"From principles is derived probability, but trust or certainty is obtained only from facts"[1]

~ Tom Stoppard

Introduction

When forming a persuasive argument, lawyers tend to rely strongly on mathematical evidence such as statistics and probability.[2] It seems that appealing to a listener's logical reasoning is dependent on the use of numbers – it is the most straightforward form of verification that simplifies the contents of the case. However, the use of probability is dependent on reliable research and data collection, as well as accurate representation and interpretation of the values. One must consider whether the variable is independent, how the probability is used and its connotation. This mathematical investigation will focus on the misuse of probability within the courtroom; specifically how invalid mathematical reasoning can achieve disastrous results. This topic was chosen out of personal interest in the subject of law. I wanted to investigate an example of how different areas of knowledge are necessary in the courtroom, and hopefully support my opinion that the law is an interdisciplinary study that requires a lot of understanding of other concepts, such as mathematics. Through this exploration, I will aim to analyze and explain the invalid use of mathematics in order to justify arguments in specific legal cases, and later on provide a hypothetical example for the correct use of maths in the courtroom.

Background Information

The phenomenon of bad maths in court reemerged during the infamous trial for the murder of British national Meredith Kercher on November 2nd, 2007 in Perugia, Italy.[3] Amanda Knox, an exchange student from Seattle, her boyfriend Raffaele Sollecito and a local bar owner Patrick Lumumba were accused of murdering Kercher, who was Knox's roommate at the time. The Italian authorities were invalidly arrogant about their arrest, announcing to the world media "case closed" just four days after the murder.[4] Soon after another suspect was revealed, Rudy Hermann Guede, who was placed at the crime scene through substantial evidence, none of which proved the involvement of the aforementioned parties. Yet the prosecution refused to admit their mistake and insisted that Amanda, Raffaele and Rudy were guilty, with Patrick having an alibi that took him out of the equation. The prosecution depended their case on two pieces of DNA evidence – a knife and a piece of a bra clasp.[5] They found a tiny trace of DNA allegedly belonging to Kercher on the blade of a knife that had a definite match to Knox's DNA on the handle. There was also a trace of Sollecito's DNA on Kercher's bra clasp that was found near the body.

The prosecution focused immensely on these traces of DNA, using them to underpin most of their case against Amanda Knox and Raffaele Sollecito. This led the court to convict them for participation in the murder of

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[1] Stoppard, Tom. "Tom Stoppard Quotes." *BrainyQuote*. Xplore Web. 3 Oct. 2016. <https://www.brainyquote.com/quotes/quotes/t/tomstoppar120725.html>.
[2] Schneps, Leila, and Coralie Colmez. "Introduction." *Math on Trial: How Numbers Get Used and Abused in the Courtroom*. New York: Basic, 2013. Print.
[3] Schneps, Leila, and Coralie Colmez. "Justice Flunks Math." *The New York Times*. The New York Times, 26 Mar. 2013. Web. 25 Sept. 2016. <http://www.nytimes.com/2013/03/27/opinion/when-judges-cant-do-math-justice-suffers.html?_r=0>.
[4] "The Amanda Knox & Raffaele Sollecito Case." *Introduction – 'Case Closed!'*. Web. 3 Oct. 2016. <http://www.amandaknoxcase.com/>.
[5] *Amanda Knox*. Dir. Rob Blackhurst and Brian McGinn. Perf. Amanda Knox, Raffaele Sollecito. *Netflix*. Netflix, 30 Sept. 2016. Web. 5 Oct. 2016. <http://www.imdb.com/title/tt5952332/>.

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Meredith Kercher, sentencing them to 26 and 25 years respectively.[6] Their lawyers launched an appeal immediately and began to strengthen their case further, overturning every allegation or piece of evidence communicated by the prosecution or the Italian authorities. They discredited a bloody footprint found on a bathmat in Meredith and Amanda's apartment, which was Raffaele's size but could not be identified to be his for certain.[7] It came to light that the ripped-off clasp of Meredith's bra was only found and collected from the floor forty-six days after the murder, strengthening the possibility of contamination from the police and forensic experts.[8] This claim was later supported by video evidence of poor forensic work, with technicians using dirty gloves to collect blood and hair samples, as well as not changing shoe covers every time they left and re-entered any room of the house.[9]

It became more and more apparent that the case against Amanda and Raffaele was heavily flawed. Many began to believe that the Italian authorities started to manipulate evidence in order to avoid admitting their mistake. The last piece of evidence remained – the kitchen knife. During the trial for the overruling of the previous decision of the case, two independent experts, Professor Carla Vecchiotti and Dr. Stefano Conti, were hired to make a final judgment on the reliability of the forensics work performed for the first trial.[10] They were the ones who discredited a majority of the evidence due to potential contamination, yet that was not a strong enough argument to explain Meredith's DNA on the blade of the knife. Therefore, the prosecution submitted a formal request to analyze the tiny sample of DNA that remained on the knife, reasoning that newer DNA technology (four years had passed since the evidence had been collected) could provide more reliable results even with such a small sample. Judge Hellmann rejected the request on September 7th, 2011, and on October 3rd, Raffaele and Amanda's conviction was overturned. [11]

Hellman's reasoning for not allowing another DNA analysis stems from the misuse of simple mathematical probability. He believed that "running the experiment independently two times and obtaining the same result twice did not increase the reliability of the result"[12], indicating a complete misunderstanding of probabilities within the concept of a 'double experiment'. Courts attempt to prove that someone is guilty or innocent beyond reasonable doubt. And even though there is no clear percentage that can illustrate this, we can safely assume that being around 98% certain is much better than being only 85% certain. It was never assessed just how confident forensic experts were that the DNA on the knife was Meredith's, however there was a high probability for that to be the case, even though the test was not very reliable due to it being performed only once. If it were to be performed again using the traces of the remaining cells on the blade, and it proved that the DNA was in fact Meredith's - then the reliability of the study would have increased, potentially strengthening the case against

[6] "The Amanda Knox & Raffaele Sollecito Case." *The Trials*. Web. 3 Oct. 2016. <http://www.amandaknoxcase.com/>.
[7] "The Amanda Knox & Raffaele Sollecito Case." *Rudy Guede's Footprint On Meredith Kercher's Bathmat*. Web. 5 Oct. 2016. <http://www.amandaknoxcase.com/rudy-guede-bathmat-footprint/>.

[8] Ibid *The Bra Clasp: Compromised Evidence*
[9] *Amanda Knox*. Dir. Rob Blackhurst and Brian McGinn. Perf. Amanda Knox, Raffaele Sollecito. *Netflix*. Netflix, 30 Sept. 2016. Web. 5 Oct. 2016. <http://www.imdb.com/title/tt5952332/>.
[10] Ibid
[11] CNN Staff. "Timeline: Meredith Kercher Murder Case." *CNN*. Cable News Network, 14 Sept. 2016. Web. 7 Oct. 2016. <http://edition.cnn.com/2011/09/28/world/europe/italy-amanda-knox-timeline/>.
[12] Schneps, Leila, and Coralie Colmez. "The Case of Meredith Kercher: The Test That Wasn't Done." *Math on Trial: How Numbers Get Used and Abused in the Courtroom*. New York: Basic, 2013. 63-86. Print.

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Amanda Knox and Raffaele Sollecito. Instead, the defendants walked free, and to this day we are still uncertain as to what happened on the night of Meredith Kercher's muder.

This case reopened the debate on the importance of having a strong foundation in mathematics when making certain probabilistic claims in court. It was another example of the consideration that simply knowing the law was not enough to exercise proper justice, and that lawyers and judges needed to maintain an interdisciplinary approach to jurisprudence.

The Maths in Knox

In the book "Math on Trial", Leila Schneps and Coralie Colmez discuss the mathematical inaccuracy of the Knox case, and use an example of a coin toss in order to explain their reasoning. When I first read their explanation, I had trouble understanding the exact mathematical method they used to calculate the final percentages and was genuinely interested in learning more about the topic myself. Therefore, I contacted Leila Schneps via email and asked for her help. Thankfully, she responded within a day with a full explanation of how she used conditional probability and Bayes Theorem in her investigation, a process I will describe throughout the next few pages.

The mathematics behind the increase in reliability of two tests counter to one can be explained through the example of a coin. If we are looking to find if a coin used in a coin toss is fair or weighted to give heads 60% of the time, we will need to perform numerous trials and interpret the results.

The first test involves flipping a coin 100 times, from which we obtain 60 heads and 40 tails. We calculate the probability of achieving these results depending on the type of coin that was used. So if a fair coin was used, we know that the probability of it landing on heads is 0.5 every time. We will use the binompdf function because we are looking for the probability of 'exactly' 60 heads tossed, rather than 'at most' or 'at least' for which we would use binomcdf. Therefore using binompdf (n, p, x) where n is the number of trials, p is the probability and x is the number of heads tossed, we find that:

$$\text{binompdf}(100, 0.5, 60) = 0.01084387$$

$$P(x = 60 \text{ with a fair coin}) \approx 0.0108$$

Now we need to find the probability of the same outcome being achieved with the biased coin. Since the coin is weighed to show heads 60% of the time, the probability of getting heads with this coin is 0.6. Therefore the binomial probability function will be the following:

$$\text{binompdf}(100, 0.6, 60) = 0.08121914$$

$$P(x = 60 \text{ with a biased coin}) \approx 0.0812$$

With the newly calculated probabilities, we need to investigate further because we are still unaware of whether the test involved a fair or biased coin. We know the probabilities of the outcome of 60 heads and 40 tails given

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that the coins were either fair or biased. However, we want to find the probability of the coin being biased given that the event happened.

$P(C \mid F) = 0.0108$ $\quad$ $P(C \mid B) = 0.0812$ $\quad$ $P(B \mid C) = ?$

Where *C* is the event that the coin was flipped 100 times and 60 heads and 40 tails were obtained, *F* is the event that the coin is fair and *B* is the event that coin is biased.

First we need to find the probability of C happening, which is the sum of the probabilities of the event happening with a fair coin and a biased coin. Assuming that there is a 50/50 chance of picking either coin:

$$P(C) = P(C \mid F) \times P(F) + P(C \mid B) \times P(B)$$
$$P(C) = (0.0108 \times 0.5) + (0.0812 \times 0.5)$$
$$P(C) = 0.0460$$

Now we can use Bayes' Theorem to find the probability of the coin being biased given the outcome of 60 heads and 40 tails.

$$P(B \mid C) = (P(C \mid B) \times P(B)) / P(C)$$
$$P(B \mid C) = (0.0812 \times 0.5) / 0.0460$$
$$P(B \mid C) = 0.88260870$$
$$P(B \mid C) \approx 0.883$$

Therefore there is an 88.3% chance that the coin was biased given that the outcome of 100 flips was 60 heads and 40 tails. Bayes' Theorem is a formula used when we want to find the conditional probability of an event, but we only know the probability of the reverse conditional probability. In this case, we knew what P (C | B) was, but we needed P (B | C). This formula is frequently used to determine the idea of an event happening given a test. For example, a DNA test might indicate that a suspect's DNA matches the DNA found at the crime scene, but that does not necessarily mean that they are guilty seeing as there is always the possibility of attaining a false positive. Bayes' Theorem looks at the probability of an event happening given the outcome of the test. So in the case of the coin, the coin flipping was the test, but we needed to find the probability that the results were achieved by a biased coin, which is the actual event. We will use the same process to perform a second test in order to indicate the change in probability and later on, how two tests are more reliable than one.

The outcome of the second test is 59 heads and 41 tails. In order to find the probability of the coin being biased given this outcome, we need the aforementioned process.

$$\text{binompdf}(100, 0.5, 59) = 0.01586907$$
$$P(x = 59 \text{ with a fair coin}) \approx 0.0159$$
$$\text{binompdf}(100, 0.6, 59) = 0.07923819$$
$$P(x = 59 \text{ with a biased coin}) \approx 0.0792$$

$P(C \mid F) = 0.0159$ $\quad$ $P(C \mid B) = 0.0792$ $\quad$ $P(B \mid C) = ?$

$$P(C) = (0.0159 \times 0.5) + (0.0792 \times 0.5)$$
$$P(C) = 0.04755000$$

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P (C) ≈ 0.0476

Bayes Theorem: P (B | C) = (P (C | B) × P (B)) / P (C)

P (B | C) = (0.0792 × 0.5) / 0.0476

P (B | C) = 0.83280757

P (B | C) ≈ 0.833

The second test has revealed that the probability of the given outcome to be reached with a biased coin is 83.3%, which is lower than the initial 88.3%. One could argue that we have not gained any certainty from repeating the test, however the event that has essentially taken place is 200 coin flips, with an outcome of 119 heads and 81 tails. Therefore, we must calculate the probability of the involvement of a biased coin whilst treating the two tests as one large test.

binompdf (200, 0.5, 119) = 0.00151867

P (x = 119 with a fair coin) ≈ 0.00152

binompdf (200, 0.6, 119) = 0.05679648

P (x = 119 with a biased coin) ≈ 0.0568

P (C | F) = 0.00152 P (C | B) = 0.0568 P (B | C) =?

P (C) = (0.00152 × 0.5) + (0.0568 × 0.5)

P (C) = 0.02916000

P (C) ≈ 0.0292

Bayes Theorem: P (B | C) = (P (C | B) × P (B)) / P (C)

P (B | C) = (0.0568 × 0.5) / 0.0292

P (B | C) = 0.97393690

P (B | C) ≈ 0.974

By interpreting the data as one large test, we found that the probability of the coin being biased is actually 97.4%. If this result were to be used in court, 97.4% provides a much better certainty than 88.3%, allowing the judge and jury to potentially rule beyond reasonable doubt that the defendant is guilty. This example illustrates the benefits of repeating a test in order to prove its reliability or potentially discredit it. The court in Italy missed the opportunity to reach such certainty when the judge rejected the second DNA analysis, leaving a few questions about what happened to Meredith Kercher unanswered. The misuse of probability in this case furthered my interest in the subject and led me to investigate another case which also experienced grave injustice due to the invalid use of mathematics.

Sally Clark

The trial of Sally Clark is considered one of the most serious mistrials in the beginning of the 21st century.[13] The poor analysis of witness statements and misinterpretation of mathematical statistics by the prosecution led to the false guilty verdict of a grieving mother who became a victim to national slander and defamation of character.

[13] Doward, Jamie. "Sally Clark Was 'let down' by Authorities." *The Guardian*. Guardian News and Media, 18 Mar. 2007. Web. 19 Nov. 2016. <https://www.theguardian.com/society/2007/mar/18/childrensservices.uknews>.

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Steve and Sally Clark were a couple of lawyers living together in London. They decided to start a family and on September 22nd, 1996, Sally gave birth to a boy named Christopher.[14] He was born to be quite fragile and weak, which alarmed the parents but doctors assured them not to worry. However, on December 13th of the same year, Sally found her son not breathing in his basket. He was rushed to the hospital where he died from an infection in the lungs, which was later revealed in the autopsy. Sally and Steve made another attempt at having a child, and on November 29th, 1997 – Harry was born.[15]

Because Harry was the sibling of a baby who had died at infancy, his health was closely monitored through a program called "Care of Next Infants" (CONI).[16] Professionals from this programme made frequent visits in order to monitor Harry's health, but no issues were detected. However on January 26th, 1998, Harry had gone limp and white under the supervision of Sally. Steve tried to resuscitate him while Sally called an ambulance, but their second child passed away at the hospital the same day. This time however, the autopsy revealed retinal hemorrhage in Harry's eyes and a broken rib – both signs of smothering.[17] Steve and Sally Clark were arrested for the murder of their two children after the aforementioned evidence was deemed strong enough to charge them for abuse. After interrogation they were set free, but the investigation continued. During this time, Sally gave birth to another child, and was arrested soon after for the double murder of her children.

During the trial, Sally's lawyers rebutted every claim made by the prosecution, scrutinizing their witnesses and making medical professionals contradict each other, as well as come to the conclusion that the babies did not necessarily die due to any abuse. Sally's competence as a mother was proven by numerous character references from friends and family. It seemed that the case was playing out in her favor, until the prosecution introduced a new witness – Roy Meadow, an expert in pediatrics and the psychology of abuse by mothers of their children. He had performed an immense amount of research into the phenomenon called "Sudden Infant Death Syndrome" (SIDS), also referred to as cot death. On the witness stand, Meadow claimed that studies have shown that the chance of a cot death in a family of the same social status as the Clarks is about 1 in 8543[18] His research was supported by evidence from a report entitled "Confidential Enquiry into Still-births and Deaths in Infancies" (CESDI), which he claimed was more comprehensive and contained more accurate statistical data than the report from CONI used by the defense.

The CESDI report states that there are three main risk factors that impact crib death: smoker in the family, unemployed parent and mother under the age of 26. However, it makes it very clear that other factors do exist; they just have not been well researched enough to be understood and announced in an official report. The table[19] below indicates the probabilities that were calculated from the statistical analysis of the CESDI report. The overall probability is simply the calculated from the amount of infants who have suffered from SIDS, regardless

[14] Schneps, Leila, and Coralie Colmez. "The Case of Sally Clark: Motherhood Under Attack." *Math on Trial: How Numbers Get Used and Abused in the Courtroom*. New York: Basic, 2013. 2-21. Print.
[15] Ibid
[16] Ibid
[17] Ibid
[18] Ibid
[19] Scheurer, Vincent. "Convicted on Statistics?" *Understanding Uncertainty*. University of Cambridge, Web. 12 Oct. 2016. <https://understandinguncertainty.org/node/545>.

of any factors present or absent. Roy Meadow cited the 'none of the factors' figure in his statement, but if we keep in mind the fact that there are other factors that can impact the chance of cot death, he should have used the general probability statistic in order to provide a more accurate response.

Different Probabilities of Death by SIDS

Table 3.6.1: SIDS rates for different factors based on the data from the CESDI SUDI Study

	SIDS rate per 1000 livebirths*	SIDS incidence in this group*
Overall rate in the study population		in 1303
Rate for groups with different factors		
Anybody smokes in the household		in 737
Nobody smokes in the household		in 5041
No waged income in the household		in 486
At least one waged income in the household		in 2088
Mother <27 years and panty		in 567
Mother > 26 years and parity		in 1882
None of these factors		in 8543
One of these factors		in 1616
Two of these factors		in 596
All three of these factors		in 214

Debunking the Maths of Roy Meadow

Roy Meadow claims that the chance that one child would suffer from Sudden Infant Death Syndrome in a family such as the Clarks, who did not smoke, were employed and above the age of 26, is 1 in 8543.[20] He then claims, that the chance of this happening twice is approximately 1 in 73 million.[21] His mathematical reasoning is the following:

Chance of it happening once: $\frac{1}{8543}$

Chance of it happening twice: $\frac{1}{8543} \times \frac{1}{8543} = \frac{1}{72982849} \approx \frac{1}{73000000}$

The issue with Meadow's reasoning is that he treats consecutive cot deaths as two independent events. He believes that if the first event took place, it does not affect the probability of the second event happening; hence he multiplies them and concludes the very small chance of 1 in 73 million. However, the very CESDI report he keeps quoting explicitly states that there are other factors that are involved in the probability of SIDS death, they just have not been discovered yet. There could be numerous environmental and genetic factors that impact the

[20] Schneps, Leila, and Coralie Colmez. "The Case of Sally Clark: Motherhood Under Attack." *Math on Trial: How Numbers Get Used and Abused in the Courtroom*. New York: Basic, 2013. 2-21. Print.
[21] Ibid

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chance of cot death, and if they were present in the death of the first child, the probability of the second child dying increases since those factors have already been effective in the previous case and are most likely still active. SIDS is not an absolute event, meaning sometimes doctors find the cause of death later and it no longer constitutes as an unexplained death.[22] Nor is this event random, further indicating that treating its probability as independent is horribly wrong. Ray Hill, a mathematics professor at Salford University, studied the data from the CESDI report, and came to the conclusion that double cot deaths within the same family are definitely not independent. He further estimated that the siblings of children who died from SIDS are between 10 and 22 times more likely than average to die the same way.[23] Now if we use the general probability, as Meadow should have, to apply this reasoning, we would find the following:

$$10 \times \frac{1}{1303} = \frac{1}{130} \qquad 22 \times \frac{1}{1303} = \frac{1}{60}$$

This indicates that the probability of the second child in the family being a victim of the second cot death is between 1 in 60 and 1 in 130. Roy Meadow completely failed to make these calculations and simply believed that the probability of SIDS remains independent in any situation.

As mentioned before, Meadow only used the statistic that applied to the absence of the three factors from the Clark family. In order to provide a better understanding to the court he should have used the 1 in 1303 probability. He would have then cited this probability of two children dying from SIDS as:

$$\frac{1}{1303} \times \frac{1}{1303} = \frac{1}{1697809} \approx \frac{1}{1700000}$$

The mathematical reasoning is still incorrect but the chance of Sally being guilty is much smaller than with the statistic Meadow used. The defense could have used that value to argue that her guilt could not be proven beyond reasonable doubt, which is the essential objective of the prosecution – to prove to the jury that the defendant is guilty beyond reasonable doubt.

This idea of reasonable doubt introduces the concept of "Prosecutor's Fallacy". Lawyers use statistics all the time in order to strengthen their case in front of a jury and are able to manipulate language in their favor. When Roy Meadow stated that the probability of two children dying in the same family from cot death is 1 in 73 million, the prosecution used that in order to come to the conclusion that the chance of the events that happened to Sally Clark happened naturally is 1 in 73 million, which means that it is almost certain that she is guilty for manipulating the situation, and hence killing her children. The jury were given two options – either the defendant committed murder or both the children died from natural but unexplained causes. With Meadow's statistic, it was justifiable for them to conclude that Sally Clark was guilty for the murder of her two children.

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[22] "Sudden Infant Death Syndrome (SIDS)." *Mayo Clinic*. Mayo Foundation for Medical Education and Research, n.d. Web. 12 Oct. 2016. <http://www.mayoclinic.org/diseases-conditions/sudden-infant-death-syndrome/basics/definition/con-20020269>.
[23] Joyce, Helen. "Beyond Reasonable Doubt." *Plus Mathematics*. University of Cambridge, 1 Sept. 2002. Web. 12 Oct. 2016. <https://plus.maths.org/content/beyond-reasonable-doubt>.

This reasoning is incorrect because courts are using the chance of a rare event happening as a justification for the chance of the defendant being innocent. The probability of a randomly chosen family who lack the three aforementioned factors losing their two children to SIDS does not help in solving the question of whether Sally is innocent. The essential idea is that the probability of innocence does not equal the probability of the rare event happening. The question that the prosecution used the 1 in 73 million to answer was "what is the probability that these deaths were natural?", when they should have been answering the question of "is it more likely for the deaths to be natural rather than murders?". This way, the court would have needed to find the probability of Sally being guilty of a double homicide, and compare the two chances.

This was somewhat done in the pre-trial proceedings, when a witness who had performed research into repetitive SIDS deaths found that from his pool of data, one third of the deaths were caused by natural causes that were detected after the autopsy declared the child a SIDS victim. Another third were true SIDS deaths, and only one third died due to child abuse.[24] Using these statistics we can conclude that there is a 2 in 3 chance that the double cot deaths happened naturally, meaning that in a randomly selected family there is a 1 in 3 chance of repetitive cot deaths being murders.

However, the court used none of this information, and Sally was sentenced to life imprisonment on November 9th, 1999. Her husband and lawyers continued to fight for her cause and a couple of years later, extraordinary evidence came to light. Harry's post-mortem medical records revealed that there were no fewer than eight different colonies of the lethal bacterium *Staphylococcus aureus* found in his nose, lungs and throat. This indicated that the child suffered from a serious bacteria infection that could have led to meningitis. These records were never revealed to the defense during Sally's trial, illustrating another level of prosecutorial misconduct. Sally Clark's sentence was overturned and she was freed on January 29th, 2003. However, her psychological state was never the same and she died from alcohol intoxication on March 16th, 2007.[25] The case of Sally Clark is a primary example of a serious misuse of mathematical theory that led to her demise. all due to a few miscalculations and misrepresentation of data.

The Perfect Case

In order to provide an example of a case where mathematical reasoning is used correctly, I created a hypothetical criminal investigation into the murder of a young woman.

Jeanine Tallbeth is found murdered in the apartment she shared with her boyfriend Christopher in Amsterdam, Netherlands. There were traces of semen in her, but no signs of sexual abuse. The investigators obviously suspected the boyfriend, but he had a strong alibi. Also, the forensics experts found something interesting in the traces of DNA left by the perpetrator. It did not match anyone they had in the criminal system, but it did show that Jeanine's murderer suffered from a very rare disease called *Hippotamusrendira.* It was a sex-linked disease

[24] Scheurer, Vincent. "Convicted on Statistics?" *Understanding Uncertainty*. University of Cambridge, Web. 12 Oct. 2016. <https://understandinguncertainty.org/node/545>.
[25] Schneps, Leila, and Coralie Colmez. "The Case of Sally Clark: Motherhood Under Attack." *Math on Trial: How Numbers Get Used and Abused in the Courtroom*. New York: Basic, 2013. 2-21. Print.

therefore it was only found in males, and caused a serious immunodeficiency. 1 in 850,000 men have it in the Netherlands, and there are approximately 7 million men in Amsterdam. The police begin to catalog the medical records of the men in Amsterdam, and find their first match who they arrest straight away. He bears no relationship with the victim, but he is still sent off to trial. Medical professionals quote the chances, stating that there is a 1 in 850,000 chance of innocence. But the defendant's lawyer has some mathematical knowledge that he uses to his advantage. He states that the probability that his client is actually guilty is 1 in 8. Seeing as there are 7 million men in Amsterdam, at least 8 of them should carry the disease. This means there are 7 other possible murderers out there. The lawyer then goes to do some basic maths of conditional probability:

$$\text{P(his client being guilty | he tested positive for the disease)} = \frac{P(guilty \cap positive)}{P(positive)}$$

$$= \frac{\frac{1}{8} \times \frac{1}{850000}}{\frac{1}{850000}} = \frac{1}{8}$$

This proves that there is only a 12.5% chance that this man is guilty of killing Jeanine, and therefore the jury cannot be sure of his guilt beyond reasonable doubt. Furthermore, a late DNA analysis shows the suspect did not match the DNA traces found in the semen. Therefore, he walks free and the police need to investigate further options. Instead of blindly sorting through medical records of every man in the city, the investigators start to target all the men that were present in Jeanine's life. With luck, they find that one of her coworkers had the medical condition in question. This provides them with enough evidence to issue a warrant for his arrest and subject him to DNA testing.

His DNA proves to be a 94% match to the semen found in Jeanine's body. His defense lawyers however are committed to proving his innocence and hence question the reliability of the DNA testing and its outcome. Experts are asked to provide some statistical analysis on the situation while a second test is performed. In a DNA test, there are essentially four different outcomes, true positive, true negative, false positive and false negative. In the interest of the investigation, the following assumptions are made about DNA testing:

P (False positive) = 0.006

P (True positive) = 0.998

P (Innocent in given situation) = 7/8 = 0.875

P (Guilty in given situation) = 1/8 = 0.125

Using conditional probability and Bayes Theorem, we can calculate the probability of the coworker being guilty given the match. In binompdf, we use 100 as the n value, seeing as that is the maximum possible match in percentage, the p value being the probability of a false or true positive, and x being the outcome of the test.

binompdf (100, 0.998, 94) = $6.32042382 \times 10^{-8}$

P (94% match from a guilty suspect) = $6.32042382 \times 10^{-8}$

Seeing as the probability of a false positive is so small, using a graphing calculator to find the binomial distribution results in an answer of 0. However, we know that the value cannot be simply 0 but is probably very close to that number, therefore in the interest of this investigation, we will assume that the value coincides with the same amount of decimal points as the result for the probability of a guilty suspect having a 94% match.

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Therefore, we will use the scientific notation of 0.1×10^{a} where a is the exponent used to signify the amount of decimal points in the value of the guilty probability. I am aware that scientific notation usually involves the multiplication of a value between 1 and 10, however in the interest of comparability of these results, the 0.1 values allows me to use the same amount of decimal points throughout the investigation. This suggests the following:

$$P\ (94\%\ \text{match from an innocent suspect}) = 0.1 \times 10^{-8}$$

$$P\ (M \mid I) = 0.1 \times 10^{-8} \qquad P\ (M \mid G) = 6.32042382 \times 10^{-8} \qquad P\ (G \mid M) = ?$$

Where M is the event of a match, G is the event of the suspect being guilty and I is the event of the suspect being innocent

$$P\ (M) = ((0.1 \times 10^{-8}) \times 0.875) + ((6.32042382 \times 10^{-8}) \times 0.125)$$

$$P\ (M) = 8.77552500 \times 10^{-9}$$

Bayes Theorem: $P\ (G \mid M) = (P\ (M \mid G) \times P\ (G))\ /\ P\ (M)$

$$P\ (G \mid M) = ((6.32042382 \times 10^{-8}) \times 0.125)\ /\ (8.77552500 \times 10^{-9})$$

$$P\ (G \mid M) = 0.9029087$$

$$P\ (G \mid M) \approx 0.903$$

Therefore, the probability that the suspect is guilty given this match is 90.3%. This is a pretty high probability but leaves room to be challenged by the defense. In the second test, there is a 92% match between the suspect's DNA and the semen, which is evidently lower than the initial match. The following statistics are composed:

$$\text{binompdf}\,(100, 0.998, 92) = 3.96248732 \times 10^{-11}$$

$$P\ (92\%\ \text{match from a guilty suspect}) = 3.96248732 \times 10^{-11}$$

$$P\ (92\%\ \text{match from an innocent suspect}) = 0.1 \times 10^{-11}$$

$$P\ (M \mid I) = 0.1 \times 10^{-11} \qquad P\ (M \mid G) = 3.96248732 \times 10^{-11} \qquad P\ (G \mid M) = ?$$

$$P\ (M) = ((0.1 \times 10^{-11}) \times 0.875) + ((3.96248732 \times 10^{-11}) \times 0.125)$$

$$P\ (M) = 5.82810915 \times 10^{-12}$$

Bayes Theorem: $P\ (G \mid M) = (P\ (M \mid G) \times P\ (G))\ /\ P\ (M)$

$$P\ (G \mid M) = ((3.96248732 \times 10^{-11}) \times 0.125)\ /\ (5.82810915 \times 10^{-12})$$

$$P\ (G \mid M) = 0.8498554$$

$$P\ (G \mid M) \approx 0.850$$

Now the probability that the suspect is guilty is 85%. The defense could naively claim that this further proves their client's innocence, however as proven with the example of the coin toss, we must interpret the two DNA tests as one large test. Consequently, we are now looking at the results as a 186% match out of the possible 200%.

$$\text{binompdf}\,(200, 0.998, 186) = 1.33200524 \times 10^{-17}$$

$$P\ (186\%\ \text{match from a guilty suspect}) = 1.33200524 \times 10^{-17}$$

$$P\ (186\%\ \text{match from an innocent suspect}) = 0.1 \times 10^{-17}$$

$$P\ (M \mid I) = 0.1 \times 10^{-17} \qquad P\ (M \mid G) = 1.33200524 \times 10^{-17} \qquad P\ (G \mid M) = ?$$

$$P\ (M) = ((0.1 \times 10^{-17}) \times 0.875) + ((1.33200524 \times 10^{-17}) \times 0.125)$$

$$P\ (M) = 2.54000655 \times 10^{-18}$$

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Bayes Theorem: $P(G \mid M) = (P(M \mid G) \times P(G)) / P(M)$

$P(G \mid M) = ((1.33200524 \times 10^{-17}) \times 0.125) / (2.54000655 \times 10^{-18})$

$P(G \mid M) = 0.65551270$

$P(G \mid M) \approx 0.656$

This value does not really make sense seeing as with such a high probability for a true positive and 186/200% of the DNA being a match, there has to be a larger probability for the suspect being guilty given this match. As mentioned before, we cannot be certain of the true probability of a match given innocence due to the fact that the probability of a false positive is so small. Therefore there is room for error seeing as we are making certain assumptions when performing these calculations. In order to rectify this issue, we can add another decimal point to the value of 0.1×10^{-17}, and hence make it 0.1×10^{-18}. Seeing as the value was so small to begin with, adding another decimal point is not a significant change. Therefore:

$$P(186\% \text{ match from a guilty suspect}) = 1.33200524 \times 10^{-17}$$

$$P(186\% \text{ match from an innocent suspect}) = 0.1 \times 10^{-18}$$

$$P(M \mid I) = 0.1 \times 10^{-18} \qquad P(M \mid G) = 1.33200524 \times 10^{-17} \qquad P(G \mid M) = ?$$

$$P(M) = ((0.1 \times 10^{-18}) \times 0.875) + ((1.33200524 \times 10^{-17}) \times 0.125)$$

$$P(M) = 1.75250655 \times 10^{-18}$$

Bayes Theorem: $P(G \mid M) = (P(M \mid G) \times P(G)) / P(M)$

$P(G \mid M) = ((1.33200524 \times 10^{-17}) \times 0.125) / (1.75250655 \times 10^{-18})$

$P(G \mid M) = 0.95007151$

$P(G \mid M) \approx 0.950$

With this result, we arrive at the conclusion that the chance of the suspect being guilty given this outcome is 95%, which makes much more sense than the initial result of 65.6%. Due to this being an investigative process, there is evidently room for some error, especially when dealing with such small numbers. However using the example provided with the coin toss, we could conclude that the following percentages are most likely to be correct: 90.3%, 85% and 95%. With these findings, the police are able to prove the connection between the suspect and the victim, and it is later discovered that Jeanine was having an affair with her coworker and after they spent the night together, she refused to leave her boyfriend for him and as a result, he killed her.

Conclusion

The mathematical incompetence within Sally Clark's case led to the prosecution and psychological destruction of an innocent, grieving mother. The misunderstanding of what constitutes an independent event as well as the misinterpretation of probabilities led the jury to completely disregard the possibility of Sally's innocence, causing an immense amount of suffering to her and her family. A few years after Sally Clark's verdict was overturned, the case of Amanda Knox and Raffaele Sollecito provided the world with another example of the misuse of mathematics in the courtroom, specifically conditional probability. The combination of these two judicial misconducts peaked my interest in the field of law and provided unquestionable evidence of the importance of mathematics in various fields of study. The applicability of mathematics within many areas of knowledge is further supported by examples such as the probability of testing falsely positive for a disease,

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indicating the importance of maths in medicine. The general idea that this exploration communicated was that mathematics should not be confined simply to solving word problems and equations. On the contrary, it is interconnected with so many subjects that it even has an impact on our use of language, as illustrated by the Prosecutor's Fallacy.

Evidently, probability and statistics are not the only mathematical concepts that are misused by the court. The book *Math on Trial* sites ten different examples of the misuse of mathematics within the law, with theories such as the Simpson's Paradox and the Birthday Problem being illustrated within cases.[26] If given the space and time, I would have liked to further investigate these mathematical problems and focus on more than one mathematical concept rather than just probability. However, this exploration provided me with the opportunity to investigate the Bayes Theorem, a very helpful formula that is not part of the standard level mathematics IB course, making this whole process more challenging and interesting. I am also very pleased with the fact that I was able to successfully contact Leila Schneps, one of the authors of the book *Math on Trial*, due to my interest in the general topic as well as my aspiration to fully understand the mathematical method involved in explaining the problems behind Amanda Knox's case. My perpetual determination to accumulate knowledge on this topic drove me to explore many different aspects, allowing me to formulate a successful example of the correct use of mathematics in cases. Even though there was room for error within my mathematical reasoning when proving the increase in reliability of performing a second DNA test, it did communicate the essential idea of the importance of repeating important tests, as well as illustrated my investigative capabilities of mathematical problems. Overall, this exploration continuously fed my growing passion for law, and provided me with knowledge that I will use unquestionably in my future studies.

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[26] Schneps, Leila, and Coralie Colmez. "Introduction." *Math on Trial: How Numbers Get Used and Abused in the Courtroom*. New York: Basic, 2013. Print.

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