Markscheme

May 2019

Mathematics

Higher level

Paper 2

16 pages

Section A

1. **METHOD 1**

equation of tangent is $y = 22.167...\,x - 14.778...$ **OR** $y - 7.389... = 22.167...\,(x-1)$ ***(M1)(A1)***

meets the x-axis when $y = 0$

$x = 0.667$

meets x -axis at $(0.667, 0)\left(=\left(\frac{2}{3}, 0\right)\right)$ ***A1A1***

Note: Award ***A1*** for $x = \frac{2}{3}$ or $x = 0.667$ seen and ***A1*** for coordinates $(x, 0)$ given.

METHOD 2

Attempt to differentiate (***M1)***

$\frac{dy}{dx} = e^{2x} + 2xe^{2x}$

when $x = 1,\ \frac{dy}{dx} = 3e^2$ (***M1)***

equation of the tangent is $y - e^2 = 3e^2(x-1)$

$y = 3e^2x - 2e^2$

meets x-axis at $x = \frac{2}{3}$

$\left(\frac{2}{3}, 0\right)$ ***A1A1***

Note: Award ***A1*** for $x = \frac{2}{3}$ or $x = 0.667$ seen and ***A1*** for coordinates $(x, 0)$ given.

Total [4 marks]

2. (a) $z = 2e^{\frac{\pi}{4}i}\left(= 2e^{0.785i}\right)$ ***A1***

Note: Accept all answers in the form $2e^{\left(\frac{\pi}{4}+2\pi n\right)i}$.

$z = 2e^{\frac{5\pi}{4}i}\left(= 2e^{3.93i}\right)$ **OR** $z = 2e^{-\frac{3\pi}{4}i}\left(= 2e^{-2.36i}\right)$ ***(M1)A1***

Note: Accept all answers in the form $2e^{\left(-\frac{3\pi}{4}+2\pi n\right)i}$.

Note: Award ***M1A0*** for correct answers in the incorrect form, *eg* $-2e^{\frac{\pi}{4}i}$.

[3 marks]

continued…

Question 2 continued

(b) $z = 1.41 + 1.41\mathrm{i}$, $z = -1.41 - 1.41\mathrm{i}$ ***A1A1***
$\left(z = \sqrt{2} + \sqrt{2}\mathrm{i}, z = -\sqrt{2} - \sqrt{2}\mathrm{i}\right)$

[2 marks]

Total [5 marks]

3. (a) (i) 6.75 ***A1***

(ii) 2.22 ***A1***

[2 marks]

(b) (i) 8.75 ***A1***

(ii) 2.22 ***A1***

[2 marks]

(c) the order is $3, 4, 6, 7, 7, 8, 9, 10$
median is currently 7 ***A1***

Note: This can be indicated by a diagram/list, rather than actually stated.

with 9 numbers the middle value (median) will be the 5th value ***R1***
which will correspond to 7 regardless of whether the position of the median moves up or down ***R1***

Note: Accept answers using data $5, 6, 8, 9, 9, 10, 11, 12$ (ie from part (b)).

[3 marks]

Total [7 marks]

4. (a) $f(x) \geq 3$ ***A1***

[1 mark]

(b) $x = \sec y + 2$ ***(M1)***

Note: Exchange of variables can take place at any point.

$\cos y = \frac{1}{x-2}$ ***(A1)***

$f^{-1}(x) = \arccos\left(\frac{1}{x-2}\right)$, $x \geq 3$ ***A1A1***

Note: Allow follow through from (a) for last ***A1*** mark which is independent of earlier marks in (b).

[4 marks]

Total [5 marks]

5. **METHOD 1**

write as $\int 1 \times (\ln x)^2 \mathrm{d}x$ ***(M1)***

$= x(\ln x)^2 - \int x \times \frac{2(\ln x)}{x} \mathrm{d}x \left(= x(\ln x)^2 - \int 2\ln x\right)$ ***M1A1***

$= x(\ln x)^2 - 2x\ln x + \int 2\mathrm{d}x$ ***(M1)(A1)***

$= x(\ln x)^2 - 2x\ln x + 2x + c$ ***A1***

METHOD 2

let $u = \ln x$ ***M1***

$\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x}$

$\int u^2 \mathrm{e}^u \mathrm{d}u$ ***A1***

$= u^2\mathrm{e}^u - \int 2u\mathrm{e}^u \mathrm{d}u$ ***M1***

$= u^2\mathrm{e}^u - 2u\mathrm{e}^u + \int 2\mathrm{e}^u \mathrm{d}u$ ***A1***

$= u^2\mathrm{e}^u - 2u\mathrm{e}^u + 2\mathrm{e}^u + c$

$= x(\ln x)^2 - 2x\ln x + 2x + c$ ***M1A1***

METHOD 3

Setting up $u = \ln x$ and $\frac{\mathrm{d}v}{\mathrm{d}x} = \ln x$ ***M1***

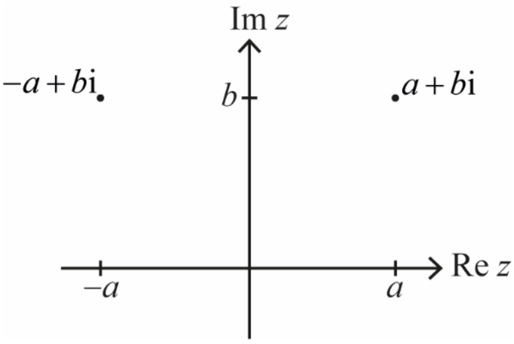
$\ln x(x\ln x - x) - \int (\ln x - 1)\mathrm{d}x$ ***M1A1***

$= x(\ln x)^2 - x\ln x - (x\ln x - x) + x + c$ ***M1A1***

$= x(\ln x)^2 - 2x\ln x + 2x + c$ ***A1***

Total [6 marks]

6. (a)



A1

Note: Award ***A1*** for z in first quadrant and z-2a its reflection in the y-axis.

[1 mark]

continued…

Question 6 continued

(b) (i) $\pi - \theta$ (or any equivalent) ***A1***

(ii) $\arg\left(\frac{z}{z-2a}\right) = \arg(z) - \arg(z-2a)$ ***(M1)***

$= 2\theta - \pi$ (or any equivalent) ***A1***

[3 marks]

(c) **METHOD 1**

if $\text{Re}\left(\frac{z}{z-2a}\right) = 0$ then $2\theta - \pi = \frac{n\pi}{2}$, ($n$ odd) ***(M1)***

$-\pi < 2\theta - \pi < 0 \Rightarrow n = -1$

$2\theta - \pi = -\frac{\pi}{2}$ ***(A1)***

$\theta = \frac{\pi}{4}$ ***A1***

METHOD 2

$\frac{a+b\text{i}}{-a+b\text{i}} = \frac{b^2 - a^2 - 2ab\text{i}}{a^2 + b^2}$ ***M1***

$\text{Re}\left(\frac{z}{z-2a}\right) = 0 \Rightarrow b^2 - a^2 = 0$

$b = a$ ***A1***

$\theta = \frac{\pi}{4}$ ***A1***

Note: Accept any equivalent, *eg* $\theta = -\frac{7\pi}{4}$.

[3 marks]

Total [7 marks]

7. volume $= \pi\int_0^9 \left(y^{\frac{1}{2}} + 1\right)^2 \text{d}y - \pi\int_1^9 (y-1)\,\text{d}y$ ***(M1)(M1)(M1)(A1)(A1)***

Note: Award ***(M1)*** for use of formula for rotating about y-axis, ***(M1)*** for finding at least one inverse, ***(M1)*** for subtracting volumes, ***(A1)(A1)*** for each correct expression, including limits.

$= 268.6\ldots - 100.5\ldots (85.5\pi - 32\pi)$

$= 168 (= 53.5\pi)$ ***A2***

Total [7 marks]

8. (a) $x < -0.414,\ x > 2.41$ ***A1A1***

$\left(x < 1-\sqrt{2},\ x > 1+\sqrt{2}\right)$

Note: Award ***A1*** for -0.414, 2.41 and ***A1*** for correct inequalities.

[2 marks]

(b) check for $n = 3$,

$16 > 9$ so true when $n = 3$ ***A1***

assume true for $n = k$

$2^{k+1} > k^2$ ***M1***

Note: Award ***M0*** for statements such as "let $n = k$".

Note: Subsequent marks after this ***M1*** are independent of this mark and can be awarded.

prove true for $n = k+1$

$2^{k+2} = 2 \times 2^{k+1}$

$> 2k^2$ ***M1***

$= k^2 + k^2$ ***(M1)***

$> k^2 + 2k + 1$ (from part (a)) ***A1***

which is true for $k \geq 3$ ***R1***

Note: Only award the **A1** or the **R1** if it is clear why. Alternate methods are possible.

$= (k+1)^2$

hence if true for $n = k$ true for $n = k+1$, true for $n = 3$ so true for all $n \geq 3$ ***R1***

Note: Only award the final ***R1*** provided at least three of the previous marks are awarded.

[7 marks]

Total [9 marks]

Section B

9. (a) (i) use of formula or Venn diagram ***(M1)***

$0.72 + 0.45 - 1$ ***(A1)***

$= 0.17$ ***A1***

(ii) $0.72 - 0.17 = 0.55$ ***A1***

[4 marks]

(b) (i) $200 \times 0.45 = 90$ ***A1***

(ii) let X be the number of customers who order cake

$X \sim \mathrm{B}(200, 0.45)$ ***(M1)***

$\mathrm{P}(X > 100) = \mathrm{P}(X \geq 101)\left(= 1 - \mathrm{P}(X \leq 100)\right)$ ***(M1)***

$= 0.0681$ ***A1***

[4 marks]

(c) (i) $0.46 \times 0.8 = 0.368$ ***A1***

(ii) **METHOD 1**

$0.368 + 0.54 \times \mathrm{P}(S \mid F) = 0.72$ ***M1A1A1***

Note: Award ***M1*** for an appropriate tree diagram. Award ***A1*** for LHS, ***A1*** for RHS.

$\mathrm{P}(S \mid F) = 0.652$ ***A1***

METHOD 2

$\mathrm{P}(S \mid F) = \dfrac{\mathrm{P}(S \cap F)}{\mathrm{P}(F)}$ ***(M1)***

$= \dfrac{0.72 - 0.368}{0.54}$ ***A1A1***

Note: Award ***A1*** for numerator, ***A1*** for denominator.

$\mathrm{P}(S \mid F) = 0.652$ ***A1***

[5 marks]

Total [13 marks]

10. (a) $3, -3$ ***A1A1***

[2 marks]

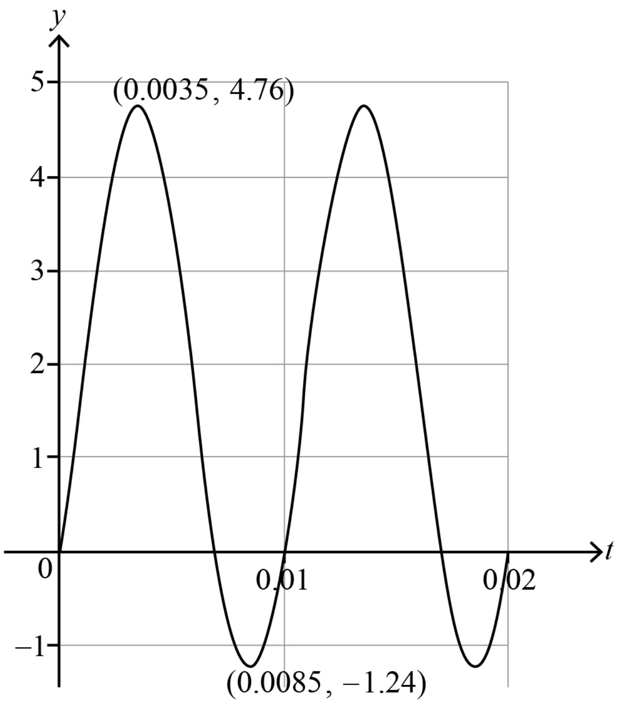
(b) stretch parallel to the y-axis (with x-axis invariant), scale factor $\frac{2}{3}$ ***A1***

translation of $\begin{pmatrix} -0.003 \\ 0 \end{pmatrix}$ (shift to the left by 0.003) ***A1***

Note: Can be done in either order.

[2 marks]

(c)



correct shape over correct domain with correct endpoints ***A1***
first maximum at $(0.0035, 4.76)$ ***A1***
first minimum at $(0.0085, -1.24)$ ***A1***

[3 marks]

(d) $p \geq 3$ between $t = 0.0016762$ and 0.0053238 and $t = 0.011676$ and 0.015324 ***(M1)(A1)***

Note: Award ***M1A1*** for either interval.

$= 0.00730$ ***A1***

[3 marks]

(e) $p_{av} = \frac{1}{0.007}\int_0^{0.007} 6\sin(100\pi t)\sin\left(100\pi(t+0.003)\right)\mathrm{d}t$ ***(M1)***

$= 2.87$ ***A1***

[2 marks]

continued…

Question 10 continued

(f) in each cycle the area under the t axis is smaller than area above the t axis ***R1***
the curve begins with the positive part of the cycle ***R1***

[2 marks]

(g) $a = \frac{4.76 - (-1.24)}{2}$ ***(M1)***

$a = 3.00$ ***A1***

$d = \frac{4.76 + (-1.24)}{2}$

$d = 1.76$ ***A1***

$b = \frac{2\pi}{0.01}$

$b = 628 (= 200\pi)$ ***A1***

$c = 0.0035 - \frac{0.01}{4}$ ***(M1)***

$c = 0.00100$ ***A1***

[6 marks]

Total [20 marks]

11. (a) recognition of the other root $= -d\mathrm{i}$ ***(A1)***

$\log_2 a + \log_2 b + \log_2 c + d\mathrm{i} - d\mathrm{i} = 3$ ***M1A1***

Note: Award ***M1*** for sum of the roots, ***A1*** for 3. Award ***A0M1A0*** for just $\log_2 a + \log_2 b + \log_2 c = 3$.

$\log_2 abc = 3$ ***(M1)***

$\Rightarrow abc = 2^3$ ***A1***

$abc = 8$ ***AG***

[5 marks]

continued…

Question 11 continued

(b) **METHOD 1**

let the geometric series be u_1, $u_1 r$, $u_1 r^2$

$(u_1 r)^3 = 8$ ***M1***

$u_1 r = 2$ ***A1***

hence one of the roots is $\log_2 2 = 1$ ***R1***

METHOD 2

$\frac{b}{a} = \frac{c}{b}$

$b^2 = ac \Rightarrow b^3 = abc = 8$ ***M1***

$b = 2$ ***A1***

hence one of the roots is $\log_2 2 = 1$ ***R1***

[3 marks]

(c) **METHOD 1**

product of the roots is $r_1 \times r_2 \times 1 \times d\text{i} \times -d\text{i} = -8d^2$ ***(M1)(A1)***

$r_1 \times r_2 = -8$ ***A1***

sum of the roots is $r_1 + r_2 + 1 + d\text{i} + -d\text{i} = 3$ ***(M1)(A1)***

$r_1 + r_2 = 2$ ***A1***

solving simultaneously ***(M1)***

$r_1 = -2$, $r_2 = 4$ ***A1A1***

METHOD 2

product of the roots $\log_2 a \times \log_2 b \times \log_2 c \times d\text{i} \times -d\text{i} = -8d^2$ ***M1A1***

$\log_2 a \times \log_2 b \times \log_2 c = -8$ ***A1***

EITHER

a, b, c can be written as $\frac{2}{r}, 2, 2r$ ***M1***

$\left(\log_2 \frac{2}{r}\right)(\log_2 2)(\log_2 2r) = -8$

attempt to solve ***M1***

$(1 - \log_2 r)(1 + \log_2 r) = -8$

$\log_2 r = \pm 3$

$r = \frac{1}{8}, 8$ ***A1A1***

continued…

Question 11 continued

OR

a, b, c can be written as $a, 2, \frac{4}{a}$ ***M1***

$(\log_2 a)(\log_2 2)\left(\log_2 \frac{4}{a}\right) = -8$

attempt to solve ***M1***

$a = \frac{1}{4}, 16$ ***A1A1***

THEN

$a,$ and c are $\frac{1}{4}, 16$ ***(A1)***

roots are $-2,\ 4$ ***A1***

[9 marks]

Total [17 marks]