Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2019

Mathematics

Higher level

Paper 2

14 pages

Section A

1. $u_1r^3 = -70$, $u_1r^6 = 8.75$ ***(M1)***

$r^3 = \frac{8.75}{-70} = -0.125$ ***(A1)***

$\Rightarrow r = -0.5$ ***(A1)***

valid attempt to find u_2 ***(M1)***

for example: $u_1 = \frac{-70}{-0.125} = 560$

$u_2 = 560 \times -0.5$

$= -280$ ***A1***

[5 marks]

2. (a) $X \sim \text{Po}(1.3)$

$\text{P}(X \geq 2) = 0.373$ ***(M1)A1***

[2 marks]

(b) $V \sim \text{B}(5, 0.373)$ ***(M1)A1***

Note: Award ***(M1)*** for recognition of binomial or equivalent, ***A1*** for correct parameters.

$\text{P}(V = 4) = 0.0608$ ***(M1)A1***

[4 marks]

Total [6 marks]

3. (a) $f(1) = 0$ ***(A1)***

$f(0) = -1$ ***A1***

[2 marks]

(b) $a = f(3)$ ***(M1)***

$\Rightarrow a = 4$ ***A1***

[2 marks]

(c) domain is $-2 \leq x \leq 6$ ***A1***

range is $-6 \leq y \leq 10$ ***A1***

[2 marks]

Total [6 marks]

4. (a) each arc has length $r\theta = 6 \times \frac{\pi}{3} = 2\pi\left(= 6.283...\right)$ ***(M1)***

perimeter is therefore $6\pi\ \left(= 18.8\right)$ (cm) ***A1***

[2 marks]

(b) area of sector, s, is $\frac{1}{2}r^2\theta = 18 \times \frac{\pi}{3} = 6\pi\left(= 18.84...\right)$ ***(A1)***

area of triangle, t, is $\frac{1}{2} \times 6 \times 3\sqrt{3} = 9\sqrt{3}\left(= 15.58...\right)$ ***(M1)(A1)***

Note: area of segment, k, is 3.261… implies area of triangle

finding $3s - 2t$ or $3k + t$ or similar

area $= 3s - 2t = 18\pi - 18\sqrt{3}\ \left(= 25.4\right)\ (\text{cm}^2)$ ***(M1)A1***

[5 marks]

Total [7 marks]

5. attempt to find coefficients in binomial expansion ***(M1)***

coefficient of x^2 : $\binom{n}{2} \times 2^{n-2}$; coefficient of x^3 : $\binom{n}{3} \times 2^{n-3}$ ***A1A1***

Note: Condone terms given rather than coefficients. Terms may be seen in an equation such as that below.

$\binom{n}{3} \times 2^{n-3} = 4\binom{n}{2} \times 2^{n-2}$ ***(A1)***

attempt to solve equation using GDC or algebraically ***(M1)***

$\binom{n}{3} = 8\binom{n}{2}$

$\frac{n!}{3!(n-3)!} = \frac{8n!}{2!(n-2)!}$

$\frac{1}{3} = \frac{8}{n-2}$

$n = 26$ ***A1***

[6 marks]

6. **METHOD 1**

one other root is $3-\text{i}$ ***A1***
let third root be α ***(M1)***
considering sum or product of roots ***(M1)***
sum of roots $=6+\alpha=\frac{37}{a}$ ***A1***
product of roots $=10\alpha=\frac{10}{a}$ ***A1***
hence $a=6$ ***A1***

[6 marks]

METHOD 2

one other root is $3-\text{i}$ ***A1***
quadratic factor will be $z^2-6z+10$ ***(M1)A1***
$P(z)=az^3-37z^2+66z-10=(z^2-6z+10)(az-1)$ ***M1***
comparing coefficients ***(M1)***
hence $a=6$ ***A1***

[6 marks]

METHOD 3

substitute $3+\text{i}$ into $P(z)$ ***(M1)***
$a(18+26\text{i})-37(8+6\text{i})+66(3+\text{i})-10=0$ ***(M1)A1***
equating real or imaginary parts or dividing ***M1***
$18a-296+198-10=0$ or $26a-222+66=0$ or $\frac{10-66(3+\text{i})+37(8+6\text{i})}{18+26\text{i}}$ ***A1***
hence $a=6$ ***A1***

[6 marks]

7. $T\sim \text{N}(11.6, 0.8^2)$

$\text{P}(T<10.7 \mid T<11)$ ***(M1)***
$=\frac{\text{P}(T<10.7 \cap T<11)}{\text{P}(T<11)}$ ***(M1)***
$=\frac{\text{P}(T<10.7)}{\text{P}(T<11)}$ ***(M1)***
$\text{P}(T<10.7)=0.1302\ldots$ ***(A1)***
$\text{P}(T<11)=0.2266\ldots$ ***(A1)***
$\text{P}(T<10.7 \mid T<11)=0.575$ ***A1***

Note: Accept only 0.575.

[6 marks]

8. (a) **METHOD 1**

$10!-2\times 9!\,(=2903040)$ ***(A1)(A1)A1***

Note: Award ***A1*** for $10!$, ***A1*** for $2\times 9!$, ***A1*** for final answer.

METHOD 2

$9\times 8\times 8!$ ***(A1)(A1)A1***

Note: Award ***A1*** for 9×8 or equivalent, ***A1*** for $8!$ and ***A1*** for answer.

[3 marks]

(b) **METHOD 1**

$8\times 7\times 8!\,(=2257920)$ ***(A1)A1***

Note: Award ***(A1)*** for 8×7, ***A1*** for final answer.

METHOD 2

$10!-2\times 8!-2\times 2\times 7\times 8!$

Note: Award ***A1*** for $10!$ minus EITHER subtracted terms and ***A1*** for final correct answer.

[2 marks]

(c) **METHOD 1**

$8\times 7\times(8!-2\times 7!)\,(=1693440)$ ***(A1)(A1)A1***

Note: Award ***(A1)*** for 8×7, ***(A1)*** for $2\times 7!$, ***A1*** for final answer.
$(8!-2\times 7!)$ can be replaced by $6\times 7!$ *or* ${}^{7}P_{2}\times 6!$ which may be awarded the second ***A1***.

METHOD 2

their answer to (a) $-2\times 8!-2\times 2\times 7\times 8!$ ***(A1)(A1)A1***

Note: Award ***A1*** for subtracting each of the terms and ***A1*** for final answer.

METHOD 3

their answer to (b) $-2\times 7\times 8!$ or equivalent ***(A1)A2***

Note: Award ***A1*** for the subtraction and ***A2*** for final answer.

[3 marks]

Total [8 marks]

Section B

9. (a) (i) $\mathrm{A}(7.47, 2.28)$ and $\mathrm{B}(43.4, -2.45)$ ***A1A1A1A1***

(ii) maximum speed is $2.45\ (\mathrm{m\,s^{-1}})$ ***A1***

[5 marks]

(b) (i) $v = 0 \Rightarrow t_1 = 25.1\,(\mathrm{s})$ ***(M1)A1***

(ii) $\int_0^{t_1} v\,\mathrm{d}t$ ***(M1)***

$= 41.0\,(\mathrm{m})$ ***A1***

(iii) $a = \frac{\mathrm{d}v}{\mathrm{d}t}$ at $t = t_1 = 25.1$ ***(M1)***

$a = -0.200\ (\mathrm{m\,s^{-2}})$ ***A1***

Note: Accept $a = -0.2$.

[6 marks]

(c) attempt to integrate between 0 and 30 ***(M1)***

Note: An unsupported answer of 38.6 can imply integrating from 0 to 30.

EITHER

$\int_0^{30} |v|\,\mathrm{d}t$ ***(A1)***

OR

$41.0 - \int_{t_1}^{30} v\,\mathrm{d}t$ ***(A1)***

THEN

$= 43.3\,(\mathrm{m})$ ***A1***

[3 marks]

Total [14 marks]

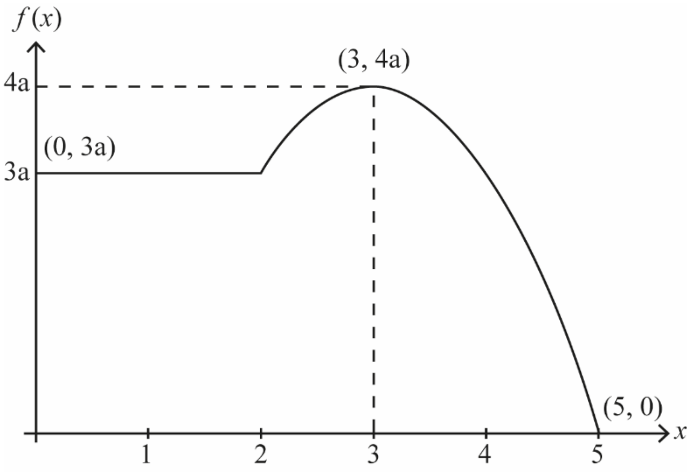
10. (a) $\left(\mathrm{P}(1 < X < 3) =\right) \int_1^2 3a\,\mathrm{d}x + a\int_2^3 -x^2 + 6x - 5\,\mathrm{d}x$ ***(M1)(A1)(A1)***

$= 3a + \frac{11}{3}a$

$= \frac{20}{3}a\left(= 6.67a\right)$ ***A1***

[4 marks]

(b)



A4

award ***A1*** for $(0,3a)$, ***A1*** for continuity at $(2,3a)$, ***A1*** for maximum at $(3,4a)$, ***A1*** for $(5,0)$

Note: Award ***A3*** if correct four points are not joined by a straight line and a quadratic curve.

[4 marks]

(c) (i) $\mathrm{P}(0 \le X \le 5) = 6a + a\int_2^5 -x^2 + 6x - 5\,\mathrm{d}x$ ***(M1)***

$= 15a$ ***(A1)***

$15a = 1$ ***(M1)***

$\Rightarrow a = \frac{1}{15}\left(= 0.0667\right)$ ***A1***

(ii) $\mathrm{E}(X) = \frac{1}{5}\int_0^2 x\,\mathrm{d}x + \frac{1}{15}\int_2^5 -x^3 + 6x^2 - 5x\,\mathrm{d}x$ ***(M1)(A1)***

$= 2.35$ ***A1***

continued…

Question 10 continued

(iii) attempt to use $\int_0^m f(x)\mathrm{d}x = 0.5$ ***(M1)***

$0.4 + a\int_2^m -x^2 + 6x - 5\,\mathrm{d}x = 0.5$ ***(A1)***

$a\int_2^m -x^2 + 6x - 5\,\mathrm{d}x = 0.1$

attempt to solve integral using GDC and/or analytically ***(M1)***

$\frac{1}{15}\left[-\frac{1}{3}x^3 + 3x^2 - 5x\right]_2^m = 0.1$

$m = 2.44$ ***A1***

[11 marks]

Total [19 marks]

11. (a) (i) valid attempt to differentiate implicitly ***(M1)***

$4x = 3\sin^2 y\cos y\frac{\mathrm{d}y}{\mathrm{d}x}$ ***A1A1***

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4x}{3\sin^2 y\cos y}$ ***A1***

(ii) at $\left(\frac{1}{4}, \frac{5\pi}{6}\right)$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4x}{3\sin^2 y\cos y} = \frac{1}{3\left(\frac{1}{2}\right)^2\left(-\frac{\sqrt{3}}{2}\right)}$ ***(M1)***

$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{8}{3\sqrt{3}}\,(= -1.54)$ ***A1***

hence equation of tangent is

$y - \frac{5\pi}{6} = -1.54\left(x - \frac{1}{4}\right)$ **OR** $y = -1.54x + 3.00$ ***(M1)A1***

Note: Accept $y = -1.54x + 3$.

[8 marks]

(b) $x = \sqrt{\frac{1}{2}\sin^3 y}$ ***(M1)***

$\int_0^\pi \sqrt{\frac{1}{2}\sin^3 y}\,\mathrm{d}y$ ***(A1)***

$= 1.24$ ***A1***

[3 marks]

continued…

Question 11 continued

(c) use of volume $= \int \pi x^2 \mathrm{d}y$ ***(M1)***

$= \int_0^{\pi} \frac{1}{2}\pi \sin^3 y \; \mathrm{d}y$ ***A1***

$= \frac{1}{2}\pi \int_0^{\pi} \left(\sin y - \sin y \cos^2 y\right) \mathrm{d}y$

Note: Condone absence of limits up to this point.

reasonable attempt to integrate ***(M1)***

$= \frac{1}{2}\pi \left[-\cos y + \frac{1}{3}\cos^3 y \right]_0^{\pi}$ ***A1A1***

Note: Award ***A1*** for correct limits (not to be awarded if previous ***M1*** has not been awarded) and ***A1*** for correct integrand.

$= \frac{1}{2}\pi\left(1 - \frac{1}{3}\right) - \frac{1}{2}\pi\left(-1 + \frac{1}{3}\right)$ ***A1***

$= \frac{2\pi}{3}$ ***AG***

Note: Do not accept decimal answer equivalent to $\frac{2\pi}{3}$.

[6 marks]

Total [17 marks]