## Mixed examination practice 14

### Short questions

1. Find a vector equation of the line passing through points $(3, -1, 1)$ and $(6, 0, 1)$. *[4 marks]*

2. The point $(3, -1, 2)$ lies on the line with equation $\frac{x+3}{2} = \frac{y-8}{-3} = \frac{z+13}{p}$.

   Find the value of $p$. *[4 marks]*

3. The vector $\boldsymbol{n} = 3\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ is normal to a plane which passes through the point $(3, -1, 2)$.

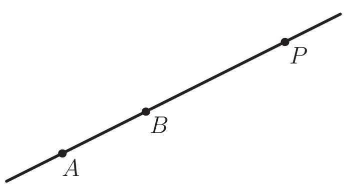
   (a) Find an equation for the plane.

   (b) Find $a$ if the point $(a, 2a, a-1)$ lies on the plane. *[6 marks]*

4. Find the coordinates of the point of intersection of the planes with equations $x - 2y + z = 5$, $2x + y + z = 1$ and $x + 2y - z = -2$. *[6 marks]*

5. Points $A\ (-1, 1, 2)$ and $B\ (3, 5, 4)$ lie on the line with equation $\boldsymbol{r} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$.

   Find the coordinates of point $P$ on the same line such that $AP = 3AB$, as shown in the diagram.




   *[5 marks]*

6. Point $A\ (-3, 0, 4)$ lies on the line $\boldsymbol{r} = -3\boldsymbol{i} + 4\boldsymbol{k} + \lambda(2\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k})$, where $\lambda$ is a real parameter. Find the coordinates of one point on the line which is 10 units from $A$. *[6 marks]*

7. Points $A\ (4, 1, 12)$ and $B\ (8, -11, 20)$ lie on the line $l$.

   (a) Find an equation of line $l$, giving the answer in parametric form.

   (b) The point $P$ is on $l$ such that $\overrightarrow{OP}$ is perpendicular to $l$. Find the coordinates of $P$. *[6 marks]*

8. (a) Given that $\boldsymbol{a} = 2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b} = \boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}$, show that $\boldsymbol{b} \times \boldsymbol{a} = 3\boldsymbol{i} + 7\boldsymbol{j} + \boldsymbol{k}$.

   Two planes have equations $\boldsymbol{r} \cdot \boldsymbol{a} = 5$ and $\boldsymbol{r} \cdot \boldsymbol{b} = 12$.

   (b) Show that the point $(2, 2, 3)$ lies in both planes.

   (c) Write down the Cartesian equation of the line of intersection of the two planes. *[6 marks]*

**9.** The plane $3x+2y-z=2$ contains the line $x-3=\frac{2y+2}{5}=\frac{z-5}{k}$.
Find the value of $k$. *[6 marks]*

**10.** (a) If $\boldsymbol{u}=\boldsymbol{i}+2\boldsymbol{j}+3\boldsymbol{k}$ and $\boldsymbol{v}=2\boldsymbol{i}-\boldsymbol{j}+2\boldsymbol{k}$ show that $\boldsymbol{u}\times\boldsymbol{v}=7\boldsymbol{i}+4\boldsymbol{j}-5\boldsymbol{k}$.

(b) Let $\boldsymbol{w}=\lambda\boldsymbol{u}+\mu\boldsymbol{v}$ where $\lambda$ and $\mu$ are scalars. Show that $\boldsymbol{w}$ is perpendicular to the line of intersection of the planes $x+2y+3z=5$ and $2x-y+2z=7$ for all values of $\lambda$ and $\mu$.

*[8 marks]*

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**11.** Find the Cartesian equation of the plane containing the two lines

$$x=\frac{3-y}{2}=z-1 \text{ and } \frac{x-2}{3}=\frac{y+1}{-3}=\frac{z-3}{5}.$$ *[8 marks]*

## Long questions

**1.** Points $A$ and $B$ have coordinates $(4, 1, 2)$ and $(0, 5, 1)$. Line $l_1$ passes through $A$ and has equation $\boldsymbol{r}_1=\begin{pmatrix}4\\1\\2\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\3\end{pmatrix}$. Line $l_2$ passes through $B$ and has equation $\boldsymbol{r}_2=\begin{pmatrix}0\\5\\1\end{pmatrix}+t\begin{pmatrix}4\\-4\\1\end{pmatrix}$.

(a) Show that the line $l_2$ also passes through $A$.

(b) Calculate the distance $AB$.

(c) Find the angle between $l_1$ and $l_2$ in degrees.

(d) Hence find the shortest distance from $B$ to $l_1$. *[10 marks]*

**2.** (a) Show that the lines $l_1:\boldsymbol{r}=\begin{pmatrix}-3\\3\\18\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\-8\end{pmatrix}$ and $l_2:\boldsymbol{r}=\begin{pmatrix}5\\0\\2\end{pmatrix}+\mu\begin{pmatrix}1\\1\\-1\end{pmatrix}$ do not intersect.

(b) Points $P$ and $Q$ lie on $l_1$ and $l_2$ respectively, such that $(PQ)$ is perpendicular to both lines.

(i) Write down $\overrightarrow{PQ}$ in terms of $\lambda$ and $\mu$.

(ii) Show that $9\mu-69\lambda+147=0$.

(iii) Find a second equation for $\lambda$ and $\mu$.

(iv) Find the coordinates of $P$ and the coordinates of $Q$.

(v) Hence find the shortest distance between $l_1$ and $l_2$. *[14 marks]*

3. Plane $\Pi$ has equation $x-2y+z=20$ and point $A$ has coordinates $(4, -1, 2)$.

(a) Write down the vector equation of the line $l$ through $A$ which is perpendicular to $\Pi$.

(b) Find the coordinates of the point of intersection of line $l$ and plane $\Pi$.

(c) Hence find the shortest distance from point $A$ to plane $\Pi$. *[10 marks]*

4. In this question, unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ point East and North, and unit vector $\boldsymbol{k}$ is vertically up. The time ($t$) is measured in minutes and the distance in kilometres.

Two aircraft move with constant velocities $\boldsymbol{v}_1 = (7\boldsymbol{i}+10\boldsymbol{j}+3\boldsymbol{k})$km/min and $\boldsymbol{v}_2 = (3\boldsymbol{i}-8\boldsymbol{j}-4\boldsymbol{k})$km/min. At $t=0$, the first aircraft is at the point with coordinates (16, 30, 3) and the second aircraft at the point with coordinates (24, 66, 12).

(a) Calculate the speed of the first aircraft.

(b) Write down the position vector of the second aircraft at the time $t$ minutes.

(c) Find the distance between the aircraft after 3 minutes.

(d) Show that there is a time when the first aircraft is vertically above the second one, and find the distance between them at that time.

5. Line $L_1$ has equation $\boldsymbol{r} = \begin{pmatrix}5\\1\\2\end{pmatrix} + t\begin{pmatrix}-1\\1\\3\end{pmatrix}$ and line $L_2$ has equation $\boldsymbol{r} = \begin{pmatrix}5\\4\\9\end{pmatrix} + s\begin{pmatrix}2\\1\\1\end{pmatrix}$.

(a) Find $\begin{pmatrix}-1\\1\\3\end{pmatrix} \times \begin{pmatrix}2\\1\\1\end{pmatrix}$.

(b) Find the coordinates of the point of intersection of the two lines.

(c) Write down a vector perpendicular to the plane containing the two lines.

(d) Hence find the Cartesian equation of the plane containing the two lines. *[10 marks]*

6. Three planes have equations:

$$\Pi_1 : 3x - y + z = 2$$

$$\Pi_2 : x + 2y - z = -1$$

$$\Pi_3 : 5x - 4y + dz = 3$$

(a) Find the value of $d$ for which the three planes do not intersect.

(b) Find the vector equation of the line $l_1$ of intersection of $\Pi_1$ and $\Pi_2$.

(c) For the value of $d$ found in part (a):

(i) Find the value of $p$ so that the point $A\,(0, 1, p)$ lines on $l_1$.

(ii) Find the vector equation of the line $l_2$ through $A$ perpendicular to $\Pi_3$.

(iii) Hence find the distance between $l_1$ and $\Pi_3$. *[17 marks]*

**7.** Line $l_1$ has Cartesian equation $\frac{x-2}{4} = \frac{y+1}{-3} = \frac{z}{3}$. Line $l_2$ is parallel to $l_1$ and passes through point $A\ (0,-1,\ 2)$.

(a) Write down a vector equation of $l_2$.

(b) Find the coordinates of the point $B$ on $l_1$ such that $(AB)$ is perpendicular to $l_1$.

(c) Hence find, to three significant figures, the shortest distance between the two lines. *[9 marks]*

**8.** Line $L$ has equation $\frac{x+5}{3} = \frac{y-1}{3} = \frac{z-2}{-1}$.

(a) Show that the point $A$ with coordinates $(4,10,-1)$ lies on $L$.

(b) Given that point $B$ has coordinates $(2,1,2)$, calculate the distance $AB$.

(c) Find the acute angle between $L$ and $(AB)$ in radians.

(d) Find the shortest distance of $B$ from $L$. *[12 marks]*

**9.** (a) The plane $\Pi_1$ has equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -3 \\ -9 \end{pmatrix}$.

The plane $\Pi_2$ has the equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

(i) For points which lie on $\Pi_1$ and $\Pi_2$, show that $\lambda = \mu$.

(ii) Hence, or otherwise, find a vector equation of the line of intersection of $\Pi_1$ and $\Pi_2$.

(b) The plane $\Pi_3$ contains the line $\frac{2-x}{3} = \frac{y}{-4} = z+1$ and is perpendicular to $3\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$. Find the cartesian equation of $\Pi_3$.

(c) Find the intersection of $\Pi_1$, $\Pi_2$ and $\Pi_3$. *[12 marks]*

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10. (a) Find the vector equation of the line $L$ through point $A\left(-2, 4, 2\right)$

parallel to the vector $\boldsymbol{l} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

(b) Point $B$ has coordinates $(2,\ 3,\ 3)$. Find the cosine of the angle between $(AB)$ and the line $L$.

(c) Calculate the distance $AB$.

(d) Point $C$ lies on $L$ and $BC$ is perpendicular to $L$. Find the exact distance $AC$. *[10 marks]*

11. Plane $\Pi$ has equation $x - 4y + 2z = 7$ and point $P$ has coordinates $(9, -7, 6)$.

(a) Show that point $R$ $(5, 1, 3)$ lies in the plane $\Pi$.

(b) Find the vector equation of the line $(PR)$.

(c) Write down the vector equation of the line through $P$ perpendicular to $\Pi$.

(d) $N$ is the foot of the perpendicular from $P$ to $\Pi$. Find the coordinates of $N$.

(e) Find the exact distance of point $P$ from the plane $\Pi$. *[14 marks]*

12. Point $A$ $(3,\ 1, -4)$ lies on line $L$ which is perpendicular to plane $\Pi : 3x - y - z = 1$.

(a) Find the Cartesian equation of $L$.

(b) Find the intersection of the line $L$ and plane $\Pi$.

(c) Point $A$ is reflected in $\Pi$. Find the coordinates of the image of $A$.

(d) Point $B$ has coordinates $(1, 1, 1)$. Show that $B$ lies in $\Pi$.

(e) Find the distance between $B$ and $L$. *[14 marks]*

13. (a) Calculate $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$.

(b) Plane $\Pi_1$ has normal vector $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ and contains point $A$ $(3, 4, -2)$.

Find the Cartesian equation of the plane.

(c) Plane $\Pi_2$ has equation $3x + y - z = 15$. Show that $\Pi_2$ contains point $A$.

(d) Write down the vector equation of the line of intersection of the two planes.

(e) A third plane, $\Pi_3$, has equation $\boldsymbol{r} \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = 12$. Find the coordinates of

the point of intersection of all three planes.

(f) Find the angle between $\Pi_1$ and $\Pi_3$ in degrees. *[17 marks]*

### Exercise 14G

**1.** (a) $\mathbf{r}=\begin{pmatrix}-3\\-3\\4\end{pmatrix}+\lambda\begin{pmatrix}2\\2\\-1\end{pmatrix}$

(b) (3,3,1) (c) 9

**2.** (c) $\mathbf{r}=\lambda\begin{pmatrix}1\\-3\\1\end{pmatrix}$

(d) $\dfrac{6\sqrt{11}}{11}$

**3.** (a) $\begin{pmatrix}-2\\3\\-1\end{pmatrix}$

(b) (ii) $(1,-3,14)$

(c) $2x-3y+z=25$

**4.** (b) $\mathbf{r}\cdot\begin{pmatrix}-1\\5\\2\end{pmatrix}=-5$

(c) $\sqrt{30}$

(d) $(8,-5,-1)$

**5.** (c) $\left(\dfrac{184}{11},-\dfrac{-32}{11},-\dfrac{-1}{11}\right)$

(d) 6.99

**6.** (a) $(10,11,-6)$

(b) $\begin{pmatrix}7\\-9\\-5\end{pmatrix}$

(c) $7x-9y-5z=1$

**7.** (a) $\begin{pmatrix}-3\\-10\\2\end{pmatrix}$ (b) 5.32

(c) $3x+10y-2z=16$

(d) $\mathbf{r}=\begin{pmatrix}-7\\-28\\11\end{pmatrix}+\lambda\begin{pmatrix}3\\10\\-2\end{pmatrix}$

(e) $(2,2,5)$; 31.9 (3*SF*)

(f) 56.5

**8.** (a) $\mathbf{r}=\begin{pmatrix}-1\\1\\4\end{pmatrix}+\lambda\begin{pmatrix}6\\1\\5\end{pmatrix}$

(c) $\begin{pmatrix}-13\\38\\8\end{pmatrix}$

(d) $-13x+38y+8z=83$

**9.** (a) $\left(\dfrac{96}{41},-\dfrac{32}{41},\dfrac{16}{41}\right)$

(b) $\dfrac{16\sqrt{41}}{41}$

**10.** (b) $\mathbf{r}=\lambda\begin{pmatrix}1\\0\\-1\end{pmatrix}$

(c) (i) $(2,0,-2)$ (ii) $(-4,0,4)$

(d) $6\sqrt{2}$

## Mixed examination practice 14

### Short questions

**1.** $\mathbf{r}=\begin{pmatrix}3\\-1\\1\end{pmatrix}+\lambda\begin{pmatrix}3\\1\\0\end{pmatrix}$

**2.** 5

**3.** (a) $3x+y-z=6$

(b) $\dfrac{5}{4}$

**4.** $\left(\dfrac{3}{2},-\dfrac{11}{6},-\dfrac{1}{6}\right)$

**5.** $(11,13,8)$

**6.** $\left(\dfrac{11}{3},\dfrac{20}{3},\dfrac{2}{3}\right)$ or $\left(\dfrac{-29}{3},\dfrac{-20}{3},\dfrac{22}{3}\right)$

**7.** (a) $x=4+\lambda, y=1-3\lambda,$
$z=12+2\lambda$

(b) $\left(\dfrac{31}{14},\dfrac{89}{14},\dfrac{59}{7}\right)$

**8.** (c) $\dfrac{x-2}{3}=\dfrac{(y-2)}{7}=z-3$

**9.** $k=8$

**11.** $7x+2y-3z=3$

### Long questions

**1.** (b) $\sqrt{33}$

(c) 45.7°

(d) 4.11

2. (b) (i) $\begin{pmatrix} \mu-2\lambda+8 \\ \mu+\lambda-3 \\ -\mu+8\lambda-16 \end{pmatrix}$

(iii) $3\mu-9\lambda+21=0$

(iv) $(1,1,2)$, $(4,-1,3)$

(v) $\sqrt{14}$

3. (a) $\boldsymbol{r}=\begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix}+\lambda\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

(b) $(6,-5,4)$

(c) $2\sqrt{6}$

4. (a) 12.6 km/min

(b) $(24+3t)\boldsymbol{i}+(66-8t)\boldsymbol{j}+(12-4t)\boldsymbol{k}$

(c) 22 km

(d) 5 km (when $t=2$)

5. (a) $\begin{pmatrix} -2 \\ 7 \\ -3 \end{pmatrix}$

(b) $(3,3,8)$

(c) $\begin{pmatrix} -2 \\ 7 \\ -3 \end{pmatrix}$

(d) $2x-7y+3z=9$

6. (a) $d=3$

(b) $\boldsymbol{r}=\begin{pmatrix} \frac{3}{7} \\ \frac{5}{7} \\ 0 \end{pmatrix}+t\begin{pmatrix} -1 \\ 4 \\ 7 \end{pmatrix}$

(c) (i) $p=3$

(ii) $\boldsymbol{r}=\begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}+\lambda\begin{pmatrix} 5 \\ -4 \\ 3 \end{pmatrix}$

(iii) $\frac{\sqrt{34}}{15}$ $(\approx 0.389)$

7. (a) $\boldsymbol{r}=\begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix}+\lambda\begin{pmatrix} 4 \\ -3 \\ 3 \end{pmatrix}$

(b) $\left(\frac{30}{17},-\frac{14}{17},-\frac{3}{17}\right)$

(c) 2.81

8. (b) $\sqrt{94}$

(c) 0.551

(d) 5.08

9. (a) (ii) $\boldsymbol{r}=\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}+\lambda\begin{pmatrix} -1 \\ -2 \\ -1 \end{pmatrix}$

(b) $3x-2y+z=5$

(c) $(2,1,1)$

10. (a) $\boldsymbol{r}=\begin{pmatrix} -2 \\ 4 \\ 2 \end{pmatrix}+\lambda\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

(b) $\frac{1}{2}$

(c) $3\sqrt{2}$ (d) $\frac{3\sqrt{2}}{2}$

11. (b) $\boldsymbol{r}=\begin{pmatrix} 9 \\ -7 \\ 6 \end{pmatrix}+\lambda\begin{pmatrix} -4 \\ 8 \\ 3 \end{pmatrix}$

(c) $\boldsymbol{r}=\begin{pmatrix} 9 \\ -7 \\ 6 \end{pmatrix}+\mu\begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}$

(d) $(7,1,2)$

(e) $\sqrt{84}$

12. (a) $\frac{x-3}{3}=\frac{y-1}{-1}=\frac{z+4}{-1}$

(b) $(0,2,-3)$

(c) $(-3,3,-2)$

(e) $3\sqrt{2}$

13. (a) $\begin{pmatrix} 0 \\ 5 \\ 5 \end{pmatrix}$

(b) $2x-y+z=0$

(d) $\boldsymbol{r}=\begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}+\lambda\begin{pmatrix} 0 \\ 5 \\ 5 \end{pmatrix}$

(e) $(3,4,0)$ (f) 47.1°