M19/5/MATHL/HP1/ENG/TZ1/XX/M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2019

Mathematics

Higher level

Paper 1

20 pages

Section A

1. $\boldsymbol{a} \bullet \boldsymbol{b} = \begin{pmatrix} 2 \\ k \\ -1 \end{pmatrix} \bullet \begin{pmatrix} -3 \\ k+2 \\ k \end{pmatrix}$

$= -6 + k(k+2) - k$ ***A1***

$\boldsymbol{a} \bullet \boldsymbol{b} = 0$ ***(M1)***

$k^2 + k - 6 = 0$

attempt at solving their quadratic equation ***(M1)***

$(k+3)(k-2) = 0$

$k = -3,\ 2$ ***A1***

Note: Attempt at solving using $|\boldsymbol{a}||\boldsymbol{b}| = |\boldsymbol{a} \times \boldsymbol{b}|$ will be ***M1A0A0A0*** if neither answer found ***M1(A1)A1A0*** for one correct answer and ***M1(A1)A1A1*** for two correct answers.

Total [4 marks]

2. attempt at binomial expansion ***M1***

$1 + \binom{11}{1}(-2x) + \binom{11}{2}(-2x)^2 + \ldots$ ***(A1)***

$\binom{11}{2} = 55$

$1 - 22x + 220x^2$ ***A1A1***

Note: ***A1*** for first two terms, ***A1*** for final term.

Note: Award ***M1(A1)A0A0*** for $(-2x)^{11} + \binom{11}{10}(-2x)^{10} + \binom{11}{9}(-2x)^9 + \ldots,$

Total [4 marks]

3. $A = P$

use of the correct formula for area and arc length ***(M1)***

perimeter is $r\theta + 2r$ ***(A1)***

Note: ***A1*** independent of previous ***M1***.

$\frac{1}{2}r^2(1) = r(1) + 2r$ ***A1***

$r^2 - 6r = 0$

$r = 6$ (as $r > 0$) ***A1***

Note: Do not award final ***A1*** if $r = 0$ is included.

Total [4 marks]

4. (a) **EITHER**

$$\frac{5\sqrt{15}}{2} = \frac{1}{2} \times 4 \times 5\sin\theta$$ ***A1***

OR

height of triangle is $\frac{5\sqrt{15}}{4}$ if using 4 as the base or $\sqrt{15}$ if using 5 as the base ***A1***

THEN

$$\sin\theta = \frac{\sqrt{15}}{4}$$ ***AG***

[1 mark]

(b) let the third side be x

$x^2 = 4^2 + 5^2 - 2 \times 4 \times 5 \times \cos\theta$ ***M1***

valid attempt to find $\cos\theta$ ***(M1)***

Note: Do not accept writing $\cos\left(\arcsin\left(\frac{\sqrt{15}}{4}\right)\right)$ as a valid method.

$$\cos\theta = \pm\sqrt{1 - \frac{15}{16}}$$

$$= \frac{1}{4}, -\frac{1}{4}$$ ***A1A1***

$$x^2 = 16 + 25 - 2 \times 4 \times 5 \times \pm\frac{1}{4}$$

$x = \sqrt{31}$ or $\sqrt{51}$ ***A1A1***

[6 marks]

Total [7 marks]

5. let $\text{OX} = x$

METHOD 1

$\frac{dx}{dt} = 24$ (or -24) ***(A1)***

$\frac{d\theta}{dt} = \frac{dx}{dt} \times \frac{d\theta}{dx}$ ***(M1)***

$3\tan\theta = x$ ***A1***

EITHER

$3\sec^2\theta = \frac{dx}{d\theta}$ ***A1***

$\frac{d\theta}{dt} = \frac{24}{3\sec^2\theta}$

attempt to substitute for $\theta = 0$ into their differential equation ***M1***

OR

$\theta = \arctan\left(\frac{x}{3}\right)$

$\frac{d\theta}{dx} = \frac{1}{3} \times \frac{1}{1 + \frac{x^2}{9}}$ ***A1***

$\frac{d\theta}{dt} = 24 \times \frac{1}{3\left(1 + \frac{x^2}{9}\right)}$

attempt to substitute for $x = 0$ into their differential equation ***M1***

THEN

$\frac{d\theta}{dt} = \frac{24}{3} = 8$ (rad s^{-1}) ***A1***

Note: Accept -8 rad s^{-1}.

continued…

Question 5 continued

METHOD 2

$\frac{\mathrm{d}x}{\mathrm{d}t}=24$ (or -24) ***(A1)***

$3\tan\theta = x$ ***A1***

attempt to differentiate implicitly with respect to t ***M1***

$3\sec^2\theta \times \frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}t}$ ***A1***

$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{24}{3\sec^2\theta}$

attempt to substitute for $\theta = 0$ into their differential equation ***M1***

$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{24}{3} = 8$ (rad s^{-1}) ***A1***

Note: Accept $-8\,\text{rad}\,\text{s}^{-1}$.

Note: Can be done by consideration of CX, use of Pythagoras.

METHOD 3

let the position of the car be at time t be $d-24t$ from O ***(A1)***

$\tan\theta = \frac{d-24t}{3}\left(=\frac{d}{3}-8t\right)$ ***M1***

Note: For $\tan\theta = \frac{24t}{3}$ award ***A0M1*** and follow through.

EITHER

attempt to differentiate implicitly with respect to t ***M1***

$\sec^2\theta\frac{\mathrm{d}\theta}{\mathrm{d}t} = -8$ ***A1***

attempt to substitute for $\theta = 0$ into their differential equation ***M1***

OR

$\theta = \arctan\left(\frac{d}{3}-8t\right)$ ***M1***

$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -\frac{8}{1+\left(\frac{d}{3}-8t\right)^2}$ ***A1***

at O, $t = \frac{d}{24}$ ***A1***

continued…

Question 5 continued

THEN

$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -8$ ***A1***

Total [6 marks]

6. (a) use of symmetry *eg* diagram ***(M1)***

$\mathrm{P}(X > \mu + 5) = 0.2$ ***A1***

[2 marks]

(b) **EITHER**

$\mathrm{P}(X < \mu + 5 \mid X > \mu - 5) = \frac{\mathrm{P}(X > \mu - 5 \cap X < \mu + 5)}{\mathrm{P}(X > \mu - 5)}$ ***(M1)***

$= \frac{\mathrm{P}(\mu - 5 < X < \mu + 5)}{\mathrm{P}(X > \mu - 5)}$ ***(A1)***

$= \frac{0.6}{0.8}$ ***A1A1***

Note: ***A1*** for denominator is independent of the previous ***A*** marks.

OR

use of diagram ***(M1)***

Note: Only award ***(M1)*** if the region $\mu - 5 < X < \mu + 5$ is indicated and used.

continued…

Question 6 continued

$\mathrm{P}(X > \mu - 5) = 0.8 \quad \mathrm{P}(\mu - 5 < X < \mu + 5) = 0.6$ ***(A1)***

Note: Probabilities can be shown on the diagram.

$= \dfrac{0.6}{0.8}$ ***M1A1***

THEN

$= \dfrac{3}{4} \ = (0.75)$ ***A1***

[5 marks]

Total [7 marks]

7. attempt at implicit differentiation ***M1***

$3y^2 \dfrac{\mathrm{d}y}{\mathrm{d}x} + 3y^2 + 6xy \dfrac{\mathrm{d}y}{\mathrm{d}x} - 3x^2 = 0$ ***A1A1***

Note: Award ***A1*** for the second & third terms, ***A1*** for the first term, fourth term & RHS equal to zero.

substitution of $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ ***M1***

$3y^2 - 3x^2 = 0$

$\Rightarrow y = \pm x$ ***A1***

substitute either variable into original equation ***M1***

$y = x \Rightarrow x^3 = 9 \Rightarrow x = \sqrt[3]{9}$ (or $y^3 = 9 \Rightarrow y = \sqrt[3]{9}$) ***A1***

$y = -x \Rightarrow x^3 = 27 \Rightarrow x = 3$ (or $y^3 = -27 \Rightarrow y = -3$) ***A1***

$\left(\sqrt[3]{9}, \sqrt[3]{9}\right)$, $(3, -3)$ ***A1***

Total [9 marks]

8. (a) 3 ***A1***

[1 mark]

(b) attempt to use definite integral of $f'(x)$ ***(M1)***

$\int_0^1 f'(x)\mathrm{d}x = 0.5$

$f(1) - f(0) = 0.5$ ***(A1)***

$f(1) = 0.5 + 3$

$= 3.5$ ***A1***

[3 marks]

continued…

Question 8 continued

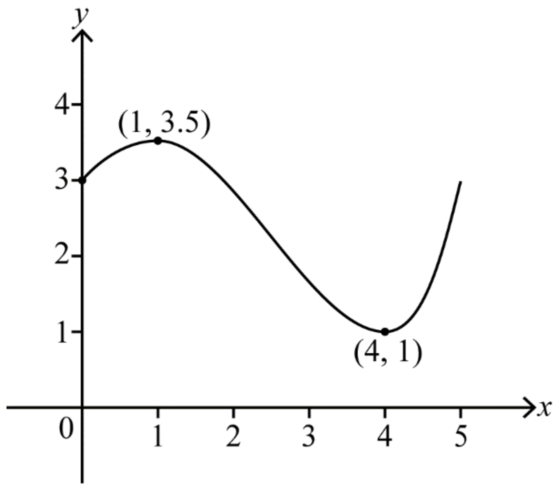
(c) $\int_1^4 f'(x)\mathrm{d}x = -2.5$ ***(A1)***

Note: ***(A1)*** is for -2.5.

$f(4) - f(1) = -2.5$
$f(4) = 3.5 - 2.5$
$= 1$ ***A1***

[2 marks]

(d)



A1A1A1

A1 for correct shape over approximately the correct domain

A1 for maximum and minimum (coordinates or horizontal lines from 3.5 and 1 are required),
A1 for y-intercept at 3

[3 marks]
Total [9 marks]

Section B

9. (a) $(\sin x+\cos x)^2=\sin^2 x+2\sin x\cos x+\cos^2 x$ ***M1A1***

Note: Do not award the ***M1*** for just $\sin^2 x+\cos^2 x$.

Note: Do not award ***A1*** if correct expression is followed by incorrect working.

$=1+\sin 2x$ ***AG***

[2 marks]

(b) $\sec 2x+\tan 2x=\frac{1}{\cos 2x}+\frac{\sin 2x}{\cos 2x}$ ***M1***

Note: ***M1*** is for an attempt to change both terms into sine and cosine forms (with the same argument) or both terms into functions of $\tan x$.

$=\frac{1+\sin 2x}{\cos 2x}$

$=\frac{(\sin x+\cos x)^2}{\cos^2 x-\sin^2 x}$ ***A1A1***

Note: Award ***A1*** for numerator, ***A1*** for denominator.

$=\frac{(\sin x+\cos x)^2}{(\cos x-\sin x)(\cos x+\sin x)}$ ***M1***

$=\frac{\cos x+\sin x}{\cos x-\sin x}$ ***AG***

Note: Apply MS in reverse if candidates have worked from RHS to LHS.

Note: Alternative method using $\tan 2x$ and $\sec 2x$ in terms of $\tan x$.

[4 marks]

(c) **METHOD 1**

$\int_0^{\frac{\pi}{6}}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)\mathrm{d}x$ ***A1***

Note: Award ***A1*** for correct expression with or without limits.

EITHER

$=\left[-\ln(\cos x-\sin x)\right]_0^{\frac{\pi}{6}}$ or $\left[\ln(\cos x-\sin x)\right]_{\frac{\pi}{6}}^{0}$ ***(M1)A1A1***

Note: Award ***M1*** for integration by inspection or substitution, ***A1*** for $\ln(\cos x-\sin x)$, ***A1*** for completely correct expression including limits.

$=-\ln\left(\cos\frac{\pi}{6}-\sin\frac{\pi}{6}\right)+\ln(\cos 0-\sin 0)$ ***M1***

Note: Award ***M1*** for substitution of limits into their integral and subtraction.

$=-\ln\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)$ ***(A1)***

continued…

Question 9 continued

OR

let $u = \cos x - \sin x$ ***M1***

$\frac{\mathrm{d}u}{\mathrm{d}x} = -\sin x - \cos x = -(\sin x + \cos x)$

$-\int_1^{\frac{\sqrt{3}}{2}-\frac{1}{2}} \left(\frac{1}{u}\right)\mathrm{d}u$ ***A1A1***

Note: Award ***A1*** for correct limits even if seen later, ***A1*** for integral.

$= \left[-\ln u\right]_1^{\frac{\sqrt{3}}{2}-\frac{1}{2}}$ or $\left[\ln u\right]_{\frac{\sqrt{3}}{2}-\frac{1}{2}}^{1}$ ***A1***

$= -\ln\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)(+\ln 1)$ ***M1***

THEN

$= \ln\left(\frac{2}{\sqrt{3}-1}\right)$ ***M1***

Note: Award ***M1*** for both putting the expression over a common denominator and for correct use of law of logarithms.

$= \ln\left(1+\sqrt{3}\right)$ ***(M1)A1***

[9 marks]

METHOD 2

$\left[\frac{1}{2}\ln\left(\tan 2x + \sec 2x\right) - \frac{1}{2}\ln\left(\cos 2x\right)\right]_0^{\frac{\pi}{6}}$ ***A1A1***

$= \frac{1}{2}\ln\left(\sqrt{3}+2\right) - \frac{1}{2}\ln\left(\frac{1}{2}\right) - 0$ ***A1A1(A1)***

$= \frac{1}{2}\ln\left(4+2\sqrt{3}\right)$ ***M1***

$= \frac{1}{2}\ln\left(\left(1+\sqrt{3}\right)^2\right)$ ***M1A1***

$= \ln\left(1+\sqrt{3}\right)$ ***A1***

[9 marks]

Total [15 marks]

10. (a) (i) $p(2) = 8 - 12 + 16 - 24$ ***(M1)***

Note: Award ***M1*** for a valid attempt at remainder theorem or polynomial division.

$= -12$ ***A1***
remainder $= -12$

(ii) $p(3) = 27 - 27 + 24 - 24 = 0$ ***A1***
remainder $= 0$

[3 marks]

(b) $x = 3$ (is a zero) ***A1***

Note: Can be seen anywhere.

EITHER

factorise to get $(x-3)\left(x^2+8\right)$ ***(M1)A1***

$x^2 + 8 \neq 0$ (for $x \in \mathbb{R}$) (or equivalent statement) ***R1***

Note: Award ***R1*** if correct two complex roots are given.

OR

$p'(x) = 3x^2 - 6x + 8$ ***A1***
attempting to show $p'(x) \neq 0$ ***M1***
eg discriminant $= 36 - 96 < 0$, completing the square
no turning points ***R1***

THEN

only one real zero (as the curve is continuous) ***AG***

[4 marks]

(c) new graph is $y = p(2x)$ ***(M1)***
stretch parallel to the x-axis (with $x = 0$ invariant), scale factor 0.5 ***A1***

[2 marks]

Note: Accept "horizontal" instead of "parallel to the x-axis".

continued…

Question 10 continued

(d) $\frac{6\lambda^3 e^{-\lambda}}{6} = \frac{3\lambda^2 e^{-\lambda}}{2} - 2\lambda e^{-\lambda} + 3e^{-\lambda}$ ***M1A1***

Note: Allow factorials in the denominator for ***A1***.

$2\lambda^3 - 3\lambda^2 + 4\lambda - 6 = 0$ ***A1***

Note: Accept any correct cubic equation without factorials and $e^{-\lambda}$.

EITHER

$4\left(2\lambda^3 - 3\lambda^2 + 4\lambda - 6\right) = 8\lambda^3 - 12\lambda^2 + 16\lambda - 24 = 0$ ***(M1)***

$2\lambda = 3$ ***(A1)***

OR

$(2\lambda - 3)\left(\lambda^2 + 2\right) = 0$ ***(M1)(A1)***

THEN

$\lambda = 1.5$ ***A1***

[6 marks]

Total [15 marks]

11. (a) (i) appreciation that two points distinct from P need to be chosen from each line ***M1***

$^4C_2 \times {}^3C_2$

$= 18$ ***A1***

(ii) **EITHER**

consider cases for triangles including P **or** triangles not including P ***M1***

$3 \times 4 + 4 \times {}^3C_2 + 3 \times {}^4C_2$ ***(A1)(A1)***

Note: Award ***A1*** for 1st term, ***A1*** for 2nd & 3rd term.

OR

consider total number of ways to select 3 points and subtract those with 3 points on the same line ***M1***

$^8C_3 - {}^5C_3 - {}^4C_3$ ***(A1)(A1)***

Note: Award ***A1*** for 1st term, ***A1*** for 2nd & 3rd term.

$56 - 10 - 4$

THEN

$= 42$ ***A1***

[6 marks]

(b) **METHOD 1**

substitution of $(4,\ 6,\ 4)$ into both equations ***(M1)***

$\lambda = 3$ and $\mu = 1$ ***A1A1***

$(4,\ 6,\ 4)$ ***AG***

METHOD 2

attempting to solve two of the three parametric equations ***M1***

$\lambda = 3$ or $\mu = 1$ ***A1***

check both of the above give $(4,\ 6,\ 4)$ ***M1AG***

Note: If they have shown the curve intersects for all three coordinates they only need to check (4,6,4) with one of "λ" or "μ".

[3 marks]

(c) $\lambda = 2$ ***A1***

[1 mark]

(d) $\overrightarrow{PA} = \begin{pmatrix} -1 \\ -2 \\ -1 \end{pmatrix},\ \overrightarrow{PB} = \begin{pmatrix} -5 \\ -6 \\ -2 \end{pmatrix}$ ***A1A1***

Note: Award ***A1A0*** if both are given as coordinates.

[2 marks]

continued…

Question 11 continued

(e) **METHOD 1**

area triangle $\text{ABP} = \frac{1}{2}\left|\overrightarrow{\text{PB}} \times \overrightarrow{\text{PA}}\right|$ ***M1***

$$\left(= \frac{1}{2}\left|\begin{pmatrix} -5 \\ -6 \\ -2 \end{pmatrix} \times \begin{pmatrix} -1 \\ -2 \\ -1 \end{pmatrix}\right|\right) = \frac{1}{2}\left|\begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}\right|$$ ***A1***

$= \frac{\sqrt{29}}{2}$ ***A1***

EITHER

$\overrightarrow{\text{PC}} = 3\overrightarrow{\text{PA}},\ \overrightarrow{\text{PD}} = 3\overrightarrow{\text{PB}}$ ***(M1)***

area triangle $\text{PCD} = 9 \times$ area triangle ABP ***(M1)A1***

$= \frac{9\sqrt{29}}{2}$ ***A1***

OR

D has coordinates $(-11, -12, -2)$ ***A1***

area triangle $\text{PCD} = \frac{1}{2}\left|\overrightarrow{\text{PD}} \times \overrightarrow{\text{PC}}\right| = \frac{1}{2}\left|\begin{pmatrix} -15 \\ -18 \\ -6 \end{pmatrix} \times \begin{pmatrix} -3 \\ -6 \\ -3 \end{pmatrix}\right|$ ***M1A1***

Note: A1 is for the correct vectors in the correct formula.

$= \frac{9\sqrt{29}}{2}$ ***A1***

THEN

area of $\text{CDBA} = \frac{9\sqrt{29}}{2} - \frac{\sqrt{29}}{2}$

$= 4\sqrt{29}$ ***A1***

[8 marks]

continued…

Question 11 continued

METHOD 2

D has coordinates $(-11, -12, -2)$ ***A1***

area $= \frac{1}{2}\left|\vec{CB} \times \vec{CA}\right| + \frac{1}{2}\left|\vec{BC} \times \vec{BD}\right|$ ***M1***

Note: Award ***M1*** for use of correct formula on appropriate non-overlapping triangles.

Note: Different triangles or vectors could be used.

$\vec{CB} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}, \ \vec{CA} = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$ ***A1***

$\vec{CB} \times \vec{CA} = \begin{pmatrix} -4 \\ 6 \\ -8 \end{pmatrix}$ ***A1***

$\vec{BC} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}, \ \vec{BD} = \begin{pmatrix} -10 \\ -12 \\ -4 \end{pmatrix}$ ***A1***

$\vec{BC} \times \vec{BD} = \begin{pmatrix} -12 \\ 18 \\ -24 \end{pmatrix}$ ***A1***

Note: Other vectors which might be used are $\vec{DA} = \begin{pmatrix} 14 \\ 16 \\ 5 \end{pmatrix}, \ \vec{BA} = \begin{pmatrix} 4 \\ 4 \\ 1 \end{pmatrix}, \vec{DC} = \begin{pmatrix} 12 \\ 12 \\ 3 \end{pmatrix}$.

Note: Previous ***A1A1A1A1*** are all dependent on the first ***M1***.

valid attempt to find a value of $\frac{1}{2}\left|a \times b\right|$ ***M1***

Note: ***M1*** independent of triangle chosen.

area $= \frac{1}{2} \times 2 \times \sqrt{29} + \frac{1}{2} \times 6 \times \sqrt{29}$

$= 4\sqrt{29}$ ***A1***

Note: Accept $\frac{1}{2}\sqrt{116} + \frac{1}{2}\sqrt{1044}$ or equivalent.

[8 marks]

Total [20 marks]