Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2015

Mathematics

Higher level

Paper 2

20 pages

Section A

1. (a) $0.818 = 0.65 + 0.48 - \mathrm{P}(A \cap B)$ ***(M1)***

$\mathrm{P}(A \cap B) = 0.312$ ***A1***

[2 marks]

(b) $\mathrm{P}(A)\,\mathrm{P}(B) = 0.312\left(= 0.48 \times 0.65\right)$ ***A1***

since $\mathrm{P}(A)\,\mathrm{P}(B) = \mathrm{P}(A \cap B)$ then A and B are independent ***R1***

> **Note:** Only award the ***R1*** if numerical values are seen. Award ***A1R1*** for a correct conditional probability approach.

[2 marks]

Total [4 marks]

2. using technology and/or by elimination (*eg* ref on GDC) ***(M1)***

$x = 1.89\left(= \frac{17}{9}\right),\ y = 1.67\left(= \frac{5}{3}\right),\ z = -2.22\left(= \frac{-20}{9}\right)$ ***A1A1A1***

[4 marks]

3. (a) $\frac{0\cdot 4 + 1\cdot k + 2\cdot 3 + 3\cdot 2 + 4\cdot 3 + 8\cdot 1}{13 + k} = 1.95 \ \left(\frac{k + 32}{k + 13} = 1.95\right)$ ***(M1)***

attempting to solve for k ***(M1)***

$k = 7$ ***A1***

[3 marks]

(b) (i) $\frac{7 + 32 + 22}{7 + 13 + 1} = 2.90\left(= \frac{61}{21}\right)$ ***(M1)A1***

(ii) standard deviation $= 4.66$ ***A1***

> **Note:** Award ***A0*** for 4.77.

[3 marks]

Total [6 marks]

4. (a) (i) $A = -3$ ***A1***

(ii) period $= \frac{2\pi}{B}$ ***(M1)***

$B = 2$ ***A1***

Note: Award as above for $A = 3$ and $B = -2$.

(iii) $C = 2$ ***A1***

[4 marks]

(b) $x = 1.74,\ 2.97$ $\left(x = \frac{1}{2}\left(\pi + \arcsin\frac{1}{3}\right), \frac{1}{2}\left(2\pi - \arcsin\frac{1}{3}\right)\right)$ ***(M1)A1***

[2 marks]

Note: Award ***(M1)A0*** if extra correct solutions *eg* $(-1.40,\ -0.170)$ are given outside the domain $0 \le x \le \pi$.

Total [6 marks]

5. (a) (i) area $= \int_2^4 \sqrt{y-2}\,\mathrm{d}y$ ***M1A1***

(ii) $= 1.886$ (4 sf only) ***A1***

[3 marks]

(b) volume $= \pi\int_2^4 (y-2)\,\mathrm{d}y$ ***(M1)***

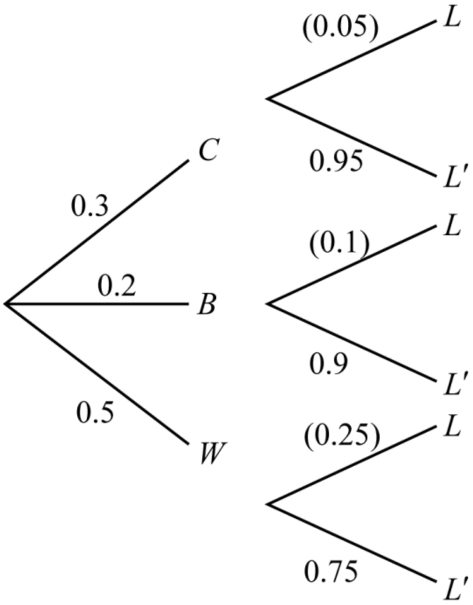
$= \pi\left[\frac{y^2}{2} - 2y\right]_2^4$ ***(A1)***

$= 2\pi$ (exact only) ***A1***

[3 marks]

Total [6 marks]

6. **EITHER**



M1A1A1

> **Note:** Award ***M1*** for a two-level tree diagram, ***A1*** for correct first level probabilities, and ***A1*** for correct second level probabilities.

OR

$$\mathrm{P}(B \mid L') = \frac{\mathrm{P}(L' \mid B)\ \mathrm{P}(B)}{\mathrm{P}(L' \mid B)\ \mathrm{P}(B) + \mathrm{P}(L' \mid C)\ \mathrm{P}(C) + \mathrm{P}(L' \mid W)\ \mathrm{P}(W)} \left(= \frac{\mathrm{P}(B \cap L')}{\mathrm{P}(L')} \right)$$ ***(M1)(A1)(A1)***

THEN

$$\mathrm{P}(B \mid L') = \frac{0.9 \times 0.2}{0.9 \times 0.2 + 0.95 \times 0.3 + 0.75 \times 0.5} \left(= \frac{0.18}{0.84} \right)$$ ***M1A1***

$$= 0.214 \left(= \frac{3}{14} \right)$$ ***A1***

[6 marks]

7. $21 = \frac{1}{2} \cdot 6 \cdot 11 \cdot \sin A$ ***(M1)***

$\sin A = \frac{7}{11}$ ***(A1)***

EITHER

$\hat{A} = 0.6897..., 2.452\ldots \left(\hat{A} = \arcsin\frac{7}{11}, \pi - \arcsin\frac{7}{11} = 39.521...^\circ, 140.478...^\circ \right)$ ***(A1)***

OR

$\cos A = \pm\frac{6\sqrt{2}}{11} (= \pm 0.771...)$ ***(A1)***

THEN

$BC^2 = 6^2 + 11^2 - 2 \cdot 6 \cdot 11 \cos A$ ***(M1)***

$BC = 16.1 \text{ or } 7.43$ ***A1A1***

Note: Award ***M1A1A0M1A1A0*** if only one correct solution is given.

[6 marks]

8. (a) $A\int_1^5 \sin(\ln x)\,dx = 1$ ***(M1)***

$A = 0.323$ (3 dp only) ***A1***

[2 marks]

(b) either a graphical approach or $f'(x) = \frac{\cos(\ln x)}{x} = 0$ ***(M1)***

$x = 4.81 \left(= e^{\frac{\pi}{2}} \right)$ ***A1***

Note: Do not award ***A1FT*** for a candidate working in degrees.

[2 marks]

(c) $P(X \le 3 | X \ge 2) = \frac{P(2 \le X \le 3)}{P(X \ge 2)} \left(= \frac{\int_2^3 \sin(\ln(x))dx}{\int_2^5 \sin(\ln(x))dx} \right)$ ***(M1)***

$= 0.288$ ***A1***

Note: Do not award ***A1FT*** for a candidate working in degrees.

[2 marks]

Total [6 marks]

9. (a) $t_1 = 1.77\text{(s)} \left(= \sqrt{\pi}\text{(s)}\right)$ and $t_2 = 2.51\text{(s)} \left(= \sqrt{2\pi}\text{(s)}\right)$ ***A1A1***

[2 marks]

(b) (i) attempting to find (graphically or analytically) the first $t_{\max}$ ***(M1)***

$t = 1.25\text{(s)} \left(= \sqrt{\frac{\pi}{2}}\text{(s)}\right)$ ***A1***

(ii) attempting to find (graphically or analytically) the first $t_{\min}$ ***(M1)***

$t = 2.17\text{(s)} \left(= \sqrt{\frac{3\pi}{2}}\text{(s)}\right)$ ***A1***

[4 marks]

(c) distance travelled $= \left|\int_{1.772\ldots}^{2.506\ldots} 1 - e^{-\sin t^2}\,dt\right|$ (or equivalent) ***(M1)***

$= 0.711\text{(m)}$ ***A1***

[2 marks]

Total [8 marks]

10. (a) $\boldsymbol{a} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ ***A1***

$\boldsymbol{b} = \frac{1}{3}\left(\begin{pmatrix} 4 \\ 16 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \end{pmatrix}\right) = \begin{pmatrix} \frac{5}{3} \\ 4 \end{pmatrix}$ ***(M1)A1***

[3 marks]

(b) **METHOD 1**

Roderick must signal in a direction vector perpendicular to Ed's path. ***(M1)***

the equation of the signal is $\boldsymbol{s} = \begin{pmatrix} 11 \\ 9 \end{pmatrix} + \lambda\begin{pmatrix} -12 \\ 5 \end{pmatrix}$ (or equivalent) ***A1***

$\begin{pmatrix} -1 \\ 4 \end{pmatrix} + \frac{t}{3}\begin{pmatrix} 5 \\ 12 \end{pmatrix} = \begin{pmatrix} 11 \\ 9 \end{pmatrix} + \lambda\begin{pmatrix} -12 \\ 5 \end{pmatrix}$ ***M1***

$\frac{5}{3}t + 12\lambda = 12$ and $4t - 5\lambda = 5$ ***M1***

$t = 2.13\left(= \frac{360}{169}\right)$ ***A1***

[5 marks]

METHOD 2

$\begin{pmatrix} 5 \\ 12 \end{pmatrix} \cdot \left(\begin{pmatrix} 11 \\ 9 \end{pmatrix} - \begin{pmatrix} -1 + \frac{5}{3}t \\ 4 + 4t \end{pmatrix}\right) = 0$ (or equivalent) ***M1A1A1***

Note: Award the ***M1*** for an attempt at a scalar product equated to zero, ***A1*** for the first factor and ***A1*** for the complete second factor.

attempting to solve for *t* ***(M1)***

$t = 2.13\left(= \frac{360}{169}\right)$ ***A1***

[5 marks]

continued…

Question 10 continued

METHOD 3

$x = \sqrt{\left(12 - \frac{5t}{3}\right)^2 + (5-4t)^2}$ (or equivalent) $\left(x^2 = \left(12 - \frac{5t}{3}\right)^2 + (5-4t)^2\right)$ ***M1A1A1***

Note: Award ***M1*** for use of Pythagoras' theorem, ***A1*** for $\left(12 - \frac{5t}{3}\right)^2$ and ***A1*** for $(5-4t)^2$.

attempting (graphically or analytically) to find *t* such that $\frac{\mathrm{d}x}{\mathrm{d}t} = 0 \left(\frac{\mathrm{d}(x^2)}{\mathrm{d}t} = 0\right)$

(M1)

$t = 2.13 \left(= \frac{360}{169}\right)$ ***A1***

[5 marks]

METHOD 4

$\cos\theta = \frac{\begin{pmatrix}12\\5\end{pmatrix}\cdot\begin{pmatrix}5\\12\end{pmatrix}}{\left|\begin{pmatrix}12\\5\end{pmatrix}\right|\left|\begin{pmatrix}5\\12\end{pmatrix}\right|} = \frac{120}{169}$ ***M1A1***

Note: Award ***M1*** for attempting to calculate the scalar product.

$\frac{120}{13} = \frac{t}{3}\left|\begin{pmatrix}5\\12\end{pmatrix}\right|$ (or equivalent) ***(A1)***

attempting to solve for *t* ***(M1)***

$t = 2.13 \left(= \frac{360}{169}\right)$ ***A1***

[5 marks]

Total [8 marks]

Section B

11. (a) (i) let W be the weight of a worker and $W \sim \mathrm{N}\left(\mu, \sigma^2\right)$

$\mathrm{P}\left(Z < \frac{62-\mu}{\sigma}\right) = 0.3$ and $\mathrm{P}\left(Z < \frac{98-\mu}{\sigma}\right) = 0.75$ ***(M1)***

$\frac{62-\mu}{\sigma} = \Phi^{-1}(0.3)\ (= -0.524\ldots)$ and

$\frac{98-\mu}{\sigma} = \Phi^{-1}(0.75)\ (= 0.674\ldots)$

or linear equivalents ***A1A1***

(ii) attempting to solve simultaneously ***(M1)***

$\mu = 77.7,\ \sigma = 30.0$ ***A1A1***

[6 marks]

(b) $\mathrm{P}(W > 100) = 0.229$ ***A1***

[1 mark]

(c) let X represent the number of workers over $100\,\mathrm{kg}$ in a lift of ten passengers

$X \sim \mathrm{B}(10,\ 0.229\ldots)$ ***(M1)***

$\mathrm{P}(X \geq 4) = 0.178$ ***A1***

[2 marks]

continued…

Question 11 continued

(d) $\mathrm{P}(X < 4 | X \geq 1) = \dfrac{\mathrm{P}(1 \leq X \leq 3)}{\mathrm{P}(X \geq 1)}$ ***M1(A1)***

Note: Award the ***M1*** for a clear indication of conditional probability.

$= 0.808$ ***A1***

[3 marks]

(e) $L \sim \mathrm{Po}(50)$ ***(M1)***

$\mathrm{P}(L > 60) = 1 - \mathrm{P}(L \leq 60)$ ***(M1)***

$= 0.0722$ ***A1***

[3 marks]

(f) 400 workers require at least 40 elevators ***(A1)***

$\mathrm{P}(L \geq 40) = 1 - \mathrm{P}(L \leq 39)$ ***(M1)***

$= 0.935$ ***A1***

[3 marks]

Total [18 marks]

Note: For Q12(a) (i) – (iii) and (b) (ii), award ***A1*** for correct endpoints and , if correct, award ***A1*** for a closed interval.
Further, award ***A1A0*** for one correct endpoint and a closed interval.

12. (a) (i) $-4 \le y \le -2$ ***A1A1***

(ii) $-5 \le y \le -1$ ***A1A1***

(iii) $-3 \le 2x - 6 \le 5$ ***(M1)***

Note: Award ***M1*** for $f\left(2x-6\right)$.

$3 \le 2x \le 11$

$\frac{3}{2} \le x \le \frac{11}{2}$ ***A1A1***

[7 marks]

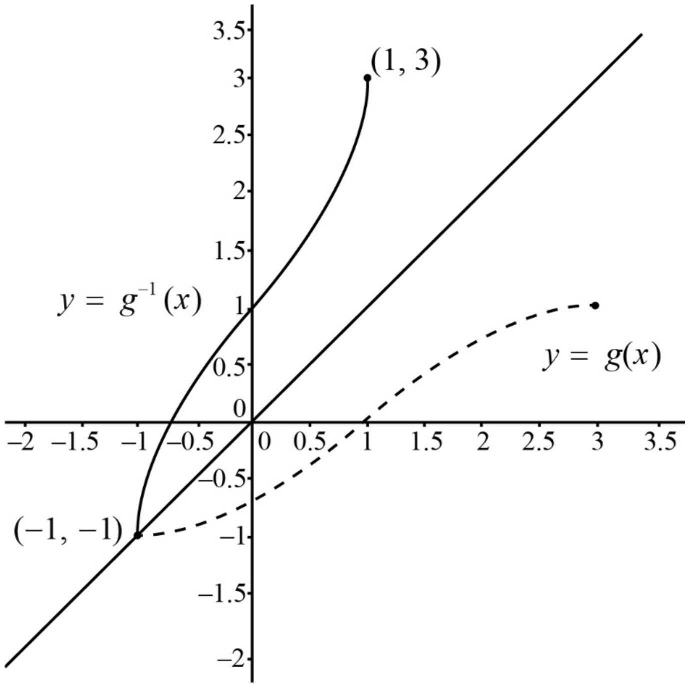
continued…

Question 12 continued

(b) (i) any valid argument *eg* f is not one to one, f is many to one, fails horizontal line test, not injective ***R1***

(ii) largest domain for the function $g(x)$ to have an inverse is $[-1, 3]$ ***A1A1***

(iii)



y-intercept indicated (coordinates not required) ***A1***
correct shape ***A1***
coordinates of end points $(1, 3)$ and $(-1, -1)$ ***A1***

Note: Do not award any of the above marks for a graph that is not one to one.

[6 marks]
continued…

Question 12 continued

(c) (i) $y = \dfrac{2x-5}{x+d}$

$(x+d)\,y = 2x-5$ ***M1***

Note: Award ***M1*** for attempting to rearrange x and y in a linear expression.

$x(y-2) = -dy-5$ ***(A1)***

$x = \dfrac{-dy-5}{y-2}$ ***(A1)***

Note: x and y can be interchanged at any stage

$h^{-1}(x) = \dfrac{-dx-5}{x-2}$ ***A1***

Note: Award ***A1*** only if $h^{-1}(x)$ is seen.

(ii) self Inverse $\Rightarrow h(x) = h^{-1}(x)$

$\dfrac{2x-5}{x+d} \equiv \dfrac{-dx-5}{x-2}$ ***(M1)***

$d = -2$ ***A1***

(iii) **METHOD 1**

$\dfrac{2k(x)-5}{k(x)-2} = \dfrac{2x}{x+1}$ ***(M1)***

$k(x) = \dfrac{x+5}{2}$ ***A1***

METHOD 2

$h^{-1}\left(\dfrac{2x}{x+1}\right) = \dfrac{2\left(\dfrac{2x}{x+1}\right)-5}{\dfrac{2x}{x+1}-2}$ ***(M1)***

$k(x) = \dfrac{x+5}{2}$ ***A1***

[8 marks]

Total [21 marks]

13. (a) $f'(x) = 30\mathrm{e}^{-\frac{x^2}{400}} \cdot -\frac{2x}{400}\left(= -\frac{3x}{20}\mathrm{e}^{-\frac{x^2}{400}}\right)$ ***M1A1***

Note: Award ***M1*** for attempting to use the chain rule.

$f''(x) = -\frac{3}{20}\mathrm{e}^{-\frac{x^2}{400}} + \frac{3x^2}{4000}\mathrm{e}^{-\frac{x^2}{400}}\left(= \frac{3}{20}\mathrm{e}^{-\frac{x^2}{400}}\left(\frac{x^2}{200} - 1\right)\right)$ ***M1A1***

Note: Award ***M1*** for attempting to use the product rule.

[4 marks]

(b) the roof function has maximum gradient when $f''(x) = 0$ ***(M1)***

Note: Award ***(M1)*** for attempting to find $f''\left(-\sqrt{200}\right)$.

EITHER

$= 0$ ***A1***

OR

$f''(x) = 0 \Rightarrow x = \pm\sqrt{200}$ ***A1***

THEN

valid argument for maximum such as reference to an appropriate graph or change in the sign of $f''(x)$ *eg* $f''(-15) = 0.010...(> 0)$ and $f''(-14) = -0.001...(< 0)$ ***R1***

$\Rightarrow x = -\sqrt{200}$ ***AG***

[3 marks]

continued…

Question 13 continued

(c) $A = 2a \cdot 30\mathrm{e}^{-\frac{a^2}{400}} \left(= 60a\mathrm{e}^{-\frac{a^2}{400}} = -400 f'(a) \right)$ ***(M1)(A1)***

EITHER

$\frac{\mathrm{d}A}{\mathrm{d}a} = 60a\mathrm{e}^{-\frac{a^2}{400}} \cdot -\frac{a}{200} + 60\mathrm{e}^{-\frac{a^2}{400}} = 0 \Rightarrow a = \sqrt{200} \quad \left(-400 f''(a) = 0 \Rightarrow a = \sqrt{200}\right)$

M1A1

OR

by symmetry *eg* $a = -\sqrt{200}$ found in (b) or $A_{\max}$ coincides with $f''(a) = 0$ ***R1***

$\Rightarrow a = \sqrt{200}$ ***A1***

THEN

$A_{\max} = 60 \cdot \sqrt{200}\mathrm{e}^{-\frac{200}{400}}$ ***M1***

$= 600\sqrt{2}\mathrm{e}^{-\frac{1}{2}}$ ***AG***

[5 marks]

(d) (i) perimeter $= 4a + 60\mathrm{e}^{-\frac{a^2}{400}}$ ***A1A1***

Note: Condone use of x.

(ii) $I(a) = \frac{4a + 60\mathrm{e}^{-\frac{a^2}{400}}}{60a\mathrm{e}^{-\frac{a^2}{400}}}$ ***(A1)***

graphing $I(a)$ or other valid method to find the minimum ***(M1)***

$a = 12.6$ ***A1***

(iii) area under roof $= \int_{-20}^{20} 30\mathrm{e}^{-\frac{x^2}{400}}\,\mathrm{d}x$ ***M1***

$= 896.18\ldots$ ***(A1)***

area of living space $= 60 \cdot (12.6\ldots) \cdot \mathrm{e}^{-\frac{(12.6\ldots)^2}{400}} = 508.56\ldots$ ***(A1)***

percentage of empty space $= 43.3\%$ ***A1***

[9 marks]

Total [21 marks]