3 : \boldsymbol{r}_3 = \begin{pmatrix} 0.744 \\ 2.256 \\ 2.512 \end{pmatrix} + \alpha\begin{pmatrix} 4 \\ 10 \\ -3 \end{pmatrix}$ ***A1***

continued…

Question 13 continued

OR

therefore $\vec{\mathrm{OQ}} = \begin{pmatrix} 1+2\mu \\ 2+\mu \\ 4+6\mu \end{pmatrix} = \begin{pmatrix} \frac{69}{125} \\ \frac{222}{125} \\ \frac{332}{125} \end{pmatrix} = \begin{pmatrix} 0.552 \\ 1.776 \\ 2.656 \end{pmatrix}$ ***A1***

therefore $L_3 : \boldsymbol{r}_3 = \begin{pmatrix} 0.552 \\ 1.776 \\ 2.656 \end{pmatrix} + \alpha \begin{pmatrix} 4 \\ 10 \\ -3 \end{pmatrix}$ ***A1***

Note:	Allow position vector(s) to be expressed in decimal or fractional form.

[10 marks]

METHOD 2

$L_3 : r_3 = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + t \begin{pmatrix} 4 \\ 10 \\ -3 \end{pmatrix}$

forming two equations as intersections with L_1 and L_2

$\begin{pmatrix} a \\ b \\ c \end{pmatrix} + t_1 \begin{pmatrix} 4 \\ 10 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$

$\begin{pmatrix} a \\ b \\ c \end{pmatrix} + t_2 \begin{pmatrix} 4 \\ 10 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix}$ ***M1A1A1***

Note:	Only award ***M1A1A1*** if two different parameters t_1, t_2 used.

attempting to solve simultaneously ***M1***

$\lambda = \frac{32}{125}(0.256),\ \mu = -\frac{28}{125}(-0.224)$ ***A1***

Note:	Award ***A1*** for either correct λ or μ.

continued…

Question 13 continued

EITHER

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0.552 \\ 1.776 \\ 2.656 \end{pmatrix}$$ ***A1***

therefore $L_3 : \boldsymbol{r}_3 = \begin{pmatrix} 0.552 \\ 1.776 \\ 2.656 \end{pmatrix} + t \begin{pmatrix} 4 \\ 10 \\ -3 \end{pmatrix}$ ***A1A1***

OR

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0.744 \\ 2.256 \\ 2.512 \end{pmatrix}$$ ***A1***

therefore $L_3 : \boldsymbol{r}_3 = \begin{pmatrix} 0.744 \\ 2.256 \\ 2.512 \end{pmatrix} + t \begin{pmatrix} 4 \\ 10 \\ -3 \end{pmatrix}$ ***A1A1***

Note: Allow position vector(s) to be expressed in decimal or fractional form.

Total [18 marks]