Practice paper 1

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

1 Find the angle between the planes $x - 2y - z = 1$ and $2x - y + z = 3$. *[Maximum mark: 5]*

2 State with a reason whether the functions are odd or even. *[Maximum mark: 6]*

a $f(x) = x\sin^3 x$

b $g(x) = \sqrt[3]{\dfrac{e^{x^2}+1}{x}}$

3 Find the following limits: *[Maximum mark: 6]*

a $\lim_{x \to 2} \frac{2x^2 - 5x + 2}{3x^2 - 4x - 4}$

b $\lim_{x \to 0} \frac{\sin 3x}{2x}$

4 The first, third and sixth terms of an arithmetic sequence form a geometric sequence.

[Maximum mark: 7]

a Express the first term in terms of the common difference of the arithmetic sequence.

b Hence show that the first, fifth and thirteenth term form another geometric sequence.

5 Eight pearls, all different in size, are to be put in a necklace on a circular thread. *[Maximum mark: 5]*

a Find the number of all the possible necklaces that can be made.

b What is the probability that a randomly formed necklace will have the two largest pearls next to each other?

6 The largest vertical cross section of a cylinder has a diagonal of 6 cm. Find the radius of the base of the cylinder so that the cylinder has a maximum volume.

[Maximum mark: 6]

7 There are two boxes with black and white marbles. The probability that Matthew selects the first box is not equal to the probability that he selects the second box. The probability that Matthew selects a white marble from the first box is twice as likely as the probability that Matthew selects the first box. The probability that Matthew selects a white marble from the second box is 0.2. *[Maximum mark: 7]*

a Draw the probability tree diagram representing the information above.

b The probability that Matthew draws a white marble is 0.44. Find the probability that Matthew selects the first box.

8 Solve the exponential equation $2 \times 25^x = 3 \times 10^x + 5 \times 4^x$. *[Maximum mark: 6]*

9 Given that $\log_5 x = p$ and $\log_2 x = q$, find the following in terms of p and q: *[Maximum mark: 6]*

a $\log_x 10$

b $\log x$

10 Solve the simultaneous equations for x and y leaving your answers in terms of α.

$$\begin{cases} \sin\alpha \cdot x - \cos\alpha \cdot y = 1 \\ \cos\alpha \cdot x + \sin\alpha \cdot y = 1 \end{cases}, \alpha \in \mathbb{R}$$

Explain why the system always has a unique solution. *[Maximum mark: 6]*

Section B

11 Consider the function $f(x) = \dfrac{x^3 - 2x^2 + 5}{x^2 - x^3}$. *[Maximum mark: 10]*

a Determine the domain of the function *f*.

b Find the horizontal asymptote of the graph of the function.

c Find any points where the graph of the function intersects its horizontal asymptote.

12 a Use the method of integration by parts to show the following formula:

$$\int_0^{\frac{\pi}{2}} \sin^n(x)\,dx = \frac{n-1}{n}\int_0^{\frac{\pi}{2}} \sin^{n-2}(x)\,dx.$$

b Hence find the integral $\int_0^{\frac{\pi}{2}} \sin^4(x)\,dx$. *[Maximum mark: 12]*

13 a Use mathematical induction to prove that $\sum_{r=1}^{n}(3r-1)=\frac{1}{2}n(3n+1)$.

b Hence or otherwise find the sum

$1-2+3+4-5+\ldots+(3n-2)-(3n-1)+3n$

[Maximum mark: 14]

14 a Solve the equation $z^2 + z + 1 = 0$, $z \in \mathbb{C}$, and write your solutions ω and ω^* in polar form. *[Maximum mark: 24]*

b Show that

i $(\omega)^2 = \omega^*$

ii $(\omega^*)^2 = \omega$

iii $(\omega)^3 = (\omega^*)^3 = 1$

c Write down and simplify the first four terms of the expansion of the following expressions.

i $(1 + \omega)^{3n}$ **ii** $(1 + \omega^*)^{3n}$

d Write the following numbers in trigonometric form.

i $1 + \omega$ **ii** $1 + \omega^*$

e Show that $\binom{3n}{0} + \binom{3n}{1} + \binom{3n}{2} + \ldots + \binom{3n}{3n} = 2^{3n}$.

f Hence show that $\binom{3n}{0} + \binom{3n}{3} + \binom{3n}{6} + \ldots + \binom{3n}{3n} = \dfrac{2^{3n} + 2\cos(n\pi)}{3}$.