Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Monday 13 May 2019 (afternoon)

2 hours

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

13 pages

2219 – 7205

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

In an arithmetic sequence, the sum of the 3rd and 8th terms is 1.
Given that the sum of the first seven terms is 35, determine the first term and the common difference.

2. [Maximum mark: 6]

Three points in three-dimensional space have coordinates $\mathrm{A}(0, 0, 2)$, $\mathrm{B}(0, 2, 0)$ and $\mathrm{C}(3, 1, 0)$.

(a) Find the vector

(i) $\overrightarrow{\mathrm{AB}}$;

(ii) $\overrightarrow{\mathrm{AC}}$. [2]

(b) Hence or otherwise, find the area of the triangle ABC. [4]

3. [Maximum mark: 5]

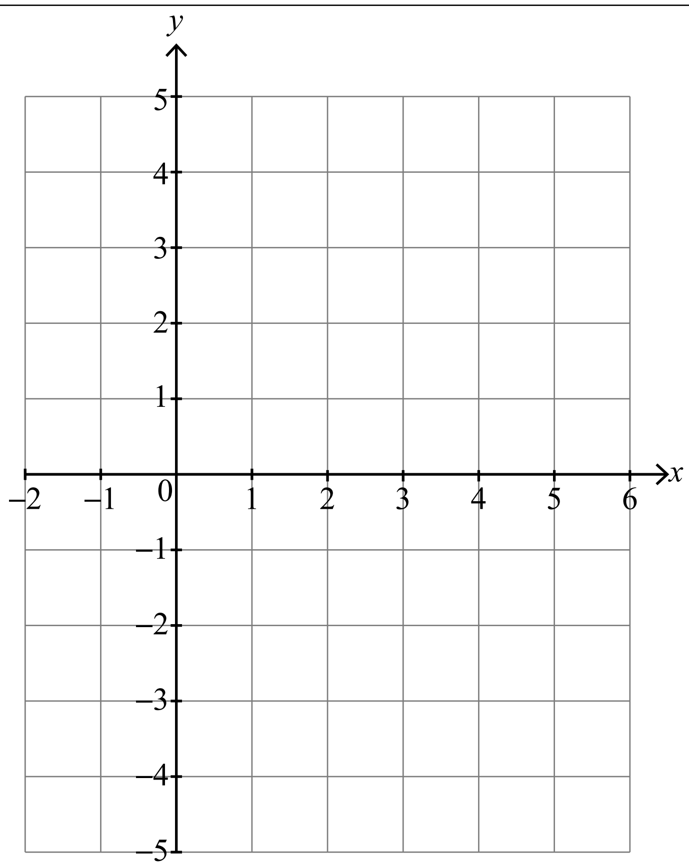
Consider the function $f(x) = x^4 - 6x^2 - 2x + 4$, $x \in \mathbb{R}$.
The graph of f is translated two units to the left to form the function $g(x)$.
Express $g(x)$ in the form $ax^4 + bx^3 + cx^2 + dx + e$ where a, b, c, d, $e \in \mathbb{Z}$.

4. [Maximum mark: 5]

Using the substitution $u = \sin x$, find $\int \frac{\cos^3 x \, dx}{\sqrt{\sin x}}$.

5. [Maximum mark: 8]

(a) Sketch the graph of $y = \dfrac{x-4}{2x-5}$, stating the equations of any asymptotes and the coordinates of any points of intersection with the axes. [5]



(b) Consider the function $f : x \to \sqrt{\dfrac{x-4}{2x-5}}$.

Write down

(i) the largest possible domain of f;

(ii) the corresponding range of f. [3]

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6. [Maximum mark: 7]

The curve C is given by the equation $y = x\tan\left(\frac{\pi xy}{4}\right)$.

(a) At the point $(1, 1)$, show that $\frac{dy}{dx} = \frac{2+\pi}{2-\pi}$. [5]

(b) Hence find the equation of the normal to C at the point $(1, 1)$. [2]

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Answers written on this page
will not be marked.

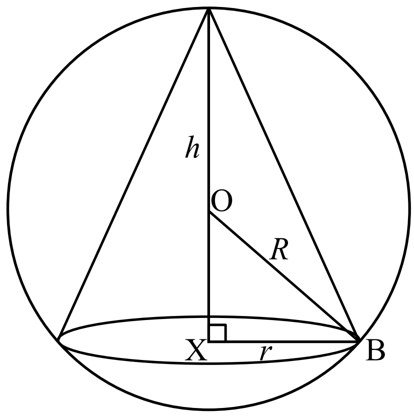
7. [Maximum mark: 7]

Solve the simultaneous equations

$$\log_2 6x = 1 + 2\log_2 y$$
$$1 + \log_6 x = \log_6(15y - 25).$$

8. [Maximum mark: 8]

A right circular cone of radius r is inscribed in a sphere with centre O and radius R as shown in the following diagram. The perpendicular height of the cone is h, X denotes the centre of its base and B a point where the cone touches the sphere.



(a) Show that the volume of the cone may be expressed by $V = \frac{\pi}{3}\left(2Rh^2 - h^3\right)$. [4]

(b) Given that there is one inscribed cone having a maximum volume, show that the volume of this cone is $\frac{32\pi R^3}{81}$. [4]

(This question continues on the following page)

(Question 8 continued)

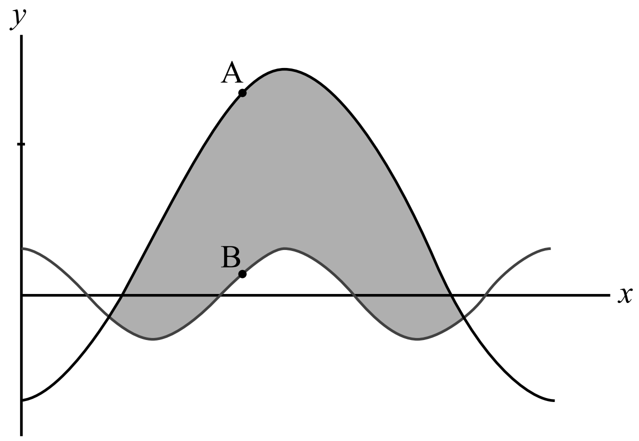
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 17]

Consider the functions f and g defined on the domain $0 < x < 2\pi$ by $f(x) = 3\cos 2x$ and $g(x) = 4 - 11\cos x$.
The following diagram shows the graphs of $y = f(x)$ and $y = g(x)$.



(a) Find the x-coordinates of the points of intersection of the two graphs. [6]

(b) Find the exact area of the shaded region, giving your answer in the form $p\pi + q\sqrt{3}$, where $p, q \in \mathbb{Q}$. [5]

At the points A and B on the diagram, the gradients of the two graphs are equal.

(c) Determine the y-coordinate of A on the graph of g. [6]

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10. [Maximum mark: 16]

The random variable X has probability density function f given by

$$f(x)=\begin{cases} k(\pi-\arcsin x) & 0\le x\le 1 \\ 0 & \text{otherwise}\end{cases}$$, where k is a positive constant.

(a) State the mode of X. [1]

(b) (i) Find $\int \arcsin x \, \mathrm{d}x$.

(ii) Hence show that $k=\frac{2}{2+\pi}$. [6]

(c) Given that $y=\left(\frac{x^2}{2}\right)\arcsin x-\left(\frac{1}{4}\right)\arcsin x+\left(\frac{x}{4}\right)\sqrt{1-x^2}$, show that

(i) $\frac{\mathrm{d}y}{\mathrm{d}x}=x\arcsin x$;

(ii) $\mathrm{E}(X)=\frac{3\pi}{4(\pi+2)}$. [9]

11. [Maximum mark: 17]

Consider the functions f and g defined by $f(x)=\ln|x|$, $x\in\mathbb{R}\setminus\{0\}$ and $g(x)=\ln|x+k|$, $x\in\mathbb{R}\setminus\{-k\}$, where $k\in\mathbb{R}$, $k>2$.

(a) Describe the transformation by which $f(x)$ is transformed to $g(x)$. [1]

(b) State the range of g. [1]

(c) Sketch the graphs of $y=f(x)$ and $y=g(x)$ on the same axes, clearly stating the points of intersection with any axes. [6]

The graphs of f and g intersect at the point P.

(d) Find the coordinates of P. [2]

The tangent to $y=f(x)$ at P passes through the origin $(0, 0)$.

(e) Determine the value of k. [7]

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