

Name:

1. (11 points) Consider the function  $f(x) = \frac{x^2 - 3x + 3}{x - 2}$ .

(a) Show that the graph of the function does not intersect the  $x$ -axis and find the  $y$ -intercept. [3]

(b) Find the asymptotes of the graph of  $y = f(x)$ . [3]

(c) Find  $f'(x)$  and hence find the coordinates of stationary points and classify these points. [3]

(d) Sketch the graph of  $y = f(x)$ . [2]

2. (5 points)

A six sided die has 3 red faces, 2 blue faces and 1 green face. The die is rolled. If it shows a red face, the player loses 2 points, if it shows a blue face, the player gains 1 point and if it shows the green face, the player gains 2 points. Let  $X$  denote the number of points the player gains.

(a) Find  $E(X)$  and  $Var(X)$  [3]

Tomasz plays the game 10 times. Each time he loses points, he swears.

(b) Find the probability that Tomasz swore at least 7 times. [2]

3. (4 points) Let  $X \sim B(n, p)$  with  $p > \frac{1}{4}$  and  $n = 10$ .

(a) Find  $p$  if  $P(X = 3) = 0.215$  (correct to 3 s.f.). [2]

(b) The value of  $n$  has now been increased, so that, for the value of  $p$  found in (a),  $P(X = 3) = 0.0468$  (correct to 3 s.f.).

Find the new value of  $n$ . [2]

4. (9 points)

A particle moves in a straight line so that its velocity in  $ms^{-1}$ , at time  $t$  seconds, is given by:

$$v(t) = \begin{cases} \sqrt{x}, & 0 \leq t \leq 9 \\ 3 - (x - 9)^2, & t > 9 \end{cases}$$

(a) Find the acceleration of the particle at  $t = 4$ . [2]

(b) Find the total distance travelled by the particle in the first 12 seconds. [2]

The particle returns to its initial position at time  $t = T$ .

(c) Find the value of  $T$ . [5]

5. (11 points)

A box contains 11 balls each with a different natural number from 1 to 11 written on it. The balls with numbers 1,2,3 and 4 are red, the remaining balls are blue.

(a) Tomasz takes 6 balls out of the box. Find the probability that:

(i) he picked at least 3 red balls. [3]

(ii) exactly two of the balls he picked are red and exactly two have even numbers written on them. [4]

(b) All the balls are returned to the box. Maria picks a ball, notes its colour and returns it to the box. She does so a total of 11 times.

(i) Find the expected number of red balls she picks. [1]

(ii) Find the probability that she picks more red balls than blue balls. [3]