Practice paper 2

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

1 Marco invests €10 000 in a bank. *[Maximum mark: 6]*

a If the interest is compounded annually at a rate of 6% per year, find the total value V of the investment after 15 years.

b If Marco decides to use a different savings contract where the same annual interest rate is compounded monthly, find the minimum number of months so that the total value of investment exceeds the value of V from part **a**.

2 Find the equation of the normal to the curve $y = \ln\sqrt{2x-1}$ at the point of intersection with the x-axis. *[Maximum mark: 5]*

3 The set of values $\{8, 5, 6, a, b\}$ has a mean value of 6.4 and the variance of 1.04. Find the values of a and b $(a < b)$. *[Maximum mark: 6]*

4 Two objects are 100 metres apart. They start moving in the same direction as given on the diagram below.

The distance traveled by the first object is given by the formula $s=\frac{1}{2}t^2, t\geq 0$. The second object moves with a constant velocity of 5 m/s. *[Maximum mark: 5]*

a Show that the time t when the first object meets the second object satisfies the equation $\frac{1}{2}t^2-5t-100=0$.

b Find the rate of change of the distance when the first object overtakes the second object.

5 A system of equations is given by $\begin{cases} \sin x + \cos y = 1 \\ x = e^{\frac{y}{2}} + 1 \end{cases}$ *[Maximum mark: 6]*

a Express y in terms x in both equations.

b Hence solve the system for $0 < x < \pi$, $0 < y < \pi$.

6 A continuous random variable X has a probability density function

$$f(x)=\begin{cases} ke^{-x^2}, 0 \le x \le 2 \\ 0, \text{otherwise} \end{cases}$$

[Maximum mark: 6]

a Explain why $k > 0$.

b Find the value of k.

c Hence find E(X).

7 Solve the simultaneous equations $\begin{cases} \mathrm{i}x + (1-\mathrm{i})y = 4 \\ (1+\mathrm{i})x - \mathrm{i}y = 2\mathrm{i} \end{cases}$ *[Maximum mark: 7]*

8 The function f is even, the function g is odd and $a > 0$. *[Maximum mark: 6]*

a Show that $\displaystyle\int_{-a}^{a} f(x)\,\mathrm{d}x = 2\int_{0}^{a} f(x)\,\mathrm{d}x$.

b Show that $\displaystyle\int_{-a}^{a} g(x)\,\mathrm{d}x = 0$.

9 Given that x_1, x_2 and x_3 are solutions of the equation $2x^3 - 3x^2 + 4x - 5 = 0$, without solving it find $x_1^2 x_2 x_3 + x_1 x_2^2 x_3 + x_1 x_2 x_3^2$. *[Maximum mark: 5]*

10 In a triangle ABC the following is given: AB = 5, BC = 6, ∠BCA + 40°. *[Maximum mark: 8]*

a Calculate the possible measures of the angle ∠CAB. Give your answer correct to the nearest tenth of a degree.

b Calculate the possible lengths of the side AC.

c Hence find the smallest possible area of the triangle ABC.

Section B

11 The function $f(x)=\dfrac{3-4\sin x}{3+2\cos x}$, $0 \le x \le 2\pi$ is given. *[Maximum mark: 15]*

a Explain why the function does not have any vertical asymptotes.

b Find the y-intercept of $y = f(x)$.

c Find the x-intercepts, p and q (where $p < q$).

d Sketch the graph of $y = f(x)$, labeling any stationary points, p, q and the y-intercept.

e Given that $g(x) = \sin(2x)$, $0 \le x \le 2\pi$, for what values of x is $f(x) > g(x)$.

f Hence or otherwise calculate the maximum value of $h(x) = f(x) - g(x)$, $0 \le x \le 2\pi$.

12 The planes $x + y + 2z = 4$ and $2y - x + z = 2$ are given. *[Maximum mark: 17]*

a Write down the normal vectors of the planes and explain why these two planes intersect.

b Find the equation of the line where these two planes intersect.

c A point $A(1, -2, 2)$ is given. Find the equation of the plane that contains the point A and is concurrent with the given planes.

d Find the perpendicular projection of the point A to the line in part **b**.

13 The production of wooden poles for oil lamp stands has a normal distribution. It is found that 74.8% of all the poles have a length less than 2 m, whilst 90.9% of all the poles have a length greater than 1.7 m. *[Maximum mark: 15]*

a Find the mean and the standard deviation of the pole length.

A pole can be used as an oil lamp stand if its length is between 1.75 and 2.15 m.

b Show that the probability that a randomly selected pole can be used as an oil lamp stand is 0.794.

Three poles are taken from the stack of produced wooden poles.

c Find the probability that at least one pole satisfies the standards of the oil lamp stand. Give your answer correct to 5 decimal places.

d Given that at least one pole satisfies the standards, find the probability that all three poles satisfy the standards of the oil lamp stand.

14 The function $f(x) = \sin\sqrt{x}$ is given. *[Maximum mark: 13]*

a Determine the domain and the range of the function f.

Find the first two positive zeroes of the function, and give your answer in terms of π.

c Calculate the area of the region enclosed by the curve and the x-axis up to the second positive zero.

d The region in part **c** is rotated for 2π about the x-axis. Find the volume of the solid generated by the revolution.