PAPER 1

1. The random variable X has probability density function f where

$$f(x) = \begin{cases} kx(x+1)(2-x), & 0 \le x \le 2 \\ 0, & \text{otherwise.} \end{cases}$$

 (a) Sketch the graph of the function. You are not required to find the coordinates of the maximum.

(1)

 (b) Find the value of k.

(5)
(Total 6 marks)

2. A continuous random variable X has probability density function

$$f(x) = \begin{cases} 0, & x < 0 \\ ae^{-ax}, & x \ge 0. \end{cases}$$

 It is known that $P(X < 1) = 1 - \dfrac{1}{\sqrt{2}}$.

 (a) Show that $a = \dfrac{1}{2}\ln 2$.

(6)

 (b) Find the median of X.

(5)

 (c) Calculate the probability that $X < 3$ given that $X > 1$.

(9)
(Total 20 marks)

3. The random variable T has the probability density function

$$f(t) = \frac{\pi}{4}\cos\left(\frac{\pi t}{2}\right), -1 \leq t \leq 1.$$

Find

(a) $P(T = 0)$;

(2)

(b) the interquartile range.

(5)
(Total 7 marks)

4. The probability density function of the random variable X is given by

$$f(x) = \begin{cases} \dfrac{k}{\sqrt{4 - x^2}}, & \text{for } 0 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of the constant k.

(5)

(b) Show that $E(X) = \dfrac{6(2 - \sqrt{3})}{\pi}$.

(7)

(c) Determine whether the median of X is less than $\dfrac{1}{2}$ or greater than $\dfrac{1}{2}$.

(8)
(Total 20 marks)

PAPER 2

5. A continuous random variable X has probability density function

$$f(x) = \begin{cases} 12x^2(1-x), & \text{for } 0 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

Find the probability that X lies between the mean and the mode.

(Total 6 marks)

6. A continuous random variable X has a probability density function given by the function $f(x)$, where

$$f(x) = \begin{cases} k(x+2)^2 & -2 \le x < 0 \\ k, & 0 \le x \le \dfrac{4}{3} \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of k.

(2)

(b) Hence find

    (i)   the mean of X;

    (ii)   the median of X.

(5)
(Total 7 marks)

7. The continuous random variable X has probability density function

$$f(x) = \frac{1}{6}x(1 + x^2) \quad \text{for } 0 \le x \le 2,$$
$$f(x) = 0 \quad\quad\quad \text{otherwise.}$$

(a) Sketch the graph of f for $0 \le x \le 2$.

(2)

(b) Write down the mode of X.

(1)

(c) Find the mean of X.

(4)

(d) Find the median of X.

(5)
(Total 12 marks)