Markscheme

November 2020

Mathematics

Higher level

Paper 2

22 pages

Section A

1. attempt to find $\hat{AOB}$ by right-angled trigonometry or the cosine rule ***(M1)***

EITHER

$\text{A}\hat{\text{O}}\text{B} = 2\arcsin\left(\frac{5.5}{15}\right)$ ***A1***

OR

$\text{A}\hat{\text{O}}\text{B} = \arccos\left(\frac{15^2 + 15^2 - 11^2}{2\times15\times15}\right)$ ***A1***

THEN

$= 0.750847...\ (= 43.0204...^\circ)$

Note: Award ***(M1)A1*** for correct calculation of $\text{A}\hat{\text{O}}\text{B}$ or $\frac{1}{2}\text{A}\hat{\text{O}}\text{B}$

shaded area = area of sector- area of triangle $\left(= \frac{1}{2}r^2\left(\theta - \sin\theta\right)\right)$ ***(M1)***

$= \frac{1}{2}\times15^2\times\left(0.750847... - \sin 0.750847...\right)$ ***(A1)***

$= 7.72$ (cm²) ***A1***

[5 marks]

2. let X be the random variable "number of books Jenna reads per week."

then $X \sim \text{Po}\left(2.6\right)$

$\text{P}\left(X \geq 4\right) = 0.264\ (0.263998\ldots)$ ***(M1)(A1)***

$0.263998... \times 52$ ***(M1)***

$= 13.7$ ***A1***

Note: Accept 14 weeks.

[4 marks]

3. (a) the principal axis is $\frac{5+(-1)}{2}(=2)$

so $p = 2$ ***A1***

the amplitude is $\frac{5-(-1)}{2}(=3)$

so $q = 3$ ***A1***

EITHER

one period is $2\left(-\frac{3\pi}{4}-\left(-\frac{9\pi}{4}\right)\right)$ ***(M1)***

$=3\pi$

$\Rightarrow \frac{2\pi}{r}=3\pi$

OR

Substituting a point eg $-1=2+\sin\left(-\frac{3\pi}{4}r\right)$

$\sin\left(-\frac{3\pi}{4}r\right)=-1\Rightarrow -\frac{3\pi}{4}r=\ldots-\frac{5\pi}{2},-\frac{\pi}{2},\frac{3\pi}{2},\ldots$

Choice of correct solution $-\frac{3\pi}{4}r=-\frac{\pi}{2}$ ***(M1)***

THEN

$\Rightarrow r=\frac{2}{3}$ ***A1***

$\left(\Rightarrow y=2+3\sin\left(\frac{2x}{3}\right)\right)$

Note: *q* and *r* can be both given as negatives for full marks

[4 marks]

(b) roots are $x = -1.09459\ldots, x = -3.617797\ldots$ ***(A1)***

$$\int_{-3.617797...}^{-1.09459...} \left(2 + 3\sin\left(\frac{2x}{3}\right)\right) dx$$ ***(M1)***

$= -1.66 (= -1.66179...)$ ***(A1)***

so area $= 1.66$ (units²) ***A1***

[4 marks]

Total [8 marks]

4. use of Binomial expansion to find a term in either $\left(\frac{1}{3x^2}-\frac{x}{2}\right)^9$, $\left(\frac{1}{3x^{7/3}}-\frac{x^{2/3}}{2}\right)^9$, $\left(\frac{1}{3}-\frac{x^3}{2}\right)^9$, $\left(\frac{1}{3x^3}-\frac{1}{2}\right)^9$ or $\left(2-3x^3\right)^9$ ***(M1)(A1)***

Note: Award ***M1*** for a product of three terms including a binomial coefficient and powers of the two terms, and ***A1*** for a correct expression of a term in the expansion.

finding the powers required to be 2 and 7 ***(M1)(A1)***

constant term is ${}^9C_2 \times \left(\frac{1}{3}\right)^2 \times \left(-\frac{1}{2}\right)^7$ ***(M1)***

Note: Ignore all x's in student's expression.

therefore term independent of x is $-\frac{1}{32}$ (= – 0.03125) ***A1***

[6 marks]

5. (a) (i) people's holidays are independent of each other ***R1***

the proportion is constant (at 0.15) ***R1***

(ii) $X \sim \mathrm{B}(16, 0.15)$

$\mathrm{P}(X \geq 3) = 0.439$ ***(M1)A1***

[4 marks]

(b) probability of at least one $= 1 -$ probability of none

$\Rightarrow 1 - 0.85^n > 0.999$ **OR** $0.85^n < 0.001$ ***(A1)***

attempt to solve inequality ***(M1)***
$n \geq 42.503...$

so least possible $n = 43$ ***A1***

[3 marks]
Total [7 marks]

6. $n=1$: $\text{LHS}=\frac{\mathrm{d}\left(x\mathrm{e}^{px}\right)}{\mathrm{d}x}=xp\mathrm{e}^{px}+\mathrm{e}^{px}=(px+1)\mathrm{e}^{px}$, $\text{RHS}=p^{0}(px+1)\mathrm{e}^{px}$

$\text{LHS}=\text{RHS}$ so true for $n=1$: ***A1***

Note: Award ***A1*** if $n=0$ is proved.

assume proposition true for $n=k$, i.e. $\frac{\mathrm{d}^{k}}{\mathrm{d}x^{k}}\left(x\mathrm{e}^{px}\right)=p^{k-1}(px+k)\mathrm{e}^{px}$ ***M1***

Notes: Do not award ***M1*** if using *n* instead of *k*. Assumption of truth must be present. Subsequent marks are not dependent on this ***M1*** mark.

$\frac{\mathrm{d}^{k+1}}{\mathrm{d}x^{k+1}}\left(x\mathrm{e}^{px}\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}^{k}}{\mathrm{d}x^{k}}\left(x\mathrm{e}^{px}\right)\right)$ ***(M1)***

$=\frac{\mathrm{d}}{\mathrm{d}x}\left(p^{k-1}(px+k)\mathrm{e}^{px}\right)$ ***M1***

$=p^{k-1}(px+k)p\mathrm{e}^{px}+\mathrm{e}^{px}\left(p^{k}\right)$

$=p^{k}(px+k)\mathrm{e}^{px}+\mathrm{e}^{px}\left(p^{k}\right)$ ***A1***

Note: Award ***A1*** for correct derivative.

$=p^{k}(px+k+1)\mathrm{e}^{px}$ ***A1***

$=p^{((k+1)-1)}\left(px+(k+1)\right)\mathrm{e}^{px}$

Note: The final ***A1*** can be awarded for either of the two lines above.

hence true for $n=1$ and $n=k$ true $\Rightarrow n=k+1$ true ***R1***

therefore true for all $n\in\mathbb{Z}^{+}$

Note: Only award the final ***R1*** if the three method marks have been awarded.

[7 marks]

7. (a) identifying two or three possible cases ***(M1)***

total number of possible groups is $\binom{7}{5}+\binom{7}{4}\binom{5}{1}+\binom{7}{3}\binom{5}{2}$ ***(A1)(A1)***

Note: Award ***A1*** for any two correct cases, ***A1*** for the other one.

$=21+(35\times5)+(35\times10)$

$=546$ ***A1***

[4 marks]

(b) **METHOD 1**

identifying at least two of the three possible cases- Gary goes, Gerwyn goes or neither goes ***(M1)***

total number of possible groups is $\binom{10}{5}+\binom{10}{4}+\binom{10}{4}$ ***(A1)***

$=252+210+210$

$=672$ ***A1***

[3 marks]

METHOD 2

identifying the overall number of groups and no. of cases where both Gary and Gerwyn go. ***(M1)***

total number of possible groups is $\binom{12}{5}-\binom{10}{3}$ ***(A1)***

$=792-120$

$=672$ ***A1***

[3 marks]
Total [7 marks]

8. (a) valid attempt to use chain rule or quotient rule ***(M1)***

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-10\mathrm{e}^{-0.5x}}{\left(3-2\mathrm{e}^{-0.5x}\right)^2} \text{ OR } \frac{\mathrm{d}y}{\mathrm{d}x}=-10\mathrm{e}^{-0.5x}\left(3-2\mathrm{e}^{-0.5x}\right)^{-2}$$ ***A1A1***

[3 marks]

Note: Award ***A1*** for numerator and ***A1*** for denominator, or ***A1*** for each part if the second alternative given.

(b) valid attempt to use chain rule $\left(\text{eg } \frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}y}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}t}\right)$ (***M1)***

$$\frac{\mathrm{d}x}{\mathrm{d}t}=-0.1\div\frac{-10\mathrm{e}^{-2}}{\left(3-2\mathrm{e}^{-2}\right)^2}\ \left(=-0.1\div-0.181676...\right)$$ or equivalent (***A1)***

$$=0.550428...$$

$\frac{\mathrm{d}x}{\mathrm{d}t}=0.550$ (ms^{-1}) ***A1***

[3 marks]
Total [6 marks]

Section B

9. (a) $X \sim \mathrm{N}\left(102, 8^2\right)$

$\mathrm{P}(X < 100) = 0.401$ ***(M1)A1***

[2 marks]

(b) $\mathrm{P}(X > w) = 0.444$ ***(M1)***

$\Rightarrow w = 103$ (g) ***A1***

[2 marks]

(c) $\mathrm{P}(X > 110 \mid X > 105) = \dfrac{\mathrm{P}(X > 110 \cap X > 105)}{\mathrm{P}(X > 105)}$ ***(M1)***

$= \dfrac{\mathrm{P}(X > 110)}{\mathrm{P}(X > 105)}$ ***(A1)***

$= \dfrac{0.15865...}{0.35383...}$

$= 0.448$ ***A1***

[3 marks]

(d) **EITHER**

$\mathrm{P}(90 < X < 114) = 0.866...$ ***(A1)***

OR

$\mathrm{P}(-1.5 < Z < 1.5) = 0.866...$ ***(A1)***

THEN

$0.866... \times 500$ ***(M1)***
$= 433$ ***A1***

[3 marks]

(e) $p = \mathrm{P}(X < 95) = 0.19078...$ ***(A1)***
recognising $Y \sim \mathrm{B}(80, p)$ ***(M1)***

now using $Y \sim \mathrm{B}(80, 0.19078...)$ ***(M1)***

$\mathrm{P}(Y \geq 20) = 0.116$ ***A1***

[4 marks]
Total [14 marks]

10. (a) $3(1-3\lambda)-(2-\lambda)+(-2+4\lambda)=-13$ (***M1***)

$\lambda = 3$ (***A1***)

$\boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} + 3\begin{pmatrix} -3 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} -8 \\ -1 \\ 10 \end{pmatrix}$ (***M1***)

so $\mathrm{P}(-8, -1, 10)$ ***A1***

Note: Do not award the final ***A1*** if a vector given instead of coordinates

[4 marks]

(b) **METHOD 1**

$\boldsymbol{r} = \mu\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$

substituting into equation of the plane ***M1***

$9\mu + \mu + \mu = -13$

$\mu = -\frac{13}{11}(= -1.18...)$ ***A1***

distance $= \frac{13\sqrt{3^2+(-1)^2+1^2}}{11}$ ***(M1)***

$= \frac{13}{\sqrt{11}}\left(= \frac{13\sqrt{11}}{11} = 3.92\right)$ ***A1***

[4 marks]

METHOD 2

choice of any point on the plane, eg $(-8,\,-1,\,10)$ to use in distance formula (***M1***)

so distance $= \dfrac{\begin{pmatrix}-8\\-1\\10\end{pmatrix}\cdot\begin{pmatrix}-3\\1\\-1\end{pmatrix}}{\sqrt{(-3)^2+1^2+(-1)^2}}$ ***A1A1***

Note: Award ***A1*** for numerator, ***A1*** for denominator.

$= \dfrac{24-1-10}{\sqrt{11}}$

$= \dfrac{13}{\sqrt{11}}\left(=\dfrac{13\sqrt{11}}{11}=3.92\right)$ ***A1***

[4 marks]

(c) **EITHER**

identify two vectors ***(A1)***

eg, $\begin{pmatrix}1\\2\\-2\end{pmatrix}$ and $\begin{pmatrix}-3\\-1\\4\end{pmatrix}$

$\boldsymbol{n} = \begin{pmatrix}1\\2\\-2\end{pmatrix}\times\begin{pmatrix}-3\\-1\\4\end{pmatrix}=\begin{pmatrix}6\\2\\5\end{pmatrix}$ (***M1***)

OR

identify three points in the plane ***(A1)***

eg $\lambda = 0,1$ gives $\begin{pmatrix}1\\2\\-2\end{pmatrix}$ and $\begin{pmatrix}-2\\1\\2\end{pmatrix}$

solving system of equations (***M1***)

THEN

$\Pi_2 : \boldsymbol{r}.\begin{pmatrix}6\\2\\5\end{pmatrix} = 0$ ***A1***

> **Note:** Accept $6x + 2y + 5z = 0$.

[3 marks]

(d) vector normal to Π_1 is eg $\boldsymbol{n_1} = \begin{pmatrix}3\\-1\\1\end{pmatrix}$

vector normal to Π_2 is eg $\boldsymbol{n_2} = \begin{pmatrix}6\\2\\5\end{pmatrix}$ ***(A1)***

required angle is θ, where $\cos\theta = \dfrac{\begin{pmatrix}3\\-1\\1\end{pmatrix}.\begin{pmatrix}6\\2\\5\end{pmatrix}}{\sqrt{11}\sqrt{65}}$ ***M1A1***

$\cos\theta = \dfrac{21}{\sqrt{11}\sqrt{65}} = 0.785...$ ***(A1)***

$\theta = 0.667526...$

$\theta = 0.668 \left(= 38.2^\circ\right)$ ***A1***

> **Note:** Award the penultimate ***(A1)*** but not the final ***A1*** for the obtuse angle $2.47406\ldots$ or 142°.

[5 marks]
Total [16 marks]

11. (a) $\frac{\pi}{6}(=0.524)$ ***A1***

$\frac{\pi}{3}(=1.05)$ ***A1***

[2 marks]

(b) attempt to use integration by parts ***M1***

$s=\int \mathrm{e}^{-3t}\sin 6t\ \mathrm{d}t$

EITHER

$=-\frac{\mathrm{e}^{-3t}\sin 6t}{3}-\int -2\mathrm{e}^{-3t}\cos 6t\ \mathrm{d}t$ ***A1***

$=-\frac{\mathrm{e}^{-3t}\sin 6t}{3}-\left(\frac{2\mathrm{e}^{-3t}\cos 6t}{3}-\int -4\mathrm{e}^{-3t}\sin 6t\ \mathrm{d}t\right)$ ***A1***

$=-\frac{\mathrm{e}^{-3t}\sin 6t}{3}-\left(\frac{2\mathrm{e}^{-3t}\cos 6t}{3}+4s\right)$

$5s=\frac{-3\mathrm{e}^{-3t}\sin 6t-6\mathrm{e}^{-3t}\cos 6t}{9}$ ***M1***

OR

$=-\frac{\mathrm{e}^{-3t}\cos 6t}{6}-\int \frac{1}{2}\mathrm{e}^{-3t}\cos 6t\ \mathrm{d}t$ ***A1***

$=-\frac{\mathrm{e}^{-3t}\cos 6t}{6}-\left(\frac{\mathrm{e}^{-3t}\sin 6t}{12}+\int \frac{1}{4}\mathrm{e}^{-3t}\sin 6t\ \mathrm{d}t\right)$ ***A1***

$=-\frac{\mathrm{e}^{-3t}\cos 6t}{6}-\left(\frac{\mathrm{e}^{-3t}\sin 6t}{12}+\frac{1}{4}s\right)$

$\frac{5}{4}s=\frac{-2\mathrm{e}^{-3t}\cos 6t-\mathrm{e}^{-3t}\sin 6t}{12}$ ***M1***

THEN

$s=-\frac{\mathrm{e}^{-3t}\left(\sin 6t+2\cos 6t\right)}{15}(+c)$ ***A1***

at $t=0, s=0\Rightarrow 0=-\frac{2}{15}+c$ ***M1***

$c=\frac{2}{15}$ ***A1***

$$s = \frac{2}{15} - \frac{e^{-3t}\left(\sin 6t + 2\cos 6t\right)}{15}$$

[7 marks]

(c) **EITHER**

substituting $t = \frac{\pi}{6}$ into their equation for s ***(M1)***

$$\left(s = \frac{2}{15} - \frac{e^{-\frac{\pi}{2}}\left(\sin \pi + 2\cos \pi\right)}{15}\right)$$

OR

using GDC to find maximum value ***(M1)***

OR

evaluating $\int_0^{\frac{\pi}{6}} v\,\mathrm{dt}$ ***(M1)***

THEN

$$= 0.161\left(= \frac{2}{15}\left(1 + e^{-\frac{\pi}{2}}\right)\right)$$ ***A1***

[2 marks]

(d) **METHOD 1**

EITHER

distance required $= \int_0^{1.5} \left| e^{-3t} \sin 6t \right| \, dt$ ***(M1)***

OR

distance required $= \int_0^{\frac{\pi}{6}} e^{-3t} \sin 6t \, dt + \left| \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} e^{-3t} \sin 6t \, dt \right| + \int_{\frac{\pi}{3}}^{1.5} e^{-3t} \sin 6t \, dt$ ***(M1)***

$(= 0.16105... + 0.033479... + 0.006806...)$

THEN

$= 0.201$ (m) ***A1***

METHOD 2

using successive minimum and maximum values on the displacement graph ***(M1)***

$0.16105... + (0.16105... - 0.12757...) + (0.13453... - 0.12757...)$

$= 0.201$ (m) ***A1***

[2 marks]

(e) (i) valid attempt to find $\frac{\mathrm{d}v}{\mathrm{d}t}$ using product rule and set $\frac{\mathrm{d}v}{\mathrm{d}t}=0$ ***M1***

$\frac{\mathrm{d}v}{\mathrm{d}t}=\mathrm{e}^{-3t}6\cos 6t-3\mathrm{e}^{-3t}\sin 6t$ ***A1***

$\frac{\mathrm{d}v}{\mathrm{d}t}=0\Rightarrow \tan 6t=2$ ***AG***

(ii) attempt to evaluate t_1, t_2, t_3 in exact form ***M1***

$6t_1=\arctan 2\left(\Rightarrow t_1=\frac{1}{6}\arctan 2\right)$

$6t_2=\pi+\arctan 2\left(\Rightarrow t_2=\frac{\pi}{6}+\frac{1}{6}\arctan 2\right)$

$6t_3=2\pi+\arctan 2\left(\Rightarrow t_3=\frac{\pi}{3}+\frac{1}{6}\arctan 2\right)$ ***A1***

Note: The ***A1*** is for any two consecutive correct, or showing that $6t_2=\pi+6t_1$ or $6t_3=\pi+6t_2$.

showing that $\sin 6t_{n+1}=-\sin 6t_n$

eg $\tan 6t=2\Rightarrow \sin 6t=\pm\frac{2}{\sqrt{5}}$ ***M1A1***

showing that $\frac{\mathrm{e}^{-3t_{n+1}}}{\mathrm{e}^{-3t_n}}=\mathrm{e}^{-\frac{\pi}{2}}$ ***M1***

eg $\mathrm{e}^{-3\left(\frac{\pi}{6}+k\right)}\div \mathrm{e}^{-3k}=\mathrm{e}^{-\frac{\pi}{2}}$

Note: Award the ***A1*** for any two consecutive terms.

$\frac{v_3}{v_2}=\frac{v_2}{v_1}=-\mathrm{e}^{-\frac{\pi}{2}}$ ***AG***

[7 marks]
Total [20 marks]