Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Tuesday 3 November 2020 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

8820 – 7201

14 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

A discrete random variable X has the probability distribution given by the following table.

x	0	1	2	3
$\mathrm{P}(X=x)$	p	$\frac{1}{4}$	$\frac{1}{6}$	q

Given that $\mathrm{E}(X)=\frac{19}{12}$, determine the value of p and the value of q.

2. [Maximum mark: 5]

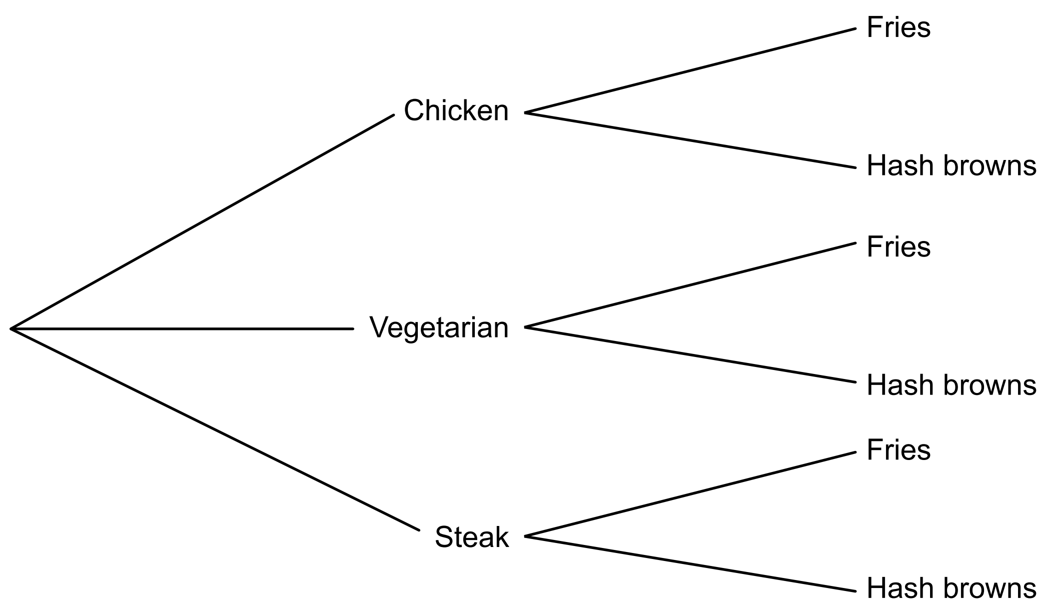
Find the equation of the tangent to the curve $y = \mathrm{e}^{2x} - 3x$ at the point where $x = 0$.

3. [Maximum mark: 7]

At Nusaybah's Breakfast Diner, three types of omelette are available to order: chicken, vegetarian and steak. Each omelette is served with either a portion of fries or hash browns. It is known that 20% of customers choose a chicken omelette, 70% choose a vegetarian omelette and 10% choose a steak omelette.

It is also known that 65% of those ordering the chicken omelette, 70% of those ordering the vegetarian omelette and 60% of those ordering the steak omelette, order fries.

The following tree diagram represents the orders made by each customer.



(a) Complete the tree diagram by adding the respective probabilities to each branch. [2]

(b) Find the probability that a randomly selected customer orders fries. [2]

(c) Find the probability that a randomly selected customer orders fries, given that they do **not** order a chicken omelette. [3]

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4. [Maximum mark: 5]

Consider the equation $\frac{2z}{3 - z^*} = \mathrm{i}$, where $z = x + \mathrm{i}y$ and $x,\ y \in \mathbb{R}$.

Find the value of x and the value of y.

5. [Maximum mark: 5]

The first term in an arithmetic sequence is 4 and the fifth term is $\log_2 625$.

Find the common difference of the sequence, expressing your answer in the form $\log_2 p$, where $p \in \mathbb{Q}$.

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6. [Maximum mark: 4]

Consider the equation $ax^2 + bx + c = 0$, where $a \neq 0$. Given that the roots of this equation are $x = \sin\theta$ and $x = \cos\theta$, show that $b^2 = a^2 + 2ac$.

7. [Maximum mark: 5]

Consider the complex numbers $z_1 = \cos\frac{11\pi}{12} + \mathrm{i}\sin\frac{11\pi}{12}$ and $z_2 = \cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}$.

(a) (i) Find $\frac{z_1}{z_2}$

(ii) Find $\frac{z_2}{z_1}$ [3]

(b) 0, $\frac{z_1}{z_2}$ and $\frac{z_2}{z_1}$ are represented by three points O, A and B respectively on an Argand diagram. Determine the area of the triangle OAB. [2]

8. [Maximum mark: 7]

(a) Show that $\frac{\sin x \tan x}{1-\cos x} \equiv 1+\frac{1}{\cos x}$, $x \neq 2n\pi$, $n \in \mathbb{Z}$. [3]

(b) Hence determine the range of values of k for which $\frac{\sin x \tan x}{1-\cos x}=k$ has no real solutions. [4]

9. [Maximum mark: 8]

By using the substitution $x = \tan u$, find the value of $\int_0^1 \frac{x^2}{\left(1+x^2\right)^3}\,\mathrm{d}x$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 20]

Consider the function $f(x) = ax^3 + bx^2 + cx + d$, where $x \in \mathbb{R}$ and $a, b, c, d \in \mathbb{R}$.

(a) (i) Write down an expression for $f'(x)$.

(ii) Hence, given that f^{-1} does not exist, show that $b^2 - 3ac > 0$. [4]

(b) Consider the function $g(x) = \frac{1}{2}x^3 - 3x^2 + 6x - 8$, where $x \in \mathbb{R}$.

(i) Show that g^{-1} exists.

(ii) $g(x)$ can be written in the form $p(x-2)^3 + q$, where $p, q \in \mathbb{R}$.
Find the value of p and the value of q.

(iii) Hence find $g^{-1}(x)$. [8]

The graph of $y = g(x)$ may be obtained by transforming the graph of $y = x^3$ using a sequence of three transformations.

(c) State each of the transformations in the order in which they are applied. [3]

(d) Sketch the graphs of $y = g(x)$ and $y = g^{-1}(x)$ on the same set of axes, indicating the points where each graph crosses the coordinate axes. [5]

11. [Maximum mark: 15]

Consider the curve C defined by $y^2 = \sin(xy)$, $y \neq 0$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y\cos(xy)}{2y - x\cos(xy)}$. [5]

(b) Prove that, when $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, $y = \pm 1$. [5]

(c) Hence find the coordinates of all points on C, for $0 < x < 4\pi$, where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. [5]

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12. [Maximum mark: 15]

Consider the function defined by $f(x) = \dfrac{kx-5}{x-k}$, where $x \in \mathbb{R}\setminus\{k\}$ and $k^2 \neq 5$.

(a) State the equation of the vertical asymptote on the graph of $y=f(x)$. [1]

(b) State the equation of the horizontal asymptote on the graph of $y=f(x)$. [1]

(c) Use an algebraic method to determine whether f is a self-inverse function. [4]

Consider the case where $k=3$.

(d) Sketch the graph of $y=f(x)$, stating clearly the equations of any asymptotes and the coordinates of any points of intersections with the coordinate axes. [3]

(e) The region bounded by the x-axis, the curve $y=f(x)$, and the lines $x=5$ and $x=7$ is rotated through 2π about the x-axis. Find the volume of the solid generated, giving your answer in the form $\pi(a+b\ln 2)$, where $a, b \in \mathbb{Z}$. [6]

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will not be marked.

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