Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Wednesday 4 November 2020 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

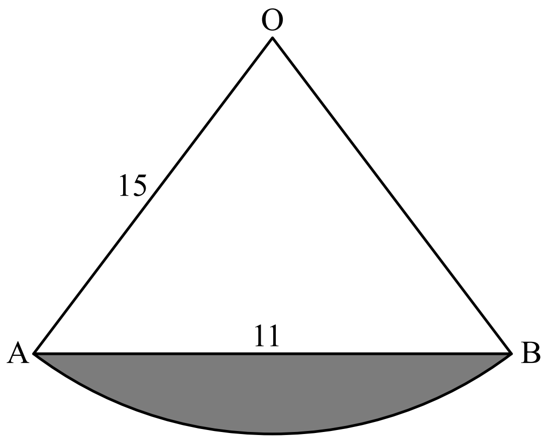
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The following diagram shows a sector OAB of radius $15\,\text{cm}$. The length of $[\text{AB}]$ is $11\,\text{cm}$.

diagram not to scale



Find the area of the shaded region.

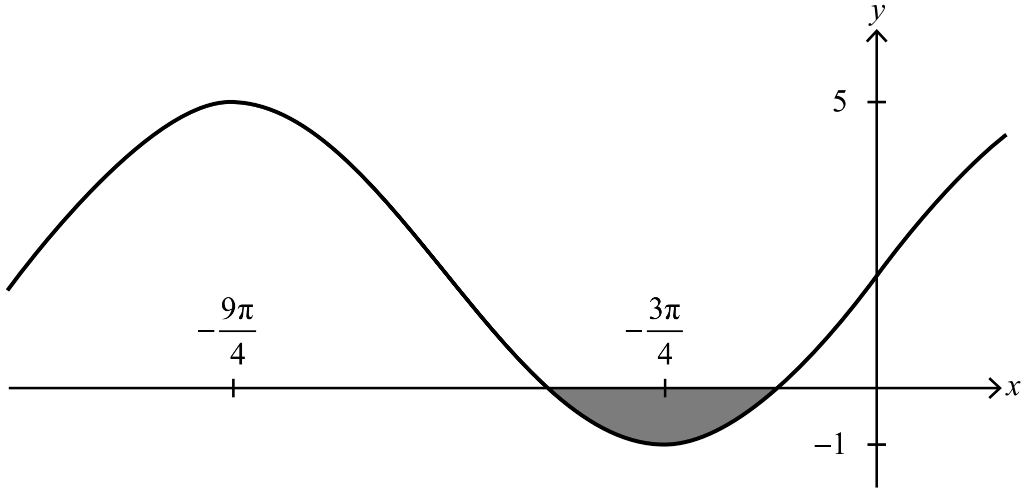
2. [Maximum mark: 4]

Jenna is a keen book reader. The number of books she reads during one week can be modelled by a Poisson distribution with mean 2.6.

Determine the expected number of weeks in one year, of 52 weeks, during which Jenna reads at least four books.

3. [Maximum mark: 8]

The following diagram shows part of the graph of $y = p + q\sin(rx)$. The graph has a local maximum point at $\left(-\frac{9\pi}{4}, 5\right)$ and a local minimum point at $\left(-\frac{3\pi}{4}, -1\right)$.



(a) Determine the values of p, q and r. [4]

(b) Hence find the area of the shaded region. [4]

4. [Maximum mark: 6]

Find the term independent of x in the expansion of $\frac{1}{x^3}\left(\frac{1}{3x^2}-\frac{x}{2}\right)^9$.

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5. [Maximum mark: 7]

A survey of British holidaymakers found that 15% of those surveyed took a holiday in the Lake District in 2019.

(a) A random sample of 16 British holidaymakers was taken. The number of people in the sample who took a holiday in the Lake District in 2019 can be modelled by a binomial distribution.

(i) State two assumptions made in order for this model to be valid.

(ii) Find the probability that at least three people from the sample took a holiday in the Lake District in 2019. [4]

(b) From a random sample of n holidaymakers, the probability that at least one of them took a holiday in the Lake District in 2019 is greater than 0.999.

Determine the least possible value of n. [3]

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6. [Maximum mark: 7]

Use mathematical induction to prove that $\frac{\mathrm{d}^n}{\mathrm{d}x^n}\left(x\mathrm{e}^{px}\right) = p^{n-1}\left(px+n\right)\mathrm{e}^{px}$ for $n \in \mathbb{Z}^+$, $p \in \mathbb{Q}$.

7. [Maximum mark: 7]

At a gathering of 12 teachers, seven are male and five are female. A group of five of these teachers go out for a meal together. Determine the possible number of groups in each of the following situations:

(a) There are more males than females in the group. [4]

(b) Two of the teachers, Gary and Gerwyn, refuse to go out for a meal together. [3]

8. [Maximum mark: 6]

A small bead is free to move along a smooth wire in the shape of the curve $y = \dfrac{10}{3 - 2\mathrm{e}^{-0.5x}}\ (x \geq 0)$.

(a) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [3]

At the point on the curve where $x = 4$, it is given that $\dfrac{\mathrm{d}y}{\mathrm{d}t} = -0.1\,\mathrm{m\,s^{-1}}$.

(b) Find the value of $\dfrac{\mathrm{d}x}{\mathrm{d}t}$ at this exact same instant. [3]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 14]

The weights, in grams, of individual packets of coffee can be modelled by a normal distribution, with mean $102\,\mathrm{g}$ and standard deviation $8\,\mathrm{g}$.

(a) Find the probability that a randomly selected packet has a weight less than $100\,\mathrm{g}$. [2]

(b) The probability that a randomly selected packet has a weight greater than w grams is 0.444. Find the value of w. [2]

(c) A packet is randomly selected. Given that the packet has a weight greater than $105\,\mathrm{g}$, find the probability that it has a weight greater than $110\,\mathrm{g}$. [3]

(d) From a random sample of 500 packets, determine the number of packets that would be expected to have a weight lying within 1.5 standard deviations of the mean. [3]

(e) Packets are delivered to supermarkets in batches of 80. Determine the probability that at least 20 packets from a randomly selected batch have a weight less than $95\,\mathrm{g}$. [4]

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10. [Maximum mark: 16]

The plane Π_1 has equation $3x - y + z = -13$ and the line L has vector equation

$$\boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ -1 \\ 4 \end{pmatrix}, \lambda \in \mathbb{R}.$$

(a) Given that L meets Π_1 at the point P, find the coordinates of P. [4]

(b) Find the shortest distance from the point $\mathrm{O}(0, 0, 0)$ to Π_1. [4]

The plane Π_2 contains the point O and the line L.

(c) Find the equation of Π_2, giving your answer in the form $\boldsymbol{r}.\boldsymbol{n} = d$. [3]

(d) Determine the acute angle between Π_1 and Π_2. [5]

Do **not** write solutions on this page.

11. [Maximum mark: 20]

A particle P moves in a straight line such that after time t seconds, its velocity, v in $\mathrm{m\,s^{-1}}$, is given by $v = \mathrm{e}^{-3t}\sin 6t$, where $0 < t < \frac{\pi}{2}$.

(a) Find the times when P comes to instantaneous rest. [2]

At time t, P has displacement $s(t)$; at time $t = 0$, $s(0) = 0$.

(b) Find an expression for s in terms of t. [7]

(c) Find the maximum displacement of P, in metres, from its initial position. [2]

(d) Find the total distance travelled by P in the first 1.5 seconds of its motion. [2]

At successive times when the acceleration of P is $0\,\mathrm{m\,s^{-2}}$, the velocities of P form a geometric sequence. The acceleration of P is zero at times t_1, t_2, t_3 where $t_1 < t_2 < t_3$ and the respective velocities are v_1, v_2, v_3.

(e) (i) Show that, at these times, $\tan 6t = 2$.

(ii) Hence show that $\frac{v_2}{v_1} = \frac{v_3}{v_2} = -\mathrm{e}^{-\frac{\pi}{2}}$. [7]

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