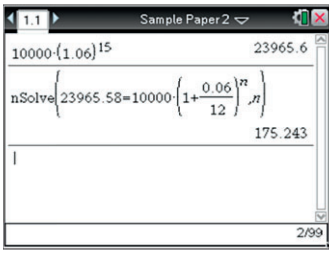
Mark scheme

Practice paper 2

1 a $V = 10000 \times 1.06^{15} = 23965.58$ (M1) (A1)

b $23965.58 = 10000 \times \left(1 + \dfrac{0.06}{12}\right)^n$ (M1) (A1) (A1)

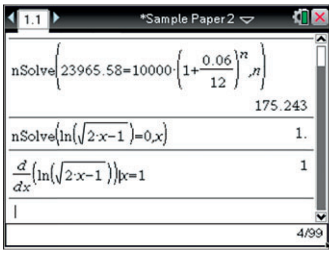
$= 175.243 = 176$ months (A1) [6 marks]



2 $\ln\sqrt{2x-1} = 0 \Rightarrow \sqrt{2x-1} = 1 \Rightarrow 2x = 2 \Rightarrow x = 1$ (A1)

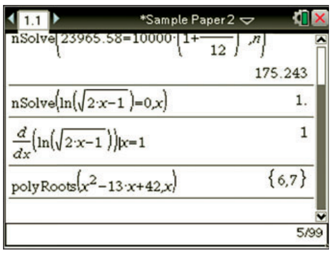
$y = \ln\sqrt{2x-1} \Rightarrow y'(1) = 1 \Rightarrow m_N = -\dfrac{1}{1} = -1$ (M1) (A1)

$N: y - 0 = -1\,(x - 1) \Rightarrow y = -x + 1$ (M1) (A1) [5 marks]



3 $\begin{cases} \dfrac{8+5+6+a+b}{5} = 6.4 \\ \dfrac{64+25+36+a^2+b^2}{5} - 6.4^2 = 1.04 \end{cases} \Rightarrow \begin{cases} a+b = 13 \\ a^2+b^2 = 85 \end{cases}$ (M1) (A1) (A1) (A1)

$\Rightarrow \begin{cases} b = 13-a \\ a^2+(13-a)^2 = 85 \end{cases} \Rightarrow \begin{cases} b = 13-a \\ a^2-13a+42 = 0 \end{cases} \Rightarrow \begin{cases} b = 7 \\ a = 6 \end{cases}$ (A1) (A1) [6 marks]

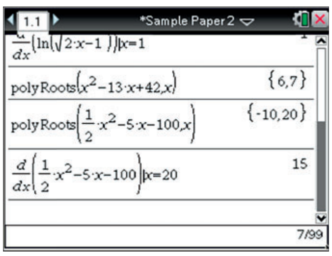


4 a $s_1 = 100 + 5t,\ s_2 = \dfrac{1}{2}t^2 \Rightarrow s = s_1 - s_2$ (M1) (A1)

$s = 0 \Rightarrow 0 = 100 + 5t - \dfrac{1}{2}t^2 \Rightarrow \dfrac{1}{2}t^2 - 5t - 100 = 0$ (A1) (AG)

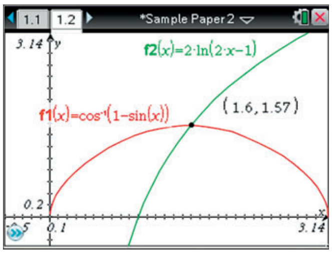
b $\dfrac{1}{2}t^2 - 5t - 100 = 0 \Rightarrow$ ~~$t_1 = -10$~~, $t_2 = 20$ (A1)

$\dfrac{ds}{dt}(20) = 15\,\text{m/s}$ (A1) [5 marks]



5 a $\begin{cases} \cos y = 1 - \sin x \\ e^{\frac{y}{2}} = x - 1 \end{cases} \Rightarrow \begin{cases} y = \arccos(1 - \sin x) \\ y = 2\ln(x - 1) \end{cases}$ (A1) (A1)

b (M1) (A1)



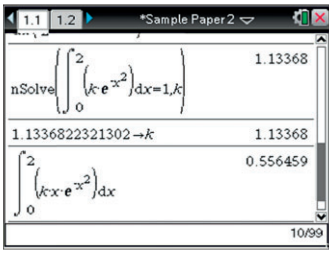
$x = 1.60$, $y = 1.57$ (A1) (A1) [6 marks]

6 a Since the probability density function must always be positive and $e^{-x^2} > 0$ for all the values of x, then $k > 0$. (R1)

b $\displaystyle\int_0^2 ke^{-x^2}\,dx = 1$ (M1) (A1)

$k = 1.13$ (A1)

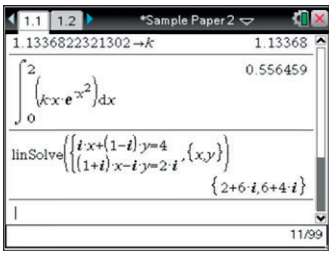
c $\displaystyle\int_0^2 kx\,e^{-x^2}\,dx = 0.556$ (M1) (A1) [6 marks]



7 $\begin{cases} ix + (1-i)y = 4 \\ (1+i)x - iy = 2i \end{cases} \Rightarrow a = i,\ b = 1-i,\ c = 1+i,\ d = -i,\ e = 4,\ f = 2i$ (A1)

$\begin{cases} x = \dfrac{ed - bf}{ad - bc} \\ y = \dfrac{af - ce}{ad - bc} \end{cases} \Rightarrow \begin{cases} x = \dfrac{4(-i) - (1-i)\cdot 2i}{i\cdot(-i) - (1+i)(1-i)} \\ y = \dfrac{i\cdot 2i - (1+i)\cdot 4}{i\cdot(-i) - (1+i)(1-i)} \end{cases}$ (M2) (A1) (A1)

$\begin{cases} x = \dfrac{-4i - 2i - 2}{1-2} \\ y = \dfrac{-2 - 4 - 4i}{1-2} \end{cases} \Rightarrow \begin{cases} x = 2 + 6i \\ y = 6 + 4i \end{cases}$ (A1) (A1) [7 marks]



8 a The function f is even therefore its antiderivative F is odd, $F(-x) = -F(x)$ for all real values. (R1)

$\displaystyle\int_{-a}^{a} f(x)\,dx = \left[F(a) - F(-a)\right] = 2 \times F(a)$ (M1)

$\displaystyle = 2 \times \Big[F(a) - \underbrace{F(0)}_{0}\Big] = 2 \times \int_0^a f(x)\,dx$ (A1)

b The function g is odd therefore its antiderivative G is even, $G(-x) = G(x)$ for all real values. (R1)

$\displaystyle\int_{-a}^{a} g(x)\,dx = \left[G(a) - G(-a)\right] = G(a) - G(a) = 0$ (M1) (A1) [6 marks]

9 Given that x_1, x_2 and x_3 are solutions of the equation

$2x^3 - 3x^2 + 4x - 5 = 0 \Rightarrow a = 2, b = -3, c = 4, d = -5$ (A1)

Using Viete's formulae

$x_1 + x_2 + x_3 = -\frac{b}{a}, x_1x_2x_3 = -\frac{d}{a}$ (M1)

$x_1^2x_2x_3 + x_1x_2^2x_3 + x_1x_2x_3^2 = x_1x_2x_3(x_1 + x_2 + x_3)$ (M1) (A1)

$= -\frac{-5}{2}\left(-\frac{-3}{2}\right) = \frac{15}{4}$ (A1) [5 marks]

10 a $\frac{\sin \text{CAB}}{\text{BC}} = \frac{\sin \text{BCA}}{\text{AB}} \Rightarrow \sin \text{CAB} = 6 \cdot \frac{\sin 40°}{5}$ (M1)

$(\angle \text{CAB})_1 = \arcsin\left(6 \cdot \frac{\sin 40°}{5}\right) = 50.5°$ (A1)

$(\angle \text{CAB})_2 = 180° - \arcsin\left(6 \cdot \frac{\sin 40°}{5}\right) = 129.5°$ (A1)

b $\frac{(\text{AC})_1}{\sin(\text{ABC})_1} = \frac{\text{BC}}{\sin \text{CAB}} \Rightarrow (\text{AC})_1 = 6 \cdot \frac{\sin 89.5°}{\sin 40°} = 9.33$ (M1) (A1)

$\frac{(\text{AC})_2}{\sin(\text{ABC})_2} = \frac{\text{BC}}{\sin \text{CAB}} \Rightarrow (\text{AC})_2 = 6 \cdot \frac{\sin 10.5°}{\sin 40°} = 1.70$ (A1)

c $\text{A} = \frac{1}{2}\text{AB} \cdot \text{BC} \cdot \sin(\text{ABC})_2$ (M1)

$= \frac{1}{2} \cdot 5 \cdot 6 \cdot \sin(10.5°) = 2.73$ (A1) [8 marks]

11 The function $f(x) = \frac{3 - 4\sin x}{3 + 2\cos x}$, $0 \le x \le 2\pi$ is given.

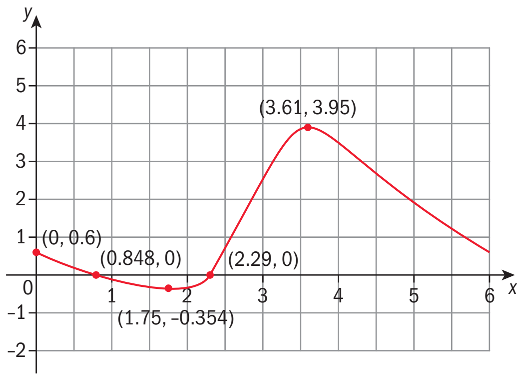
a A vertical asymptote occurs when the denominator equals 0. (R1)

$3 + 2\cos x = 0 \Rightarrow \cos x = -\frac{3}{2} < -1 \Rightarrow x \in \varnothing$ (A1)

b $x = 0 \Rightarrow f(0) = \frac{3}{5} = 0.6$ (A1)

c $y = 0 \Rightarrow 3 - 4\sin x = 0 \Rightarrow \sin x = \frac{3}{4} \Rightarrow p = 0.848, q = 2.29$ (M1) (A1) (A1)

d

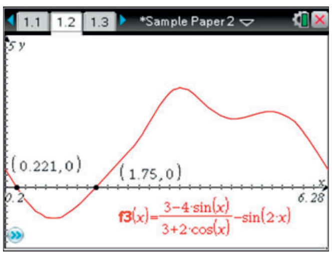


Shape (A1)

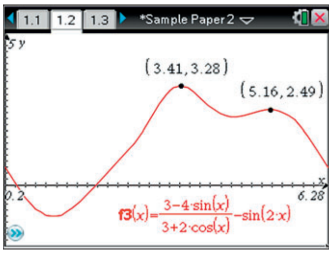
Zeroes (A1)

Stationary points (A1)

e $f(x) - g(x) > 0$ (M1)



$x \in [0, 0.221[\cup]1.75, 6.28]$ (A1) (A1) f

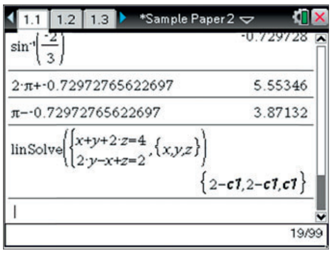


M_1 (3.41, 3.28), M_2 (5.16, 2.49) (M1) (A1)

Since the function is continuous on the domain $y_{max} = 3.28$ (A1) [15 marks]

12 a $\boldsymbol{n}_1 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \boldsymbol{n}_2 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \Rightarrow \boldsymbol{n}_1 \neq k \cdot \boldsymbol{n}_2,\ k \in \mathbb{R}$ (A1) (R1)

b $\begin{cases} x + y + 2z = 4 \\ -x + 2y + z = 2 \end{cases}$ (M1) (A1)



$x = 2 - t,\ y = 2 - t,\ z = t,\ t \in \mathrm{R}$ (A1) (A1) (A1)

c We take a point P(2, 2, 0) that lies on the line and the direction vector

$\boldsymbol{d} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ to find the equation of the plane $\prod$.

$\boldsymbol{AP} \times \boldsymbol{d} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \\ -3 \end{pmatrix} \Rightarrow \boldsymbol{n}_3 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ (M1) (A1) (A1)

$\mathrm{A} \in \prod \quad 2 \cdot 1 + 1 \cdot (-2) + 3 \cdot 2 = d \Rightarrow d = 6$ (M1) (A1)

$\prod: 2x + y + 3z = 6$ (A1)

d The direction vector of the line is the normal vector of the plane Ω and we include the point A.

$\mathrm{A} \in \Omega \quad 1 \cdot 1 + 1 \cdot (-2) + 1 \cdot 2 = d \Rightarrow d = -3$ (M1)

$\Omega: x + y - z = -3$ (A1)

$2 - t + 2 - t - t = -3 \Rightarrow 7 = 3t \Rightarrow t = \frac{7}{3}$ (A1)

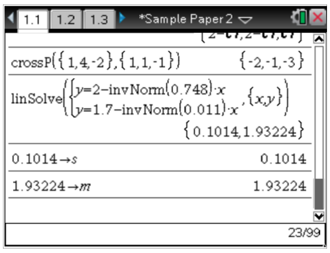
$\Rightarrow \mathrm{A}'\left(-\frac{1}{3}, -\frac{1}{3}, \frac{7}{3}\right)$ (A1) [17 marks]

13 a $X \sim N(\mu, \sigma)$

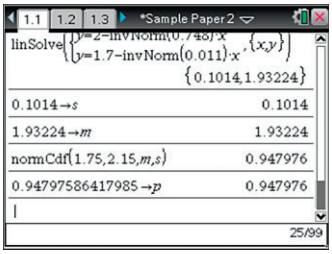
$\begin{cases} \mathrm{P}(X < 2) = 0.748 \\ \mathrm{P}(X > 1.7) = 0.909 \end{cases} \Rightarrow \begin{cases} \mathrm{P}(X < 2) = 0.748 \\ \mathrm{P}(X < 1.7) = 0.011 \end{cases}$ (M1) (A1) (A1)

$\begin{cases} \dfrac{2 - \mu}{\sigma} = \Phi^{-1}(0.748) \\ \dfrac{1.7 - \mu}{\sigma} = \Phi^{-1}(0.011) \end{cases} \Rightarrow \begin{cases} \mu = 2 - \Phi^{-1}(0.748) \cdot \sigma \\ \mu = 1.7 - \Phi^{-1}(0.011) \cdot \sigma \end{cases}$ (A1) (A1)

$\mu = 1.93,\ \sigma = 0.101$ (A1) (A1)



b P(1.75 < X < 2.15) = 0.794 (M1) (A1) (AG)



c A: "At least one pole satisfies the standards."

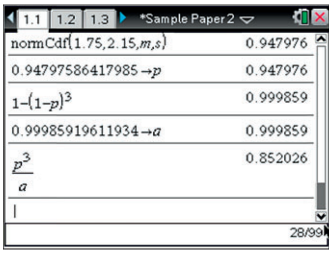
P(A) = 1 – P (A′) = 1 – $(1 - 0.948)^3$ (M1) (A1)

= 0.99986 (A1)

d B:"All three poles satisfy the standards."

$P(B|A) = \frac{P(B)}{P(A)}$ (M1) (A1)

$= \frac{0.948^3}{0.99986} = 0.852$ (A1) [15 marks]



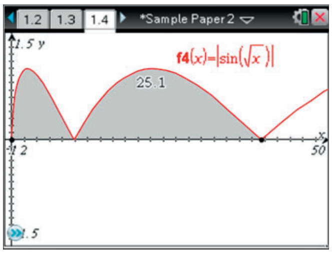
14 a $D(f) = \{x \in \mathbb{R}: x \geq 0)$ (A1)

$R(f) = [-1, 1]$ (A1)

b $\sin\sqrt{x} = 0 \Rightarrow \begin{cases} \sqrt{x} = \pi \Rightarrow x_1 = \pi^2 \\ \sqrt{x} = 2\pi \Rightarrow x_2 = 4\pi^2 \end{cases}$ (M1) (A1) (A1)

c $A = \int_0^{4\pi^2} \left|\sin\sqrt{x}\right| dx$ (M1) (A1) (A1)

= 25.1 (A1)



d $V = \int_0^{4\pi^2} \left(\sin\sqrt{x}\right)^2 dx$ (M1) (A1) (A1)

= 62.0 (A1) [13 marks]