Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Monday 18 November 2019 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The probability distribution of a discrete random variable, X, is given by the following table, where N and p are constants.

x	1	5	10	N
$\mathrm{P}(X=x)$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{5}$	p

(a) Find the value of p. [2]

(b) Given that $\mathrm{E}(X)=10$, find the value of N. [3]

2. [Maximum mark: 6]

Given that $\int_0^{\ln k} e^{2x}\,dx = 12$, find the value of k.

3. [Maximum mark: 5]

Three planes have equations:

$$\begin{aligned} 2x - y + z &= 5 \\ x + 3y - z &= 4 \\ 3x - 5y + az &= b \end{aligned} \quad \text{, where } a, b \in \mathbb{R}.$$

Find the set of values of a and b such that the three planes have no points of intersection.

4. [Maximum mark: 7]

A and B are acute angles such that $\cos A = \frac{2}{3}$ and $\sin B = \frac{1}{3}$.

Show that $\cos(2A + B) = -\frac{2\sqrt{2}}{27} - \frac{4\sqrt{5}}{27}$.

5. [Maximum mark: 7]

Consider the equation $z^4 = -4$, where $z \in \mathbb{C}$.

(a) Solve the equation, giving the solutions in the form $a + \mathrm{i}b$, where $a, b \in \mathbb{R}$. [5]

(b) The solutions form the vertices of a polygon in the complex plane. Find the area of the polygon. [2]

6. [Maximum mark: 7]

Consider the function $f(x) = x\mathrm{e}^{2x}$, where $x \in \mathbb{R}$. The n^{th} derivative of $f(x)$ is denoted by $f^{(n)}(x)$.

Prove, by mathematical induction, that $f^{(n)}(x) = (2^n x + n2^{n-1})\mathrm{e}^{2x}$, $n \in \mathbb{Z}^+$.

7. [Maximum mark: 7]

(a) Write $2x - x^2$ in the form $a(x-h)^2 + k$, where $a, h, k \in \mathbb{R}$. [2]

(b) Hence, find the value of $\int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{\sqrt{2x - x^2}}\, \mathrm{d}x$. [5]

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8. [Maximum mark: 6]

A straight line, L_θ, has vector equation $\boldsymbol{r} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ \sin\theta \\ \cos\theta \end{pmatrix}$, $\lambda, \theta \in \mathbb{R}$.

The plane, Π_p, has equation $x = p$, $p \in \mathbb{R}$.

Show that the angle between L_θ and Π_p is independent of both θ and p.

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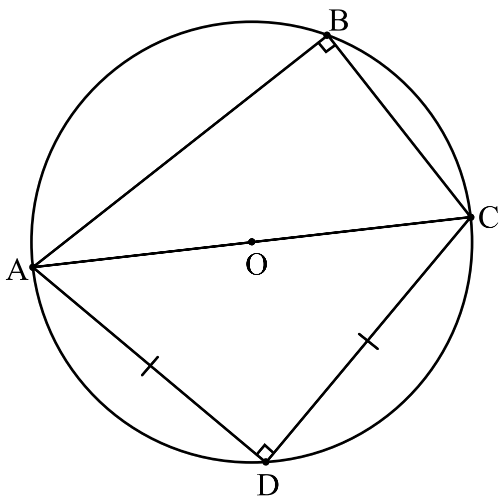
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 14]

(a) Given that $\cos 75^\circ = q$, show that $\cos 105^\circ = -q$. [1]

In the following diagram, the points A, B, C and D are on the circumference of a circle with centre O and radius r. [AC] is a diameter of the circle. $\mathrm{BC} = r$, $\mathrm{AD} = \mathrm{CD}$ and $\mathrm{A\hat{B}C} = \mathrm{A\hat{D}C} = 90^\circ$.



(b) Show that $\mathrm{B\hat{A}D} = 75^\circ$. [3]

(c) (i) By considering triangle ABD, show that $\mathrm{BD}^2 = 5r^2 - 2r^2 q\sqrt{6}$.

(ii) By considering triangle CBD, find another expression for BD^2 in terms of r and q. [7]

(d) Use your answers to part (c) to show that $\cos 75^\circ = \dfrac{1}{\sqrt{6}+\sqrt{2}}$. [3]

Do **not** write solutions on this page.

10. [Maximum mark: 19]

Consider $f\left(x\right)=\frac{2x-4}{x^2-1}$, $-1<x<1$.

(a) (i) Find $f'(x)$.

(ii) Show that, if $f'(x)=0$, then $x=2-\sqrt{3}$. [5]

(b) For the graph of $y=f(x)$,

(i) find the coordinates of the y-intercept;

(ii) show that there are no x-intercepts;

(iii) sketch the graph, showing clearly any asymptotic behaviour. [5]

(c) Show that $\frac{3}{x+1}-\frac{1}{x-1}=\frac{2x-4}{x^2-1}$. [2]

(d) The area enclosed by the graph of $y=f(x)$ and the line $y=4$ can be expressed as $\ln v$. Find the value of v. [7]

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11. [Maximum mark: 17]

Points $\text{A}(0, 0, 10)$, $\text{B}(0, 10, 0)$, $\text{C}(10, 0, 0)$, $V(p, p, p)$ form the vertices of a tetrahedron.

(a) (i) Show that $\overrightarrow{\text{AB}} \times \overrightarrow{\text{AV}} = -10\begin{pmatrix} 10-2p \\ p \\ p \end{pmatrix}$ and find a similar expression for $\overrightarrow{\text{AC}} \times \overrightarrow{\text{AV}}$.

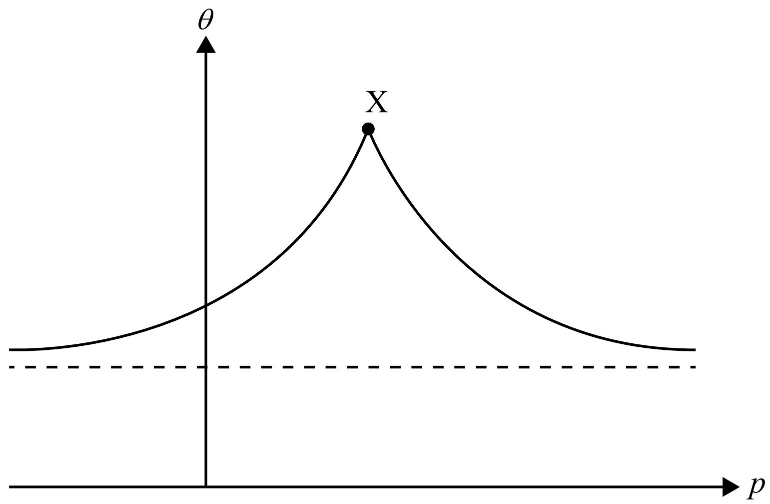
(ii) Hence, show that, if the angle between the faces ABV and ACV is θ, then $\cos\theta = \dfrac{p(3p-20)}{6p^2 - 40p + 100}$. [8]

(b) Consider the case where the faces ABV and ACV are perpendicular.

(i) Find the two possible coordinates of V.

(ii) Comment on the positions of V in relation to the plane ABC. [4]

(c) The following diagram shows the graph of θ against p. The maximum point is shown by X.



(i) At X, find the value of p and the value of θ.

(ii) Find the equation of the horizontal asymptote of the graph. [5]