Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Tuesday 19 November 2019 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

A geometric sequence has $u_4 = -70$ and $u_7 = 8.75$. Find the second term of the sequence.

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2. [Maximum mark: 6]

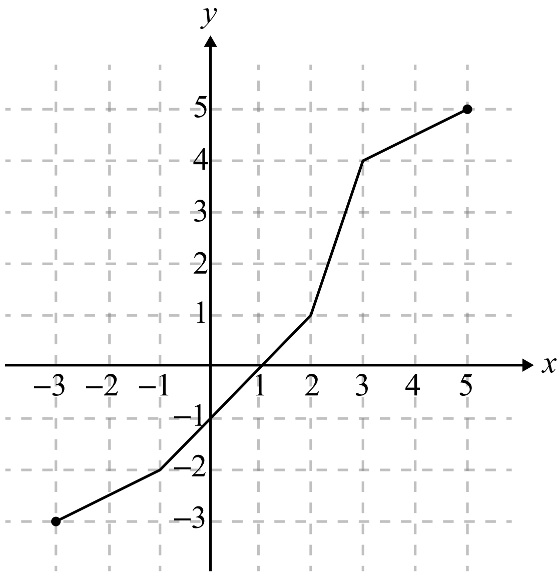
The number of marathons that Audrey runs in any given year can be modelled by a Poisson distribution with mean 1.3.

(a) Calculate the probability that Audrey will run at least two marathons in a particular year. [2]

(b) Find the probability that she will run at least two marathons in exactly four out of the following five years. [4]

3. [Maximum mark: 6]

The following diagram shows the graph of $y = f(x),\ -3 \le x \le 5$.



(a) Find the value of $(f \circ f)(1)$. [2]

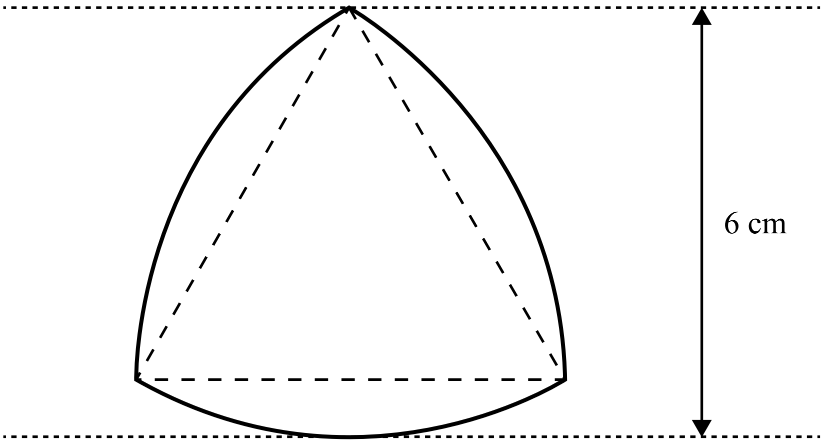
(b) Given that $f^{-1}(a) = 3$, determine the value of a. [2]

(c) Given that $g(x) = 2f(x-1)$, find the domain and range of g. [2]

4. [Maximum mark: 7]

The following shape consists of three arcs of a circle, each with centre at the opposite vertex of an equilateral triangle as shown in the diagram.

diagram not to scale



For this shape, calculate

(a) the perimeter; [2]

(b) the area. [5]

5. [Maximum mark: 6]

Consider the expansion of $(2+x)^n$, where $n \geq 3$ and $n \in \mathbb{Z}$.

The coefficient of x^3 is four times the coefficient of x^2. Find the value of n.

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6. [Maximum mark: 6]

Let $P(z) = az^3 - 37z^2 + 66z - 10$, where $z \in \mathbb{C}$ and $a \in \mathbb{Z}$.

One of the roots of $P(z) = 0$ is $3 + \mathrm{i}$. Find the value of a.

7. [Maximum mark: 6]

Runners in an athletics club have season's best times for the $100\,\text{m}$, which can be modelled by a normal distribution with mean 11.6 seconds and standard deviation 0.8 seconds. To qualify for a particular competition a runner must have a season's best time of under 11 seconds. A runner from this club who has qualified for the competition is selected at random. Find the probability that he has a season's best time of under 10.7 seconds.

8. [Maximum mark: 8]

Eight boys and two girls sit on a bench. Determine the number of possible arrangements, given that

(a) the girls do not sit together; [3]

(b) the girls do not sit on either end; [2]

(c) the girls do not sit on either end and do not sit together. [3]

Do **not** write solutions on this page.

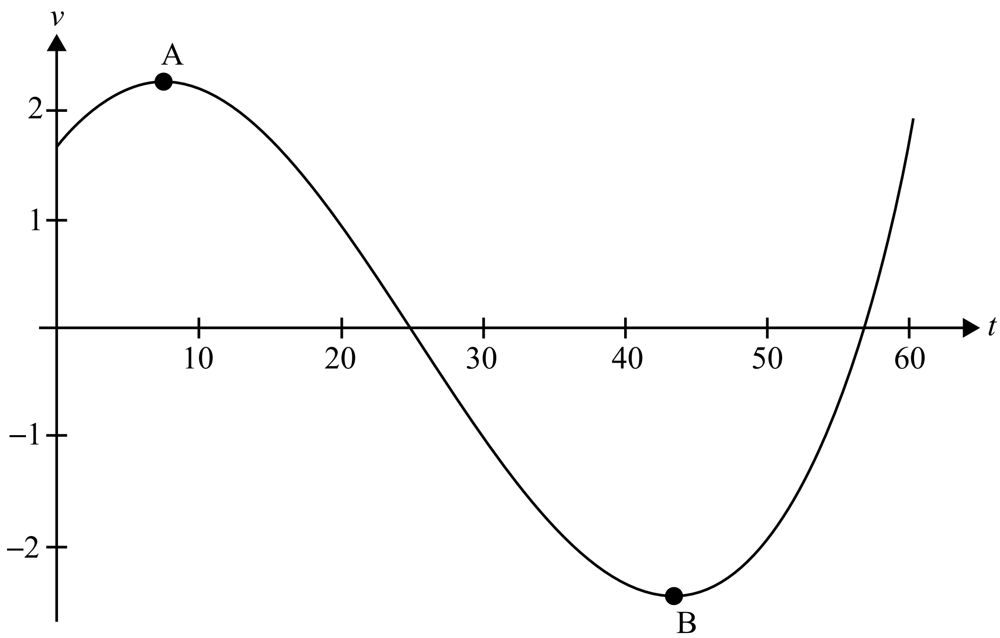
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 14]

A body moves in a straight line such that its velocity, $v\ \mathrm{m\,s^{-1}}$, after t seconds is given by $v = 2\sin\left(\frac{t}{10}+\frac{\pi}{5}\right)\csc\left(\frac{t}{30}+\frac{\pi}{4}\right)$ for $0 \le t \le 60$.

The following diagram shows the graph of v against t. Point A is a local maximum and point B is a local minimum.



(a) (i) Determine the coordinates of point A and the coordinates of point B.

(ii) Hence, write down the maximum speed of the body. [5]

(b) The body first comes to rest at time $t = t_1$. Find

(i) the value of t_1;

(ii) the distance travelled between $t = 0$ and $t = t_1$;

(iii) the acceleration when $t = t_1$. [6]

(c) Find the distance travelled in the first 30 seconds. [3]

Do **not** write solutions on this page.

10. [Maximum mark: 19]

A random variable X has probability density function

$$f(x)=\begin{cases} 3a & , \quad 0 \le x < 2 \\ a(x-5)(1-x) & , \quad 2 \le x \le b \\ 0 & , \quad \text{otherwise} \end{cases} \quad a, b \in \mathbb{R}^+, 3 < b \le 5.$$

(a) Find, in terms of a, the probability that X lies between 1 and 3. [4]

Consider the case where $b = 5$.

(b) Sketch the graph of f. State the coordinates of the end points and any local maximum or minimum points, giving your answers in terms of a. [4]

(c) Find the value of

(i) a;

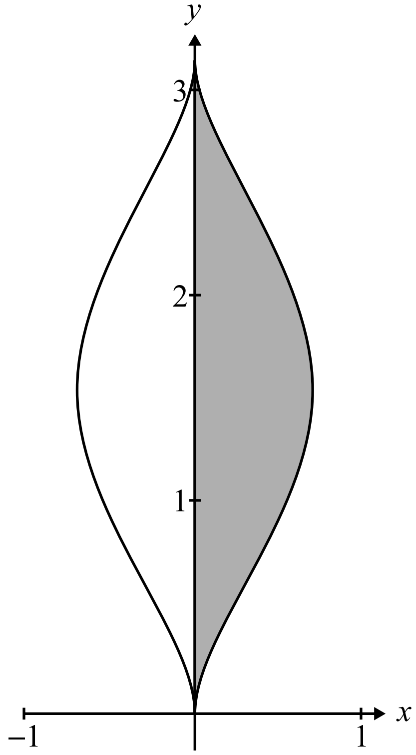
(ii) $\mathrm{E}(X)$;

(iii) the median of X. [11]

Do **not** write solutions on this page.

11. [Maximum mark: 17]

The following diagram shows part of the graph of $2x^2 = \sin^3 y$ for $0 \le y \le \pi$.



(a) (i) Using implicit differentiation, find an expression for $\frac{dy}{dx}$.

(ii) Find the equation of the tangent to the curve at the point $\left(\frac{1}{4}, \frac{5\pi}{6}\right)$. [8]

The shaded region R is the area bounded by the curve, the y-axis and the lines $y = 0$ and $y = \pi$.

(b) Find the area of R. [3]

The region R is now rotated about the y-axis, through 2π radians, to form a solid.

(c) By writing $\sin^3 y$ as $(1 - \cos^2 y) \sin y$, show that the volume of the solid formed is $\frac{2\pi}{3}$. [6]